# Air Force Cadet-Career Problem Optimization Model Formulation

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**Primary Sets and Indices**

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|---|---|
| $i \in \mathcal{I}$ | the set of all cadets |
| $j \in \mathcal{J}$ | the set of all AFSCs |
| $k \in \mathcal{K}$ | the set of all AFSC objectives |
| $l \in \mathcal{L}_{jk}$ | the set of all breakpoints for objective $k$'s function for AFSC $j$ |

**Subsets and Indices**

| | |
|---|---|
| $\mathcal{K}^D \subseteq \mathcal{K}$ | the set of AFSC objectives that seek some proportion of cadets with certain demographics. The objectives to balance the "USAFA Proportion", the AFOCD tiers, etc. are contained here. |
| $\mathcal{I}^{\mathrm{C}} \subseteq \mathcal{I}$ | the set of cadets with constrained minimum values |
| $\mathcal{I}^E_j \subseteq \mathcal{I}$ | the set of cadets that are eligible for AFSC $j$ |
| $\mathcal{I}^D_{jk} \subseteq \mathcal{I}^E_j$ | the set of cadets with the demographic that corresponds to objective $k \in \mathcal{K}^D$ and are eligible for AFSC $j$. Depending on the value of $k$, this could be the set of USAFA, male, or minority cadets that are eligible for AFSC $j$ or the set of cadets with specific AFOCD tier degrees for AFSC $j$ |
| $\mathcal{J}^{\mathrm{C}} \subseteq \mathcal{J}$ | the set of AFSCs with constrained minimum values |
| $\mathcal{J}^E_i \subseteq \mathcal{J}$ | the set of AFSCs for which cadet $i$ is eligible |
| $\mathcal{K}^A_j \subseteq \mathcal{K}$ | the set of objectives that apply to AFSC $j$ |
| $\mathcal{K}^{\mathrm{C}}_j \subseteq \mathcal{K}^A_j$ | the set of objectives with constrained minimum values for AFSC $j$ |
| $\mathcal{I}^R \subseteq \mathcal{I}$ | the set of all rated-eligible cadets by source of commissioning (SOC) |
| $\mathcal{J}^R \subseteq \mathcal{J}$ | the set of all rated AFSCs |
| $\mathcal{I}^{SF} \subseteq \mathcal{I}$ | the set of all cadets that MUST be classified to the Space Force |
| $\mathcal{J}^{SF} \subseteq \mathcal{J}$ | the set of all Space Force AFSCs (SFSCs) |
| $\mathcal{I}^{Reserved} \subseteq \mathcal{I}$ | the set of cadets with reserved AFSCs (See Algorithm 1) |
| $\mathcal{J}^{Reserved}_i \subseteq \mathcal{J}$ | the set of AFSCs that cadet $i$ MUST be assigned to (See Algorithm 1) |
| $\mathcal{I}^{U1} \subseteq \mathcal{I}$ | the set of all USAFA Cadets |
| $\mathcal{I}^{R2} \subseteq \mathcal{I}$ | the set of all ROTC Cadets |
| $\mathcal{I}^{Alt}_j \subseteq \mathcal{I}^E_j$ | the set of all cadets on AFSC $j$'s alternate list |
| $\mathcal{I}^{Pref}_{ji} \subseteq \mathcal{I}^E_j$ | the set of all cadets that are more preferred by AFSC $j$ than cadet $i$ |
| $\mathcal{J}^{Pref}_{ji} \subseteq \mathcal{J}^E_i$ | the set of all AFSCs that are more preferred by cadet $i$ than AFSC $j$ |

**Fixed Parameters**

| | |
|---|---|
| $merit_i$ | percentile of cadet $i$ |
| $cadet\_utility_{ij}$ | utility that cadet $i$ has assigned for AFSC $j$ |
| $afsc\_utility_{ij}$ | utility that AFSC $j$ has assigned for cadet $i$ |
| $quota_j$ | the minimum number of cadets to be assigned to AFSC $j$ |
| $quota_j^e$ | the estimated number of cadets to be assigned to AFSC $j$ |

**VFT Value Function Parameters**

| | |
|---|---|
| $f_{jk}(\cdot)$ | value function chosen by AFSC $j$ for objective $k$ |
| $r_{jk}$ | number of breakpoints for objective $k$'s function for AFSC $j$ |
| $a_{jkl}$ | measure at breakpoint $l$ for objective $k$'s function for AFSC $j$ |
| $\hat{f}_{jkl}$ | value at breakpoint $l$ for objective $k$'s function for AFSC $j$ |

**VFT Weight Parameters**

| | |
|---|---|
| $objective\_weight_{jk}$ | the weight on objective $k$ for AFSC $j$ |
| $afsc\_weight_j$ | the weight on AFSC $j$ relative to the other AFSCs |
| $cadet\_weight_i$ | the weight on cadet $i$ relative to the other cadets |
| $afscs\_overall\_weight$ | the weight on the AFSCs as a whole relative to the cadets |
| $cadets\_overall\_weight$ | the weight on the cadets as a whole relative to the AFSCs |

**VFT/GUO Shared Value Parameters**

| | |
|---|---|
| $objective\_min_{jk}$ | the minimum measure of objective $k$ for AFSC $j$ |
| $objective\_max_{jk}$ | the maximum measure of objective $k$ for AFSC $j$ |
| $cadet\_value\_min_i$ | the minimum value for cadet $i$ |
| $global\_utility_{ij}$ | combined afsc/cadet utility on AFSC $j$ for cadet $i$ $cadets\_overall\_weight \cdot cadet\_utility_{ij}$ $+ afscs\_overall\_weight \cdot afsc\_utility_{ij}$ |

**Space Force and AF Rated Parameters**

| | |
|---|---|
| $sf\_merit\_up$ | upper bound on average OM for the Space Force |
| $sf\_merit\_lo$ | lower bound on average OM for the Space Force |
| $sf\_quota\_up^{U1}$ | upper bound on number of USAFA cadets entering SFSCs |
| $sf\_quota\_lo^{U1}$ | lower bound on number of USAFA cadets entering SFSCs |
| $sf\_quota\_up^{R2}$ | upper bound on number of ROTC cadets entering SFSCs |
| $sf\_quota\_lo^{R2}$ | lower bound on number of ROTC cadets entering SFSCs |

**Decision Variables**

| | |
|---|---|
| $\boldsymbol{x}_{ij}$ | 1 if cadet $i$ is assigned to AFSC $j$, and 0 otherwise |

**AFSC Objective Auxiliary Variables**

| | |
|---|---|
| $\boldsymbol{count}_j$ | number of cadets assigned to AFSC $j$, $\sum_{i \in \mathcal{I}_j^E} \boldsymbol{x}_{ij}$ |
| $\boldsymbol{usafa\_count}_j$ | number of USAFA cadets assigned to AFSC $j$, $\sum_{i \in \mathcal{I}_{jk}^D} \boldsymbol{x}_{ij}$ where $k =$ "USAFA Proportion" |
| $\boldsymbol{rotc\_count}_j$ | number of ROTC cadets assigned to AFSC $j$, $\boldsymbol{count}_j - \boldsymbol{usafa\_count}_j$ |
| $\boldsymbol{measure}_{jk}$ | real measure of objective $k$ for AFSC $j$, $\begin{cases} \dfrac{\sum_{i \in \mathcal{I}_{jk}^D} \boldsymbol{x}_{ij}}{\boldsymbol{count}_j}, & \text{if } k \in \mathcal{K}^D \\ \dfrac{\sum_{i \in \mathcal{I}_j^E} merit_i \cdot \boldsymbol{x}_{ij}}{\boldsymbol{count}_j}, & \text{if } k = \text{"Balance Merit"} \\ \dfrac{\sum_{i \in \mathcal{I}_j^E} cadet\_utility_{ij} \cdot \boldsymbol{x}_{ij}}{\boldsymbol{count}_j}, & \text{if } k = \text{"Maximize Utility"} \\ \boldsymbol{count}_j, & \text{if } k = \text{"Meet Combined Quota"} \\ \boldsymbol{usafa\_count}_j, & \text{if } k = \text{"Meet USAFA Quota"} \\ \boldsymbol{rotc\_count}_j, & \text{if } k = \text{"Meet ROTC Quota"} \end{cases}$ |
| $\boldsymbol{value}_{jk}$ | value of objective $k$ for AFSC $j$ obtained through value function $f_{jk}(\cdot)$, $f_{jk}(\boldsymbol{measure}_{jk})$ |

**AFSC Objective Value Function Variables**

| | |
|---|---|
| $\boldsymbol{\lambda}_{jkl}$ | percentage of the line segment between breakpoints $l$ and $l+1$ that $\boldsymbol{measure}_{jk}$ has yet to travel along the piece-wise function |
| $\boldsymbol{y}_{jkl}$ | 1 if $\boldsymbol{measure}_{jk}$ is on the line segment between breakpoints $l$ and $l+1$; 0 otherwise |

**VFT Additional Auxiliary Variables**

| | |
|---|---|
| $\boldsymbol{afsc\_value}_j$ | the weighted sum of all objective values for AFSC $j$, $\sum_{k \in \mathcal{K}_j^A} objective\_weight_{jk} \cdot \boldsymbol{value}_{jk}$ |
| $\boldsymbol{cadet\_value}_i$ | the utility obtained for cadet $i$, $\sum_{j \in \mathcal{J}_i^E} cadet\_utility_{ij} \cdot \boldsymbol{x}_{ij}$ |
| $\boldsymbol{afscs\_overall\_value}$ | the weighted sum of all AFSC values, $\sum_{j \in \mathcal{J}} afsc\_weight_j \cdot \boldsymbol{afsc\_value}_j$ |
| $\boldsymbol{cadets\_overall\_value}$ | the weighted sum of all cadet utilities, $\sum_{i \in \mathcal{I}} cadet\_weight_i \cdot \boldsymbol{cadet\_value}_i$ |

**Objective Function Variables**

| | |
|---|---|
| $\boldsymbol{Z}^{VFT}$ | VFT Model objective value, $afscs\_overall\_weight \cdot \boldsymbol{afscs\_overall\_value} +$ $cadets\_overall\_weight \cdot \boldsymbol{cadets\_overall\_value}$ |
| $\boldsymbol{Z}^{GUO}$ | GUO Model objective value, $\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}_i^E} global\_utility_{ij} \cdot \boldsymbol{x}_{ij}$ |

**Basic Formulation**

$$\text{maximize} \quad \boldsymbol{Z}^{VFT} \quad or \quad \boldsymbol{Z}^{GUO} \tag{1a}$$

$$\text{subject to} \quad \sum_{j \in \mathcal{J}_i^E} \boldsymbol{x}_{ij} = 1 \quad \forall i \in \mathcal{I} \tag{1b}$$

$$\boldsymbol{measure}_{jk} \geq objective\_min_{jk} \quad \forall j \in \mathcal{J},\ k \in \mathcal{K}_j^{\mathrm{C}} \tag{1c}$$

$$\boldsymbol{measure}_{jk} \leq objective\_max_{jk} \quad \forall j \in \mathcal{J},\ k \in \mathcal{K}_j^{\mathrm{C}} \tag{1d}$$

$$\boldsymbol{cadet\_value}_i \geq cadet\_value\_min_i \quad \forall i \in \mathcal{I}^C \tag{1e}$$

$$\boldsymbol{x}_{ij} \in \{0,1\} \quad \forall i \in \mathcal{I},\ j \in \mathcal{J}_i^E \tag{1f}$$

**Value Function Constraints (Strictly VFT)**

$$\boldsymbol{measure}_{jk} = \sum_{l \in \mathcal{L}_{jk}} a_{jkl} \cdot \boldsymbol{\lambda}_{jkl} \quad \forall j \in \mathcal{J},\ k \in \mathcal{K}_j^A \tag{2a}$$

$$\boldsymbol{value}_{jk} = \sum_{l \in \mathcal{L}_{jk}} \hat{f}_{jkl} \cdot \boldsymbol{\lambda}_{jkl} \quad \forall j \in \mathcal{J},\ k \in \mathcal{K}_j^A \tag{2b}$$

$$\boldsymbol{\lambda}_{jk1} \leq \boldsymbol{y}_{jk1} \quad \forall j \in \mathcal{J},\ k \in \mathcal{K}_j^A \tag{2c}$$

$$\boldsymbol{\lambda}_{jkl} \leq \boldsymbol{y}_{jk(l-1)} + \boldsymbol{y}_{jkl} \quad \forall j \in \mathcal{J},\ k \in \mathcal{K}_j^A,\ l \in \{2, ..., r_{jk} - 1\} \tag{2d}$$

$$\boldsymbol{\lambda}_{jkr_{jk}} \leq \boldsymbol{y}_{jk(r_{jk}-1)} \quad \forall j \in \mathcal{J},\ k \in \mathcal{K}_j^A \tag{2e}$$

$$\sum_{l=1}^{r_{jk}-1} \boldsymbol{y}_{jkl} = 1 \quad \forall j \in \mathcal{J},\ k \in \mathcal{K}_j^A \tag{2f}$$

$$\sum_{l \in \mathcal{L}_{jk}} \boldsymbol{\lambda}_{jkl} = 1 \quad \forall j \in \mathcal{J},\ k \in \mathcal{K}_j^A \tag{2g}$$

$$0 \leq \boldsymbol{\lambda}_{jkl} \leq 1 \quad \forall j \in \mathcal{J},\ k \in \mathcal{K}_j^A,\ l \in \mathcal{L}_{jk} \tag{2h}$$

$$\boldsymbol{y}_{jkl} \in \{0, 1\} \quad \forall j \in \mathcal{J},\ k \in \mathcal{K}_j^A,\ l \in \{1, ..., r_{jk} - 1\} \tag{2i}$$

**Space Force Constraints**

$$\sum_{j \in \mathcal{J}^{SF}} \sum_{i \in \mathcal{I}_j^E} \text{merit}_i \cdot \boldsymbol{x}_{ij} \leq sf\_merit\_up \cdot \sum_{j \in \mathcal{J}^{SF}} \sum_{i \in \mathcal{I}_j^E} \boldsymbol{x}_{ij} \tag{3a}$$

$$\sum_{j \in \mathcal{J}^{SF}} \sum_{i \in \mathcal{I}_j^E} \text{merit}_i \cdot \boldsymbol{x}_{ij} \geq sf\_merit\_lo \cdot \sum_{j \in \mathcal{J}^{SF}} \sum_{i \in \mathcal{I}_j^E} \boldsymbol{x}_{ij} \tag{3b}$$

$$sf\_quota\_lo^{U1} \leq \sum_{j \in \mathcal{J}^{SF}} \sum_{i \in \mathcal{I}_j^E \cap \mathcal{I}^{U1}} \boldsymbol{x}_{ij} \leq sf\_quota\_up^{U1} \tag{3c}$$

$$sf\_quota\_lo^{R2} \leq \sum_{j \in \mathcal{J}^{SF}} \sum_{i \in \mathcal{I}_j^E \cap \mathcal{I}^{R2}} \boldsymbol{x}_{ij} \leq sf\_quota\_up^{R2} \tag{3d}$$

$$\sum_{j \in \mathcal{J}^{SF} \cap \mathcal{J}_i^E} \boldsymbol{x}_{ij} = 1 \quad \forall i \in \mathcal{I}^{SF} \tag{3e}$$

**Rated Reserved/Alternate List Constraints**

$$\sum_{j \in \mathcal{J}_i^{Reserved}} \boldsymbol{x}_{ij} = 1 \quad \forall i \in \mathcal{I}^{Reserved} \tag{4a}$$

$$quota_j^{U1} \cdot (1 - \sum_{j' \in \mathcal{J}_{ji}^{Pref}} \boldsymbol{x}_{ij'}) \leq \sum_{i' \in \mathcal{I}_{ji}^{Pref}} \boldsymbol{x}_{i'j} \quad \forall j \in \mathcal{J}^R,\ i \in \mathcal{I}_j^{Alt} \cap \mathcal{I}^{U1} \tag{4b}$$

$$quota_j^{R2} \cdot (1 - \sum_{j' \in \mathcal{J}_{ji}^{Pref}} \boldsymbol{x}_{ij'}) \leq \sum_{i' \in \mathcal{I}_{ji}^{Pref}} \boldsymbol{x}_{i'j} \quad \forall j \in \mathcal{J}^R,\ i \in \mathcal{I}_j^{Alt} \cap \mathcal{I}^{R2} \tag{4c}$$

**Algorithm 1** Source of Commissioning (SOC) Rated Board Algorithm

Initialize:
    $M^R \leftarrow \emptyset$ (empty set: rated cadet/AFSC matched pair)
    $E^R \leftarrow \emptyset$ (empty set: cadets that have exhausted all preferences and cannot be matched)
Input:
    $\mathcal{I}^R$ all rated eligible cadets from this SOC and their preferences
    $\mathcal{J}^R$ all rated AFSCs and their preferences
Output:
    $\mathcal{J}^{Fixed}$ cadet-AFSC fixed pairs
    $\mathcal{J}^{Reserved}$ AFSCs that cadets are reserved for
    $\mathcal{I}^{Reserved}$ cadets that have reserved AFSCs
**while** $|M^R| + |E^R| < |\mathcal{I}^R|$ **do**
    $M^R \leftarrow \emptyset$ (reset matches)
    **for** AFSC $j \in \mathcal{J}^R$ **do**
        $count \leftarrow 0$ (number of matches this iteration)
        **for** cadet $i \in \mathcal{I}_j^E \cap \mathcal{I}^R \setminus E^R$ (in order of AFSC $j$'s preference) **do**
            $j' \leftarrow$ cadet $i$'s most preferred, non-rejected AFSC
            **if** $j' = j$ and $count < quota_j$ **then**
                $M^R \leftarrow M^R \cup \{(i, j)\}$
                $count \leftarrow count + 1$
            **else if** $j' = j$ and $count = quota_j$ **then**
                AFSC $j$ reject cadet $i$
                **if** cadet $i$ rejected by all preferred rated AFSCs **then**
                    $E^R \leftarrow E^R \cup \{(i)\}$
**for** cadet, AFSC $(i, j) \in M^R$ **do**
    **if** AFSC $j$ is cadet $i$'s ***true*** most preferred AFSC **then**
        $\mathcal{J}_i^{Fixed} \leftarrow j$, ($\boldsymbol{x}_{ij}$ is fixed at 1 in VFT/GUO models)
    **else**
        $\mathcal{I}^{Reserved} \leftarrow \mathcal{I}^{Reserved} \cup \{(i)\}$
        $\mathcal{J}_i^{Reserved} \leftarrow$ set of cadet $i$'s more preferred AFSCs up to and including AFSC $j$

**Algorithm 2** Source of Commissioning (SOC) Rated Alternates Algorithm

Input:
$\mathcal{I}^R$ all rated eligible cadets from this SOC and their preferences
$\mathcal{J}^R$ all rated AFSCs and their preferences
$\mathcal{I}_j^{Res}$ cadets currently reserved to each rated AFSC
$\mathcal{I}_j^{Match}$ cadets currently matched to each rated AFSC
Output:
$\mathcal{I}_j^{Alt}$ cadets on each rated AFSC $j$'s alternate list
$\mathcal{I}_{ji}^{Pref}$ cadets that are more preferred by AFSC $j$ than cadet $i$
$\mathcal{J}_{ji}^{Pref}$ AFSCs that are more preferred by cadet $i$ than AFSC $j$
$iterating = True$
**while** *iterating* **do**
    **for** AFSC $j \in \mathcal{J}^R$ **do**
        Obtain $\mathcal{I}_j^{Res}$ and $\mathcal{I}_j^{Match}$
        $n^A = |\mathcal{I}_j^{Res}|$ (number of alternates is based on number of reserved slots)
        $\mathcal{I}_j^{Alt} \leftarrow \emptyset$ (start with empty set of cadets on this iteration)
        **for** cadet $i \in \mathcal{I}_j^E \cap \mathcal{I}^R \setminus I^{Fixed}$ (in order of AFSC $j$'s preference) **do**
            **if** cadet not reserved for $j$ or something better **then**
                **if** $|\mathcal{I}_j^{Alt}| < n^A$ (under alternate slot capacity) **then**
                $\mathcal{I}_j^{Alt} \leftarrow \mathcal{I}_j^{Alt} \cup \{(i)\}$
    **for** AFSC $j \in \mathcal{J}^R$ **do**
        **for** cadet $i \in \mathcal{I}_j^{Res}$ (in order of AFSC $j$'s preference) **do**
            **if** $i$ must be matched to $j$ (it's an inevitable match) **then**
                $\mathcal{I}_j^{Res} \leftarrow \mathcal{I}_j^{Res} \setminus \{(i)\}$
                $\mathcal{I}_j^{Match} \leftarrow \mathcal{I}_j^{Match} \cup \{(i)\}$
    **if** $\mathcal{I}_j^{Alt}$ stops changing between iterations **then**
        $iterating = False$

**Base/Training Sets and Indices**

| | |
|---|---|
| $b \in \mathcal{B}$ | the set of all bases |
| $c \in \mathcal{C}_j$ | the set of all training courses for AFSC $j$ |
| $d \in \mathcal{D}_i$ | the set of all AFSC outcome states that cadet $i$ has designated |
| $\mathcal{J}_{id}^{State} \subseteq \mathcal{J}_i^E$ | the set of AFSCs that, if assigned, would put cadet $i$ into state $d$ |
| $\mathcal{J}^B \subseteq \mathcal{J}$ | the set of AFSCs that have bases that need to be assigned |
| $\mathcal{B}_j^A \subseteq \mathcal{B}$ | the set of bases that AFSC $j$ may assign cadets to |
| $\mathcal{B}_i^E \subseteq \mathcal{B}$ | the set of bases that cadet $i$ may be assigned to (based on $\mathcal{J}_i^E$) |
| $\mathcal{B}_{id}^{State} \subseteq \mathcal{B}_i^E$ | the set of all bases that cadet $i$ can be assigned to in state $d$ based on its set of AFSCs ($\mathcal{J}_{id}^{State}$) |
| $\mathcal{C}_{ij}^E \subseteq \mathcal{C}_j$ | the set of courses that cadet $i$ is available to take with AFSC $j$ |
| $\mathcal{I}_{jc}^A \subseteq \mathcal{I}_j^E$ | the set of cadets that are available to take course $c$ with AFSC $j$ |

**Base/Training Parameters**

| | |
|---|---|
| $base\_utility_{ib}$ | the utility value that cadet $i$ has placed on base $b$ |
| $course\_utility_{ijc}$ | the utility value that cadet $i$ has for AFSC $j$'s course $c$ (calculated) |
| $w_{id}^A$ | the weight that cadet $i$ has on AFSCs in state $d$ |
| $w_{id}^B$ | the weight that cadet $i$ has on bases in state $d$ |
| $w_{id}^C$ | the weight that cadet $i$ has on courses in state $d$ |
| $u_{id}^S$ | the maximum utility that cadet $i$ receives from state $d$ |
| $M$ | sufficiently large number (big M) |
| $lo_{jb}^B$, $hi_{jb}^B$ | the minimum and maximum number of cadets, respectfully, that are to be assigned to base $b$ for AFSC $j$ |
| $lo_{jc}^C$, $hi_{jc}^C$ | the minimum and maximum number of cadets, respectfully, that are to be assigned to course $c$ for AFSC $j$ |

**Base/Training Decision Variables**

| | |
|---|---|
| $\boldsymbol{v}_{ib}$ | 1 if cadet $i$ is assigned to base $b$, and 0 otherwise |
| $\boldsymbol{q}_{ijc}$ | 1 if cadet $i$ is assigned to AFSC $j$'s course $c$, and 0 otherwise |
| $\boldsymbol{cadet\_value}_i$ | the value that cadet $i$ receives from their assignment outcome (revised variable for use in VFT/GUO models) |

**Base/Training Auxiliary Variables**

| | |
|---|---|
| $\boldsymbol{u}_{id}^A$ | the utility that cadet $i$ receives from their assigned AFSC in state $d$ $\frac{1}{u_{id}^S} \cdot \sum_{j \in \mathcal{J}_{id}^{State}} cadet\_utility_{ij} \cdot \boldsymbol{x}_{ij}$ |
| $\boldsymbol{u}_{id}^B$ | the utility that cadet $i$ receives from their assigned base in state $d$ $\sum_{b \in \mathcal{B}_{id}^{State}} base\_utility_{ib} \cdot \boldsymbol{v}_{ib}$ |
| $\boldsymbol{u}_{id}^C$ | the utility that cadet $i$ receives from their assigned course in state $d$ $\sum_{j \in \mathcal{J}_{id}^{State}} \sum_{c \in \mathcal{C}_{ij}^E} course\_utility_{ijc} \cdot \boldsymbol{q}_{ijc}$ |

**Base/Training Objective Function Revision**

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|---|---|
| $\boldsymbol{Z}^{GUO}$ | revised objective function variable for the GUO model |

$$cadets\_overall\_weight \cdot \sum_{i \in \mathcal{I}} cadet\_weight_i \cdot \boldsymbol{cadet\_value}_i + \frac{1}{N} \cdot afscs\_overall\_weight \cdot \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}_i^E} afsc\_utility_{ij} \cdot \boldsymbol{x}_{ij}$$

**Base/Training Constraints**

$$\boldsymbol{cadet\_value}_i \;\leq\; u_{id}^S(w_{id}^A \boldsymbol{u}_{id}^A + w_{id}^B \boldsymbol{u}_{id}^B + w_{id}^C \boldsymbol{u}_{id}^C) + M(1 - \sum_{j \in \mathcal{J}_{id}^{State}} \boldsymbol{x}_{ij}) \quad \forall i \in \mathcal{I},\ d \in \mathcal{D}_i \tag{5a}$$

$$\sum_{b \in \mathcal{B}_i^E} \boldsymbol{v}_{ib} \;=\; \sum_{j \in \mathcal{J}^B \cap \mathcal{J}_i^E} \boldsymbol{x}_{ij} \quad \forall i \in \mathcal{I} \tag{5b}$$

$$\boldsymbol{x}_{ij} \;\leq\; \sum_{b \in \mathcal{B}_j^A} \boldsymbol{v}_{ib} \quad \forall i \in \mathcal{I},\ j \in \mathcal{J}^B \cap \mathcal{J}_i^E \tag{5c}$$

$$\boldsymbol{x}_{ij} \;=\; \sum_{c \in \mathcal{C}_{ij}^E} \boldsymbol{q}_{ijc} \quad \forall i \in \mathcal{I},\ j \in \mathcal{J}_i^E \tag{5d}$$

$$lo_{jb}^B \;\leq\; \sum_{i \in \mathcal{I}_j^E} \boldsymbol{v}_{ib} \;\leq\; hi_{jb}^B \quad \forall j \in \mathcal{J}^B,\ b \in \mathcal{B}_j^A \tag{5e}$$

$$lo_{jc}^C \;\leq\; \sum_{i \in \mathcal{I}_{jc}^A} \boldsymbol{q}_{ijc} \;\leq\; hi_{jc}^C \quad \forall j \in \mathcal{J},\ c \in \mathcal{C}_j \tag{5f}$$

$$0 \;\leq\; \boldsymbol{cadet\_value}_i \;\leq\; 1 \quad \forall i \in \mathcal{I} \tag{5g}$$

$$\boldsymbol{v}_{ib} \;\in \{0,1\} \quad \forall i \in \mathcal{I},\ b \in \mathcal{B}_i^E \tag{5h}$$

$$\boldsymbol{q}_{ijc} \;\in \{0,1\} \quad \forall i \in \mathcal{I},\ j \in \mathcal{J}_i^E,\ c \in \mathcal{C}_{ij}^E \tag{5i}$$