Homework 1

David Mathews

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Part A

I made two main changes to this code. The first being a slight edit to the included libraries so the code would compile properly on my personal machine.

```
#include<stdlib.h>
```

The second change is the Euler method function. This was changed to be the following.

```
void PROPAGATE::euler_cromer(const PARAMS& p){
  avel=avel-(p.gbl*ang+p.kappa*avel-p.F_D*sin(2*pi*time/p.T_D))*dt;
  ang=ang+avel*dt;
  time+=dt;
};
```

Part B

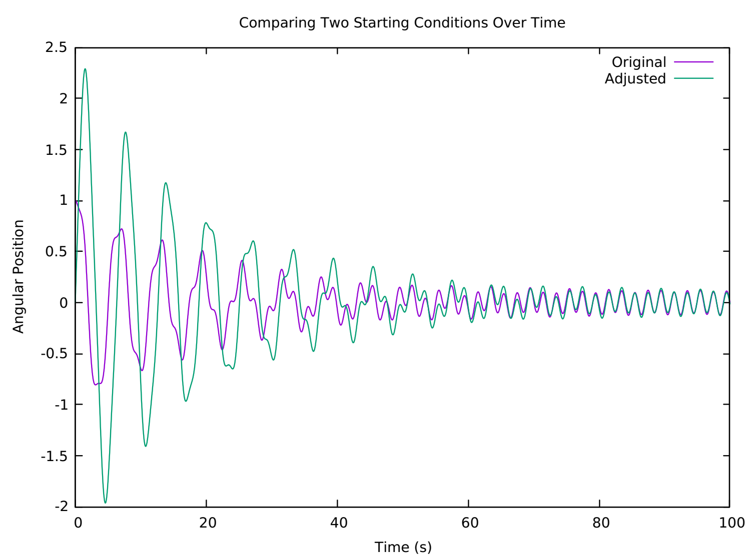

Figure 1: Two different initial conditions evolving over time

In this plot, we can see that the initial behavior is substantially different between the two different initial conditions. One of the conditions starts from rest while stretched out, and the other starts from the origin with a high initial velocity. Despite these differences, over time they both evened out in amplitude and matched in frequency.

For the analytic solution, the following equation was used to determine the amplitude. For reference, the variable w is defined as $w^2 = g/l = 1.0$ and $T_D = 2 * pi/t_d$ where t_d is the value given in this assignment of 2.0.

$$A = \frac{F_D}{\sqrt{((w^2 - T_D^2)^2 + k^2 * T_D^2)}} \tag{1}$$

Combining with the term for the phase difference, $tan(\phi) = \frac{-k*T_D}{w^2 - T_D^2}$ gives the following equation for the analytic solution.

$$\theta(t) = \frac{F_D}{\sqrt{((w^2 - T_D^2)^2 + k^2 * T_D^2)}} * sin([T_D - arctan(\frac{-k * T_D}{w^2 - T_D^2})] * t) \tag{2}$$

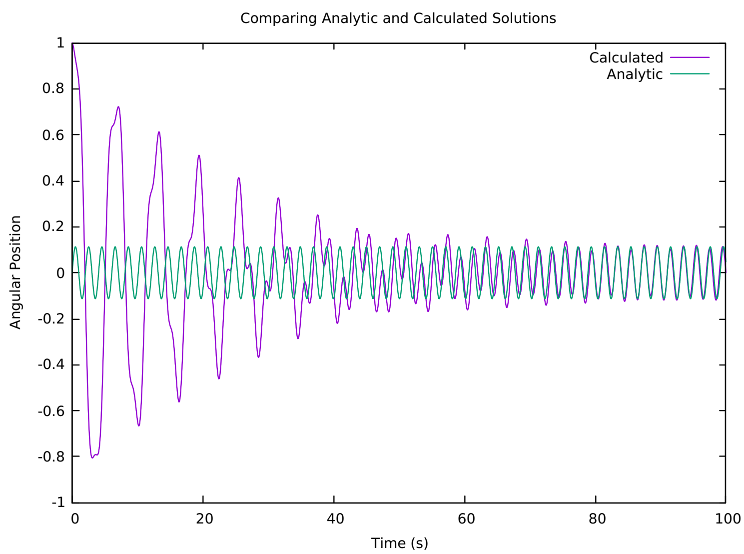

Figure 2: Analytic and Calculated Solution for Long Term Behavior

As shown in Figure 2 the long term behavior matches up perfectly with what we expect from the analytic solution.

Part C

Using the previously defined equation for amplitude, $\frac{F_D}{\sqrt{((w^2 - T_D^2)^2 + k^2 * T_D^2)}}$, we can expect resonance to occur whenever the value of w is approximately equal to T_D. As we are given that $w = 1$, we expect that at $T_D = w = 1$ or $t_d = 2pi$ we will have resonance. The amplitude was calculated over a range of values. This graph is shown below. As expected, the greatest amplitutde occured when $t_d = 2 * pi$ which is when $w = T_D$.

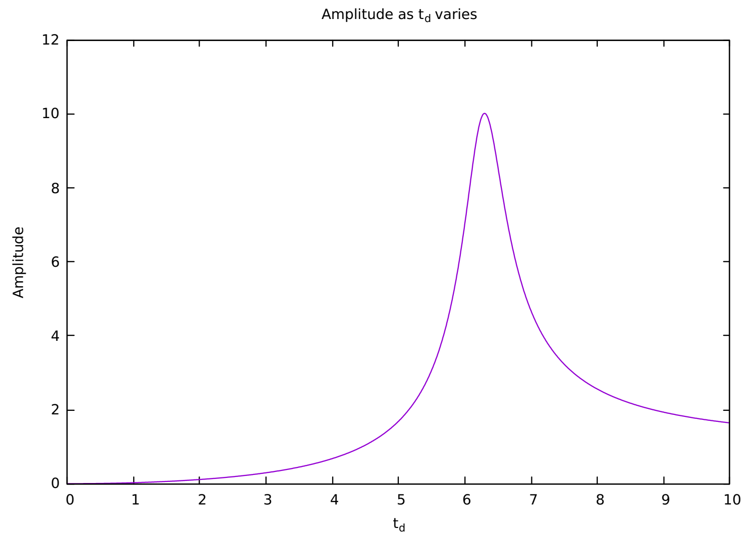

Figure 3: Long term amplitude as a function of t_d