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**Problem Set 5 *Solutions***

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**Question 1.** To obtain a consistent estimate of the causal effect of family size on female labor supply, some authors have suggested using twins on their first birth as an instrument for the number of children in the household. A twin birth is arguably random and by definition, the realization of a twin increases the number of children in the household, relative to a singleton birth. The Stata dataset *twins1sta.dta* was created from the 1980 Public Use Micro Sample 5% Census data files, and includes women aged 21-40 with at least one child. The 1980 PUMS identifies a person's age at the time of the census and their quarter of birth. We can infer that any two children in the household with the same age and quarter of birth are twins. There are roughly 6,000 first births to mothers that are twins. While there are over 800,000 observations in the original data set, a random sample of 6,500 non-twin births has been retained, for a total of about 12,500 observations. **(50 points)**

- (a) What fraction of mothers in the sample worked in the previous year? What is the average weeks worked among women that worked? What is the median labor earnings for women who worked? **(3 points)**

**See attached log. 60.4% of these mothers worked. Those who did work worked an average of 38.3 weeks with median earnings \$5,505 (this was 1979).**

- (b) Construct an indicator variable *second* that equals 1 for women that have two or more children (and zero otherwise). What fraction of women had two or more children? Estimate a simple bivariate regression where *weeks* of work is regressed on *second*. Interpret the slope coefficient in words. Explain why this regression is likely to suffer from omitted variables bias, and speculate on the direction of the bias. **(5 points)**

**See attached log. 85.5% of mothers had at least two children. The slope estimate of -6.8 tells us that women with 2 or more children worked 6.8 fewer weeks, on average, than those with 1 child. This regression likely suffers from omitted variables bias since the decision to have more children is endogenous. If women who expect to earn less in the labor market decide to stay home and raise more children, for example, this would produce the same negative association.**

- (c) Try using twins on first birth (*twin1st*) as an instrument for *second* in the main regression model of interest. That is, estimate the first-stage and reduced-form regression models, then calculate the Wald estimate. (Again, *weeks* of work is the outcome of interest). Interpret the slope coefficients in both regressions, and compare the IV (Wald)

estimate to the OLS. What is the  $R^2$  from the regression of *second* on *twin1st*? (5 points)

See the first stage and reduced form regressions in the attached log. The Wald estimate is the reduced form (-0.99) divided by the first stage (0.275), or -3.6. This is nearly half the size of the OLS estimate in absolute value, which makes sense if we believe OLS overstates the effect of family size on labor market participation (i.e., it reflects the influence of omitted variables associated with lower labor market participation).

The first stage slope coefficient tells us that mothers with twins on their first birth were 27.5 percentage points more likely to (ultimately) have 2 or more children than mothers who did not have twins. The reduced form slope coefficient tells us that mothers with first birth twins worked about 1 week less, on average, than mothers who did not have twins. The first stage slope coefficient (0.275) is not equal to 1.0 since many women who did *not* have twins went on to have 2 or more children. The  $R^2$  from the first stage is 0.15.

- (d) Repeat part (c) but use 2SLS and compare your results. Estimate the model a second time but allow for heteroskedasticity by using the heteroskedasticity-robust standard errors. Does this change your inference about the slope coefficient  $\beta$ ? (4 points)

See attached log. The coefficient of -3.6 on *second* is identical to the Wald estimate in part (c). The heteroskedasticity-robust standard errors are virtually the same as the traditional standard errors, leading to the same inference.

- (e) Carefully state the assumptions required for interpreting  $\hat{\beta}_{IV}$  in this case as an estimate of the causal effect of having two or more children on mothers' labor supply. (4 points)

The assumptions required for causal inference are: (1) instrument relevance: non-zero covariance between the instrument and explanatory variable ( $\text{Cov}(Z, X) \neq 0$ ), and (2) the independence/exclusion restriction: no covariance between the instrument and error term in the structural equation ( $\text{Cov}(Z, u) = 0$ ). In this application, there must be a significant association between having twins on the first birth and the propensity to have two or more children; the first stage regression provides strong evidence for this. Independence means the instrument (twins on first birth) is uncorrelated with other factors in the error term of the weeks worked equation. This seems unlikely, if some women are systematically more likely to have twins on their first birth (e.g., women who use IVF).

- (f) You are concerned that twin births are not entirely random, and convey some informa-

tion about the mother. Regress the following seven variables (individually) on *twin1st* and interpret your results: mother's education, age at first birth, current age, married, white, Black, other race. (You will need to create dummy variables for the last three in this list). Which of these have statistically significant relationships with *twin1st*? Are they meaningful in size? **(5 points)**

Coefficient estimates and standard errors are shown below. Twin births are positively related to mother's education, both parents' age, and mother's race = Black. Twin births are negatively related to married status and mother's race = white. All of these coefficient estimates are statistically significant, and many are meaningful in size. For example, mothers with twins on the first birth have 0.127 more years of education, on average. More years of education are related to labor market outcomes as well (e.g., weeks of work, earnings).

	educm	agefst	agem	married	white	black	other
	b/se	b/se	b/se	b/se	b/se	b/se	b/se
twin1st	0.127**	0.749***	0.521***	-0.015*	-0.034***	0.033***	0.001
	(0.045)	(0.064)	(0.087)	(0.007)	(0.006)	(0.006)	(0.003)

- (g) Now expand your 2SLS models in part (d) to include the covariates listed in (f). Interpret and compare your findings to the model without covariates. **(5 points)**

See attached log. With the covariates, the coefficient of -3.84 on *second* is similar to the model without covariates. There is a slight difference since the covariates are correlated with the twins instrument.

- (h) You remain concerned that the covariates do not fully account for correlation between the instrument and the error term, which could lead to inconsistency. This remaining correlation would be especially problematic if the instruments were weak. Conduct a weak instruments test following part (g) and report your conclusion. **(4 points)**

The first stage F statistic is very large (see log). Inconsistency could be a problem in the presence of weak instruments, but this does not appear to be a concern here.

- (i) OLS would be preferable if in fact family size (as represented here by *second*) were exogenous. Explain why. Conduct a test for endogeneity following the models in part (g) and report your conclusion. **(4 points)**

See attached log. The null hypothesis in the Durbin-Wu-Hausman test is that the explanatory variable of interest (*second*) is exogenous. The large test statistic and small p-value leads us to reject this hypothesis, suggesting that IV is appropriate.

- (j) Create three new dummy variables that indicate whether the mother's age at first birth was before age 20, between ages 20 and 24 (inclusive), or above age 24. Call these *age1st1*, *age1st2*, and *age1st3*. Next, create variables called *twin1st1*, *twin1st2*, and *twin1st3* that are interactions between the *age1st* variables and *twin1st*. Estimate a first stage regression that includes all of the covariates in (f), the three new *age1st* dummy variables and the three interactions. (Leave out the original *agefst*). Explain why the interaction terms can be considered instruments, and why they (might) improve upon the original single instrument *twin1st*.

Use an F-test to test two different hypotheses. First, test whether the coefficients on all three instruments are the same. Then, test whether the coefficients on all three instruments are zero. (Use the `test` command after `regress`). (5 points)

See attached log. For comparison, the original first stage had a coefficient on *twin1st* of 0.285. The new first stage includes the new “age at first birth” dummies (with one category necessarily omitted) and the new instruments: interactions between the age at first birth dummies and twins on first birth. First, notice that women who are older at their first birth are less likely to have second children. Second, notice that the effect of having twins on having 2+ children is larger for older women. This makes sense if the counterfactual (older women who don't have twins on their first birth) are less likely to have 2+ children. Both F tests reject the null hypothesis. So there is strong evidence that the effect of twins differs by age at first birth, and strong evidence that the instruments jointly explain variation in *second*.

- (k) Finally, estimate the 2SLS model from part (g) but using the new set of three instruments created in (j). How does your result compare to that in part (g), if at all? Compare both the point estimate and standard error. Conduct a test of over-identifying restrictions. What is the degrees of freedom for this test, and what is the conclusion? (6 points)

The first stage and 2SLS estimates are reported below. The 2SLS coefficient estimate for *second* is -3.37 with a standard error of 1.36. This is very similar to the results in part (g). The overid test is also shown. There are 2 degrees of freedom, the total number of additional restrictions. (Three instruments minus one endogenous explanatory variable). We cannot reject the null hypothesis that the model is appropriately specified.

**Question 2.** This problem will examine the role of measurement error using the dataset *cps87.dta* on Github. These data are a subsample of working men from the Current Population Survey of 1987. **(16 points)**

- (a) First create a variable that is the natural log of weekly earnings (*lnweekly*) and regress this on the individual's years of education (*years\_educ*). What is the estimated slope coefficient and standard error? **(2 points)**

**See log.** The estimated slope coefficient on *years\_educ* is 0.074, with a standard error of 0.0012. The interpretation is a predicted 7.4% increase in weekly earnings with every additional year of education.

- (b) Now create a “random noise” variable drawn from the standard normal distribution: `gen v=rnormal(0,1)`. Add this random noise to the years of education variable to create an education variable measured with classical measurement error (call it *years\_educ2*). What are the means and standard deviations of *years\_educ*, *years\_educ2*, and *v*? **(2 points)**

**See log.** The mean of the original years of education variable is 13.16. The mean of the new (noisy) education variable is 13.17, only slightly higher. In expectation, the new variable should have the same mean, but my mean for *v* turned out to be a little higher than 0. By construction, the standard deviation of *v* is close to 1. The standard deviation of the original education variable is 2.80 years, while the standard deviation of the new variable is 2.96 years. Note the increase is not 1; that is, adding a random variable *v* with a standard deviation of 1 does not increase the standard deviation by 1. Why? Let years of education be *x*. If *x* and *v* are uncorrelated, we know that  $\text{Var}(x + v) = \text{Var}(x) + \text{Var}(v)$ . However, it is not the case that  $\text{SD}(x + v) = \text{SD}(x) + \text{SD}(v)$ .

- (c) In our model of measurement error, we distinguished between the observed (noisy) measure  $x^*$ , the true measure *x* and the random noise  $e_0$ . Here, those variables are *years\_educ2*, *years\_educ*, and *v*. Regress log weekly earnings on *years\_educ2* rather than *years\_educ*. What is the estimated slope coefficient and standard error, and how does it compare to part (a)? Does this change make sense to you? Explain. **(2 points)**

**See log.** The estimated slope coefficient on the noisy measure of education is 0.066, with a standard error of 0.0011. That the slope coefficient is smaller in absolute value than the one in part (1) is expected, since classical measurement error in the explanatory variable will attenuate the slope estimate (that is, bias it toward zero).

- (d) Calculate the “reliability ratio” (or attenuation factor) below. How does it compare to the ratio of slope coefficients in (c) and (a)? **(2 points)**

$$\frac{\sigma_x^2}{\sigma_x^2 + \sigma_v^2}$$

See log. The attenuation factor is 0.888, which is approximately the ratio of the slopes in (c) and (a):  $0.0659/0.0741 = 0.889$ .

- (e) Repeat parts (b)-(d) but with a “noisier”  $v$  term: `gen v2=normal(0,2)`. How does this change the estimated slope coefficient, standard error, and reliability ratio when regressing log weekly earnings on the mis-measured education variable? (4 points)

See log. The estimated coefficient is now 0.048, with a standard error of 0.001. The slope estimate is attenuated further toward zero. Accordingly, the reliability ratio is smaller, at 0.657.

- (f) Finally, create a mis-measured version of log weekly earnings: `gen y2=lnweekly+v`. Regress this on the (correct) measure of education, `years_educ`. How do the slope coefficient and standard error compare with earlier results? (4 points)

See attached log. The slope coefficient of 0.070 is now close to the original OLS estimate of 0.074, and the standard error (0.0028) is higher than the original (0.0012). This is expected since classical measurement error in the dependent variable does not bias the OLS estimator, but does make it less precise.

**Question 3.** A researcher has collected data on alcohol consumption for 50 students each from 100 different colleges. The outcome of interest ( $y_i$ ) is the number of drinks consumed in the past 30 days. The researchers have developed an index ( $x_i$ ) that represents the strictness of a college’s alcohol use policy with higher values meaning a more strict policy. The authors are interested in the following model:

$$y_i = \beta_0 + \beta_1 x_i + u_i$$

The researchers are concerned about measurement error in  $y_i$ . In particular, they believe that students at schools with stricter alcohol policies may be less likely to report actual drinking because they are not supposed to drink. In this case, let  $y_i$  be actual consumption and  $y_i^*$  be reported consumption:  $y_i^* = y_i + e_i$ . We will assume that  $E(u_i) = 0$  and that  $Cov(x_i, u_i) = 0$ , but the measurement error is systematic such that  $Cov(e_i, x_i) < 0$ . In this case, with this form of measurement error, will the OLS estimate generated from a regression of  $y_i^*$  on  $x_i$  still be unbiased and consistent? If not, is the estimate biased upward or downward? Explain. (6 points)

Since we are forced to use the mismeasured  $y_i^*$ , the regression we are estimating is:

$$y_i^* = \beta_0 + \beta_1 x_i + \underbrace{u_i + e_i}_{v_i}$$

Using the OVB formula, in large samples we know that the OLS estimator  $\hat{\beta}_1$  converges in probability to:

$$\text{plim } \hat{\beta}_1 = \beta_1 + \frac{\text{Cov}(x_i, v_i)}{\text{Var}(x_i)}$$

or:

$$\text{plim } \hat{\beta}_1 = \beta_1 + \frac{\text{Cov}(x_i, e_i)}{\text{Var}(x_i)}$$

Since we assume the covariance between  $x$  and  $u$  is zero. If we believe there is a negative covariance between  $x$  (strictness of the alcohol policy) and  $e$  (measurement error in  $y$ ), then the second term is negative. If we also believe that  $\beta_1 < 0$ —the true relationship between strictness of alcohol policies and drinking is negative—then our estimated  $\beta_1$  will be “too negative”.

Put another way, we are regressing reported alcohol consumption on the strictness of a college’s alcohol use policy. If this relationship works as hypothesized, then  $\beta_1 < 0$ . That is, stricter alcohol policies reduce alcohol consumption. However, we believe that students in stricter environments are also more likely to under-report alcohol consumption. If this is the case, the relationship between alcohol consumption and the strictness of a college’s alcohol use policy will be overstated. It will appear that the policies are more effective than they are.

**Question 4.** You are conducting a randomized experiment of an intervention designed to improve graduation rates among a vulnerable student population. Assume 50% of your study sample is offered the intervention and 50% is not. In your population, assume that 60% of individuals are “compliers,” 30% are “always takers,” and 10% are “never-takers.” (There are no defiers). These three groups have mean potential outcomes as shown in the table below. **(12 points)**

Table 1: Mean potential outcomes (graduation rates)			
	Compliers	Always-takers	Never-takers
$D_i = 1$	0.62	0.85	0.55
$D_i = 0$	0.55	0.70	0.50
Treatment effect	<b>0.07</b>	<b>0.15</b>	<b>0.05</b>

- (a) Calculate the intent-to-treat (ITT) effect of the intervention. **(4 points)**

Let  $Z_i$  indicate treatment assignment. The ITT is  $E(Y_i|Z_i = 1) - E(Y_i|Z_i = 0)$ .

Presuming random assignment worked, the  $Z_i = 1$  group will consist of compliers, always-takers, and never-takers in their same proportion as in the population (60%, 30%, and 10%). The average graduation rate among this group would be:  $(0.62 * 0.60) + (0.85 * 0.30) + (0.50 * 0.10) = 0.677$ . Note the 0.62, 0.85, and 0.50 correspond to the *actual* treatment status ( $D_i$ ) observed in these groups when  $Z_i = 1$ .

Similarly, the  $Z_i = 0$  group will consist of compliers, always-takers, and never-takers in the same proportions as above. The average graduation rate among this group would be:  $(0.55 * 0.60) + (0.70 * 0.30) + (0.50 * 0.10) = 0.635$ .

Putting these two together, the ITT is  $0.677 - 0.635 = 0.042$ .

- (b) Calculate the first stage, and show that the IV (Wald) estimate equals the treatment effect for the compliers. (In other words, it is a LATE for the compliers). **(4 points)**

The first stage is  $E(D_i|Z_i = 1) - E(D_i|Z_i = 0)$ . In the  $Z_i = 1$  group, 90% receive the intervention (everyone but the never-takers), so this is the first term. In the  $Z_i = 0$  group, 30% receive the intervention (the always-takers), so this is the second term. The first stage is therefore:  $0.90 - 0.30 = 0.60$ .

The Wald estimate is the ITT/first stage, or  $0.042/0.6 = 0.07$ . This is the same as the treatment effect for the compliers shown in the table. Why is this the case? Notice that the graduation rates for the always-takers and



never-takers cancel out in the ITT (they are the same value, on average, in the  $Z_i = 1$  and  $Z_i = 0$  group.) Compliers only represent 60% of the ITT, however. (The ITT equals some value for the compliers and zero for the other two groups). Dividing by 0.6 gives you the treatment effect specific to the compliers.

- (c) Using the information in the table, what is the TOT? What is the ATE in the population? (4 points)

The TOT would be the average treatment effect for those treated. In this example, among those with  $Z_i = 1$ , the treated include the compliers and always-takers. Among those with  $Z_i = 0$ , the treated group includes the always-takers. Suppose the population were of size 100. The treated would include 30 compliers ( $50 \times 0.6$ ) and 30 always-takers ( $50 \times 0.3 + 50 \times 0.3$ ). In other words, the treated would be an even split of compliers and always-takers. (That's not always the case, it just worked out that way here). Generally, the TOT would be a weighted average of the treatment effects for these two treated groups:  $(1/2) \times 0.07 + (1/2) \times 0.15 = 0.11$

The ATE would be the average treatment effect in the *population*. This would be a weighted average of treatment effects across the three groups:  $(0.60 \times 0.07) + (0.30 \times 0.15) + (0.10 \times 0.05) = 0.092$

This is a good illustration of how the LATE can differ from the TOT and ATE in the population.

```

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. // *****
. // LP0-8852 Problem set 5 solutions
. // Last updated: November 12, 2021
. // *****
.
. // Question 1
. // ****
. // (a)
. // ****
.
. clear
. estimates drop _all
. use https://github.com/spcorcor18/LP0-8852/raw/main/data/twins1sta.dta
.
. sum worked
      Variable |      Obs      Mean   Std. Dev.      Min      Max
-----+-----
      worked |    12,500    .60456   .4889646         0         1
. sum weeks if worked==1
      Variable |      Obs      Mean   Std. Dev.      Min      Max
-----+-----
      weeks |     7,557   38.30899   16.53096         1        52
. sum lincome if worked==1,det
      moms labor income, 1979
-----+-----
      Percentiles      Smallest
1%              0              0
5%              45              0
10%             415              0      Obs              7,557
25%            2005              0      Sum of Wgt.         7,557
50%            5505              0      Mean              6475.015
                                Largest      Std. Dev.         5680.504
75%            9645            58515
90%           14005            60005      Variance           3.23e+07
95%           17005            70005      Skewness            1.727431
99%           23005            75000      Kurtosis            11.62867
. nmissing
.
. // ****
. // (b)
. // ****

```

```

. tabulate kids,miss
# of kids |
ever born |
to mom |      Freq.      Percent      Cum.
-----+-----
      1 |      1,808      14.46      14.46
      2 |      5,958      47.66      62.13
      3 |      3,248      25.98      88.11
      4 |      1,054       8.43      96.54
      5 |        318       2.54      99.09
      6 |         75       0.60      99.69
      7 |         24       0.19      99.88
      8 |          11       0.09      99.97
      9 |           3       0.02      99.99
     10 |           1       0.01     100.00
-----+-----
      Total |      12,500     100.00
. gen byte second=kids>=2
. tabulate second
second |      Freq.      Percent      Cum.
-----+-----
      0 |      1,808      14.46      14.46
      1 |     10,692      85.54     100.00
-----+-----
      Total |      12,500     100.00
. _eststo ols: reg weeks second
Source |      SS      df      MS      Number of obs      =      12,500
-----+-----
      Model |  71801.5838         1   71801.5838      F(1, 12498)      =      140.68
      Residual |  6378669.1     12,498   510.375188      Prob > F      =      0.0000
-----+-----
      Total |  6450470.68     12,499   516.078941      R-squared      =      0.0111
                                         Adj R-squared   =      0.0111
                                         Root MSE      =      22.591
-----+-----
      weeks |      Coef.   Std. Err.      t    P>|t|      [95% Conf. Interval]
-----+-----
      second | -6.813862   .5744749    -11.86   0.000    -7.939921    -5.687803
      _cons |  28.98838   .531307     54.56   0.000    27.94694    30.02983
-----+-----

.
. // ****
. // (c)
. // ****
. // Wald estimate
. reg weeks twin1st
Source |      SS      df      MS      Number of obs      =      12,500
-----+-----
      Model |  3054.30028         1   3054.30028      F(1, 12498)      =      5.92
      Residual |  6447416.38     12,498   515.875851      Prob > F      =      0.0150
-----+-----
      Total |  6450470.68     12,499   516.078941      R-squared      =      0.0005
                                         Adj R-squared   =      0.0004
                                         Root MSE      =      22.713
-----+-----
      weeks |      Coef.   Std. Err.      t    P>|t|      [95% Conf. Interval]
-----+-----
      twin1st | -.990038   .4068821     -2.43   0.015    -1.78759    -.1924865
      _cons |  23.62865   .279916     84.41   0.000    23.07997    24.17732
-----+-----

. scalar rf=_b[twin1st]

```

```

. reg second twin1st

```

Source	SS	df	MS	Number of obs	=	12,500
Model	234.976907	1	234.976907	F(1, 12498)	=	2239.20
Residual	1311.51397	12,498	.104937908	Prob > F	=	0.0000
				R-squared	=	0.1519
				Adj R-squared	=	0.1519
Total	1546.49088	12,499	.123729169	Root MSE	=	.32394

  

second	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
twin1st	.2746051	.0058031	47.32	0.000	.2632301	.2859801
_cons	.7253949	.0039923	181.70	0.000	.7175694	.7332204

```

. scalar fs=_b[twin1st]
. display rf/fs
-3.6053155

```

```

. // ****
. // (d)
. // ****
. // 2SLS
. _eststo iv1: ivregress 2sls weeks (second=twin1st)
Instrumental variables (2SLS) regression

```

Number of obs	=	12,500
Wald chi2(1)	=	5.97
Prob > chi2	=	0.0145
R-squared	=	0.0087
Root MSE	=	22.618

weeks	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
second	-3.605315	1.475498	-2.44	0.015	-6.497239	-.7133917
_cons	26.24392	1.278193	20.53	0.000	23.73871	28.74913

```

Instrumented:  second
Instruments:  twin1st
. _eststo iv1r: ivregress 2sls weeks (second=twin1st), robust
Instrumental variables (2SLS) regression

```

Number of obs	=	12,500
Wald chi2(1)	=	5.96
Prob > chi2	=	0.0146
R-squared	=	0.0087
Root MSE	=	22.618

		Robust				
weeks	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
second	-3.605315	1.476209	-2.44	0.015	-6.498632	-.7119987
_cons	26.24392	1.276994	20.55	0.000	23.74106	28.74679

```

Instrumented:  second
Instruments:  twin1st

```

```

. // ****
. // (f)
. // ****
. gen white=race==1
. gen black=race==2
. gen other=race==3

```

```

. foreach j in educm agefst agem married white black other {
2.   _eststo cov'j': reg 'j' twin1st
3.   }

```

Source	SS	df	MS	Number of obs	=	12,500
				F(1, 12498)	=	8.01
Model	50.1389213	1	50.1389213	Prob > F	=	0.0047
Residual	78274.9424	12,498	6.26299747	R-squared	=	0.0006
				Adj R-squared	=	0.0006
Total	78325.0813	12,499	6.26650782	Root MSE	=	2.5026

educm	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
twin1st	.126848	.0448319	2.83	0.005	.0389705	.2147254
_cons	12.46173	.0308423	404.05	0.000	12.40127	12.52218

Source	SS	df	MS	Number of obs	=	12,500
				F(1, 12498)	=	135.53
Model	1746.73053	1	1746.73053	Prob > F	=	0.0000
Residual	161071.047	12,498	12.8877458	R-squared	=	0.0107
				Adj R-squared	=	0.0106
Total	162817.777	12,499	13.0264643	Root MSE	=	3.59

agefst	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
twin1st	.748702	.0643109	11.64	0.000	.6226427	.8747612
_cons	21.28341	.0442429	481.06	0.000	21.19669	21.37014

Source	SS	df	MS	Number of obs	=	12,500
				F(1, 12498)	=	35.90
Model	845.129424	1	845.129424	Prob > F	=	0.0000
Residual	294226.049	12,498	23.5418507	R-squared	=	0.0029
				Adj R-squared	=	0.0028
Total	295071.179	12,499	23.6075829	Root MSE	=	4.852

agem	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
twin1st	.520784	.0869193	5.99	0.000	.3504088	.6911592
_cons	30.77688	.0597964	514.69	0.000	30.65967	30.89409

Source	SS	df	MS	Number of obs	=	12,500
				F(1, 12498)	=	5.15
Model	.732045576	1	.732045576	Prob > F	=	0.0232
Residual	1774.87203	12,498	.142012485	R-squared	=	0.0004
				Adj R-squared	=	0.0003
Total	1775.60408	12,499	.142059691	Root MSE	=	.37685

married	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
twin1st	-.0153273	.0067509	-2.27	0.023	-.02856	-.0020946
_cons	.8358141	.0046443	179.97	0.000	.8267106	.8449176

Source	SS	df	MS	Number of obs	=	12,500
				F(1, 12498)	=	27.44
Model	3.56574038	1	3.56574038	Prob > F	=	0.0000
Residual	1624.29218	12,498	.129964169	R-squared	=	0.0022
				Adj R-squared	=	0.0021
Total	1627.85792	12,499	.130239053	Root MSE	=	.36051

white	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
-------	-------	-----------	---	------	----------------------	--

Source	SS	df	MS	Number of obs	=	12,500
				F(1, 12498)	=	31.66
Model	3.45703454	1	3.45703454	Prob > F	=	0.0000
Residual	1364.85529	12,498	.109205896	R-squared	=	0.0025
				Adj R-squared	=	0.0024
Total	1368.31232	12,499	.109473743	Root MSE	=	.33046

black	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
twin1st	.0333079	.00592	5.63	0.000	.0217039	.044912
_cons	.109356	.0040727	26.85	0.000	.101373	.1173391

Source	SS	df	MS	Number of obs	=	12,500
				F(1, 12498)	=	0.03
Model	.000841382	1	.000841382	Prob > F	=	0.8623
Residual	349.631159	12,498	.027974969	R-squared	=	0.0000
				Adj R-squared	=	-0.0001
Total	349.632	12,499	.027972798	Root MSE	=	.16726

other	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
twin1st	.0005196	.0029963	0.17	0.862	-.0053535	.0063928
_cons	.0285541	.0020613	13.85	0.000	.0245136	.0325945

. estimates table cov\*, se(%5.3f) b(%8.3f) style(columns)

Variable	coveducm	covage-t	covagem	covmar~d	covwhite
twin1st	0.127	0.749	0.521	-0.015	-0.034
	0.045	0.064	0.087	0.007	0.006
_cons	12.462	21.283	30.777	0.836	0.862
	0.031	0.044	0.060	0.005	0.004

legend: b/se

Variable	covblack	covother
twin1st	0.033	0.001
	0.006	0.003
_cons	0.109	0.029
	0.004	0.002

legend: b/se

```
.
. // ****
. // (g)
. // ****
. // 2SLS with covariates
```

```
. _eststo iv2: ivregress 2sls weeks educm agefst agem married black other (second=twin1st)
Instrumental variables (2SLS) regression      Number of obs   =    12,500
                                             Wald chi2(7)     =    799.03
                                             Prob > chi2      =    0.0000
                                             R-squared       =    0.0713
                                             Root MSE       =    21.892
```

weeks	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
second	-3.840711	1.388089	-2.77	0.006	-6.561314	-1.120107
educm	1.338171	.0850866	15.73	0.000	1.171404	1.504938
agefst	-1.00932	.0702044	-14.38	0.000	-1.146918	-.8717218
agem	.893219	.052759	16.93	0.000	.7898133	.9966247
married	-6.005684	.5624385	-10.68	0.000	-7.108044	-4.903325
black	2.761305	.6253911	4.42	0.000	1.535561	3.987049
other	2.651669	1.174782	2.26	0.024	.3491376	4.9542
_cons	8.371989	1.810752	4.62	0.000	4.822981	11.921

Instrumented: second

Instruments: educm agefst agem married black other twin1st

```
. _eststo iv2r: ivregress 2sls weeks educm agefst agem married black other (second=twin1st
> ), robust
```

```
Instrumental variables (2SLS) regression      Number of obs   =    12,500
                                             Wald chi2(7)     =    871.98
                                             Prob > chi2      =    0.0000
                                             R-squared       =    0.0713
                                             Root MSE       =    21.892
```

		Robust					
	weeks	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
	second	-3.840711	1.388178	-2.77	0.006	-6.56149	-1.119931
	educm	1.338171	.0824623	16.23	0.000	1.176548	1.499794
	agefst	-1.00932	.0703404	-14.35	0.000	-1.147185	-.8714552
	agem	.893219	.0521858	17.12	0.000	.7909367	.9955014
	married	-6.005684	.5608533	-10.71	0.000	-7.104937	-4.906432
	black	2.761305	.6359378	4.34	0.000	1.51489	4.007721
	other	2.651669	1.189649	2.23	0.026	.3199998	4.983338
	_cons	8.371989	1.77135	4.73	0.000	4.900208	11.84377

Instrumented: second

Instruments: educm agefst agem married black other twin1st

```
.
. // ****
. // (h)
. // ****
. // F-test for weak instruments
. quietly ivregress 2sls weeks educm agefst agem married black other (second=twin1st)
```

```

. estat firststage
First-stage regression summary statistics
-----
Variable | R-sq.      Adjusted R-sq.      Partial R-sq.      F(1,12492)      Prob > F
-----+-----
second | 0.2354      0.2350      0.1738      2627.73      0.0000
-----

Minimum eigenvalue statistic = 2627.73
Critical Values      # of endogenous regressors:      1
Ho: Instruments are weak      # of excluded instruments:      1
-----

2SLS relative bias | 5%      10%      20%      30%
                    | (not available)
-----+-----

2SLS Size of nominal 5% Wald test | 10%      15%      20%      25%
LIML Size of nominal 5% Wald test | 16.38      8.96      6.66      5.53
LIML Size of nominal 5% Wald test | 16.38      8.96      6.66      5.53
-----

.
. quietly ivregress 2sls weeks educm agefst agem married black other (second=twin1st), rob
> ust
. estat firststage
First-stage regression summary statistics
-----
Variable | R-sq.      Adjusted R-sq.      Partial R-sq.      Robust F(1,12492)      Prob > F
-----+-----
second | 0.2354      0.2350      0.1738      2779.11      0.0000
-----

.
. // ****
. // (i)
. // ****
. // Endogeneity test
. quietly ivregress 2sls weeks educm agefst agem married black other (second=twin1st)
. estat endog
Tests of endogeneity
Ho: variables are exogenous
Durbin (score) chi2(1)      = 18.5511 (p = 0.0000)
Wu-Hausman F(1,12491)      = 18.5653 (p = 0.0000)
.
. quietly ivregress 2sls weeks educm agefst agem married black other (second=twin1st), rob
> ust
. estat endog
Tests of endogeneity
Ho: variables are exogenous
Robust score chi2(1)      = 18.5198 (p = 0.0000)
Robust regression F(1,12491) = 18.5472 (p = 0.0000)
.
. // ****
. // (j)
. // ****
. gen agefst1=(agefst<20)
. gen agefst2=(agefst>=20 & agefst<=24)
. gen agefst3=(agefst>24)
.
. gen twin1st1=(agefst1*twin1st)
. gen twin1st2=(agefst2*twin1st)

```



```

. gen twin1st3=(agefst3*twin1st)
.
. reg second twin1st1 twin1st2 twin1st3 educm agefst2 agefst3 agem married black other

```

Source	SS	df	MS	Number of obs	=	12,500
				F(10, 12489)	=	384.50
Model	364.042163	10	36.4042163	Prob > F	=	0.0000
Residual	1182.44872	12,489	.094679215	R-squared	=	0.2354
				Adj R-squared	=	0.2348
Total	1546.49088	12,499	.123729169	Root MSE	=	.3077

  

second	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
twin1st1	.2272634	.010031	22.66	0.000	.2076012	.2469256
twin1st2	.2617009	.007974	32.82	0.000	.2460705	.2773312
twin1st3	.4141127	.0121261	34.15	0.000	.3903436	.4378817
educm	-.0040774	.0011842	-3.44	0.001	-.0063986	-.0017563
agefst2	-.0997083	.0088162	-11.31	0.000	-.1169895	-.0824271
agefst3	-.2954771	.0119809	-24.66	0.000	-.3189615	-.2719926
agem	.0179598	.0006236	28.80	0.000	.0167375	.0191822
married	.0939062	.0077174	12.17	0.000	.0787789	.1090336
black	-.0268822	.0088008	-3.05	0.002	-.0441332	-.0096311
other	.0011631	.0165179	0.07	0.944	-.0312145	.0335407
_cons	.2470463	.0234947	10.51	0.000	.2009929	.2930996

  

```

. test twin1st1=twin1st2=twin1st3
( 1) twin1st1 - twin1st2 = 0
( 2) twin1st1 - twin1st3 = 0
      F( 2, 12489) =    77.48
      Prob > F =    0.0000
. test twin1st1 twin1st2 twin1st3
( 1) twin1st1 = 0
( 2) twin1st2 = 0
( 3) twin1st3 = 0
      F( 3, 12489) =   916.69
      Prob > F =    0.0000
.
. // ****
. // (k)
. // ****

```

```
. _eststo iv3: ivregress 2sls weeks educm agefst2 agefst3 agem married black other ///
> (second=twin1st1 twin1st2 twin1st3), first
```

First-stage regressions

					Number of obs	=	12,500
					F( 10, 12489)	=	384.50
					Prob > F	=	0.0000
					R-squared	=	0.2354
					Adj R-squared	=	0.2348
					Root MSE	=	0.3077
-----							
second		Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
-----							
educm		-.0040774	.0011842	-3.44	0.001	-.0063986	-.0017563
agefst2		-.0997083	.0088162	-11.31	0.000	-.1169895	-.0824271
agefst3		-.2954771	.0119809	-24.66	0.000	-.3189615	-.2719926
agem		.0179598	.0006236	28.80	0.000	.0167375	.0191822
married		.0939062	.0077174	12.17	0.000	.0787789	.1090336
black		-.0268822	.0088008	-3.05	0.002	-.0441332	-.0096311
other		.0011631	.0165179	0.07	0.944	-.0312145	.0335407
twin1st1		.2272634	.010031	22.66	0.000	.2076012	.2469256
twin1st2		.2617009	.007974	32.82	0.000	.2460705	.2773312
twin1st3		.4141127	.0121261	34.15	0.000	.3903436	.4378817
cons		.2470463	.0234947	10.51	0.000	.2009929	.2930996

Instrumental variables (2SLS) regression

Number of obs	=	12,500
Wald chi2(8)	=	759.80
Prob > chi2	=	0.0000
R-squared	=	0.0671
Root MSE	=	21.941

weeks		Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
-----							
second		-3.370982	1.359771	-2.48	0.013	-6.036083	-.7058805
educm		1.26522	.0847011	14.94	0.000	1.099209	1.431231
agefst2		-3.65137	.494139	-7.39	0.000	-4.619865	-2.682876
agefst3		-8.971728	.6795455	-13.20	0.000	-10.30361	-7.639843
agem		.8275766	.0510731	16.20	0.000	.7274751	.927678
married		-6.164801	.5626361	-10.96	0.000	-7.267548	-5.062055
black		2.976924	.626407	4.75	0.000	1.749188	4.204659
other		2.477429	1.177288	2.10	0.035	.1699871	4.78487
cons		-7.189732	1.711086	-4.20	0.000	-10.5434	-3.836065

Instrumented: second

Instruments: educm agefst2 agefst3 agem married black other twin1st1  
twin1st2 twin1st3

. estat overid

Tests of overidentifying restrictions:

Sargan (score) chi2(2) = 4.32266 (p = 0.1152)

Basman chi2(2) = 4.32035 (p = 0.1153)

```
. estat first
```

```
First-stage regression summary statistics
```

Variable	R-sq.	Adjusted R-sq.	Partial R-sq.	F(3,12489)	Prob > F
second	0.2354	0.2348	0.1805	916.693	0.0000

```
Minimum eigenvalue statistic = 916.693
```

```
Critical Values # of endogenous regressors: 1
```

```
Ho: Instruments are weak # of excluded instruments: 3
```

	5%	10%	20%	30%
2SLS relative bias	13.91	9.08	6.46	5.39

	10%	15%	20%	25%
2SLS Size of nominal 5% Wald test	22.30	12.83	9.54	7.80
LIML Size of nominal 5% Wald test	6.46	4.36	3.69	3.32

```
. estimates table ols iv*, b(%4.3f) se(%4.3f)
```

Variable	ols	iv1	iv1r	iv2	iv2r	iv3
second	-6.814	-3.605	-3.605	-3.841	-3.841	-3.371
	0.574	1.475	1.476	1.388	1.388	1.360
educm				1.338	1.338	1.265
				0.085	0.082	0.085
agefst				-1.009	-1.009	
				0.070	0.070	
agem				0.893	0.893	0.828
				0.053	0.052	0.051
married				-6.006	-6.006	-6.165
				0.562	0.561	0.563
black				2.761	2.761	2.977
				0.625	0.636	0.626
other				2.652	2.652	2.477
				1.175	1.190	1.177
agefst2						-3.651
						0.494
agefst3						-8.972
						0.680
_cons	28.988	26.244	26.244	8.372	8.372	-7.190
	0.531	1.278	1.277	1.811	1.771	1.711

legend: b/se

```
.
.
. // Question 2
. // ****
. // (a)
. // ****
. clear
. estimates drop _all
. use https://github.com/spcorcor18/LP0-8852/raw/main/data/cps87.dta
.
. gen lnweekly = ln(weekly_earn)
```

```
. _eststo parta: reg lnweekly years_educ
```

Source	SS	df	MS	Number of obs	=	19,906
Model	854.28055	1	854.28055	F(1, 19904)	=	3877.62
Residual	4385.05814	19,904	.220310397	Prob > F	=	0.0000
				R-squared	=	0.1631
				Adj R-squared	=	0.1630
Total	5239.33869	19,905	.263217216	Root MSE	=	.46937

  

lnweekly	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
years_educ	.0741141	.0011902	62.27	0.000	.0717813 .076447
_cons	5.091872	.0160138	317.97	0.000	5.060484 5.123261

```
.
. // ****
. // (b)
. // ****
. // random noise drawn from N(0,1)
. gen v=rnormal(0,1)
. gen years_educ2 = years_educ + v
. sum years_educ years_educ2 v
```

Variable	Obs	Mean	Std. Dev.	Min	Max
years_educ	19,906	13.16126	2.795234	0	18
years_educ2	19,906	13.17125	2.956716	-1.768029	21.19708
v	19,906	.0099873	.9943866	-4.468302	4.327502

```
.
. // ****
. // (c)
. // ****
. // regress lnweekly on noisy educ
. _eststo partc: reg lnweekly years_educ2
```

Source	SS	df	MS	Number of obs	=	19,906
Model	755.421105	1	755.421105	F(1, 19904)	=	3353.30
Residual	4483.91758	19,904	.22527721	Prob > F	=	0.0000
				R-squared	=	0.1442
				Adj R-squared	=	0.1441
Total	5239.33869	19,905	.263217216	Root MSE	=	.47463

  

lnweekly	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
years_educ2	.0658876	.0011378	57.91	0.000	.0636574 .0681178
_cons	5.199485	.0153593	338.52	0.000	5.16938 5.229591

```
.
. // ****
. // (d)
. // ****
. // reliability ratio
. sum years_educ
```

Variable	Obs	Mean	Std. Dev.	Min	Max
years_educ	19,906	13.16126	2.795234	0	18

```
. local varx=r(Var)
. sum v
```

Variable	Obs	Mean	Std. Dev.	Min	Max
v	19,906	.0099873	.9943866	-4.468302	4.327502

```
. local varv=r(Var)
```

```

. display 'varx'/'('varx' + 'varv')
.88766309
.
. reg lnweekly years_educ

```

Source	SS	df	MS	Number of obs	=	19,906
Model	854.28055	1	854.28055	F(1, 19904)	=	3877.62
Residual	4385.05814	19,904	.220310397	Prob > F	=	0.0000
				R-squared	=	0.1631
				Adj R-squared	=	0.1630
Total	5239.33869	19,905	.263217216	Root MSE	=	.46937

```

-----
lnweekly |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-----+-----
years_educ |   .0741141   .0011902    62.27   0.000    .0717813    .076447
_cons      |   5.091872   .0160138   317.97   0.000    5.060484    5.123261
-----+-----
. display _b[years_educ]*('varx'/'('varx' + 'varv'))
.06578838
.
. // ****
. // (e)
. // ****
. // "noisier" term drawn from N(0,2)
. gen v2=rnormal(0,2)
. gen years_educ3 = years_educ + v2
. sum years_educ years_educ3 v2

```

Variable	Obs	Mean	Std. Dev.	Min	Max
years_educ	19,906	13.16126	2.795234	0	18
years_educ3	19,906	13.15989	3.438949	-5.131371	24.68635
v2	19,906	-.0013695	2.018955	-8.005374	7.888378

```

. _eststo parte: reg lnweekly years_educ3

```

Source	SS	df	MS	Number of obs	=	19,906
Model	542.557441	1	542.557441	F(1, 19904)	=	2299.25
Residual	4696.78125	19,904	.235971727	Prob > F	=	0.0000
				R-squared	=	0.1036
				Adj R-squared	=	0.1035
Total	5239.33869	19,905	.263217216	Root MSE	=	.48577

```

-----
lnweekly |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-----+-----
years_educ3 |   .0480083   .0010012    47.95   0.000    .0460458    .0499707
_cons       |   5.435524   .0136182   399.14   0.000    5.408831    5.462217
-----+-----
.
. // reliability ratio
. sum years_educ

```

Variable	Obs	Mean	Std. Dev.	Min	Max
years_educ	19,906	13.16126	2.795234	0	18

```

. local varx=r(Var)
. sum v2

```

Variable	Obs	Mean	Std. Dev.	Min	Max
v2	19,906	-.0013695	2.018955	-8.005374	7.888378

```

. local varv2=r(Var)
. display 'varx'/'('varx' + 'varv2')
.65716169
.

```

```
. reg lnweekly years_educ
```

Source	SS	df	MS	Number of obs	=	19,906
Model	854.28055	1	854.28055	F(1, 19904)	=	3877.62
Residual	4385.05814	19,904	.220310397	Prob > F	=	0.0000
				R-squared	=	0.1631
				Adj R-squared	=	0.1630
Total	5239.33869	19,905	.263217216	Root MSE	=	.46937

  

lnweekly	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
years_educ	.0741141	.0011902	62.27	0.000	.0717813 .076447
_cons	5.091872	.0160138	317.97	0.000	5.060484 5.123261

```
. display _b[years_educ]*('varx'/(('varx' + 'varv2')))
```

```
.04870497
```

```
.
. // ****
. // (f)
. // ****
. // mis-measured dependent variable
. gen y2=lnweekly + v
. _eststo partf: reg y2 years_educ
```

Source	SS	df	MS	Number of obs	=	19,906
Model	768.102804	1	768.102804	F(1, 19904)	=	636.19
Residual	24030.8772	19,904	1.20733909	Prob > F	=	0.0000
				R-squared	=	0.0310
				Adj R-squared	=	0.0309
Total	24798.98	19,905	1.24586687	Root MSE	=	1.0988

  

y2	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
years_educ	.0702766	.0027862	25.22	0.000	.0648153 .0757378
_cons	5.152367	.037488	137.44	0.000	5.078887 5.225847

```
. estimates table part*, b(%4.3f) se(%4.3f)
```

Variable	parta	partc	parte	partf
years_educ	0.074			0.070
	0.001			0.003
years_educ2		0.066		
		0.001		
years_educ3			0.048	
			0.001	
_cons	5.092	5.199	5.436	5.152
	0.016	0.015	0.014	0.037

legend: b/se

```
.
.
.
. capture log close
```