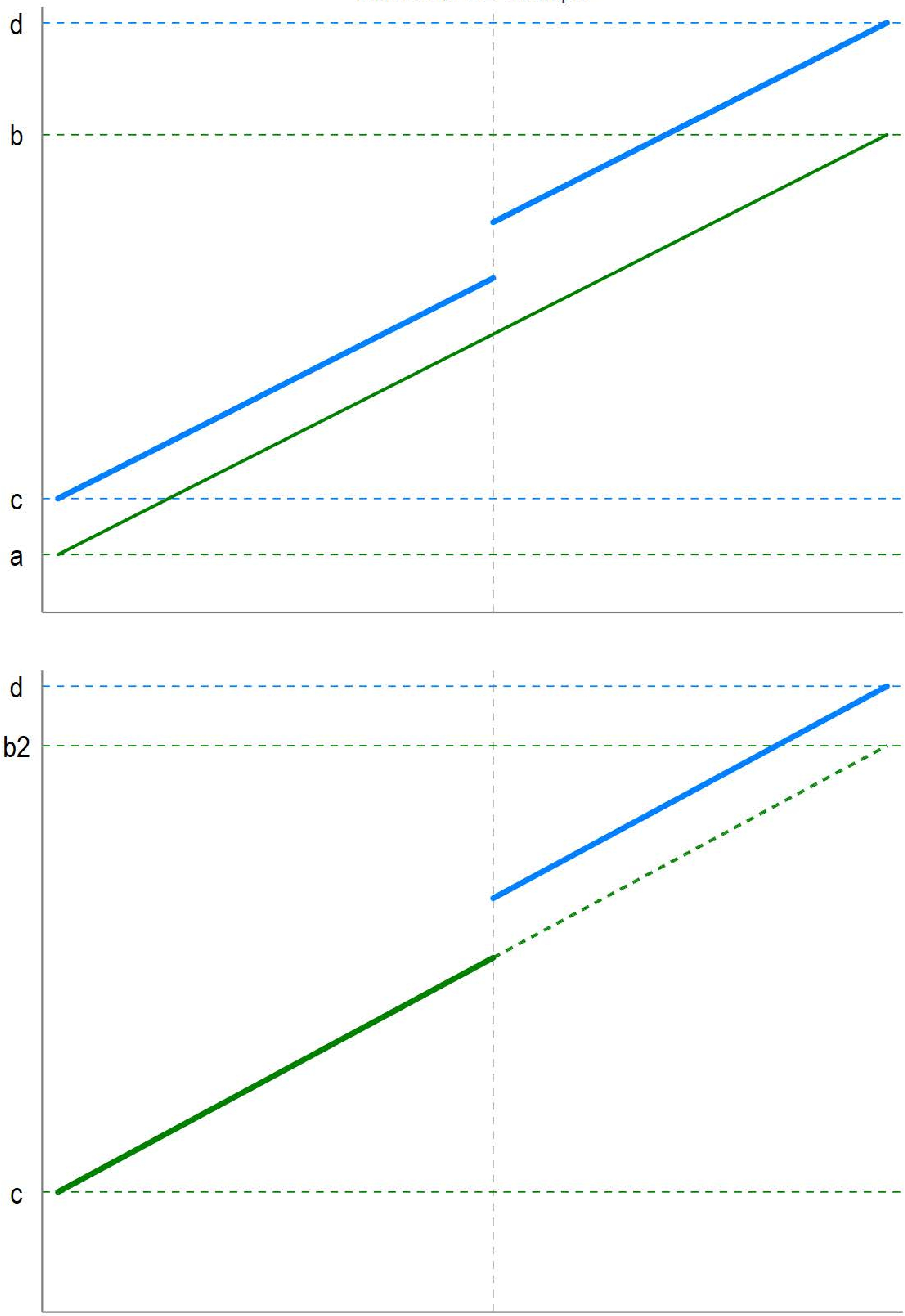
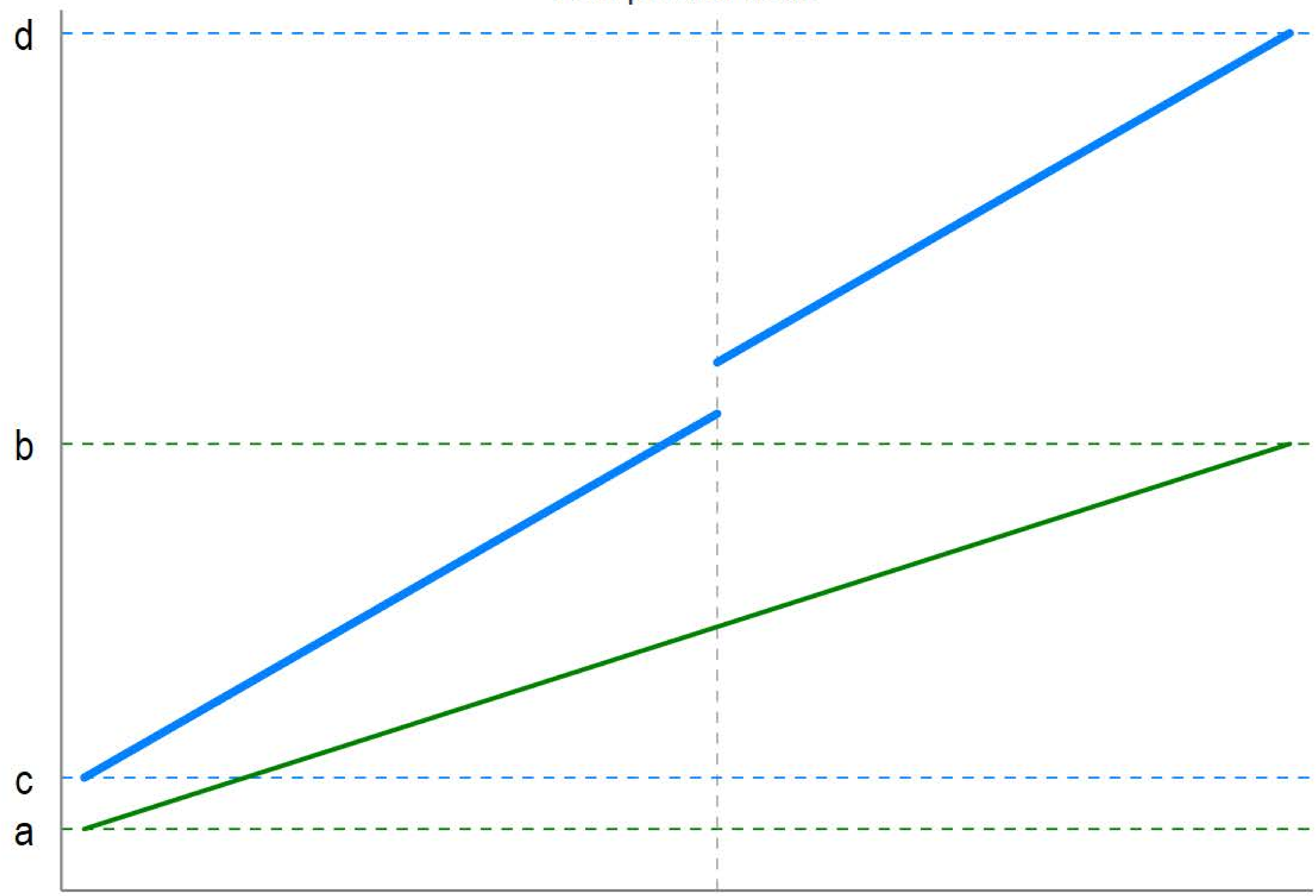
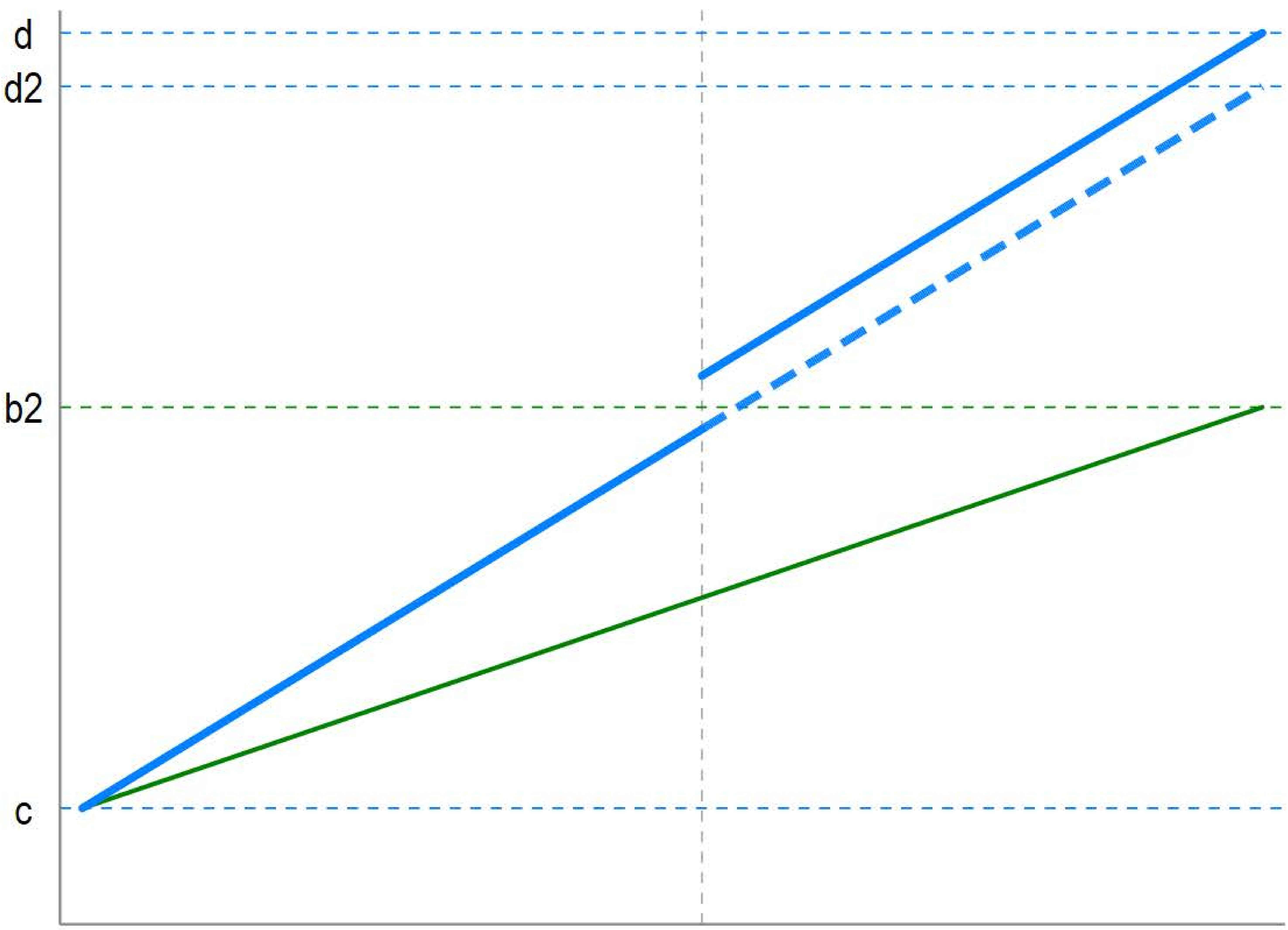
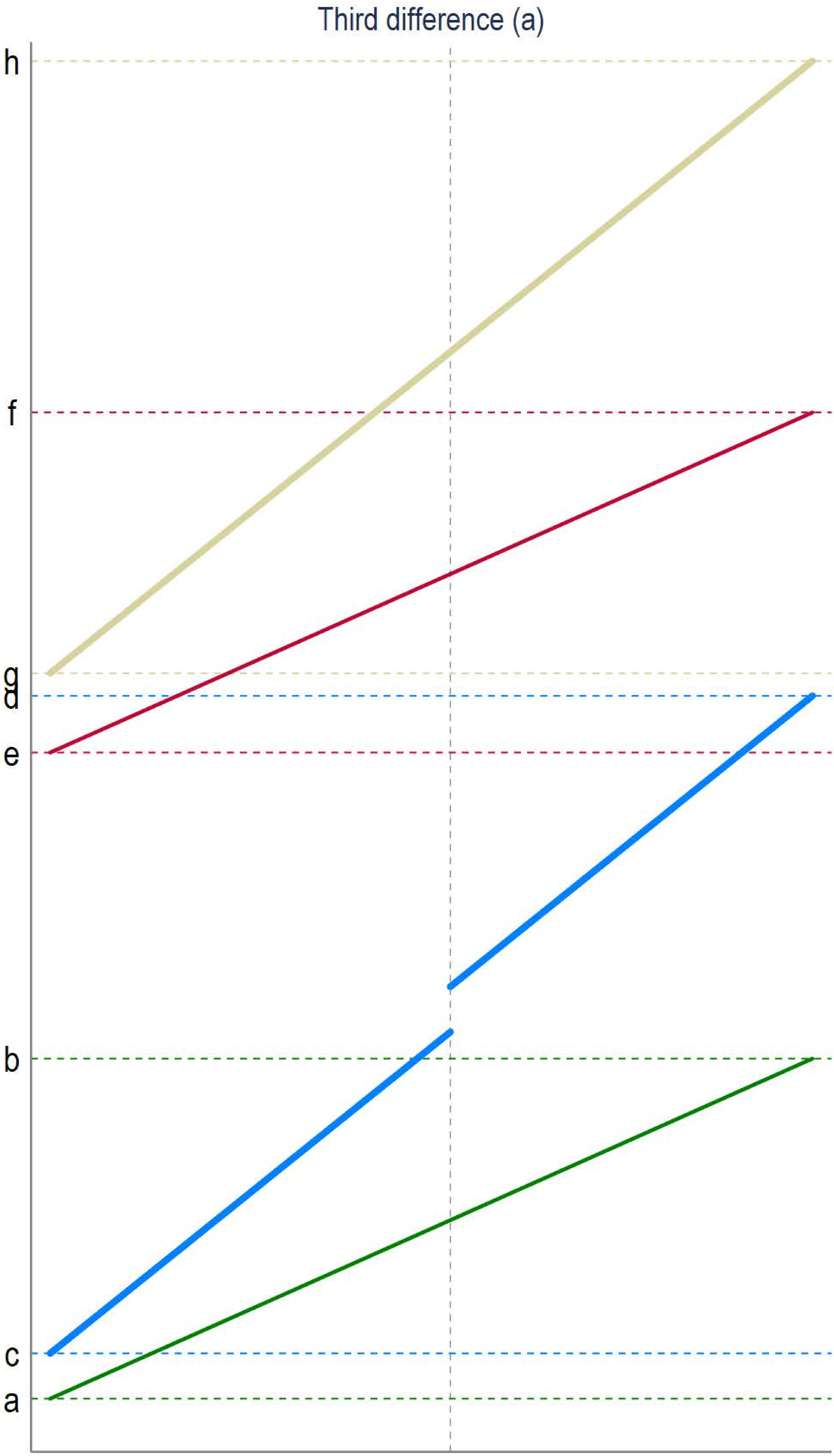
Traditional DD example

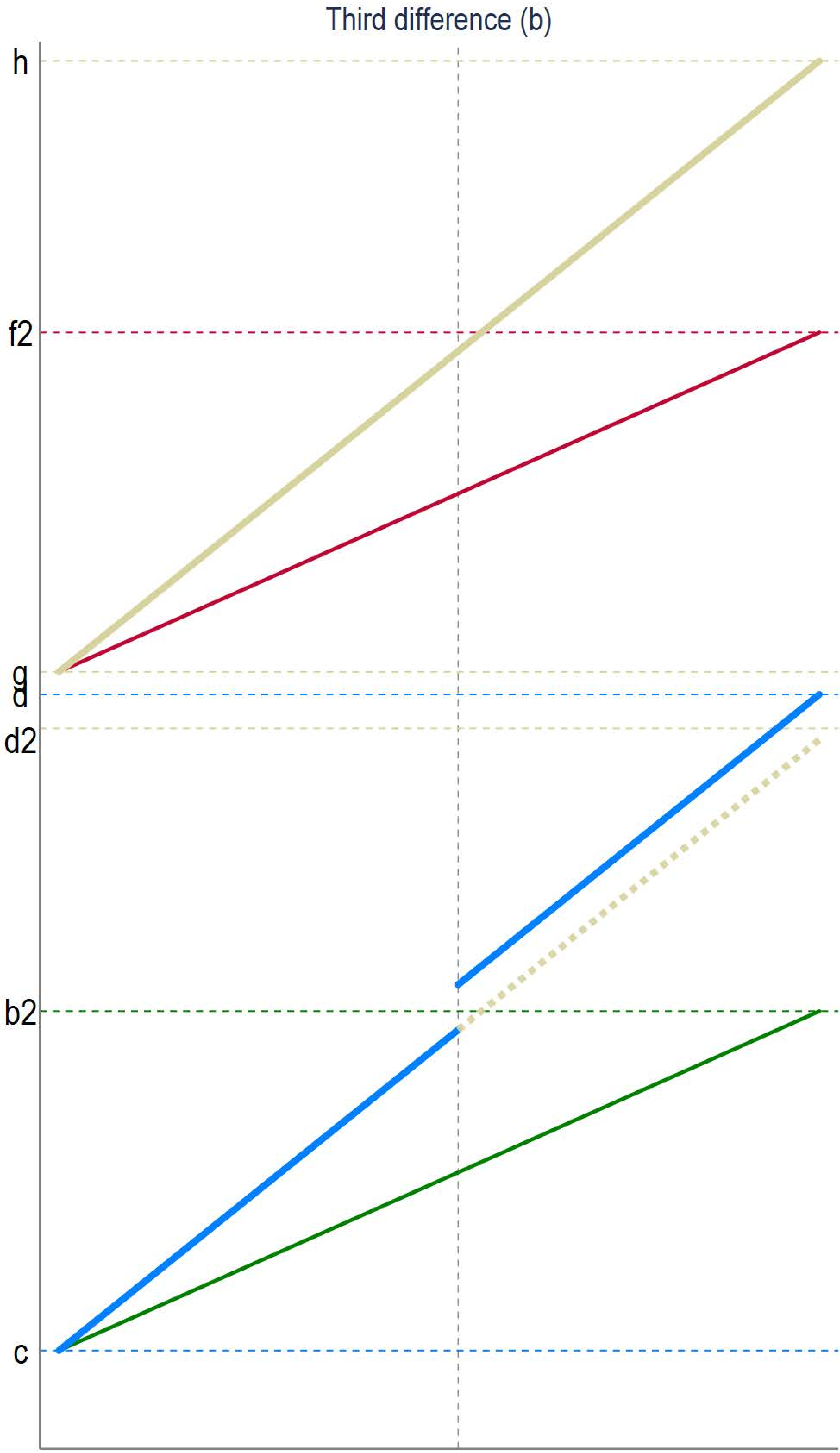


Non-parallel trend






Third difference (a)
h
f
g
d
e
b
c
a


Third difference (b)
h
f2
g
d
d2
b2
c

Triple differences - visual representation

Graph 1 top panel

- Traditional 2x2 difference-in-difference: treated group (blue/thick line) and untreated group (green/thin line), observed before and after treatment.
- **DD = (d - c) - (b - a)**
- If the parallel trends assumption holds--as it is assumed to here--DD measures the treatment effect.

Graph 1 bottom panel

- An alternative way of looking at the top panel. This graph is the same as the top, except that the untreated (green/thin) line has been shifted up to meet the treated (blue/thick) line. The dashed green line represents the counterfactual for the treated group.
- **DD = (d - b2)**
- Note b2 is just b + (c-a), the "post" value adjusted for baseline differences.

Graph 2 top panel

- Similar to Graph 1, but now we have **non-parallel trends!**
- We could calculate **DD = (d - c) - (b - a)**, but this would no longer measure the treatment effect, since the parallel trends assumption no longer holds. DD reflects <u>both</u> the treatment effect and the differential trend.

Graph 2 bottom panel

- An alternative way of looking at the top panel. The graph is the same as the top, except that the untreated (green/thin) line has been shifted up to meet the treated (blue/thick) line.
- DD = (d - b2), but again this reflects both the treatment effect (d-d2) and the differential trend (d2-b2). Extending the treated (blue/thick) line into the post period (the blue dotted line) allows us to see how much of DD is due to the differential trend. Unfortunately, we can't separately identify these in the data.

Graph 3

- Note that this is *one graph* with a long y-axis. The bottom part of the graph is the same non-parallel trends case from Graph 2. The top part introduces *two additional groups:* one (yellow/thick) with the same time trend as the treated group, and another (red/thin) with the same time trend as the untreated group. Neither of these two additional groups were treated, so there's no break in their line.
- The two additional groups will help us measure the differential time trend: (h - g) - (f - e)

Graph 4

- An alternative way of looking at Graph 3. Here the untreated (green/thin) line has been shifted up to meet the treated (blue/thick) line, and the red line has been shifted up to meet the yellow line.
- The bottom part of the graph again shows that the DD [(d - c) - (b - a)] reflects both the treatment effect and the differential time trend. Now, however, we can measure the differential time trend using the yellow and red groups.
- **Triple difference = d - d2 = (d - b2) - (h - f2)**
- **Written another way = [(d - c) - (b - a)] - [(h - g) - (f - e)]**
- The assumption here is that the differential change over time [(h - g) - (f - e)] reflects what would have happened to the treatment and control groups over time, in the absence of treatment.

Let's try to map Graph 3 onto the triple difference regression equation:

*b0 + b1*POST + b2*G + b3*D + b4*(G*POST) + b5*(D*POST) + b6*(G*D) + b7*(G*D*Post)*

D = 1 for Blue, Yellow; D = 0 for Red, Green
G = 1 for Blue, Green; G = 0 for Red, Yellow

<u>Green</u>
Pre: b0 + b2 [a]
Post: b0 + b1 + b2 + b4 [b]
Diff: b1 + b4 [b - a]

<u>Blue</u>
Pre: b0 + b2 + b3 + b6 [c]
Post: b0 + b1 + b2 + b3 + b4 + b5 + b6 + b7 [d]
Diff: b1 + b4 + b5 + b7 [d - c]

Blue diff - Green diff = b5 + b7 [(d-c) - (b-a)]

<u>Red</u>
Pre: b0 [e]
Post: b0 + b1 [f]
Diff: b1 [f - e]

<u>Yellow</u>
Pre: b0 + b3 [g]
Post: b0 + b1 + b3 + b5 [h]
Diff: b1 + b5 [h - g]

Yellow diff - Red diff = b5 [(h-g) - (f-e)]
Triple diff = b7 [(d - c) - (b -a)] - [(h - g) - (f - e)]