Vanderbilt University
Leadership, Policy and Organizations
Class Number 9522
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**Diagnosing and Fixing Common Problems With Regression**

# Introduction

There are three ways to get a good intuitive grasp of whether there might be some issues with your model fit:

1. Plot the data
2. Plot the data
3. Plot the data

# Collinearity

The test to use for collinearity in Stata is `vif`. The results of the VIF (Variance Inflation Factor) test states whether inflation has been increased because the covariate is correlated with the other regressors. The informal rule with VIF's is that 10 is large, while 20 is unacceptable.

Stata's vif command can be run after a regression to check for collinearity.

```
.
. estat vif

    Variable |       VIF       1/VIF
-------------+----------------------
          iq |     52.76    0.018955
       test3 |     52.24    0.019143
           s |      1.60    0.623911
         kww |      1.31    0.761735
         med |      1.18    0.848345
        expr |      1.15    0.867604
      tenure |      1.10    0.907839
         rns |      1.06    0.944157
        smsa |      1.05    0.948668
-------------+----------------------
    Mean VIF |     12.61
```

It looks like there's a big problem with the iq variable and the test3 variable. I first do an F test to see if they both belong in the model.

```
. test test3 iq

 ( 1)  test3 = 0
```

```
  ( 2)  iq = 0

       F(  2,   748) =    6.65
            Prob > F =    0.0014
```

They are jointly significant, so I need to choose one to eliminate. In this case I drop test3, re-run the model, and ask again for variance inflation factors.

```
. local controls kww iq expr tenure rns smsa med

.
. reg `y´ `x´ `controls´

      Source |       SS       df       MS              Number of obs =     758
-------------+------------------------------           F(  8,   749) =   53.43
       Model |  50.6098566     8  6.32623207           Prob > F      =  0.0000
    Residual |  88.6762933   749  .118392915           R-squared     =  0.3634
-------------+------------------------------           Adj R-squared =  0.3566
       Total |   139.28615   757  .183997556           Root MSE      =  .34408

------------------------------------------------------------------------------
          lw |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           s |   .0878976   .0070921    12.39   0.000     .0739749    .1018203
         kww |   .0030669   .0019596     1.57   0.118    -.0007801    .0069139
          iq |   .0028559   .0011028     2.59   0.010      .000691    .0050208
        expr |   .0386396   .0063668     6.07   0.000     .0261407    .0511385
      tenure |   .0322462   .0078371     4.11   0.000     .0168608    .0476315
         rns |  -.0720075   .0289852    -2.48   0.013    -.1289095   -.0151055
        smsa |   .1302547   .0281144     4.63   0.000     .0750623    .1854471
         med |   .0055788    .004952     1.13   0.260    -.0041427    .0153004
       _cons |   3.840349   .1126832    34.08   0.000     3.619136    4.061561
------------------------------------------------------------------------------

. eststo full_model_a, title("Model 2:No Test 3")

.
. estat vif

    Variable |       VIF       1/VIF
-------------+----------------------
           s |      1.60    0.624254
          iq |      1.44    0.693400
         kww |      1.31    0.763785
         med |      1.18    0.848802
        expr |      1.15    0.870279
      tenure |      1.10    0.909065
         rns |      1.06    0.945150
        smsa |      1.05    0.949180
-------------+----------------------
    Mean VIF |      1.24
```

Much better! All vifs are now in an acceptable range.

Remember that as a practical matter, collinearity does not bias your estimates, it just makes them inefficient. It's usually just not that serious a problem— but it's worth checking to see if you're dealing with an extreme case.

## Quick Exercise

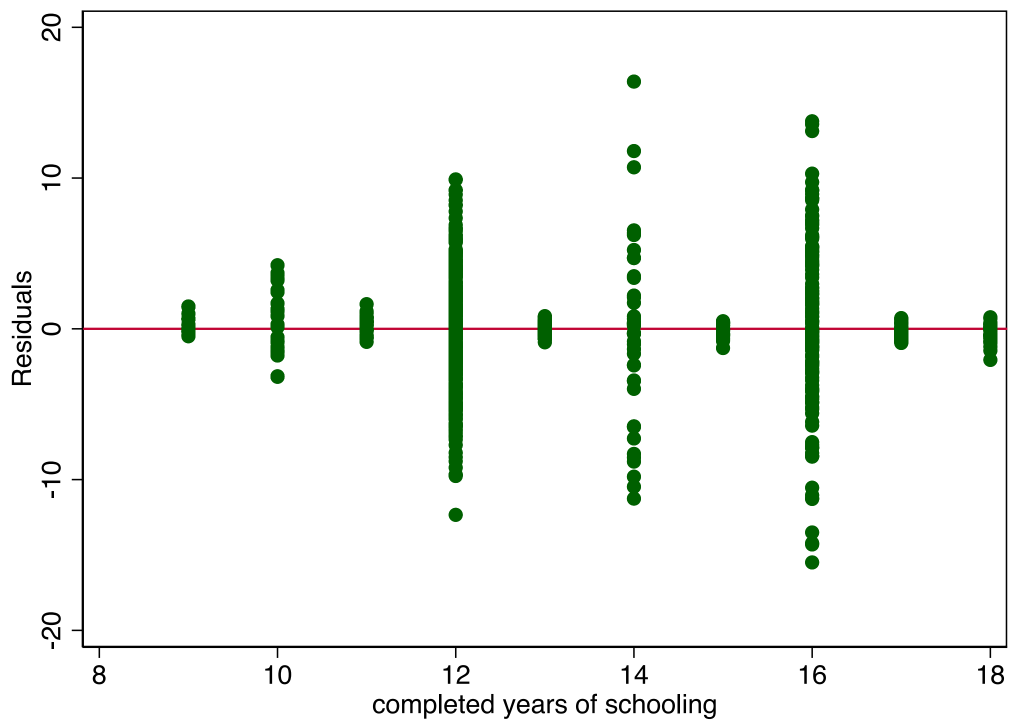
What happens to VIFs when a variable of your choice is removed?

## Heteroskedasticity

Heteroskedasticity implies that the error terms are not identically distributed, but instead may be related in some way to the regressors or another factor. In this situation, our estimates are unbiased, but our distributional assumptions about our *variance* estimates no longer hold, and standard tests of significance don't work.

Figure 1 shows the residuals for a regression of log wages on various covariates plotted against years of experience. This shows a highly heteroskedastic pattern in the residuals.

Figure 1: Residuals Plotted Against Years of Experience




There are several ways of attempting to diagnose heteroskedastic results. The Breusch-Pagan command as an omnibus test looks at whether the variance of the squared residuals is related to any of the regressors.We get this by regressing the square of the residuals on the covariates and conducting an F test (or an LM test of the form $nR^2$). If significant, then the covariates predict the residuals, which indicatres a violation. In Stata we get this result by using the hettest command. Another option is the White test. The White test follows the same basic form as the BP test, but instead uses the test statistic $nR^2$ from the regression of the square of the residuals on the covariates, their squares, and their cross-products. Like the BP test, this test statistic has been shown to be distributed $\chi^2$ with degrees of freedom equal to the number of regressors in the auxiliary regression.

```
Breusch-Pagan / Cook-Weisberg test for heteroskedasticity
         Ho: Constant variance
         Variables: fitted values of lw_het
```

```
         chi2(1)      =     4.69
         Prob > chi2  =   0.0304

.
. estat hettest s expr

Breusch-Pagan / Cook-Weisberg test for heteroskedasticity
         Ho: Constant variance
         Variables: s expr

         chi2(2)      =    21.50
         Prob > chi2  =   0.0000

.
. /*White Test */
.
. estat imtest, white

White´s test for Ho: homoskedasticity
         against Ha: unrestricted heteroskedasticity

         chi2(42)     =     87.39
         Prob > chi2  =    0.0000

Cameron & Trivedi´s decomposition of IM-test

---------------------------------------------------
              Source |       chi2     df      p
---------------------+-----------------------------
  Heteroskedasticity |      87.39     42    0.0000
            Skewness |       7.80      8    0.4534
            Kurtosis |      15.12      1    0.0001
---------------------+-----------------------------
               Total |     110.31     51    0.0000
---------------------------------------------------
```

In the first BP test above, the squared residuals are regressed on the predicted values from the regression. In the second test, the squared residuals are regressed on all of the covariates. And in the last, the squared residuals are regressed on a specified subset of the covariates. This last can help to identify the source of the non iid error terms.

You can correct for heteroskedasticity in a number of ways. The most straightforward is to use robust standard errors, which take into account possible non iid errors. If you suspect the errors in clusters (such as schools) are correlated, you can calculated clustered standard errors, with clustering at the group level. (N.B. My example below is artificial, and honestly, incorrect).

```
. /*Robust s.e.´s*/
. reg lw_het `x´ `controls´, robust

Linear regression                                      Number of obs =     758
                                                       F(  8,   749) =    2.37
                                                       Prob > F      =  0.0158
                                                       R-squared     =  0.0202
                                                       Root MSE      =  3.9241

------------------------------------------------------------------------------
             |               Robust
      lw_het |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           s |   .2513847   .0746952     3.37   0.001     .1047479    .3980215
         kww |  -.0103424   .0237815    -0.43   0.664    -.0570288     .036344
          iq |  -.0189512     .01152    -1.65   0.100    -.0415666    .0036642
        expr |  -.0359087   .0682226    -0.53   0.599     -.169839    .0980215
```

```
      tenure |   .1580198   .0947811     1.67   0.096    -.0280485    .3440881
         rns |   .3126439   .3020458     1.04   0.301    -.2803131    .9056009
        smsa |   .3692457   .3106519     1.19   0.235    -.2406063    .9790977
         med |   -.028291   .0568756    -0.50   0.619    -.1399456    .0833635
       _cons |   4.345555   1.177585     3.69   0.000     2.033796    6.657315
------------------------------------------------------------------------------

. eststo full_model_robust, title("Model 2: Robust SE")

.
. /*Clustered se.´s*/
. reg `y´ `x´ `controls´, cluster(med)

Linear regression                                      Number of obs =     758
                                                       F(  8,    18) =  126.22
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.3634
                                                       Root MSE      =  .34408

                                 (Std. Err. adjusted for 19 clusters in med)
------------------------------------------------------------------------------
             |               Robust
          lw |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           s |   .0878976   .0090694     9.69   0.000     .0688436    .1069516
         kww |   .0030669   .0017164     1.79   0.091    -.0005391    .0066729
          iq |   .0028559   .0008181     3.49   0.003      .001137    .0045747
        expr |   .0386396   .0068981     5.60   0.000     .0241472     .053132
      tenure |   .0322462   .0049196     6.55   0.000     .0219105    .0425818
         rns |  -.0720075   .0236252    -3.05   0.007    -.1216422   -.0223729
        smsa |   .1302547    .014499     8.98   0.000     .0997935    .1607159
         med |   .0055788    .003547     1.57   0.133    -.0018732    .0130309
       _cons |   3.840349   .1197726    32.06   0.000     3.588716    4.091982
------------------------------------------------------------------------------

. eststo full_model_cluster, title("Model 2: Cluster SE")

.
```

Angrist and Pischke suggest that we should assume that there IS heteroskedasticity in the residuals, unless we can specifically prove that there's none. Basically, you should always use robust standard errors (or other appropriate variance estimation technique).

### Quick Exercise

Is there heteroskedasticity when we don't use the (made up) log wage variable, but rather the real one?

## Data Scaling

The results of regression are invariant to linear transforms, but sensitive to non-linear transforms. Changing the latter changes the functional form of the regression model.

Results can be scaled in any number of ways. The most common is standardized coefficients, which scales each coefficient as follows:

$$\widehat{\beta_j^s} = \widehat{\beta_j}\frac{SD(x_j)}{SD(y)}$$

In the results below I show the standardized coefficients for the independent variables in the model, then transform the years of schooling variable and re-run the model. While the coefficient estimates change, the t-statistic and standardized coefficient does not.

```
.
. /*Data Scaling*/
.
. reg `y´ `x´ `controls´

      Source |       SS       df       MS              Number of obs =     758
-------------+------------------------------           F(  8,   749) =   53.43
       Model |  50.6098566     8  6.32623207           Prob > F      =  0.0000
    Residual |  88.6762933   749  .118392915           R-squared     =  0.3634
-------------+------------------------------           Adj R-squared =  0.3566
       Total |   139.28615   757  .183997556           Root MSE      =  .34408

------------------------------------------------------------------------------
          lw |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           s |   .0878976   .0070921    12.39   0.000     .0739749    .1018203
         kww |   .0030669   .0019596     1.57   0.118    -.0007801    .0069139
          iq |   .0028559   .0011028     2.59   0.010      .000691    .0050208
        expr |   .0386396   .0063668     6.07   0.000     .0261407    .0511385
      tenure |   .0322462   .0078371     4.11   0.000     .0168608    .0476315
         rns |  -.0720075   .0289852    -2.48   0.013    -.1289095   -.0151055
        smsa |   .1302547   .0281144     4.63   0.000     .0750623    .1854471
         med |   .0055788    .004952     1.13   0.260    -.0041427    .0153004
       _cons |   3.840349   .1126832    34.08   0.000     3.619136    4.061561
------------------------------------------------------------------------------

.
. reg `y´ `x´ `controls´, beta

      Source |       SS       df       MS              Number of obs =     758
-------------+------------------------------           F(  8,   749) =   53.43
       Model |  50.6098566     8  6.32623207           Prob > F      =  0.0000
    Residual |  88.6762933   749  .118392915           R-squared     =  0.3634
-------------+------------------------------           Adj R-squared =  0.3566
       Total |   139.28615   757  .183997556           Root MSE      =  .34408

------------------------------------------------------------------------------
          lw |      Coef.   Std. Err.      t    P>|t|                     Beta
-------------+----------------------------------------------------------------
           s |   .0878976   .0070921    12.39   0.000                 .4573323
         kww |   .0030669   .0019596     1.57   0.118                 .0522099
          iq |   .0028559   .0011028     2.59   0.010                 .0906704
        expr |   .0386396   .0063668     6.07   0.000                 .1896665
      tenure |   .0322462   .0078371     4.11   0.000                 .1258147
         rns |  -.0720075   .0289852    -2.48   0.013                -.0745005
        smsa |   .1302547   .0281144     4.63   0.000                 .1386435
         med |   .0055788    .004952     1.13   0.260                 .0356506
       _cons |   3.840349   .1126832    34.08   0.000                        .
------------------------------------------------------------------------------

. eststo full_model_beta, title("Model 2: Standardized Coefficients")

.
. gen expr_new=1+2*s

.
. local x expr_new

.
. reg `y´ `x´ `controls´, beta

      Source |       SS       df       MS              Number of obs =     758
-------------+------------------------------           F(  8,   749) =   53.43
       Model |  50.6098566     8  6.32623207           Prob > F      =  0.0000
    Residual |  88.6762933   749  .118392915           R-squared     =  0.3634
-------------+------------------------------           Adj R-squared =  0.3566
```

```
       Total |   139.28615   757  .183997556           Root MSE      =  .34408

------------------------------------------------------------------------------
          lw |      Coef.   Std. Err.      t    P>|t|                     Beta
-------------+----------------------------------------------------------------
    expr_new |   .0439488    .003546    12.39   0.000                 .4573323
         kww |   .0030669   .0019596     1.57   0.118                 .0522099
          iq |   .0028559   .0011028     2.59   0.010                 .0906704
        expr |   .0386396   .0063668     6.07   0.000                 .1896665
      tenure |   .0322462   .0078371     4.11   0.000                 .1258147
         rns |  -.0720075   .0289852    -2.48   0.013                -.0745005
        smsa |   .1302547   .0281144     4.63   0.000                 .1386435
         med |   .0055788    .004952     1.13   0.260                 .0356506
       _cons |     3.7964   .1134835    33.45   0.000                        .
------------------------------------------------------------------------------
```

Rescaling on Z scores can make sense in many applications, but it is not a way to compare coefficients.

### Quick Exercise

Rescale the parental education variable using both a linear and a non-linear transform and check to see what difference it makes in the results.

## Functional Form

In checking on functional form, your best bet is almost always using graphical approaches. Below I plot the wages as a function of years of schooling, then plot the a line based on local linear regression.

Next I plot both a lowess line and a linear fit to see what the results of a simple regression might look like.

### Quick Exercise

Using a similar approach to the one I use in the do file, check on the functional form of the relationship between wages and kww scores.

### The log transformation

We covered this previously. Remember that the log transformation is a good idea when the dependent variable is on some kind of exponential scale.

Figure 2: Wages as a function of schooling

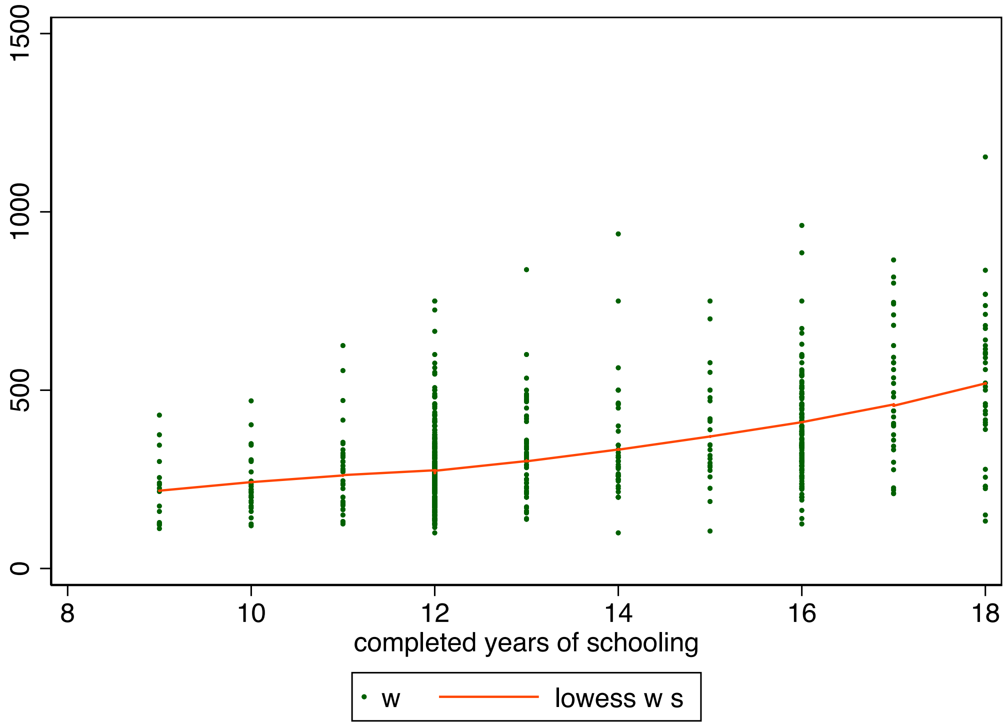



Figure 3: Wages as a function of schooling, linear fit to data

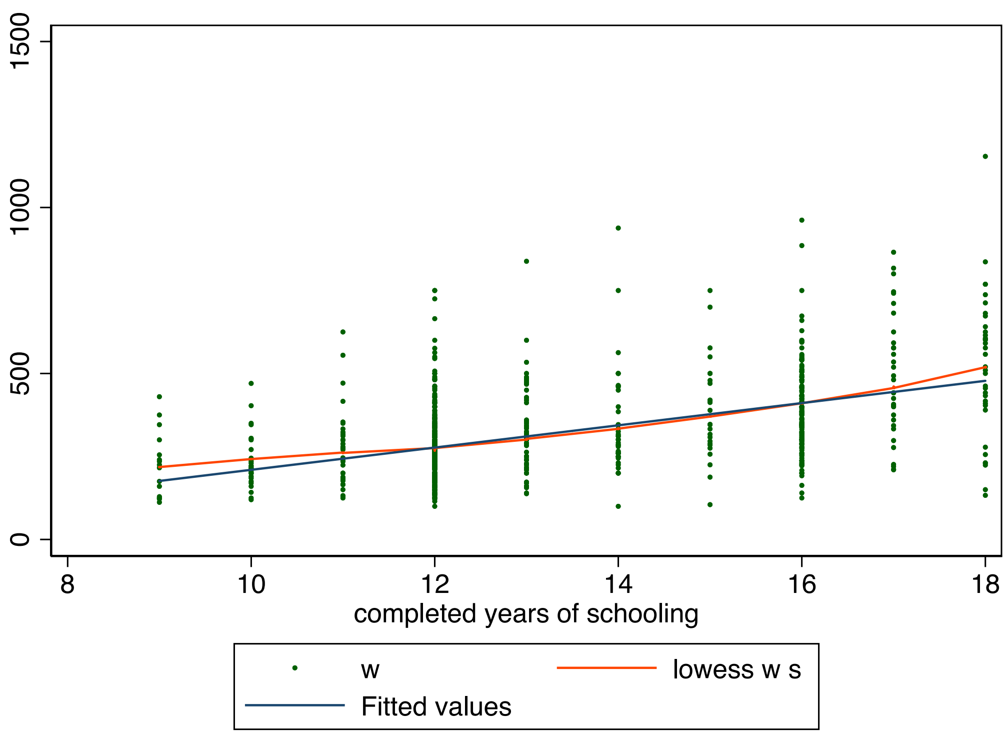
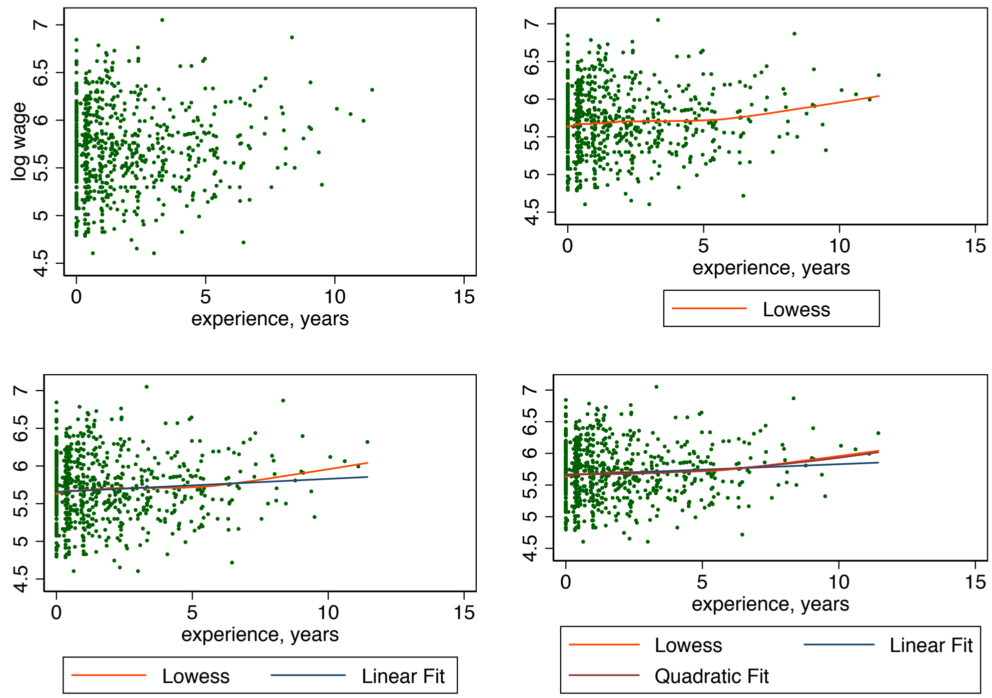


Figure 4: Wages as a function of schooling, multiple functional forms




# Influential Observations

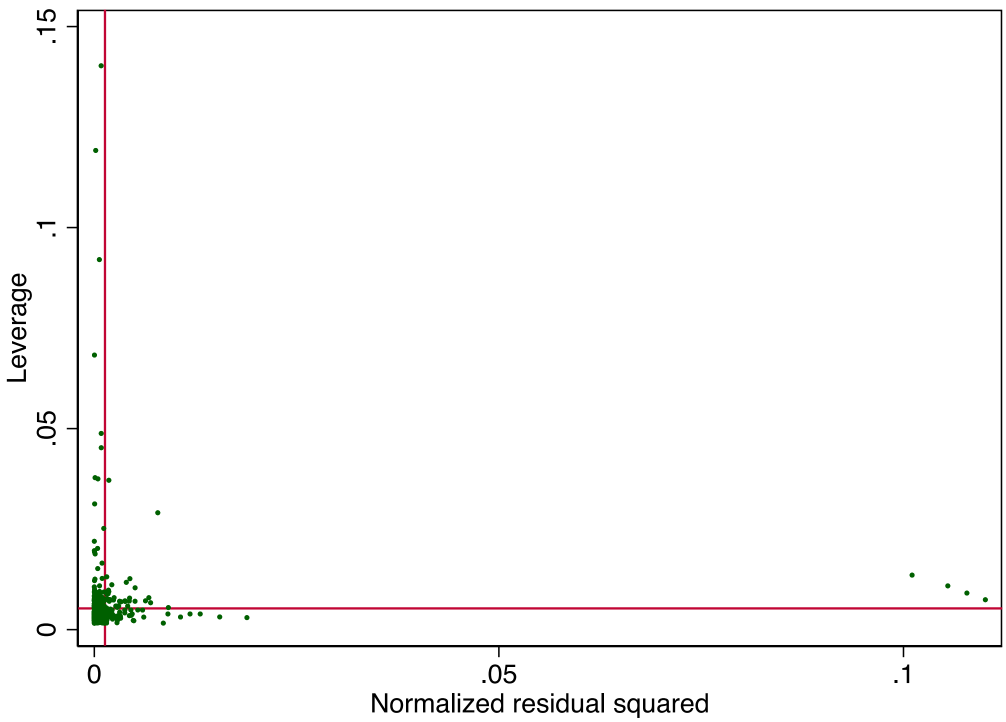
Regression is quite sensitive to outliers. A data point can "pull" the regression line quite far away given its distance from that line. We test for influential measures using several different measures including leverage, dfits, cooks D, or dfbeta.

- Leverage is measured in the scale of the dv and is the basic measure of influence from a residual.
- The dfits statistic compares the standardized residual for every observation on the scale of the standard error of the regression.
- Cook's d combines information regarding leverage—how influential the observation is on the results— and the size of the residual—how far off the prediction the actual result is.
- Dfbetas are calculated for every unit AND every coefficient, and state how far the coefficient would move should that case be excluded.

A leverage plot is a good place to being examining the data for influential observations.

The leverage plot shows the leverage of each observation on the y axis and the square of the residual on the x axis. The red lines on each axis are the cutoff points for "large" leverage or residual stats.

Figure 5: Leverage Plot




For each of the measures leverage, dfits, cooks' D and dfbeta, the procedure is the same. Calculate the measure, then look for observations that exceed a given informally set level. Here's that procedure applied to leverage.

For DFits, the measure is given by:

$$DFITS_j = r_j \sqrt{\frac{h_j}{1 - h_j}}$$

Where $r_j$ is a studentized residual:

$$r_j = \frac{\epsilon_j}{s_{(j)}} \sqrt{1 - h_j}$$

The informal rule for the cutoff for dfits is $|DFITS_j| > 2\sqrt{k/N}$.

For Cook's D, the calculation is:

$$D_i = \frac{\sum_{i=1}^{n} (\hat{y}_i - \hat{y}_{i(w)})^2}{pMSE}$$

Where $\hat{y}_i$ is the prediction with observation $i$ and $\hat{y}_{i(w)}$ is the prediction *without* observation $i$.

The informal rule for cooks D is to investigate values over $4/n$.

Last, the calculation for DFBETA is given by:

$$DFBETA_j = \frac{r_j v_j}{\sqrt{v^2(1-h_j}}$$

This tells you how much a regression coefficient would change if unit $j$ was excluded. The cutoff suggested for DFBETA is $2/\sqrt{n}$.

```
. /* Leverage and outliers  */
.
. predict lev if e(sample), leverage

.
. predict resid if e(sample), resid

.
. gen  resid2=resid^2

.
. gsort -lev

.
. list w_influence s lev resid2 in 1/10

     +------------------------------------+
     | w_infl~e    s        lev     resid2 |
     |------------------------------------|
  1. | 555.0176   11   .1402453   16845.92 |
  2. | 401.0154   12   .1191845   3480.702 |
  3. | 430.0924    9   .0920351   12456.28 |
  4. | 454.8647   12   .0682902    395.947 |
  5. | 204.9979   10   .0488069   16943.56 |
     |------------------------------------|
  6. | 288.0114   12    .045244   17041.28 |
  7. | 367.9695   12    .037804   1923.356 |
  8. | 375.0278    9   .0375033   8861.435 |
  9. | 600.0421   12   .0371506   35667.26 |
 10. | 332.9526   11   .0312744   839.5032 |
     +------------------------------------+
```

## Quick Exercise

Run the same regression without the schooling variable. Check for outliers using both graphical and tabular methods.