Assignment 3

Drew Murray

10/20/2022

```
(echo = TRUE)
## [1] TRUE
T1 <- read.table("C:/Users/dgmur/Downloads/T1-9.dat")
colnames(T1) <- c("Country","100m","200m","400m","800m","1500m","3000m","Marathon")</pre>
round(apply(T1[,-1],2,mean),2)
                                            1500m
                                                      3000m Marathon
##
       100m
                 200m
                          400m
                                    800m
##
      11.36
                23.12
                         51.99
                                    2.02
                                             4.19
                                                       9.08
                                                              153.62
round(cov(T1[,-1]),2)
```

```
##
           100m
                 200m
                       400m 800m 1500m 3000m Marathon
                                                4.33
## 100m
           0.16
                 0.34
                       0.89 0.03 0.08 0.23
## 200m
           0.34
                 0.86
                       2.19 0.07
                                 0.20
                                       0.55
                                               10.38
## 400m
           0.89 2.19 6.75 0.18 0.51 1.43
                                               28.90
## 800m
           0.03
                 0.07
                       0.18 0.01 0.02 0.06
                                                1.22
## 1500m
           0.08 0.20 0.51 0.02 0.07 0.22
                                                3.54
## 3000m
           0.23 0.55 1.43 0.06 0.22 0.66
                                               10.71
## Marathon 4.33 10.38 28.90 1.22 3.54 10.71
                                              270.27
```

```
round(cor(T1[,-1]),2)
```

```
##
            100m 200m 400m 800m 1500m 3000m Marathon
## 100m
            1.00 0.94 0.87 0.81 0.78
                                       0.73
                                                0.67
            0.94 1.00 0.91 0.82 0.80
## 200m
                                       0.73
                                                0.68
            0.87 0.91 1.00 0.81 0.72
## 400m
                                       0.67
                                                0.68
## 800m
            0.81 0.82 0.81 1.00 0.91
                                       0.87
                                                0.85
## 1500m
            0.78 0.80 0.72 0.91 1.00
                                       0.97
                                                0.79
## 3000m
            0.73 0.73 0.67 0.87 0.97
                                       1.00
                                                0.80
## Marathon 0.67 0.68 0.68 0.85 0.79 0.80
                                                1.00
```

```
#B
track = as.matrix(T1[,-1])
colnames(track) <- c("100m","200m","400m","800m","1500m","3000m","Marathon")</pre>
V1 <- track[,1]/3+track[,2]/6+track[,3]/12
V2<- track[,4]/2.4+track[,5]/4.5+track[,6]/9
V3 <- track[,7]/60
V1[c(15,25,21,14)]
## [1] 12.28000 11.90250 12.69333 12.11000
V2[c(15,25,21,14)]
## [1] 2.978611 2.650833 2.970278 2.725000
V3[c(15,25,21,14)]
## [1] 2.774333 2.370500 2.855500 2.489000
#C
mean(V1)
## [1] 11.97144
mean(V2)
## [1] 2.782629
mean(V3)
## [1] 2.560321
var(cbind(V1,V2,V3))
##
                         V2
## V1 0.23678853 0.07004779 0.09307002
## V2 0.07004779 0.03350372 0.04140643
## V3 0.09307002 0.04140643 0.07507504
```

```
#D
xbar <- colMeans(track)</pre>
S <- cov(track)</pre>
x \leftarrow c(1/3,1/6,1/12,0,0,0,0)
y \leftarrow c(0,0,0,1/2.4,1/4.5,1/9,0)
z \leftarrow c(0,0,0,0,0,0,1/60)
meanV1<- x%*%xbar
meanV1
##
             [,1]
## [1,] 11.97144
meanV2 <- y%*%xbar
meanV2
##
             [,1]
## [1,] 2.782629
meanV3 <- z%*%xbar
meanV3
##
             [,1]
## [1,] 2.560321
varV1 <- x%*%S%*%x
varV1
              [,1]
## [1,] 0.2367885
varV2 <- y%*%S%*%y
varV2
##
               [,1]
## [1,] 0.03350372
varV3 <- z%*%S%*%z
varV3
##
               [,1]
```

[1,] 0.07507504

```
covV1V2<- x%*%S%*%y
covV1V2
##
              [,1]
## [1,] 0.07004779
covV1V3 <- x%*%S%*%z
covV1V3
##
              [,1]
## [1,] 0.09307002
covV2V3 <- y%*%S%*%z
covV2V3
##
              [,1]
## [1,] 0.04140643
X \leftarrow rbind(x,y,z)
MatMean<- X%*%xbar
MatMean
##
         [,1]
## x 11.971435
## y 2.782629
## z 2.560321
MatCov <- X%*%S%*%t(X)
MatCov
## x 0.23678853 0.07004779 0.09307002
## y 0.07004779 0.03350372 0.04140643
## z 0.09307002 0.04140643 0.07507504
```

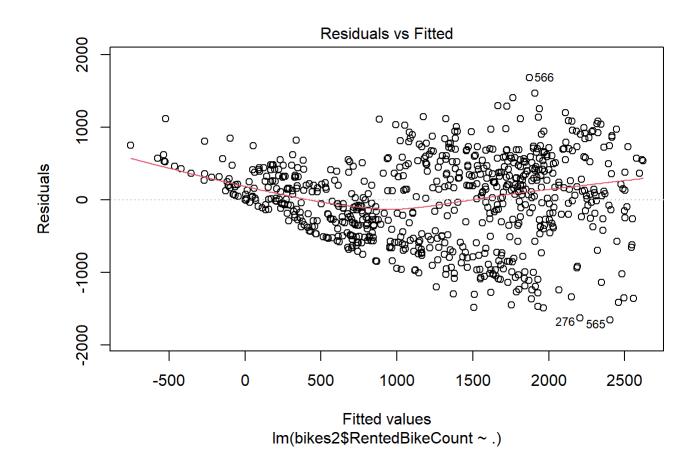
```
#E
#Showed the same results as part C.
bikes2 <- read.csv("C:/Users/dgmur/Downloads/SeoulBikes_Fl2022.csv")
dim(bikes2)</pre>
```

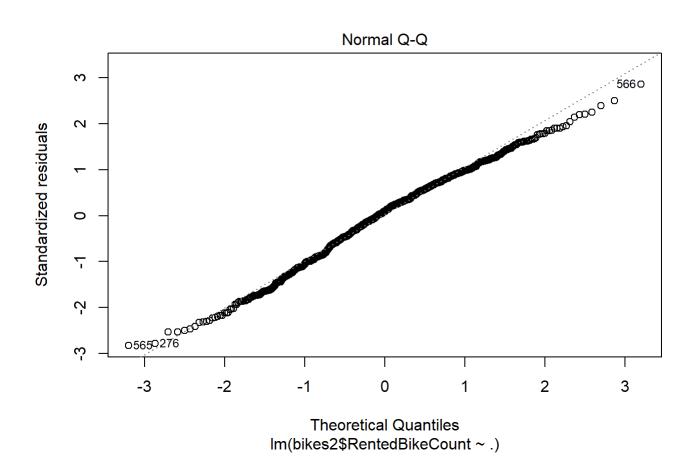
```
## [1] 730 16
```

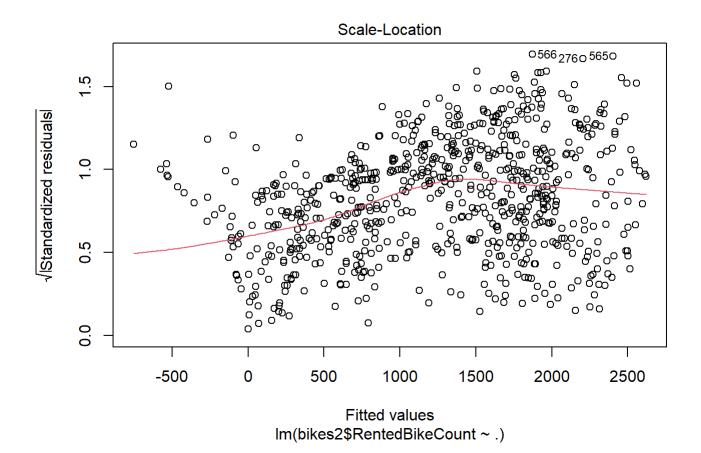
```
View(bikes2)
bikes2 <- bikes2[,c(4:16)]
View(bikes2)
bikes2$Holiday<-factor(bikes2$Holiday)
bikes2$Seasons <- as.factor(bikes2$Seasons)
bikes2$Functioning.Day<- factor(bikes2$Functioning.Day)
bikes2$Time<- factor(bikes2$Time)
#A
lm1 <- lm(bikes2$RentedBikeCount~., data= bikes2)
summary(lm1)</pre>
```

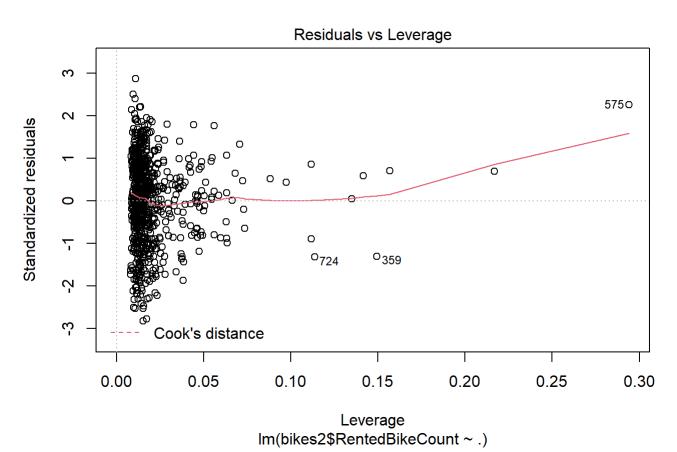
```
##
## Call:
## lm(formula = bikes2$RentedBikeCount ~ ., data = bikes2)
##
## Residuals:
##
       Min
                      Median
                  1Q
                                    3Q
                                            Max
                               423.16 1682.70
##
  -1657.94 -379.36
                       55.29
##
## Coefficients:
##
                        Estimate Std. Error t value Pr(>|t|)
                      1266.52970 575.05965
                                              2.202 0.027953 *
## (Intercept)
## Temperature
                       -45.46624
                                   21.00316 -2.165 0.030738 *
## Humidity
                       -26.92340
                                   6.22476 -4.325 1.74e-05 ***
## WindSpeed
                       13.02980
                                   26.00716
                                             0.501 0.616520
## Visibility
                         0.00403
                                   0.04784
                                              0.084 0.932890
## DewPointTemp
                       76.03379
                                   22.57952
                                             3.367 0.000799 ***
## SolarRadiation
                      550.55773
                                   97.15453
                                              5.667 2.11e-08 ***
## Rainfall
                       -74.51760
                                   15.73064 -4.737 2.62e-06 ***
## Snowfall
                                   57.35246
                                             0.079 0.937323
                         4.51150
## SeasonsSpring
                                   72.73163 -6.463 1.90e-10 ***
                      -470.07335
## SeasonsSummer
                      -519.76455
                                   87.53538 -5.938 4.51e-09 ***
## SeasonsWinter
                      -781.32605
                                   93.22146 -8.381 2.77e-16 ***
## HolidayNo Holiday
                                            5.301 1.54e-07 ***
                      540.94438
                                 102.04515
                                 127.14174 13.561 < 2e-16 ***
## Functioning.DayYes 1724.11080
## TimeMorning
                      -237.60832
                                   61.53045 -3.862 0.000123 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 590 on 715 degrees of freedom
## Multiple R-squared: 0.6112, Adjusted R-squared: 0.6035
## F-statistic: 80.27 on 14 and 715 DF, p-value: < 2.2e-16
```

```
plot(lm1)
```



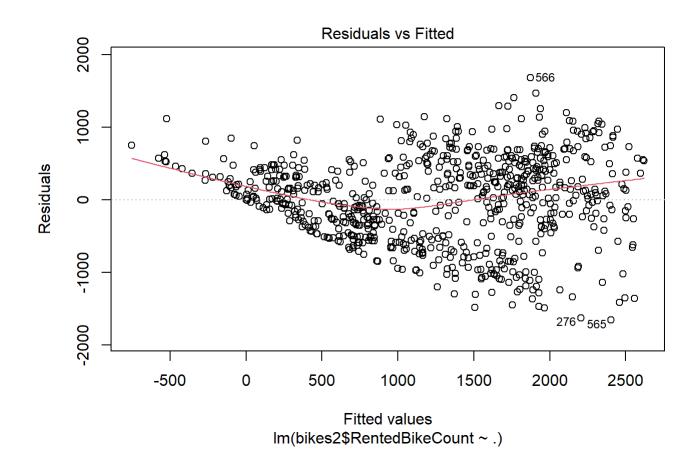


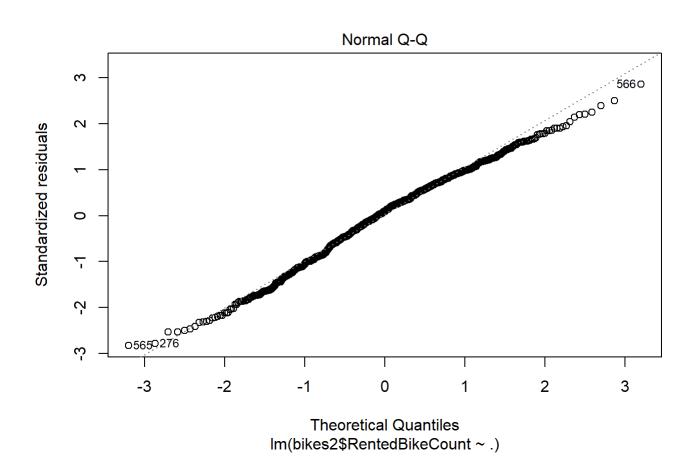


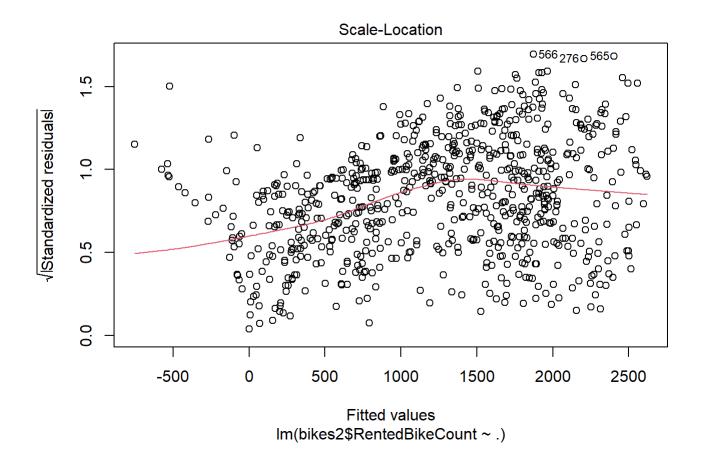


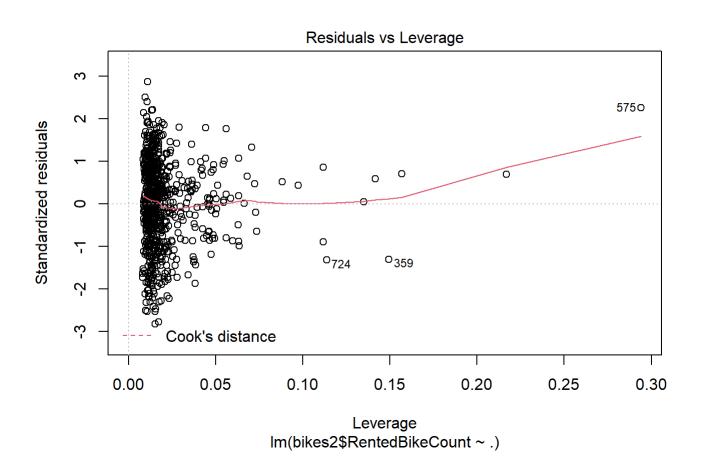
Residuals vs Fitted plot shows heteroscedasticity, qqplot steers away from the line once it hits between quartile 1 and 2. A log transformation on one of the predictor variables would fix this issue.

plot(lm1)



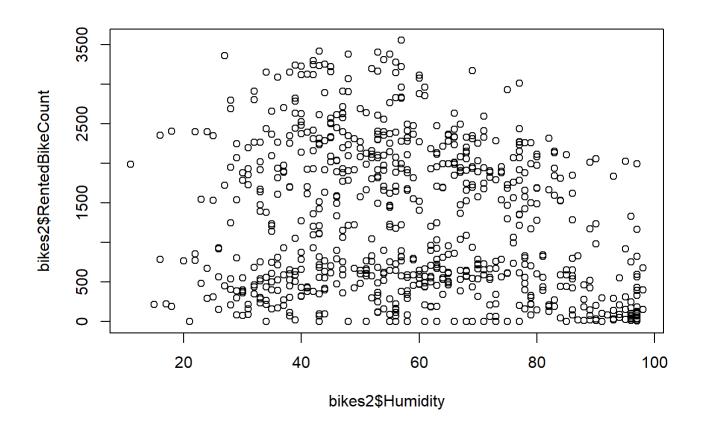


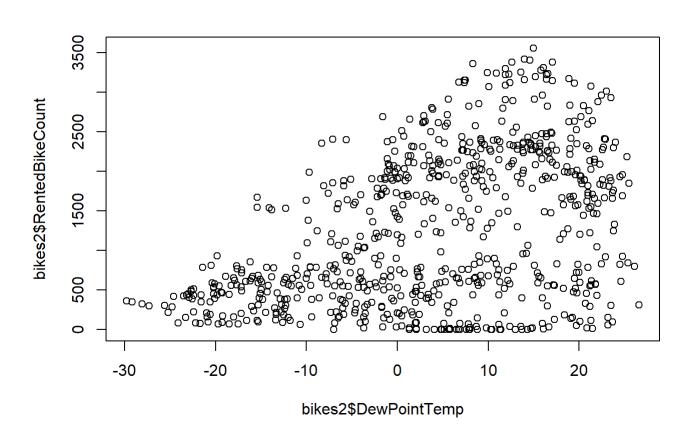


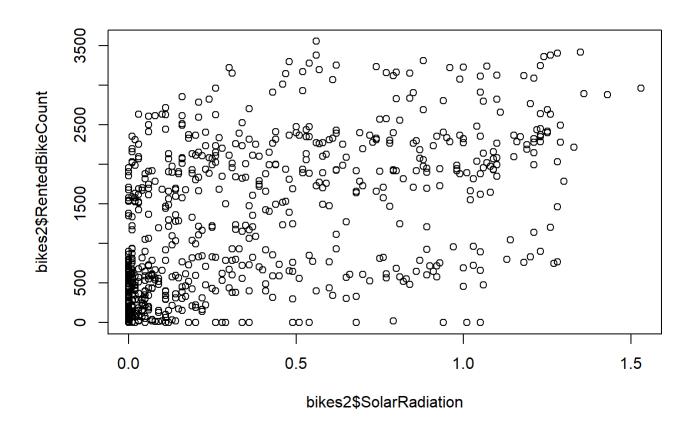


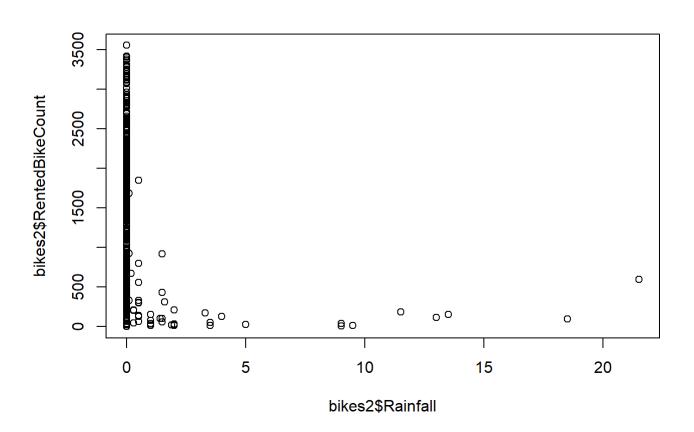
plot(bikes2\$RentedBikeCount~bikes2\$Humidity+bikes2\$DewPointTemp+bikes2\$SolarRadiation+bikes2\$Rai
nfall+bikes2\$Seasons

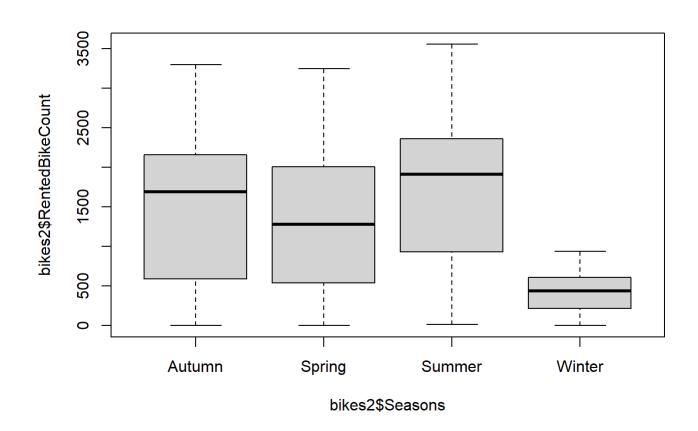
+bikes2\$Holiday+bikes2\$Functioning.Day+bikes2\$Time, data = bikes2)

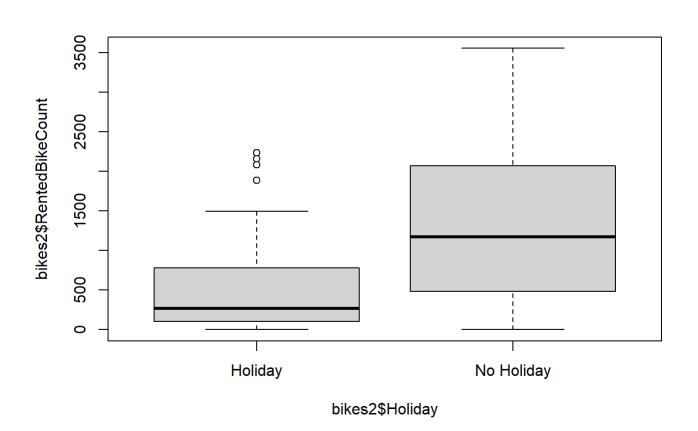


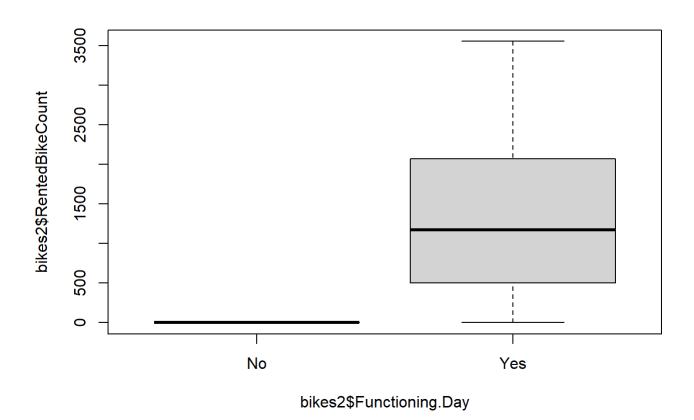


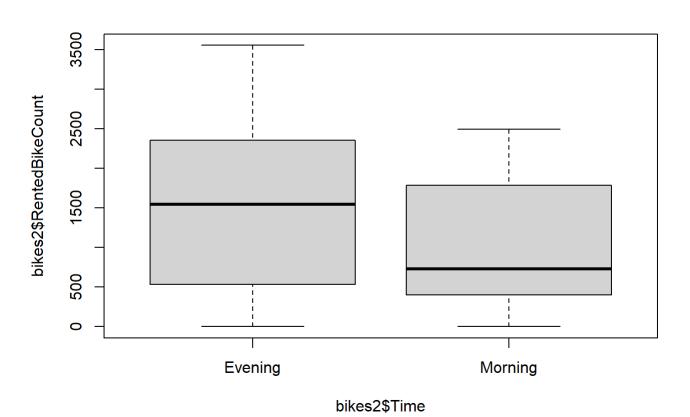




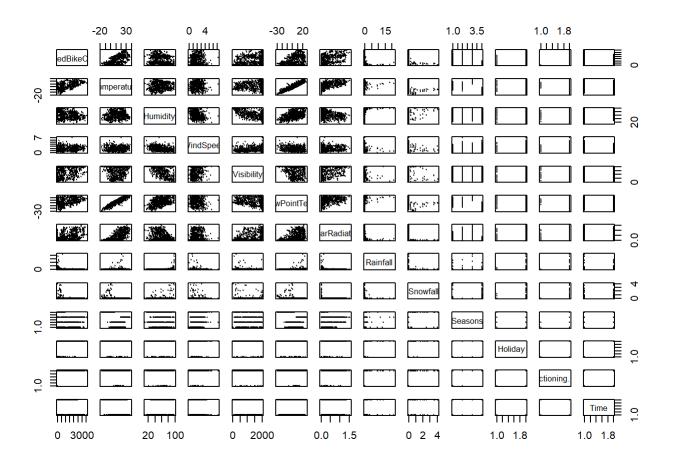








pairs(bikes2, labels = names(bikes2), cex = 0.1)



library(leaps)

Warning: package 'leaps' was built under R version 4.1.3

regfit_full = regsubsets(bikes2\$RentedBikeCount~., data = bikes2)
summary(regfit_full)

```
## Subset selection object
## Call: regsubsets.formula(bikes2$RentedBikeCount ~ ., data = bikes2)
## 14 Variables (and intercept)
                     Forced in Forced out
##
                         FALSE
## Temperature
                                    FALSE
## Humidity
                         FALSE
                                    FALSE
## WindSpeed
                         FALSE
                                    FALSE
## Visibility
                         FALSE
                                    FALSE
## DewPointTemp
                         FALSE
                                    FALSE
## SolarRadiation
                         FALSE
                                    FALSE
## Rainfall
                         FALSE
                                    FALSE
## Snowfall
                         FALSE
                                    FALSE
## SeasonsSpring
                         FALSE
                                    FALSE
## SeasonsSummer
                         FALSE
                                    FALSE
## SeasonsWinter
                         FALSE
                                    FALSE
## HolidayNo Holiday
                                    FALSE
                         FALSE
## Functioning.DayYes
                         FALSE
                                    FALSE
## TimeMorning
                         FALSE
                                    FALSE
## 1 subsets of each size up to 8
## Selection Algorithm: exhaustive
##
            Temperature Humidity WindSpeed Visibility DewPointTemp SolarRadiation
                                                     .....
## 1 ( 1 ) "*"
                       "*"
                                .. ..
                                          . .
                                                     "*"
                                                                  .. ..
## 2 (1)""
                       "*"
                                .. ..
                                          .....
                                                     "*"
     (1)""
## 3
                                                     "*"
                       "*"
     (1)""
## 5
     (1)""
                       "*"
                                                     "*"
## 6
    (1)""
                       "*"
                                11 11
                       "*"
     (1)""
## 7
     (1)""
                       "*"
## 8
##
            Rainfall Snowfall SeasonsSpring SeasonsSummer SeasonsWinter
     (1)""
## 1
     (1)""
## 2
     (1)""
                             .. ..
## 3
     (1)""
## 4
     (1)"*"
## 5
     (1)"*"
## 6
     (1)"*"
## 7
    (1)"*"
                    "*"
                                           "*"
## 8
##
            HolidayNo Holiday Functioning.DayYes TimeMorning
     (1)""
## 1
                             .. ..
                                                .. ..
## 2
    (1)""
     (1)""
## 3
     (1)""
                             "*"
## 4
     (1)""
## 5
                             " * "
     (1)"*"
## 6
     (1)"*"
                             " * "
## 7
## 8 (1) "*"
                             "*"
```

```
reg <- summary(regfit_full)
names(reg)</pre>
```

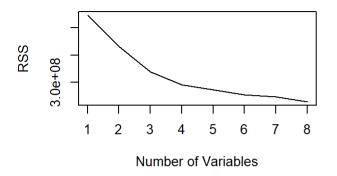
```
## [1] "which" "rsq" "rss" "adjr2" "cp" "bic" "outmat" "obj"
```

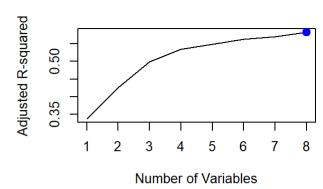
```
par(mfrow=c(2,2))
plot(reg$rss, xlab = "Number of Variables", ylab = "RSS", type = "1")

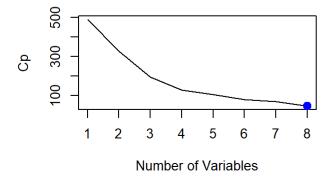
plot(reg$adjr2, xlab = "Number of Variables", ylab= "Adjusted R-squared", type = "1")
adjr2max = which.max(reg$adjr2)
points(adjr2max, reg$adjr2[adjr2max], col="blue", cex = 2, pch=20)

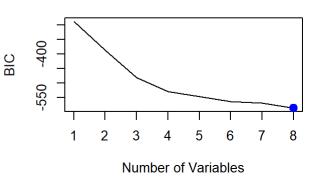
plot(reg$cp, xlab = "Number of Variables", ylab= "Cp", type ="1")
cpmin = which.min(reg$cp)
points(cpmin,reg$cp[cpmin], col="blue", cex=2, pch=20)

plot(reg$bic, xlab = "Number of Variables", ylab = "BIC", type="1")
bicmin = which.min(reg$bic)
points(bicmin, reg$bic[bicmin], col="blue", cex=2, pch=20)
```



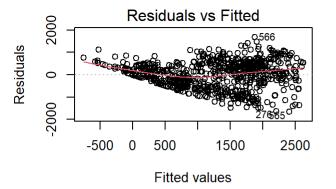


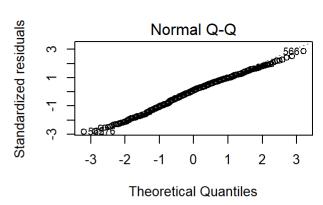


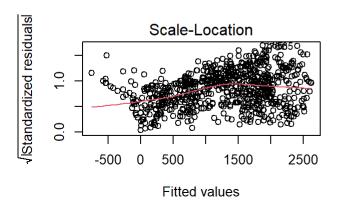


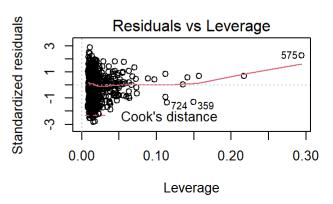
coef(regfit_full,8)

```
Rainfall
##
          (Intercept)
                                  Humidity
                                                  DewPointTemp
            706.52873
                                                                         -78.75892
##
                                 -24.58723
                                                      43.85470
##
        SeasonsSpring
                            SeasonsSummer
                                                 SeasonsWinter
                                                                HolidayNo Holiday
            -316.16994
                                -420.48498
                                                    -667.49369
                                                                         559.13015
##
## Functioning.DayYes
           1706.17969
##
```









```
## fit lwr upr
## 1 1146.693 -63.59837 2356.985
```

```
#3
```

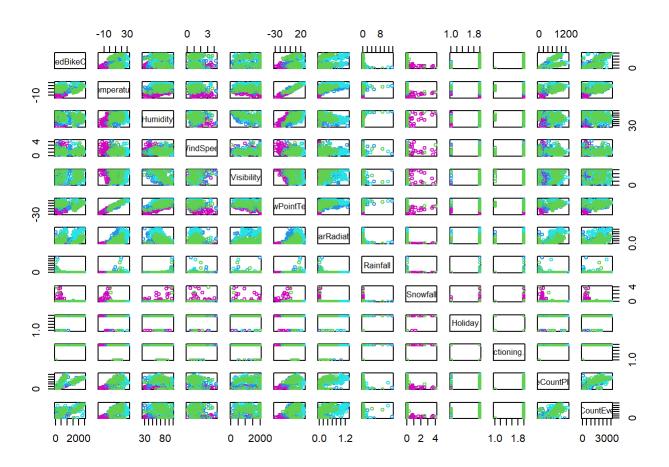
bikes3 <- read.csv("C:/Users/dgmur/Downloads/SeoulBikesVer2_Fl2022.csv")
dim(bikes3)</pre>

```
## [1] 365 17
```

```
bikes3$Seasons<- as.factor(bikes3$Seasons)
bikes3$Holiday <- as.factor(bikes3$Holiday)
bikes3$Functioning.Day <- as.factor(bikes3$Functioning.Day)

bikes3 <- bikes3[,c(4:17)]

pairs(bikes3[,c(1:9,11:14)], label=names(bikes3[,c(1:9,11:14)]),cex=0.75, col=2+as.numeric(bikes3$Seasons))</pre>
```



names(bikes3[10])

[1] "Seasons"

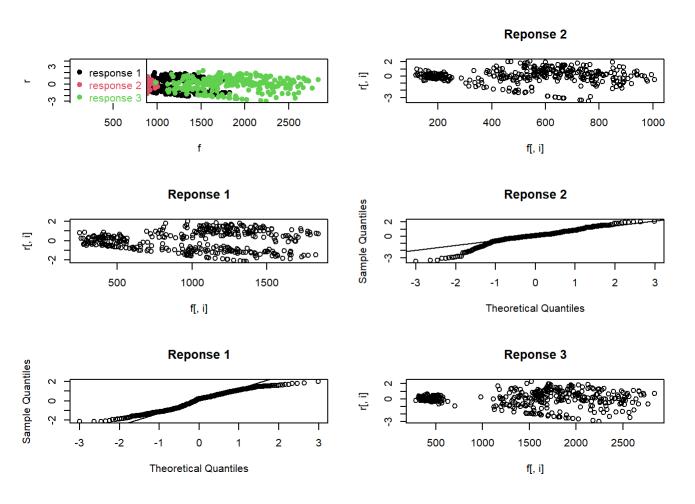
```
##
   Response RentedBikeCount :
##
                  Df
                        Sum Sq Mean Sq F value
                                                   Pr(>F)
                   1 40358746 40358746 98.6585 < 2.2e-16 ***
## Temperature
## Humidity
                   1
                       8129664 8129664 19.8733 1.110e-05 ***
## WindSpeed
                   1
                          2818
                                   2818 0.0069 0.9339030
                          1777
                                   1777 0.0043 0.9474863
## Visibility
                   1
## SolarRadiation
                   1
                       5370405 5370405 13.1282 0.0003332 ***
                   3 11635416 3878472 9.4811 4.848e-06 ***
## Seasons
## Residuals
                 356 145630770
                                 409075
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
##
   Response BikeCountPlus2:
##
                  Df
                       Sum Sq Mean Sq F value
                                                   Pr(>F)
## Temperature
                   1 12374911 12374911 259.6806 < 2.2e-16 ***
                   1 1902007 1902007 39.9126 7.932e-10 ***
## Humidity
## WindSpeed
                       147770
                                         3.1009
                                                  0.07911 .
                   1
                                147770
## Visibility
                   1
                        40585
                                 40585
                                         0.8517
                                                  0.35671
## SolarRadiation
                   1 2776087 2776087 58.2546 2.134e-13 ***
                               1257476 26.3874 1.959e-15 ***
## Seasons
                   3 3772427
## Residuals
                 356 16964953
                                 47654
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
   Response BikeCountEvening:
##
##
                  Df
                                 Mean Sq F value
                        Sum Sq
                                                     Pr(>F)
## Temperature
                   1 129164247 129164247 226.3397 < 2.2e-16 ***
                                 6669541 11.6873 0.0007022 ***
## Humidity
                   1
                       6669541
## WindSpeed
                   1
                          5104
                                    5104
                                           0.0089 0.9247088
## Visibility
                                 1100014
                                          1.9276 0.1658895
                   1
                       1100014
## SolarRadiation
                   1 19931795 19931795 34.9273 8.013e-09 ***
## Seasons
                   3 25616518
                                 8538839 14.9630 3.406e-09 ***
## Residuals
                 356 203156902
                                  570665
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

```
summary(manova(lm.multi), test = c("Wilks"))
```

11/18/22, 5:02 PM

```
Assignment 3
##
                   Df
                       Wilks approx F num Df den Df
                                                       Pr(>F)
                                           3 354.00 < 2.2e-16 ***
## Temperature
                  1 0.53589 102.195
                                          3 354.00 5.551e-09 ***
## Humidity
                   1 0.88994
                              14.594
## WindSpeed
                   1 0.98603
                              1.672
                                           3 354.00
                                                       0.1728
                              1.205 3 354.00 0.3076
21.033 3 354.00 1.463e-12 ***
## Visibility
                  1 0.98989
## SolarRadiation 1 0.84872
                                           9 861.69 3.509e-13 ***
## Seasons
                   3 0.80230
                                9.070
## Residuals
                 356
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
library(car)
## Warning: package 'car' was built under R version 4.1.3
## Loading required package: carData
## Warning: package 'carData' was built under R version 4.1.3
Manova(lm.multi, type="II", test=c("Wilks"))
##
## Type II MANOVA Tests: Wilks test statistic
##
                 Df test stat approx F num Df den Df
                                                      Pr(>F)
## Temperature
                  1
                     0.99461
                                0.6400
                                            3 354.00
                                                      0.58968
## Humidity
                  1 0.97994 2.4149
                                            3 354.00
                                                      0.06632 .
## WindSpeed
                1 0.98942 1.2617
                                            3 354.00
                                                       0.28737
## Visibility 1 0.98952
## Visibility 1 0.84764 21.2108 ## SolarRadiation 1 0.84764 21.2108
                                1.2493
                                            3 354.00
                                                       0.29173
                                            3 354.00 1.171e-12 ***
## Seasons
                  3 0.80230
                                9.0697
                                            9 861.69 3.509e-13 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

```
rstandard.mlm <- function(model){</pre>
  Q <- with(model, qr.qy(qr, diag(1, nrow = nrow(qr$qr), ncol = qr$rank)))
  hii <- rowSums(Q^2)
  RSS <- colSums(model$residuals^2)</pre>
  sigma <- sqrt(RSS/model$df.residual)</pre>
  pointwise_sd <- outer(sqrt(1-hii), sigma)</pre>
  model$residuals/pointwise_sd
}
f<- fitted(lm.multi);</pre>
r<- rstandard(lm.multi);</pre>
par(mfcol=c(3,2))
a \leftarrow plot(f, r, col = as.numeric(col(f)), pch = 19, ylim = c(-3, 4))
legend("topleft", legend = paste0("response ", 1:ncol(f)), pch = 19,
       col = 1:ncol(f), text.col = 1:ncol(f))
for(i in 1:ncol(f)){
  plot(f[,i],r[,i], main = paste("Reponse",i))
  qqnorm(r[,i], main = paste("Reponse", i))
  qqline(r[,i])
}
```



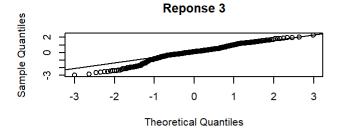
```
##
   Response RentedBikeCount :
##
                  Df
                         Sum Sq Mean Sq F value
                                                   Pr(>F)
## Temperature
                   1 40358746 40358746 99.0865 < 2.2e-16 ***
                       8129664 8129664 19.9595 1.062e-05 ***
## Humidity
                   1
                       5324622 5324622 13.0727 0.0003426 ***
## SolarRadiation
                  1
## Seasons
                   3 11500164 3833388 9.4115 5.309e-06 ***
## Residuals
                 358 145816400
                                 407308
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
   Response BikeCountPlus2 :
##
                  Df
                       Sum Sq Mean Sq F value
                                                  Pr(>F)
## Temperature
                   1 12374911 12374911 257.380 < 2.2e-16 ***
## Humidity
                   1 1902007 1902007 39.559 9.282e-10 ***
## SolarRadiation
                   1 2787828 2787828 57.983 2.377e-13 ***
## Seasons
                    3 3701248
                              1233749 25.660 4.658e-15 ***
                 358 17212745
## Residuals
                                 48080
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
   Response BikeCountEvening:
##
                  Df
                                 Mean Sq F value
                        Sum Sq
                                                    Pr(>F)
                   1 129164247 129164247 225.914 < 2.2e-16 ***
## Temperature
                                 6669541 11.665 0.0007098 ***
## Humidity
                   1
                       6669541
## SolarRadiation 1 20728763 20728763 36.255 4.291e-09 ***
## Seasons
                   3 24398197
                                 8132732 14.225 8.888e-09 ***
## Residuals
                 358 204683373
                                  571741
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '* 0.05 '.' 0.1 ' ' 1
```

```
summary(manova(Nlm.multi), test = c("Wilks"))
```

```
##
                  Df
                       Wilks approx F num Df den Df
                                                      Pr(>F)
## Temperature
                   1 0.53923 101.400
                                          3 356.00 < 2.2e-16 ***
                                          3 356.00 6.004e-09 ***
## Humidity
                   1 0.89092
                              14.529
## SolarRadiation
                   1 0.84847
                              21.194
                                         3 356.00 1.181e-12 ***
## Seasons
                   3 0.80564
                               8.941
                                          9 866.56 5.654e-13 ***
## Residuals
                 358
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

```
library(car)
Manova(Nlm.multi, type="II", test=c("Wilks"))
```

```
##
## Type II MANOVA Tests: Wilks test statistic
##
                  Df test stat approx F num Df den Df
                                                         Pr(>F)
                       0.99564
                                 0.5191
                                             3 356.00
                                                        0.66936
## Temperature
## Humidity
                   1
                       0.97467
                                 3.0840
                                             3 356.00
                                                        0.02741 *
## SolarRadiation 1
                       0.84650 21.5178
                                             3 356.00 7.873e-13 ***
                                             9 866.56 5.654e-13 ***
## Seasons
                   3
                       0.80564
                                 8.9411
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```



shows lack of normality in each of the responses for the qqplots. A log transformation on one or more of the predictor variables may fix this issue.

```
#C
x1<- c(1, Temperature=12.1, Humidity=29, SolarRadiation=2.26, SeasonsSpring=1, SeasonsSummer=0, Seaso
nsWinter=0)
x1</pre>
```

```
Humidity SolarRadiation
##
                      Temperature
                                                                   SeasonsSpring
##
             1.00
                            12.10
                                            29.00
                                                             2.26
                                                                             1.00
##
    SeasonsSummer
                    SeasonsWinter
                             0.00
             0.00
##
```

```
h.Beta <- Nlm.multi$coeff
h.Beta</pre>
```

```
##
                  RentedBikeCount BikeCountPlus2 BikeCountEvening
## (Intercept)
                     1393.6422623
                                      635.4826330
                                                      1455.3299530
## Temperature
                        0.9199711
                                        0.1228168
                                                         7.2503531
## Humidity
                       -4.8995808
                                       -1.3913855
                                                         -0.2964755
## SolarRadiation
                      556.1318554
                                      393.5483466
                                                      1017.0309208
## SeasonsSpring
                     -319.9633232
                                     -167.0434712
                                                      -326.1688311
## SeasonsSummer
                      -40.5792335
                                      -90.0289049
                                                      -180.2825661
## SeasonsWinter
                     -686.0779732
                                     -371.5310146
                                                      -972.9115205
```

Х

```
p<- x1%*%h.Beta

X<-model.matrix(~Temperature+Humidity+SolarRadiation+Seasons, data = bikes3)

m=3

r=nrow(h.Beta)-1

Resid=lm.multi$residuals

n=nrow(Resid)
h.Beta<- as.vector(h.Beta)
hat.sigma = t(Resid)%*%Resid/n
hat.sigma</pre>
```

```
## RentedBikeCount BikeCountPlus2 BikeCountEvening
## RentedBikeCount 398988.41 72593.13 329508.54
## BikeCountPlus2 72593.13 46479.32 98767.32
## BikeCountEvening 329508.54 98767.32 556594.25
```

```
## Response 1 287.2362 4111.925
## Response 2 666.2909 1971.698
## Response 3 1248.099 5765.466
```