IS609 - Anatomy of a Simple Global Futures Trend-Following Strategy

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2017-05-21

Contents

Abstract

Systematic traders employ fully systematic strategies to manage their investments. As a result of the fully defined algorithmic nature of such strategies, it is possible to determine their exact responses to any conceivable set of market conditions. Consequently, sensitivity analysis can be conducted to systematically uncover undesirable strategy behavior and enhance strategy robustness by adding controls to reduce exposure during periods of poor performance / unfavorable market conditions, or increase exposure during periods of strong performance / favorable market conditions.

In this report, we formulate both a simple systematic trend-following strategy (i.e., trading model) to simulate investment decisions, and a market model to simulate the evolution of instrument prices. We then map the relationship between market model parameters and strategy performance under a particular set of trading model parameters to explore the sensitivity of the strategy to different market conditions. We focus, in particular, on identifying the performance impact of changes in both serial dependence in price variability and changes in the trend.

The sensitivities derived provide an effective set of metrics for for determining the fundamental profile of the simple trading strategy and suggest an explanation for the functions of trading model components commonly found in trend-following strategies.

keywords: trend-following, Monte Carlo, sensitivity analysis

Chapter 1

Introduction

For the class of market participants employing fully systematic approaches to manage their investments, it is possible to determine the exact responses of their strategies to any conceivable set of market conditions. As a result, they can conduct sensitivity analysis to systematically uncover undesirable strategy behavior and enhance strategy robustness.

Systematic traders generally use sensitivity analysis to identify the set conditions under which the *trading system* will operate within acceptable bounds. In the this report, we refer to this set of conditions as the *operational domain* of the strategy (for a specific set of trading model parameters). The broader the spectrum of market conditions over which a trading system can perform within acceptable performance bounds (i.e. the broader the operational domain of the strategy), the more *robust* the system.

In general, the operational domain of a trading strategy can be broadened through the introduction of feedback and feed-forward risk controls. Feedback risk controls operate to reduce the impact of unpredictable phenomena or events on strategy performance, while feed-forward controls exploit regularities in market structure to make local predictions that aid in the enhancement of strategy performance. We use feedback controls when poor trading performance is not driven by something we can predict. We use feed-forward controls when we understand the drivers of poor performance and there is enough persistence in the market conditions for us to effectively anticipate future poor performance.

In the following sections, a simple systematic investment approach - a so called trend-following strategy - is explored through the use of Monte Carlo simulation. In particular, a market model is specified and used to generate realistic realizations of financial instrument prices across of broad spectrum of market conditions. Sensitivity analysis is then conducted, mapping the relationship between market model parameters and the strategy performance under a particular set of trading model parameters.

The market model (i.e., the model used to simulate instrument prices) has been designed to capture a set of essential stylized facts believed to be critical to the effective functioning of the strategy. As a model is a simplification of reality by definition, we do not attempt to reproduce all empirical stylized facts. We also limit the complexity and scope of the work by focusing on the instrument-level strategy. Portfolio-level meta-strategies that determine how to allocate across instrument-level strategy instances are not explored.

1.1 Methodology

Typically, systematic traders *backtest* the strategies that they employ (i.e., they use historical data to evaluate potential performance). Such backtesting allows systematic traders to determine the response of a strategy to the exact mix of market conditions that actually occurred, but not the response of a strategy to conditions that have not yet occurred or that may occur in different proportions in the future. Typically, the longer the historical period used, the more varied the market conditions, and the more likely that historical data can be used to build a relatively complete picture of the operational domain.

There are two main ways to supplement the historical data available for testing, namely market model-based Monte Carlo simulation, and Monte Carlo resampling. In this report, we focus on the former approach to explore the characteristics of a simple trend-following strategy.

In order to simulate financial prices, a market model is designed, implemented, and calibrated to financial market data. The market model reproduces key well-established stylized facts, particularly focusing on time-varying, serially dependent price variability. Trading strategy sensitivities are created by simulating price and true range scenarios - consisting of many realizations - for a range of key market model parameters, then computing the performance of the trading strategy for all realizations under each scenario.

In the following sub-section of Chapter 1, we provide a description of the data acquisition and transformation process (Section 1.2). Instrument-specific details for each of the global futures markets in the instrument universe under study can be found in Appendix A. In subsequent Chapters, we provide an overview of the stylized facts (Chapter 2), then detail the market model (Chapter 3) and the trading model (Chapter 4) used to generate sensitivities (Chapter 5).

1.2 Data Acquisition and Transformation

Futures contracts for 115 distinct markets over the period between 1999-01-01 and 2017-05-05 were acquired from CSI. For each instrument, a *back-adjustment* process was used to build a continuous series, then a *volatility-normalized* total return index accounting for changes in prices and the impact of rolls was constructed[1] (see Figure 1.1).

True range - a commonly used measure of the daily price range of a financial instrument that accounts for gaps from the close of the previous period to open of the current period - is defined as follows:

$$R_t = \max[P_{t,H} - P_{t,L}, \mathrm{abs}(P_{t,H} - P_{t-1}), \mathrm{abs}(P_{t,L} - P_{t-1})] \tag{1.1}$$

where $P_{t,H}$ and $P_{t,L}$ are the current daily high and low prices respectively, and P_{t-1} is the previous close price.

The index for each instrument represents the total return on a re-balanced position sized to equate a move of 4 units of price variability (i.e., average true range) to a 1% loss[2]. Positions are rebalanced at each roll. Use of the volatility-normalized total return index facilitates comparison of model parameters across the instrument universe[3].

The total return index for each of the global futures markets is shown in the upper half of Figure 1.1, while the corresponding true range for the volatility-normalized total return indices is depicted in the lower half of the figure.

[1]It is important to note that the total return of a long-term position taken using futures instruments must account for so-called 'rolls'. As futures contracts have finite maturities, to take a long-term position, a trader must trade a series of individual futures contracts. Traders must repeatedly extend the maturity of their positions by executing spread trades that close positions nearing expiry and and open equivalent positions in contracts with greater maturity. The process of converting a position about to expire into a position with an expiry further into the future (thereby extending the maturity) is commonly referred to as a 'roll'. The maturity profile associated with particular roll parameters is an important determinant of total return in futures-based strategies.

[2]The huge 2015 spike that appears in the true range plot above was caused by the actions of the Swiss central bank and is not a data error. Although such moves are rare, they do happen.

[3]The volatility-normalization process was also used to meet the conditions of the data agreement with the vendor.

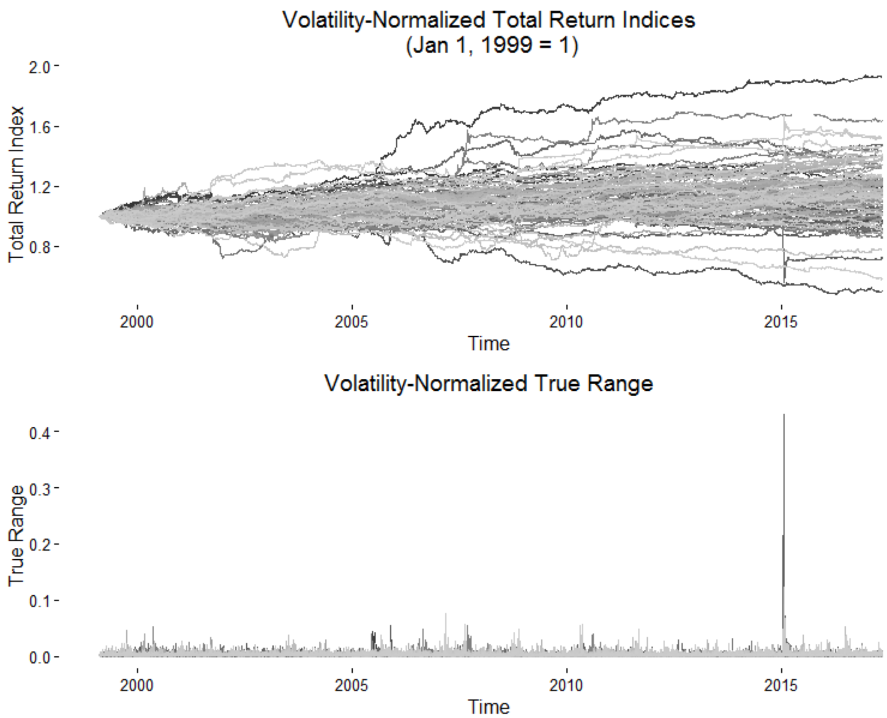


Figure 1.1: Global Futures - Volatility-Normalized Total Return Indices

1.3 Software

This report has been written using the **bookdown** package (Xie, 2016), which was built on top of R Markdown and **knitr** (Xie, 2015).

All of the code used to generate the report is available on the project github page.

buildGlobalFuturesDataset.R: Extracts data from a MySQL database and builds the R data set.

establishStylizedFacts.R: Used to explore the stylized facts

calibrateMarketModelLRD_log.R: Calibrates the market model

selectTradingModelParameters.R: Used to explore the trading model parameters via brute force search.

simulateMarketModelLRD_rerun_2.R: Simulates the market model and computes the sensitivities

As a single run of the simulation produces nearly 28 gigabytes of data, it is not possible to make the full result set available on github. Key results and data have been uploaded to github.

The project .Rmd file assembles the visualization and text comprising the report.

Chapter 2

Stylized Facts

2.1 Literature Review - Stylized Facts

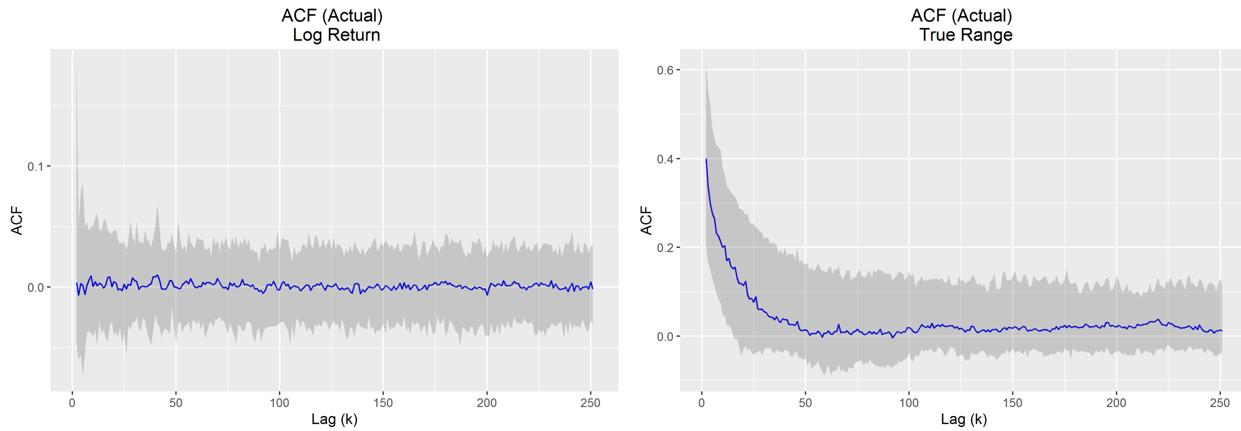
There exists a vast literature on the empirical characteristics of financial markets, documenting extensively the basic stylized facts. A similarly broad literature also exists on the derivation of financial derivative sensitivities. To price and risk manage products with path-dependent payoffs similar to a trend-following strategy, Monte Carlo simulation is often required. Despite a seemly obvious link between the analysis of systematic trading strategies and the analysis of replication strategies used to manufacture financial derivative products, little published work exists leveraging the findings in these two areas of research to the analysis of systematic trading strategies.

Although the scope of this report does not allow for a detailed exploration of the stylized facts, a number of comprehensive surveys exist ((Bollerslev et al., 1992), (Brock and de Lima, 1996), (Pagan, 1996), (Cont, 2001), (Farmer and Geanakoplos, 2009), (Gourieroux and Jasiak, 2001), (Rao and Maddala, 1996), (Pagan, 1996), and (Shephard, 1996))

The most basic and commonly agreed upon facts upon which we rely in this report are as follows: 1) Price returns of financial instruments show insignificant serial correlation; 2) The unconditional distributions of returns are heavy-tailed, and; 3) Price variability for all financial instruments is both time-varying and serially dependent.

2.1.1 Serial Dependence

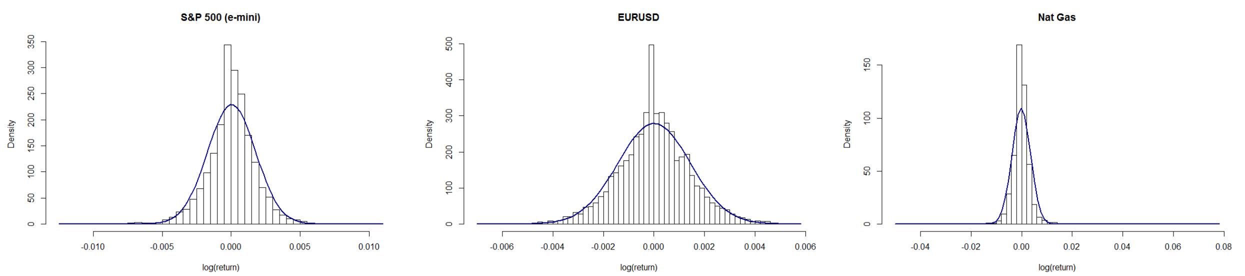
The figure below was derived by computing the autocorrelation functions (ACFs) for the log returns and the true ranges for each instrument in the universe under study, then computing the median and 95% confidence interval.



Notice that the autocorrelation of the true range decays very slowly. A similar pattern is found for all common measures of price variability. In the model calibration section of Chapter 3 we outline the scaling law that describes the pattern in the autocorrelation and specify a model to simulate this behavior.

2.1.2 Heavy-Tailed Returns Distributions

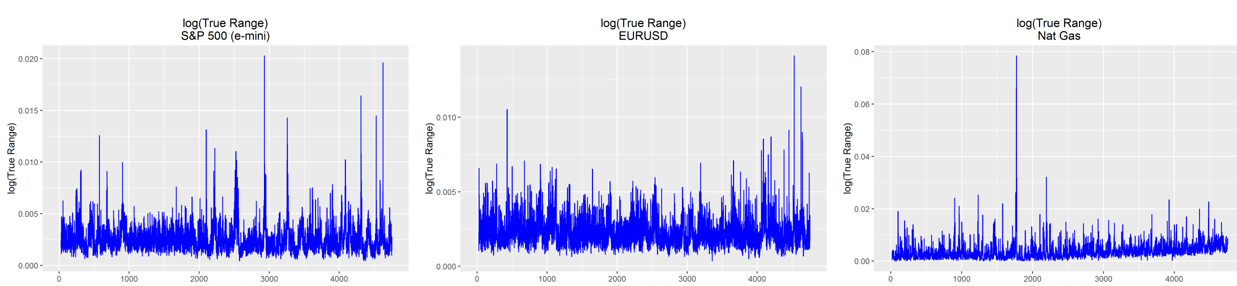
The figure below illustrates the heavy-tailed distributions of log returns for three instruments from the instrument universe under study. A normal distribution is superimposed over the histogram for each instrument.



A similar pattern is found for the log returns of all global futures instruments.

2.1.3 Time-varying Price Variability

The illustration below shows the true range for the same three instruments as those depicted in the figure directly above.



It is clear that the true range varies over time.

Again, the pattern persists across all global futures instruments.

Chapter 3

Market Model

We define and use a simple discrete time model to simulate a broad set of market conditions. Each scenario consists of realizations of both price and true range (defined in Chapter 1, Equation (1.1)).

3.1 Model Specification

The following discrete time process is used to generate price realizations for a single instrument:

$$P_t = P_{t-1} \exp\left(\mu \Delta t + \sigma_t \epsilon_t\right) \tag{3.1}$$

where $t = 1 \dots T$, $\Delta t = 1/T$, $\epsilon_t \sim N(0, 1)$, σ_t is the time-varying volatility, and μ is the constant drift for the instrument over time period, T.

The volatility is a function of the observed true range, R_t (Chou, 2005):

$$\sigma_t = \sqrt{\frac{\pi}{8}} R_t \tag{3.2}$$

Similar to the work of (Chou, 2005) and (Brunetti and Lildholdt, 2007), we extend a model originally designed for the modeling of duration time series to time-varying price range. Following (Beran et al., 2015), the true range at time, t, is given by:

$$R_t = v \lambda_t \eta_t \tag{3.3}$$

where v is a scale parameter ($v > 0$), λ_t is the conditional mean of the true range ($\lambda_t > 0$) divided by v, and η_t is an independent and identically distributed (i.i.d) log-normal random variable.

After subtracting the unconditional mean, $\log(v)$, the log true range is represented as a zero mean FARIMA(p,d,q) process

$$Z_t = \log(R_t) - \log(v) = \log(\lambda_t) + e_t \tag{3.4}$$

with innovations, $e = \log(\eta_t)$, given by

$$(1 - B)^d \phi(B) Z_t = \psi(B) e_t \tag{3.5}$$

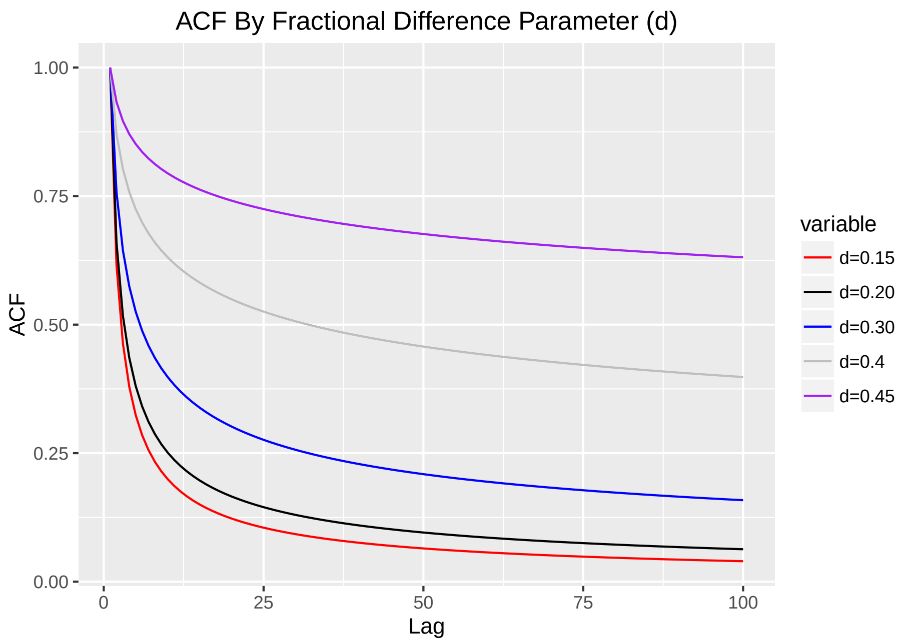


Figure 3.1: Long-Memory - Hyperbolic ACF

where d is the long memory parameter ($0 < d < 0.5$), B is the back-shift (or lag operator)[1], and $\phi(z) = 1 - \phi_1 z - \ldots - \phi_p z^p$ and $\psi(z) = 1 + \psi_1 + \ldots + \psi_q z^q$ are MA- and AR-polynomials with all roots outside the unit circle[2].

Denoting $\log(\lambda_t)$ as ζ_t and rearranging, we can see that the conditional mean of Z_t is given by

$$\zeta_t = \log(\lambda_t) = \left[\phi^{-1}(B)\psi(B)(1-B)^{-d} - 1\right]e_t \tag{3.6}$$

where $E(\zeta_t) = 0$

The autocorrelations of Z_t - which follow a scaling law - exhibit a hyperbolic decay (See Figure 3.1), the speed of which depends upon the parameter d

$$\rho(k) \sim c_\rho^Z |k|^{2d-1} \tag{3.7}$$

where $c_\rho^Z > 0$ is a constant.

For non-integer values of d, the autocorrelation function (ACF) decays hyperbolically to zero according to (3.7) (See Figure 3.1).

The general class of models defined by (3.4) and (3.5) are referred to as *exponential FARIMA (EFARIMA)* models in the literature (Beran et al., 2015). Where e_t are normally distributed, the model is referred to as a *Gaussian EFARIMA* model.

Setting $p = 0$ and $q = 0$, the innovations, $e = \log(\eta_t)$, simplify to

[1] The back-shift operator is used for notational convenience. $B^m x_t = x_{t-m}$. B notation allows (even infinite) distributed lags to be represented concisely.

[2] Wold's decomposition - a fundamental theorem in time series analysis - states that every weakly stationary, purely non-deterministic, stochastic process $(x_t - \mu)$ can be written as a linear combination (or linear filter) or a sequence of uncorrelated random variables. See [@Mills1999-bo] for an highly readable, but informal introduction.

$$(1 - B)^d Z_t = e_t \tag{3.8}$$

This simpler specification is particularly useful for sensitivity analysis. Equation (3.4) is denoted EFARIMA(0,d,0).

Our market model has two sources of uncertainty, namely ϵ and e. Bursts in volatility driven by the true range process can generate price momentum that looks very similar to that observed in real markets.

3.2 Model Calibration

For each instrument in the universe under study, we fit an EFARIMA(0,d,0) model with log-normal errors. We then use the cross-section of parameters to define the starting range of parameters for use in our sensitivity analysis.

Given the definition of our market model (defined above), we observe two processes - P_t and R_t.

We assume that R_t $(t = 1, 2, \ldots, T)$ is generated by an EFARIMA process with an unknown parameter vector

$$\theta_R = \left(v, \sigma_e^2, d, \phi_1, \ldots, \phi_p, \psi_1, \ldots, \psi_q\right)^T \tag{3.9}$$

For the EFARIMA(0,d,0) model, this parameter set reduces to

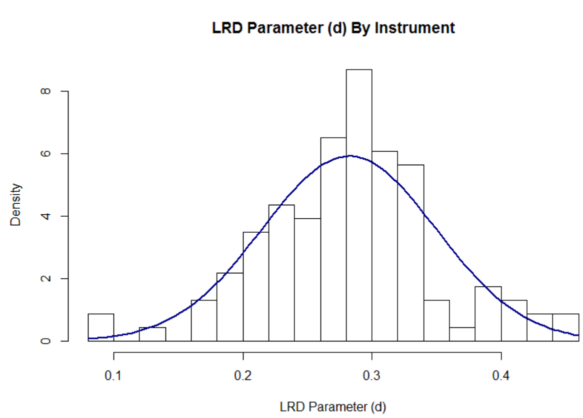
$$\theta_R = \left(v, \sigma_e^2, d\right)^T \tag{3.10}$$

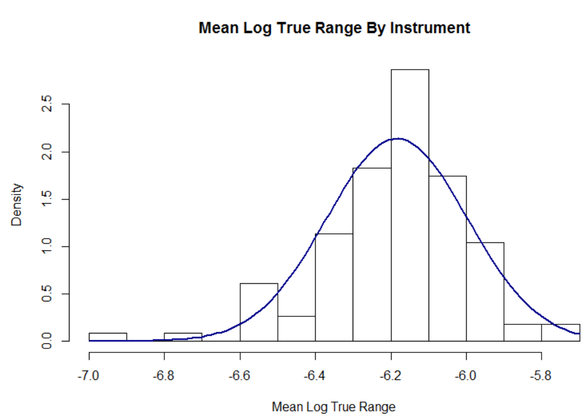
Given that - by assumption - $Z_t = \log R_t - \log v$ is a centered Gaussian FARIMA process, maximum likelihood estimation (MLE) can be used to estimate the parameters (See (Mills, 1999),(Fox and Taqqu, 1986),(Giraitis and Surgailis, 1990),(Beran, 1995), and (Haslett and Raftery, 1989)). This allows us to use standard, widely-available, estimation software to calibrate the model ((McLeod et al., 2007) and (Veenstra and McLeod, 2015)).

We assume that the price process is a function the volatility, which is in turn a function of R_t.

We employ the ARFIMA R package (Veenstra and McLeod, 2015) to estimate the parameters of true range for each instrument in the universe under study (See Tables 3.1, 3.2, 3.3 and ,3.4).

For the vast majority of the instruments in the universe under study, the estimated d parameter is between 0.15 and 0.35.





As is evident from the calibration tables, all parameters are highly significant.

Table 3.1: Global Futures - Instrument Details

	Instrument Name	d	log(V)	var(e)	d p-Value	var(e) p-value	mu
1	Brent Crude Oil	0.2355	-6.2819	0.1489	0	0	0.0092
2	Crude Oil	0.2209	-6.3405	0.1700	0	0	0.0122
3	Ethanol	0.2809	-6.7159	0.4139	0	0	0.0571
4	Gas Oil	0.2198	-6.3044	0.1542	0	0	0.0096
5	Gas-RBOB	0.2011	-6.1794	0.1672	0	0	0.0022
6	Heating Oil	0.2449	-6.2499	0.1439	0	0	0.0099
7	Nat Gas	0.2886	-6.1400	0.1594	0	0	-0.0022
8	WTI Crude Oil	0.2177	-6.2201	0.3278	0	0	-0.0024
9	ECX EUA Emissions	0.3822	-5.7152	0.1031	0	0	-0.0142
10	Nat Gas	0.3720	-5.8487	0.4072	0	0	-0.0386
11	AUDUSD	0.2638	-6.1850	0.1400	0	0	0.0067
12	CADUSD	0.2694	-6.1393	0.1334	0	0	0.0020
13	CHFUSD	0.2222	-6.1448	0.1587	0	0	0.0034
14	EURUSD	0.2132	-6.1638	0.1455	0	0	0.0014
15	GBPUSD	0.2373	-6.1258	0.1397	0	0	0.0004
16	JPYUSD	0.2847	-6.0951	0.1754	0	0	-0.0058
17	NZDUSD	0.1830	-6.2719	0.3233	0	0	0.0123
18	US Dollar Index	0.1830	-6.0317	0.1807	0	0	-0.0018
19	EURCHF	0.3317	-6.1226	0.5683	0	0	-0.0181
20	EURGBP	0.2030	-5.9885	0.4755	0	0	-0.0013
21	EURJPY	0.2507	-6.3500	0.4758	0	0	0.0084
22	BRLUSD	0.2252	-6.5292	0.7088	0	0	0.0223
23	CZKUSD	0.0844	-6.5137	1.1831	0	0	-0.0043
24	HUFUSD	0.0897	-6.4835	1.1712	0	0	-0.0073
25	MXNUSD	0.3382	-6.1907	0.1611	0	0	0.0050
26	PLNUSD	0.1315	-6.5222	1.1359	0	0	0.0062
27	RUBUSD	0.2953	-6.1630	0.3264	0	0	0.0046
28	ZARUSD	0.1748	-6.2179	0.6859	0	0	-0.0049
29	USDKRW	0.2730	-6.0789	0.2116	0	0	-0.0022

Table 3.2: Global Futures - Instrument Details

	Instrument Name	d	log(V)	var(e)	d p-Value	var(e) p-value	mu
30	Corn	0.3198	-6.0545	0.1720	0	0	-0.0009
31	Oats	0.3307	-6.2312	0.2129	0	0	0.0118
32	Rough Rice	0.2860	-6.0389	0.2080	0	0	-0.0084
33	Soybean Meal	0.3132	-6.3220	0.1647	0	0	0.0203
34	Soybean Oil	0.2440	-6.1338	0.1459	0	0	0.0053
35	Soybeans	0.2783	-6.2305	0.1561	0	0	0.0164
36	Wheat	0.2748	-6.0158	0.1416	0	0	-0.0066
37	Corn	0.2860	-6.5743	0.6159	0	0	0.0274
38	Milling Wheat	0.3447	-6.5546	0.6575	0	0	0.0339
39	Rapeseed	0.2804	-6.5859	0.5275	0	0	0.0166
40	Wheat	0.3195	-6.2202	0.4898	0	0	0.0065
41	Dow Jones Industrial (mini)	0.3304	-6.1172	0.1621	0	0	0.0105
42	MSCI EAFE (mini)	0.3136	-6.1508	0.2195	0	0	0.0100
43	Nasdaq 100 (e-mini)	0.3325	-6.1532	0.1491	0	0	0.0136
44	Russell 2000 (mini)	0.3029	-6.1539	0.1477	0	0	0.0125
45	SP 500 (e-mini)	0.3359	-6.1039	0.1596	0	0	0.0089
46	Belgian 20	0.2390	-6.0888	0.2724	0	0	0.0123
47	CAC 40	0.2930	-6.1944	0.1899	0	0	0.0070
48	DAX	0.2998	-6.2708	0.2117	0	0	0.0081
49	DJ Euro STOXX 50	0.3090	-6.0845	0.1636	0	0	0.0058
50	EOE (Amsterdam)	0.2994	-5.9945	0.1449	0	0	0.0028
51	FTSE 100	0.3368	-5.9297	0.1181	0	0	0.0040
52	IBEX 35	0.2771	-6.2074	0.1749	0	0	0.0072
53	MIB FTSE	0.3057	-6.2135	0.1974	0	0	0.0011
54	Nikkei 225	0.2941	-6.3303	0.3148	0	0	0.0055
55	OMX	0.3860	-6.9382	0.1650	0	0	0.0124
56	SPTSE 60	0.2485	-6.0678	0.1485	0	0	0.0113
57	SMI	0.3235	-5.9756	0.1364	0	0	0.0080
58	SPI 200	0.2745	-6.3048	0.2375	0	0	0.0099
59	TOPIX	0.2845	-6.2746	0.2545	0	0	0.0074
60	MSCI EM (mini)	0.2801	-6.0982	0.1943	0	0	-0.0027
61	MSCI Taiwan	0.2195	-6.1515	0.1817	0	0	0.0092
62	SP CNX Nifty	0.1867	-6.3298	0.4275	0	0	0.0175
63	Hang Seng	0.1864	-6.2192	0.1769	0	0	0.0128
64	Hang Seng (mini)	0.1814	-6.1844	0.1782	0	0	0.0128
65	Hang Seng China Enterprises	0.2633	-6.2258	0.1530	0	0	0.0158
66	IPC	0.3258	-5.8855	0.1262	0	0	0.0064
67	KOSPI 200	0.2357	-6.5445	0.4424	0	0	0.0126

Table 3.3: Global Futures - Instrument Details

	Instrument Name	d	log(V)	var(e)	d p-Value	var(e) p-value	mu
68	Feeder Cattle	0.2522	-6.1331	0.1549	0	0	0.0109
69	Lean Hogs	0.2451	-5.9264	0.1445	0	0	-0.0064
70	Live Cattle	0.2083	-6.1276	0.1491	0	0	0.0088
71	Copper	0.2635	-6.1975	0.1522	0	0	0.0091
72	Gold	0.3005	-6.1800	0.1894	0	0	0.0108
73	Palladium	0.3813	-6.2387	0.2212	0	0	0.0111
74	Platinum	0.3442	-6.2491	0.1652	0	0	0.0110
75	Silver	0.3047	-6.1876	0.1795	0	0	0.0066
76	USD Deliverable Swap 10yr	0.2990	-6.0867	0.2080	0	0	0.0123
77	USD Deliverable Swap 5yr	0.2849	-6.0717	0.2412	0	0	0.0070
78	USD Govt 10yr	0.2797	-6.2127	0.1532	0	0	0.0173
79	USD Govt 15-30yr	0.2606	-6.1806	0.1457	0	0	0.0141
80	USD Govt 2yr	0.2951	-6.2539	0.1923	0	0	0.0209
81	USD Govt 30yr	0.3198	-6.1727	0.1455	0	0	0.0156
82	USD Govt 5yr	0.2751	-6.2401	0.1649	0	0	0.0176
83	AUD Govt 10yr	0.1748	-6.3502	0.3921	0	0	0.0076
84	AUD Govt 3yr	0.1689	-6.3336	0.4123	0	0	0.0084
85	CAD Govt 10yr	0.2198	-6.1027	0.2192	0	0	0.0182
86	CHF Govt 10yr	0.2652	-6.3925	0.3511	0	0	0.0200
87	DEM Govt 10yr	0.2364	-6.4194	0.2785	0	0	0.0172
88	DEM Govt 2yr	0.2896	-6.3881	0.3155	0	0	0.0179
89	DEM Govt 5yr	0.2472	-6.4036	0.2799	0	0	0.0186
90	FRF Govt 10yr	0.3146	-6.0980	0.1477	0	0	0.0307
91	GBP Govt 10yr	0.2338	-6.0316	0.1308	0	0	0.0132
92	ITL Govt 10yr	0.4065	-5.9488	0.1303	0	0	0.0201
93	ITL Govt 2yr	0.4559	-6.0190	0.2678	0	0	0.0294
94	JPY Govt 10yr (mini)	0.3342	-6.3402	0.2904	0	0	0.0182
95	KRW Govt 10yr	0.2950	-6.3422	0.2746	0	0	0.0275

Table 3.4: Global Futures - Instrument Details

	Instrument Name	d	log(V)	var(e)	d p-Value	var(e) p-value	mu
96	Butter	0.2603	-6.0707	0.8538	0	0	0.0164
97	Cocoa	0.2437	-6.1100	0.1751	0	0	0.0011
98	Coffee	0.2895	-6.0532	0.1854	0	0	-0.0044
99	Cotton 2	0.3031	-6.1075	0.1810	0	0	-0.0023
100	Lumber	0.2773	-5.9161	0.1449	0	0	-0.0138
101	Milk-Class III Fluid	0.3337	-6.1052	0.3525	0	0	0.0072
102	Orange Juice	0.2462	-6.1736	0.2549	0	0	0.0069
103	Robusta Coffee	0.3565	-6.0909	0.2177	0	0	0.0004
104	Sugar 11	0.3112	-6.2256	0.1673	0	0	0.0063
105	Sugar 5	0.2975	-6.2787	0.2071	0	0	0.0126
106	TSR20 Rubber	0.3321	-6.1874	0.3907	0	0	0.0106
107	Cocoa	0.3086	-5.9108	0.1417	0	0	0.0039
108	USD STIR	0.4512	-6.1231	0.2431	0	0	0.0303
109	AUD STIR	0.2282	-6.0767	0.3378	0	0	0.0081
110	CAD STIR	0.3843	-5.9982	0.2806	0	0	0.0265
111	CHF STIR	0.4136	-6.0595	0.2454	0	0	0.0332
112	EUR STIR	0.4227	-5.7489	0.2022	0	0	0.0162
113	GBP STIR	0.3300	-5.9621	0.2374	0	0	0.0244
114	VIX	0.4345	-5.9535	0.2370	0	0	-0.0443
115	VSTOXX (mini)	0.4162	-5.9015	0.1454	0	0	-0.0248

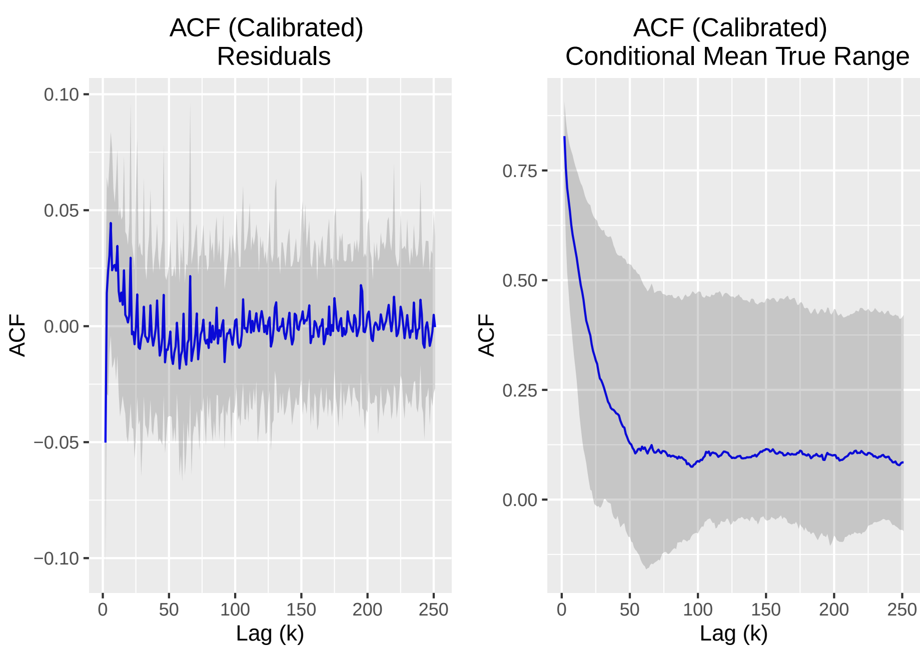


Figure 3.2: Calibrated EFARIMA(0,1,01) - ACF

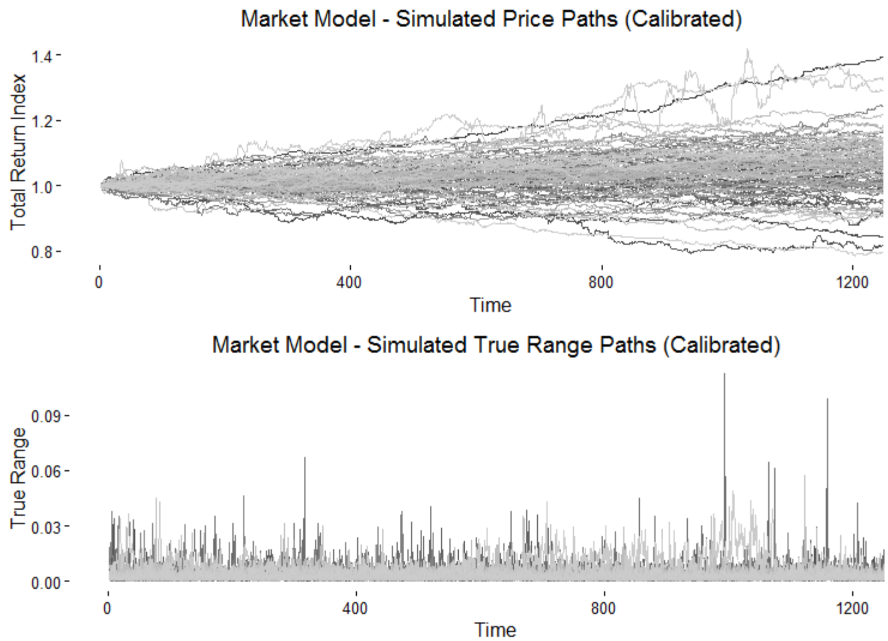


Figure 3.3: Market Model - Simulated Paths (Calibrated to Global Futures)

The EFARIMA(0,d,0) model for true range provides a framework for forecasting the conditional mean true range. These predictions could replace the EMA smoothed true range in the simple trend-following system (detailed in the next Chapter), potentially significantly improving the effectiveness of the the position-sizing and trailing stop loss components of the trading system.

It is evident from Figure 3.2 that model residuals retain some short-term memory, suggesting that a EFARIMA(p,d,q) model could provide a better fit. To reduce the dimension of our sensitivity analysis, we use the simpler model (recognizing that it captures the broader long memory but could likely be improved with a higher order model).

Using the market model, we simulate a single price and true range path for each instrument and plot the results to provide an intuitive means of checking the realism of the model (Figure 3.3). The simulated paths appear similar to the actual paths (shown in Chapter 1)

Chapter 4

Trading Model

4.1 Model Specification

We implement a very simple version of a common systematic trend-following strategy (Faith, 2007)[1]. The instrument-level logic of the trading system has a several core components: 1) The *entry signal*, determines timing for initiating a position (either long or short) in a particular instrument; 2) The *position sizing* algorithm determines the size of a position; and, 3) The *trailing stop loss* determines the timing of a exit from a position[2].

Both the position size and the distance of the trailing stop from the current price level are functions of the true range, R_t (defined in Chapter 1, Equation (1.1)).

Filters are commonly used to smooth price series. We use exponentially weighted moving averages (EMAs) to smooth both price and the true range time series.

The core rules of our simple trading model are detailed briefly in the next two sub-sections.

4.1.1 Long Position

At t, if the fast $\mathrm{EMA}_{t-1,F}$ is *above* the slow $\mathrm{EMA}_{t-1,S}$ and we have no position, we enter a *long* position of p_t units:

$$p_t = \mathrm{floor}\left[\frac{f \times A_{t-1}}{\max[\mathrm{ATR}_{t-1} \times M, L]}\right] \tag{4.1}$$

where f is the fraction of account size plus accrued realized P&L, A, risked per bet, ATR_{t-1} is the EMA of the true range for the previous time step, M is the risk multiplier, and L is the ATR floor.

We set our initial stop loss level M units of ATR *below* the entry price level, p_t. For each subsequent time, t, we update our stop level as follows:

$$s_t = \max[P_t - \mathrm{ATR}_{t-1} \times M, s_{t-1}] \tag{4.2}$$

We exit our long position if the price, p_t moves below the stop loss level, s_{t-1}.

[1]See the reference for the definition of the EMA

[2]Although trend-following models used in practice have a layer of controls at the portfolio level, in this paper we focus only on the instrument-level components of the strategy.

4.1.2 Short Position

At t, if the fast $\mathrm{EMA}_{t-1,F}$ is *below* the slow $\mathrm{EMA}_{t-1,S}$ and we have no position, we enter a *short* position of p_t units:

$$p_t = -\mathrm{floor}\left[\frac{f \times A_{t-1}}{\max[\mathrm{ATR}_{t-1} \times M, L]}\right] \tag{4.3}$$

We set our initial stop loss level M units of ATR *above* the entry price level, p_t. For each subsequent time, t, we update our stop level as follows:

$$S_t = \min[P_t - \mathrm{ATR}_{t-1} \times M, S_{t-1}] \tag{4.4}$$

Regardless of whether we are long or short, for each trade we budget for a loss of f percent of our account size plus accrued realized P&L. The effectiveness of this crude risk budgeting system is a function of the characteristics of the true range. Serial dependence in the true range can transform this simple mechanism from a feedback control to a feed-forward control.

4.2 Parameter Selection

Chapter 5

Sensitivity Analysis

In Chapter 3, we specified a market model and calibrated it to each instrument in the global futures universe under study. In Chapter 4, we outlined the rules of a simple trading strategy and explored the parameter space. In this Chapter, we create sensitivities by simulating price and true range scenarios for a set of market model parameters and computing the performance of the trading model under each scenario.

The parameter space of the combined market and trading models is vast. To reduce the dimension of the problem, an initial study was conducted to coarsely explore the impact of different trading model parameters on the strategy backtest results (See parameter selection section). A set of trading model parameters was selected from stable areas of the response curves[1].

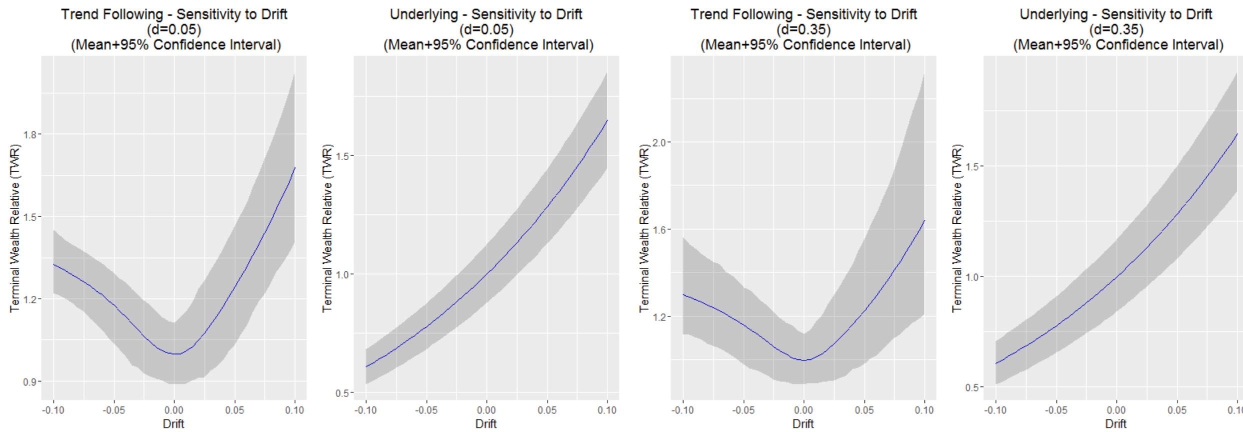
Following the selection of the trading model parameters, the range of market parameters observed over the entire instrument universe under study was examined and used to determine realistic starting parameter ranges for sensitivity analysis. These ranges were then extended to account for realistic conditions that may be observed in in the future. Once ranges were selected, another coarse study was conducted to determine which market model parameters had the largest impact on performance. Based on these results, the drift (μ) and d parameters were selected for the final sensitivity analysis. 1000 paths, each with a 1250 day length (roughly 5 years), were used for all simulations. The strategy performance measure (TWR) is defined in Appendix B. A single sensitivity simulation run varying only the drift and d parameters, but holding all other variables constant, generates just under of 28 gigabytes of simulated market model input and trading model output.

5.0.1 Sensitivity to Trend Conditions

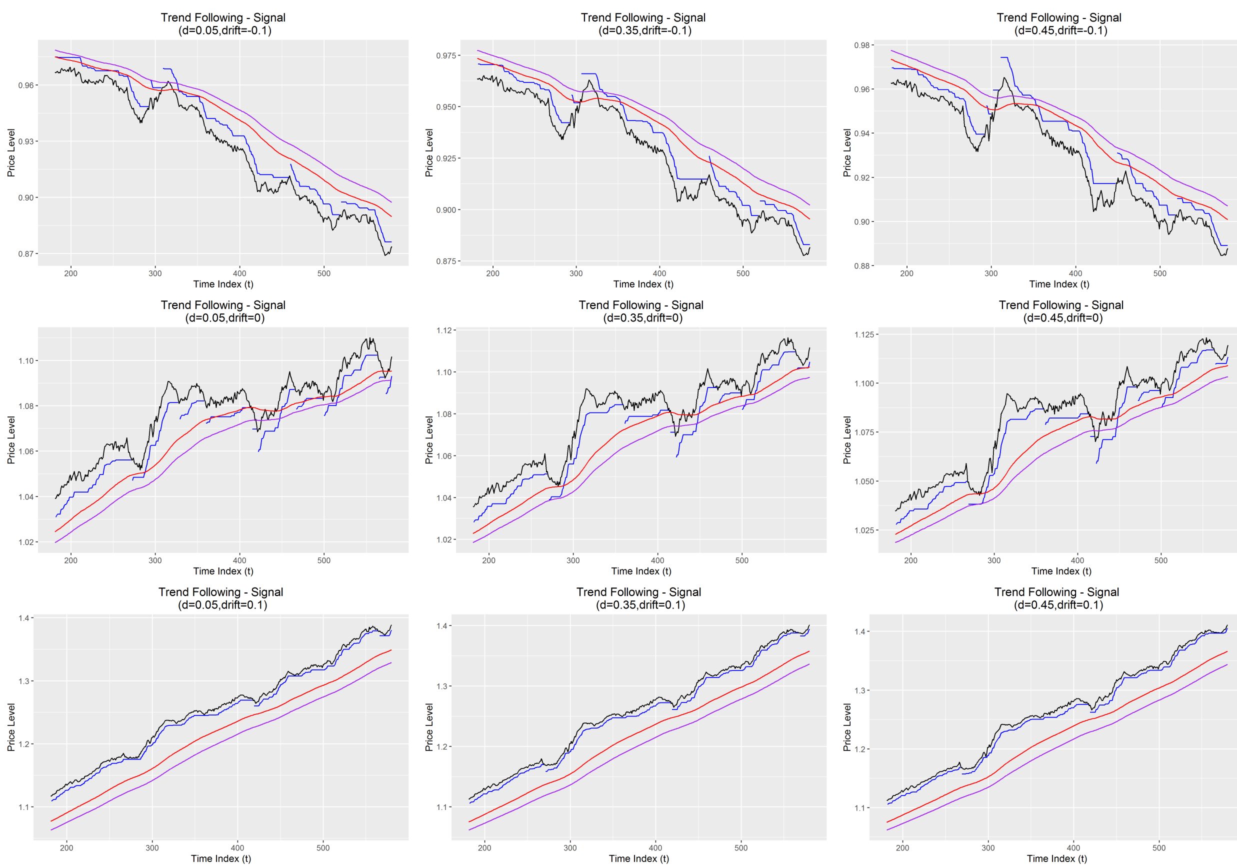
Trend-following strategies operate on the premise that the emergence of a trend in a particular instrument can not be predicted. The system is designed to maintain a position in an instrument as long as it is trending and exit the position when the trend has reversed beyond a multiple of the typical daily range. Any predictability in the characteristics of true range, is thus expected to enable strategy enhancement.

First we use our market model defined above to determine the sensitivity of the strategy to trends of different magnitudes by computing trading model performance under different drift rates (μ).

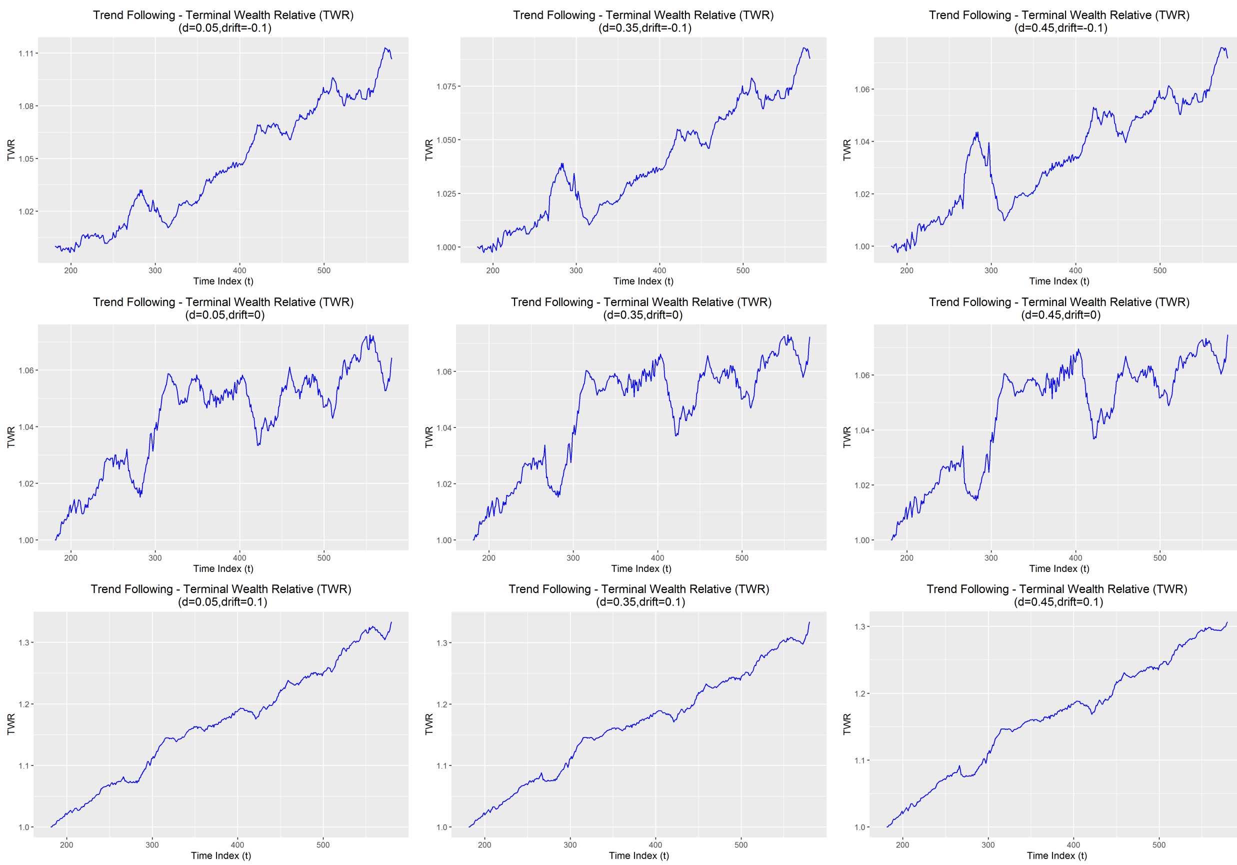
[1] The process required to select robust trading parameters is beyond the scope of this report. An extensive literature associated with a number of disciplines, including machine learning, addressing over-fitting and robust parameter selection exists



The profile that emerges (See Figure directly above) from this sensitivity analysis of the strategy performance with respect to changes in the drift (μ) illustrates the essence of the strategy. From the profile, it is clear that as the price moves up or down strongly, the strategy performance increases. The less variability around the trend, the better the strategy performance. Choppy, sideways movement in prices produces a condition where the strategy repeatedly enters and gets stopped out, generating losses for roughly half of the paths. Increasing the strength of long range dependence in true range by increasing d increases the dispersion of results, particularly for very favorable trend conditions (i.e., high drift,μ).



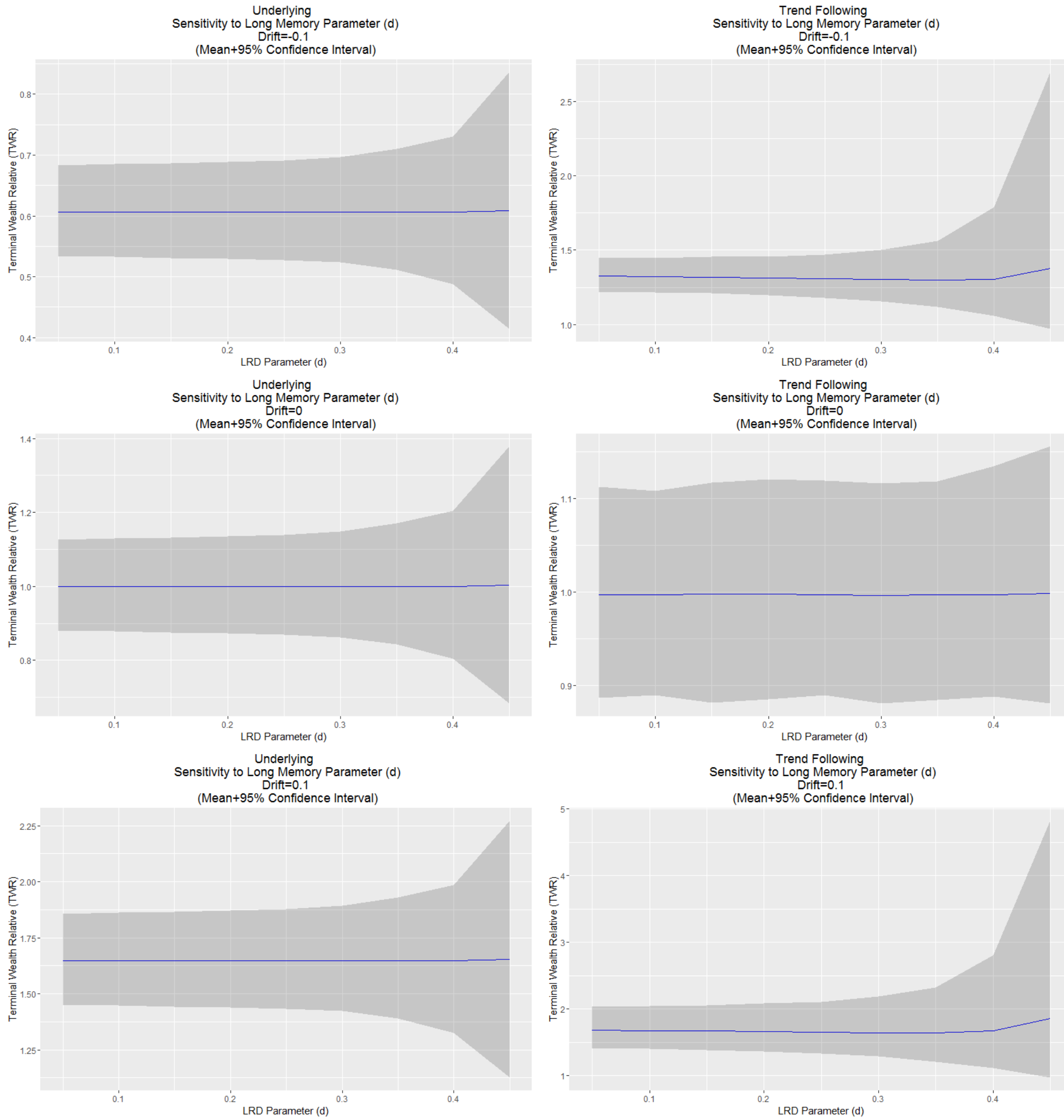
Examining sample price paths with the trailing stop and signal superimposed, it is possible to get some intuition about the result. As we increase the strength of long memory in the true range, the price paths get visibly more volatile (See Figure directly above). The stop is the blue line that ratchets behind the price depicted in black. The red line depicts the fast EMA, while the purple depicts the slow EMA. As we reduce the drift (in absolute value terms), the strategy gets stopped out more often. Performance is better on the long side than the short side because price is unbounded on the positive side, but bounded by zero on the negative side.



Focusing on the terminal wealth relative curves (See Appendix B for details), one can see that increasing variation around the trend reduces performance.The impact of increasing d is largest during strong trend conditions.

5.0.2 Sensitivity to Serial Dependence in True Range

Given the observed long range serial dependence in the true range, a natural question arises as to the sensitivity of the performance of our simple trading model to the strength of autocorrelation. To determine the link between strategy performance and autocorrelation we perturb the d parameter, generate price and true range scenarios, then evaluate strategy performance under each scenario.



Counter-intuitively, increasing the long range dependence in true range increases the dispersion of outcomes and - for the most part - *worsens* performance. There is a slight improvement in both median and mean performance as we *decrease d*. Although it is difficult to determine the source of this effect without a significant amount of additional research, it seems likely that it is possible to redesign components of the trading system to exploit the regularity in true range driven by long range dependence. The model that has been developed in this report should provide a reasonable starting point for such an endeavor.

Chapter 6

Conclusion

In this report, we formulated both a simple systematic trend-following strategy (i.e., trading model) to simulate investment decisions, and a market model to simulate the evolution of instrument prices. We explored the sensitivity of our strategy to different market conditions (for a particular set of trading model parameters) and provided a map between the market model parameters for each scenario representing a particular market condition and strategy performance. In particular, we focused on identifying the performance impact of changes in 1) serial dependence in price variability, and; 2) changes in the trend.

The sensitivities derived provide an effective visual depiction of the fundamental profile of the simple trading strategy and suggest an explanation for the functions of trading model components commonly found in trend-following strategies. The long range serial dependence in the true range appears to *worsen* performance of the simple classic trend-following strategy. During periods of strong performance, the dispersion of trading outcomes increases significantly as long range serial dependence increases.

More research is required to determine whether a slightly more complex feed-forward controller could be created to improve performance of the strategy by exploiting long memory in the true range.

An extension of our simple single instrument market market model to a multiple instrument model could provide useful sensitivity analysis relating to the cross-dependence between instruments.

Appendix A

Global Futures - Instrument Details

The global futures instrument universe used in this report is defined as follows:

Table A.1: Global Futures - Instrument Details

	Instrument Name	Group Name	Multiplier	Currency
1	Brent Crude Oil	Energy	1000	USD
2	Crude Oil	Energy	1000	USD
3	Ethanol	Energy	29000	USD
4	Gas Oil	Energy	100	USD
5	Gas-RBOB	Energy	42000	USD
6	Heating Oil	Energy	42000	USD
7	Nat Gas	Energy	10000	USD
8	WTI Crude Oil	Energy	1000	USD
9	ECX EUA Emissions	Energy	1000	EUR
10	Nat Gas	Energy	300	GBP
11	AUDUSD	FX	100000	USD
12	CADUSD	FX	100000	USD
13	CHFUSD	FX	125000	USD
14	EURUSD	FX	125000	USD
15	GBPUSD	FX	62500	USD
16	JPYUSD	FX	125000	USD
17	NZDUSD	FX	100000	USD
18	US Dollar Index	FX	1000	USD
19	EURCHF	FX	125000	CHF
20	EURGBP	FX	125000	GBP
21	EURJPY	FX	125000	JPY
22	BRLUSD	FX (Emerging)	100000	USD
23	CZKUSD	FX (Emerging)	40000	USD
24	HUFUSD	FX (Emerging)	300000	USD
25	MXNUSD	FX (Emerging)	500000	USD
26	PLNUSD	FX (Emerging)	500000	USD
27	RUBUSD	FX (Emerging)	2500000	USD
28	ZARUSD	FX (Emerging)	500000	USD
29	USDKRW	FX (Emerging)	10000	KRW

Table A.2: Global Futures - Instrument Details

	Instrument Name	Group Name	Multiplier	Currency
30	Corn	Grain	50	USD
31	Oats	Grain	50	USD
32	Rough Rice	Grain	2000	USD
33	Soybean Meal	Grain	100	USD
34	Soybean Oil	Grain	600	USD
35	Soybeans	Grain	50	USD
36	Wheat	Grain	50	USD
37	Corn	Grain	50	EUR
38	Milling Wheat	Grain	50	EUR
39	Rapeseed	Grain	50	EUR
40	Wheat	Grain	100	GBP
41	Dow Jones Industrial (mini)	Index	5	USD
42	MSCI EAFE (mini)	Index	50	USD
43	Nasdaq 100 (e-mini)	Index	20	USD
44	Russell 2000 (mini)	Index	100	USD
45	SP 500 (e-mini)	Index	50	USD
46	Belgian 20	Index	10	EUR
47	CAC 40	Index	10	EUR
48	DAX	Index	25	EUR
49	DJ Euro STOXX 50	Index	10	EUR
50	EOE (Amsterdam)	Index	200	EUR
51	FTSE 100	Index	10	GBP
52	IBEX 35	Index	10	EUR
53	MIB FTSE	Index	5	EUR
54	Nikkei 225	Index	500	JPY
55	OMX	Index	100	SEK
56	SPTSE 60	Index	200	CAD
57	SMI	Index	10	CHF
58	SPI 200	Index	25	AUD
59	TOPIX	Index	10000	JPY
60	MSCI EM (mini)	Index (Emerging)	50	USD
61	MSCI Taiwan	Index (Emerging)	100	USD
62	SP CNX Nifty	Index (Emerging)	2	USD
63	Hang Seng	Index (Emerging)	50	HKD
64	Hang Seng (mini)	Index (Emerging)	10	HKD
65	Hang Seng China Enterprises	Index (Emerging)	50	HKD
66	IPC	Index (Emerging)	10	MXN
67	KOSPI 200	Index (Emerging)	500000	KRW

Table A.3: Global Futures - Instrument Details

	Instrument Name	Group Name	Multiplier	Currency
68	Feeder Cattle	Meat	500	USD
69	Lean Hogs	Meat	400	USD
70	Live Cattle	Meat	400	USD
71	Copper	Metal	250	USD
72	Gold	Metal	100	USD
73	Palladium	Metal	100	USD
74	Platinum	Metal	50	USD
75	Silver	Metal	50	USD
76	USD Deliverable Swap 10yr	Rates	1000	USD
77	USD Deliverable Swap 5yr	Rates	1000	USD
78	USD Govt 10yr	Rates	1000	USD
79	USD Govt 15-30yr	Rates	1000	USD
80	USD Govt 2yr	Rates	2000	USD
81	USD Govt 30yr	Rates	1000	USD
82	USD Govt 5yr	Rates	1000	USD
83	AUD Govt 10yr	Rates	8000	AUD
84	AUD Govt 3yr	Rates	2800	AUD
85	CAD Govt 10yr	Rates	1000	CAD
86	CHF Govt 10yr	Rates	1000	CHF
87	DEM Govt 10yr	Rates	1000	EUR
88	DEM Govt 2yr	Rates	1000	EUR
89	DEM Govt 5yr	Rates	1000	EUR
90	FRF Govt 10yr	Rates	1000	EUR
91	GBP Govt 10yr	Rates	1000	GBP
92	ITL Govt 10yr	Rates	1000	EUR
93	ITL Govt 2yr	Rates	1000	EUR
94	JPY Govt 10yr (mini)	Rates	100000	JPY
95	KRW Govt 10yr	Rates	1000000	KRW

Table A.4: Global Futures - Instrument Details

	Instrument Name	Group Name	Multiplier	Currency
96	Butter	Soft	200	USD
97	Cocoa	Soft	10	USD
98	Coffee	Soft	375	USD
99	Cotton 2	Soft	500	USD
100	Lumber	Soft	110	USD
101	Milk-Class III Fluid	Soft	2000	USD
102	Orange Juice	Soft	150	USD
103	Robusta Coffee	Soft	10	USD
104	Sugar 11	Soft	1120	USD
105	Sugar 5	Soft	50	USD
106	TSR20 Rubber	Soft	50	USD
107	Cocoa	Soft	10	GBP
108	USD STIR	STIR	2500	USD
109	AUD STIR	STIR	2400	AUD
110	CAD STIR	STIR	2500	CAD
111	CHF STIR	STIR	2500	CHF
112	EUR STIR	STIR	2500	EUR
113	GBP STIR	STIR	1250	GBP
114	VIX	Volatility	1000	USD
115	VSTOXX (mini)	Volatility	100	EUR

Appendix B

Objective Function: Evaluation Measure(s) for Performance

In order to meet our objectives, we must have measures to quantify both strategy performance and the breadth of the operational domain.

We can define the *terminal wealth relative* as the multiplier that we apply to our starting equity to get our ending equity. In other words, the terminal wealth relative is one plus our total return.

$$\mathrm{TWR}_T = \prod_{t=1}^{T} (1 + r_t) = \prod_{t=1}^{T} (\mathrm{HPR}_t) \tag{B.1}$$

Where:

r_t is our return over period t

HPR_t is our holding period return or one plus our return over the t^{th} period

TWR_T is our terminal wealth relative or one plus our total return over T periods

We can approximate our TWR_T with the following formula:

$$\mathrm{aTWR}_T = \left(\sqrt{(\mathrm{AHPR}_T^2 - \mathrm{SDHPR}_T^2)}\right)^T = \mathrm{EGM}^T \tag{B.2}$$

Where:

N is the number of sub-periods over which we have returns

aTWR_T is the approximate terminal wealth relative (i.e., one plus the approximate total return over the T periods)

HPR_t is the holding period return (i.e., the return over the t^{th} period)

AHPR_T is arithmetic average of the holding period returns over the T periods:

$$\mathrm{AHPR}_T = \frac{1}{T} \sum_{t=1}^{T} (\mathrm{HPR}_t) \tag{B.3}$$

SDHPR_T is the standard deviation of the holding period returns over the T periods:

$$\mathrm{SDHPR}_T = \frac{1}{T-1}\sum_{t=1}^{T}(\mathrm{AHPR}_T - \mathrm{HPR}_t)^2 \tag{B.4}$$

EGM_T is the estimated geometric mean (EGM) over the T periods

$$\mathrm{EGM}_T = \sqrt{\left(\mathrm{AHPR}_T^2 - \mathrm{SDHPR}_T^2\right)} \tag{B.5}$$

Equation (B.2) illustrates that:

[1] If AHPR_T is less than or equal to 1, then regardless of the other two variables, SDHPR_T and T, our result can be no greater than 1 (i.e., our total return will be less than or equal to zero).

[2] If AHPR_T is less than 1, then as T approaches infinity, TWR_T approaches zero. This means that if AHPR_T is less than 1, we will eventually go broke.

[3] If AHPR_T is greater than 1, increasing T increases our TWR_T.

[4] If we reduce our SDHPR_T more than we reduce our AHPR_T our TWR_T will rise.

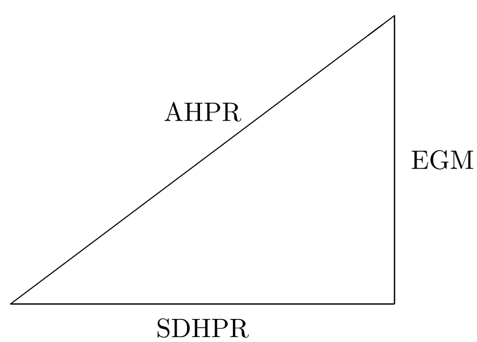
Reducing variability or increasing average return by the same amount has an *identical* impact on compound return.

We can use equation (B.2) to understand how changes in the average return, return variability, or both impact our compounded return.

EGM_T - which is composed of AHPR_T^2 and SDHPR_T^2 - is our primary measure of trading strategy performance. All other performance metrics are a function of these three metrics.

The breadth of the operational domain will be measured as the span of parameters over which strategy performance is acceptable, where acceptable is defined using a vector of strategy performance metrics, including EGM_T, AHPR_T^2, and SDHPR_T^2.

We can visualize the relationship between EGM, AHPR, and SDHPR based on a rearragement of (B.5):



$$\mathrm{AHPR}_T^2 = \mathrm{EGM}_T^2 + \mathrm{SDHPR}_T^2 \tag{B.6}$$

The above diagram makes it clear that any decrease in SDHPR is equivalent to an increase in AHPR.

B.0.1 Extending to a Porfolio

We can also extend this result to show how the cross-dependence between strategies/investments - which ultimately drives portfolio variation - impacts compound return.

B.0.1.1 Portfolio Return

Portfolio return $r_{P,t}$ is a function of the weights and the returns of portfolio investment components. We define the portfolio return for I component investments for the period t given the period returns $r_{i,t}$ and portfolio weights $w_{i,t}$ for each component investment i as:

$$r_{P,t} = \sum_{i=1}^{I} (r_{i,t} w_{i,t}) \tag{B.7}$$

Letting W_t be a vector of portfolio component weights for period t, ' denote the transpose operator, and R_t be a vector of the period t component returns, we can use matrix notation to define the portfolio return as follows:

$$r_{P,t} = W'R \tag{B.8}$$

The holding period return (HPR) for the portfolio is one plus the portfolio return for the period t:

$$\text{HPR}_{P,t} = 1 + \sum_{i=1}^{I} (r_{i,t} w_{i,t}) = 1 + r_{P,t} \tag{B.9}$$

The portfolio holding period return is the factor by which we multiply the starting value of the portfolio to get the ending value of the portfolio, given the period returns and weights of each component investment.

Similarly, we define the terminal wealth relative (TWR) as the factor by which we multiply the starting value of the portfolio to get the ending value of the portfolio given the return streams and weights for a sequence of periods between one and T:

$$\text{TWR}_{P,T} = \prod_{1=t}^{T} \left(1 + \left(\sum_{i=1}^{I} (r_{i,t} w_{i,t})\right)\right) = \prod_{t=1}^{T} \text{HPR}_{P,t} \tag{B.10}$$

We define the portfolio compounded return for the interval from period 1 to T as the portfolio terminal wealth relative minus one:

$$r_{P,T} = \left(\prod_{1=t}^{T} \left(1 + \left(\sum_{i=1}^{I} (r_{i,t} w_{i,t})\right)\right)\right) - 1 = \left(\prod_{1=t}^{T} (1 + r_{P,t})\right) - 1 = \left(\prod_{1=t}^{T} \text{HPR}_{P,t}\right) - 1 = \text{TWR}_{P,T} - 1 \tag{B.11}$$

B.0.1.2 Portfolio Return Variability

Assuming that *standardized* component returns are normally distributed, and thus that portfolio returns are multivariate normally distributed, we can define the standard deviation of the portfolio *standardized* returns using matrix notation as:

$$\sigma_{P,T} = \sqrt{Var\left(W_t{}'R_t\right)} = \sqrt{W_t{}'\Sigma W_t} \tag{B.12}$$

Where W_t is a vector of portfolio component weights for period t, ' denotes the transpose operator, R_t is a vector of the period t component returns, and Σ is the return covariance matrix.

In the portfolio context, EGM_T is also a function of the return covariance. Reducing the return covariance - keeping all other return properties the same - thus reduces portfolio variation, increasing EGM_T.

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