BPY504 Assignment 1

Due: Friday October 31, 2025 5pm

A note about sign conventions: We define object distance u as positive when the object is on the side of the incoming light, and image distance v as positive when the image is on the outgoing side. A negative v therefore corresponds to a *virtual* image located on the same side as the object. Magnification $m = -v/u$ is positive for upright (virtual) images and negative for inverted (real) images.

1 A coin in water

A coin is resting on the bottom of a tank of water ($n = 1.33$) that is 1 meter deep. On top of the water floats a layer of of benzene ($n = 1.5$) that is 20 cm thick.

(a) Looking down nearly perpendicularly, how far beneath the topmost surface does the coin appear?

(b) Draw a ray diagram to explain your answer.

2 A fish in water

A fish looking straight up toward the smooth surface of a pond receives a cone of rays and sees a circle of light filled with the images of sky and birds and whatever else is up there. This bright circular field is surrounded by darkness. Explain what is happening and compute the cone angle.

3 Lenses and images

(a) An object initially at a distance of 10 m moves to a distance of 50 cm in front of a lens. In the process, its image distance doubles. Determine the focal length of the lens.

(b) A firefly is present 20 cm in front of a thin lens. The real image of the firefly is three times its size. Calculate the image's position and the lens focal length. Repeat the above exercise if the image formed was virtual.

(c) A glowing figurine 5 cm tall is standing 15 cm in front of a thin convex lens whose focal length is 20 cm. Locate and describe the image of the figurine. Can you project it on a screen?

4 Compound lenses

(a) A compound lens is composed of two thin lenses separated by 10 cm. The first of these has a focal length of $+20$ cm, and the second a focal length of -20 cm. What is the focal length of the combination? Draw a ray diagram of the system.

(b) A blade of grass standing 10 mm tall is 150 mm in front of a thin positive lens having a 100 mm focal length; 250 mm behind that first lens is a thin negative lens with a focal length of -75 mm. Show that the first lens forms an image 300 mm behind it and describe that image. What is its magnification?

(c) Now, prove that the final image of the blade of grass formed by both lenses is located 150 mm behind the negative lens. What is the total magnification of the combination?

## 5 Single spherical refraction and translation

Light travels from air ($n_1 = 1$) into glass ($n_2 = 1.5$) through a spherical surface of radius $R = 50$ mm. After refraction, it propagates a distance $d = 20$ mm through the glass.

(a) Write the ABCD matrix for the combined system (refraction + translation) using the (y, θ) convention.

(b) For an object at distance u from the vertex in air, determine the imaging condition (value of u for which $B = 0$) and the image distance v.

(c) Compute the transverse magnification m.

6 Fun with ABCD matrices

The ABCD matrix, as defined in class, for a thin lens of focal length f is $\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$. For the propagation of a ray through a medium of index of refraction n, with air on either side of the medium, the matrix is $\begin{pmatrix} 1 & d/n \\ 0 & 1 \end{pmatrix}$.

(a) Derive the ABCD matrix for a thick lens made of material n_2 surrounded by a liquid of index n_1. Let the lens have curvatures R_1 and R_2 and thickness d.
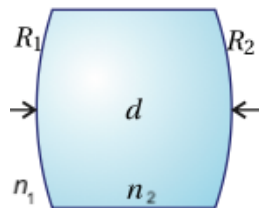
(b) Now, let $n_1 = 1$, i.e. the lens is in air. Show what the matrix reduces to that of a thin lens when the thickness d goes to zero.

7   An unknown lens system

A complicated lens element is represented by an unknown ABCD matrix. An object placed a distance d_1 before the lens element causes an image to appear a distance d_2 after the unknown element.

Suppose that when $d_1 = l$, we find that $d_2 = 2l$. Also, suppose that when $d_1 = 2l$, we find that $d_2 = 3l/2$ with magnification $M = -1/2$. What is the ABCD matrix for the unknown element?