Curso de $\LaTeX$

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Sesión

Cuestiones prácticas

El profesor

¿Qué es LaTeX?

LaTeX es un sistema de preparación de documentos, utilizado en documentos científicos y técnicos.

LaTeX ¡**no** es un procesador de textos!
Nos permite separar el contenido del continente,
dejando el formato a un lado.

Por eso, LaTeX se escribe en documentos de texto "sin formato" con una cabecera.
La cabecera dice cómo será el formato (tipo de letra, espaciados, márgenes, títulos...).

¿Por qué LaTeX?

¿Quién lo usa?

1. Las principales revistas del mundo: Nature, Science, PNAS, PLOS, ...
2. Todas las revistas de Matemáticas
3. Los profesores en sus apuntes (en la UAM y en todas partes)

ARTICLE

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Undecidability of the spectral gap

Toby S. Cubitt[1,2], David Perez-Garcia[3,4] & Michael M. Wolf[5]

Theorem 1
We can explicitly construct a dimension d, $d^2 \times d^2$ matrices A, B, C and D, and a rational number $\beta > 0$, which can be chosen to be as small as desired, such that
(i) A is Hermitian, with matrix elements in $\mathbb{Z} + \beta\mathbb{Z} + \frac{\beta}{\sqrt{2}}\mathbb{Z}$;
(ii) B and C have integer matrix elements; and
(iii) D is Hermitian, with matrix elements in $\{0, 1, \beta\}$.
For each positive integer n, define the local interactions of a translationally invariant, nearest-neighbour Hamiltonian $H(n)$ on a 2D square lattice as

$$h_1(n) = \alpha(n)\Pi$$
$$h_{\mathrm{row}} = D$$
$$h_{\mathrm{col}} = A + \beta\left(\mathrm{e}^{i\pi\varphi(n)}B + \mathrm{e}^{-i\pi\varphi(n)}B^\dagger + \mathrm{e}^{i\pi 2^{-|\varphi(n)|}}C + \mathrm{e}^{-i\pi 2^{-|\varphi(n)|}}C^\dagger\right)$$

where $\varphi(n) = n/2^{|n|-1}$ is the rational number whose binary fraction expansion contains the binary digits of n after the decimal point, $|\varphi(n)|$ denotes the number of digits in this expansion, $\alpha(n) \le \beta$ is an algebraic number that is computable from n, Π is a projector and the daggers denote Hermitian conjugation. Then
(i) the local interaction strength is ≤ 1 (that is, $\|h_1(n)\|$, $\|h_{\mathrm{row}}\|$, $\|h_{\mathrm{col}}(n)\| \le 1$);
(ii) if the universal Turing machine halts on input n, the Hamiltonian $H(n)$ is gapped with $\gamma \ge 1$; and

$$h(\varphi)^{(i,j)} = |0\rangle\langle 0|^{(i)} \otimes (\mathbb{1} - |0\rangle\langle 0|)^{(j)} + h_{\mathrm{u}}^{(i,j)}(\varphi) \otimes \mathbb{1}_{\mathrm{d}}^{(i,j)} + \mathbb{1}_{\mathrm{u}}^{(i,j)} \otimes h_{\mathrm{d}}^{(i,j)} \quad (1)$$

The spectrum of the new Hamiltonian H is

$$\mathrm{spec}H = \{0\} \cup \{\mathrm{spec}H_{\mathrm{u}}(\varphi) + \mathrm{spec}H_{\mathrm{d}}\} \cup S \quad (2)$$

with $S \ge 1$ (see Supplementary Information for details). Recalling that we chose H_{d} to be gapless, we see immediately from equation (2) that if the ground state energy density of H_{u} tends to zero from below (so that $\lambda_0(H_{\mathrm{u}}) < 0$), then $H(\varphi)$ is gapless; if H_{u} has a strictly positive ground state energy density (so that $\lambda_0(H_{\mathrm{u}})$ diverges to $+\infty$), then it has a spectral gap ≥ 1, as required (see Fig. 2).
This construction is rather general: by choosing different h_{d}, we obtain undecidability of any physical property that distinguishes a Hamiltonian from a gapped system with a unique product ground state.

Encoding computation in ground states

To construct the Hamiltonian $H_{\mathrm{u}}(\varphi)$, we encode the halting problem into the local interactions $h_{\mathrm{u}}(\varphi)$ of the Hamiltonian. The halting problem concerns the dynamics of a classical system—a Turing machine. To relate it to the ground state energy density—a static property of a quantum system—we construct a Hamiltonian whose ground state encodes the entire history of the computation carried out by the Turing

Estructura del curso

Procedimiento de evaluación

Bibliografía