

# Curso de L<sup>A</sup>T<sub>E</sub>X

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# Sesión

## Cuestiones prácticas

# El profesor

# ¿Qué es $\text{\LaTeX}$ ?

$\text{\LaTeX}$  es un sistema de preparación de documentos, utilizado en documentos científicos y técnicos.

$\text{\LaTeX}$  **¡no** es un procesador de textos!

Nos permite separar el contenido del continente, dejando el formato a un lado.

Por eso,  $\text{\LaTeX}$  se escribe en documentos de texto “sin formato” con una cabecera.

La cabecera dice cómo será el formato (tipo de letra, espaciados, márgenes, títulos...).

- 1 Las principales revistas del mundo: Nature, Science, PNAS, PLOS, ...
- 2 Todas las revistas de Matemáticas
- 3 Los profesores en sus apuntes (en la UAM y en todas partes)

## ARTICLE

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# Undecidability of the spectral gap

Toby S. Cubitt<sup>1,2</sup>, David Perez-Garcia<sup>3,4</sup> & Michael M. Wolf<sup>5</sup>

### Theorem 1

We can explicitly construct a dimension  $d, d^2 \times d^2$  matrices  $A, B, C$  and  $D$ , and a rational number  $\beta > 0$ , which can be chosen to be as small as desired, such that

- (i)  $A$  is Hermitian, with matrix elements in  $\mathbb{Z} + \beta\mathbb{Z} + \frac{\beta}{\sqrt{2}}\mathbb{Z}$ ;
- (ii)  $B$  and  $C$  have integer matrix elements; and
- (iii)  $D$  is Hermitian, with matrix elements in  $\{0, 1, \beta\}$ .

For each positive integer  $n$ , define the local interactions of a translationally invariant, nearest-neighbour Hamiltonian  $H(n)$  on a 2D square lattice as

$$\begin{aligned} h_1(n) &= \alpha(n)II \\ h_{\text{new}} &= D \\ h_{\text{old}} &= A + \beta(e^{i\pi\varphi(n)}B + e^{-i\pi\varphi(n)}B^\dagger + e^{i\pi 2^{-|\varphi(n)|}}C + e^{-i\pi 2^{-|\varphi(n)|}}C^\dagger) \end{aligned}$$

where  $\varphi(n) = n/2^{|\alpha|-1}$  is the rational number whose binary fraction expansion contains the binary digits of  $n$  after the decimal point,  $|\varphi(n)|$  denotes the number of digits in this expansion,  $\alpha(n) \leq \beta$  is an algebraic number that is computable from  $n$ ,  $II$  is a projector and the daggers denote Hermitian conjugation. Then

- (i) the local interaction strength is  $\leq 1$  (that is,  $\|h_1(n)\|, \|h_{\text{new}}\|, \|h_{\text{old}}(n)\| \leq 1$ );
- (ii) if the universal Turing machine halts on input  $n$ , the Hamiltonian  $H(n)$  is gapped with  $\gamma \geq 1$ ; and

$$h(\varphi)^{(i,j)} = |0\rangle\langle 0|^{(i)} \otimes (1 - |0\rangle\langle 0|)^{(j)} + h_{\text{a}}^{(i,j)}(\varphi) \otimes 1_{\text{d}}^{(i,j)} + 1_{\text{a}}^{(i,j)} \otimes h_{\text{d}}^{(i,j)} \quad (1)$$

The spectrum of the new Hamiltonian  $H$  is

$$\text{spec}H = \{0\} \cup \{\text{spec}H_{\text{a}}(\varphi) + \text{spec}H_{\text{d}}\} \cup S \quad (2)$$

with  $S \geq 1$  (see Supplementary Information for details). Recalling that we chose  $H_{\text{d}}$  to be gapless, we see immediately from equation (2) that if the ground state energy density of  $H_{\text{a}}$  tends to zero from below (so that  $\lambda_0(H_{\text{a}}) < 0$ ), then  $H(\varphi)$  is gapless; if  $H_{\text{a}}$  has a strictly positive ground state energy density (so that  $\lambda_0(H_{\text{a}})$  diverges to  $+\infty$ ), then it has a spectral gap  $\geq 1$ , as required (see Fig. 2).

This construction is rather general: by choosing different  $h_{\text{d}}$ , we obtain undecidability of any physical property that distinguishes a Hamiltonian from a gapped system with a unique product ground state.

### Encoding computation in ground states

To construct the Hamiltonian  $H_{\text{d}}(\varphi)$ , we encode the halting problem into the local interactions  $h_{\text{d}}(\varphi)$  of the Hamiltonian. The halting problem concerns the dynamics of a classical system—a Turing machine. To relate it to the ground state energy density—a static property of a quantum system—we construct a Hamiltonian whose ground state encodes the entire history of the computation carried out by the Turing

# Estructura del curso

# Procedimiento de evaluación

# Bibliografía