

Curso de L^AT_EX

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Sesión

Cuestiones prácticas

El profesor

¿Qué es L^AT_EX?

L^AT_EX es un sistema de preparación de documentos, utilizado en documentos científicos y técnicos.

L^AT_EX ¡no es un procesador de textos!

Nos permite separar el contenido del continente, dejando el formato a un lado.

Por eso, L^AT_EX se escribe en documentos de texto “sin formato” con una cabecera.

La cabecera dice cómo será el formato (tipo de letra, espaciados, márgenes, títulos...).

¿Por qué L^AT_EX?

¿Quién lo usa?

- 1 Las principales revistas del mundo: Nature, Science, PNAS, PLOS, ...
- 2 Todas las revistas de Matemáticas
- 3 Los profesores en sus apuntes (en la UAM y en todas partes)

ARTICLE

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Undecidability of the spectral gap

Toby S. Cubitt^{1,2}, David Perez-Garcia^{3,4} & Michael M. Wolf⁵

Theorem 1

We can explicitly construct a dimension d , $d^2 \times d^2$ matrices A , B , C and D , and a rational number $\beta > 0$, which can be chosen to be as small as desired, such that

- (i) A is Hermitian, with matrix elements in $\mathbb{Z} + \beta\mathbb{Z} + \frac{\beta}{\sqrt{2}}\mathbb{Z}$;
- (ii) B and C have integer matrix elements; and
- (iii) D is Hermitian, with matrix elements in $[0, 1, \beta]$.

For each positive integer n , define the local interactions of a translationally invariant, nearest-neighbour Hamiltonian $H(n)$ on a 2D square lattice as

$$\begin{aligned} h(n) &= \alpha(n)I \\ h_{\text{row}} &= D \\ h_{\text{col}} &= A + \beta(e^{i\pi\varphi(n)}B + e^{-i\pi\varphi(n)}B^\dagger + e^{i\pi 2^{-l(\alpha)}}C + e^{-i\pi 2^{-l(\alpha)}}C^\dagger) \end{aligned}$$

where $\varphi(n) = n/2^{m-1}$ is the rational number whose binary fraction expansion contains the binary digits of n after the decimal point, $|_\varphi(n)|$ denotes the number of digits in this expansion, $\alpha(n) \leq \beta$ is an algebraic number that is computable from n , I is a projector and the daggers denote Hermitian conjugation. Then

- (i) the local interaction strength is ≤ 1 (that is, $\|h_1(n)\| \cdot \|h_{\text{row}}\| \cdot \|h_{\text{col}}\| \leq 1$);
- (ii) if the universal Turing machine halts on input n , the Hamiltonian $H(n)$ is gapped with $\gamma \geq 1$; and

$$\begin{aligned} h(\varphi)^{(i,j)} &= |0\rangle\langle 0|^{(i)} \otimes (\mathbf{1} - |0\rangle\langle 0|)^{(j)} + h_a^{(i,j)}(\varphi) \otimes \mathbf{1}_d^{(i,j)} \\ &\quad + \mathbf{1}_d^{(i,j)} \otimes h_d^{(i,j)} \end{aligned} \quad (1)$$

The spectrum of the new Hamiltonian H is

$$\text{spec}H = \{0\} \cup [\text{spec}H_a(\varphi) + \text{spec}H_d] \cup S \quad (2)$$

with $S \geq 1$ (see Supplementary Information for details). Recalling that we chose H_d to be gapless, we see immediately from equation (2) that if the ground state energy density of H_a tends to zero from below (so that $\lambda_0(H_a) < 0$), then $H(\varphi)$ is gapless; if H_a has a strictly positive ground state energy density (so that $\lambda_0(H_a)$ diverges to $+\infty$), then it has a spectral gap ≥ 1 , as required (see Fig. 2).

This construction is rather general: by choosing different h_d , we obtain undecidability of any physical property that distinguishes a Hamiltonian from a gapped system with a unique product ground state.

Encoding computation in ground states

To construct the Hamiltonian $H_a(\varphi)$, we encode the halting problem into the local interactions $h_a(\varphi)$ of the Hamiltonian. The halting problem concerns the dynamics of a classical system—a Turing machine. To relate it to the ground state energy density—a strict property of a quantum system—we construct a Hamiltonian whose ground state encodes the entire history of the computation carried out by the Turing

Estructura del curso

Procedimiento de evaluación

Bibliografía