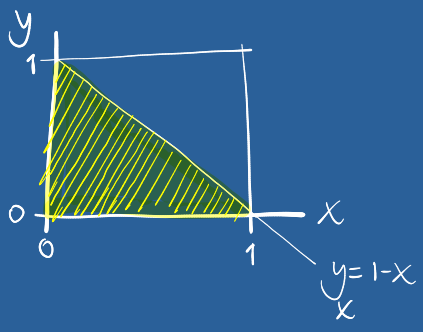
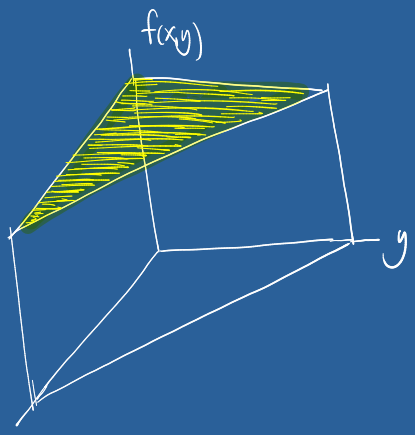
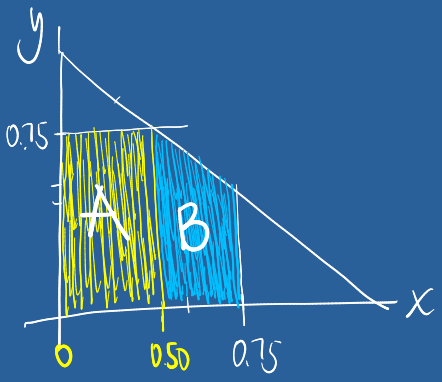
PUNTO 1

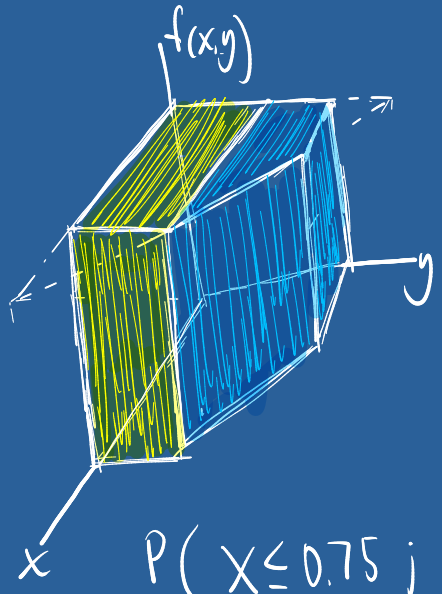
$$f(x,y)=\begin{cases}2, & 0\le x\le 1,\ 0\le y\le 1,\ x+y\le 1\\ 0 & \text{en cualquier otro caso}\end{cases}$$





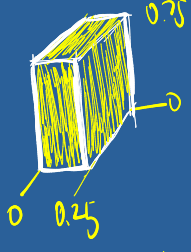
a) $P(X\le 0.75\ ;\ Y\le 0.75)$





$$P(X\le 0.75\ ;\ Y\le 0.75)=V_A+V_B$$

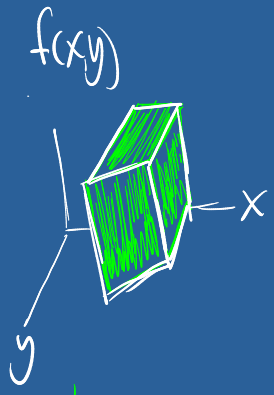
$$\frac{3}{8}+\frac{1}{2}=\frac{7}{8}$$



$$\int_0^{0.25}\int_0^{0.75} 2\,dy\,dx$$

$$=\int_0^{1/4}\left(2y\Big|_0^{3/4}\right)dx$$

$$=\int_0^{1/4}\frac{3}{2}\,dx=\frac{3}{2}x\Big|_0^{1/4}=\frac{3}{8}$$

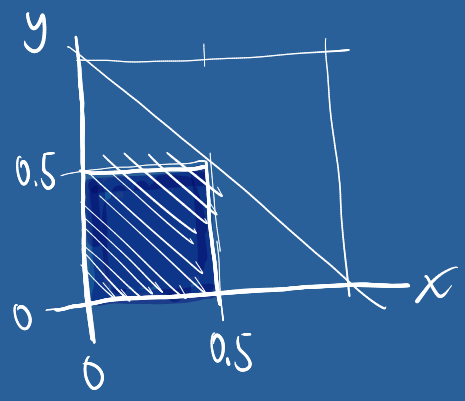


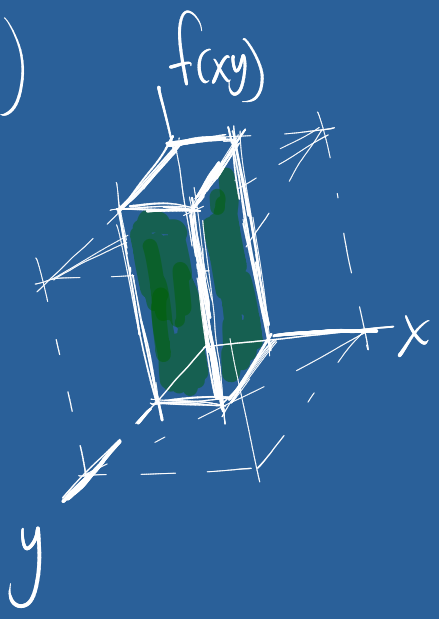
$$\int_{0.25}^{0.75}\int_0^{1-x} 2\,dy\,dx=$$

$$\int_{1/4}^{3/4}\left(2y\Big|_0^{1-x}\right)dx=\int_{1/4}^{3/4}2(1-x)\,dx=$$

$$2\left(x-\frac{x^2}{2}\right)\Big|_{1/4}^{3/4}=1/2$$

b) $P(X \leq 0.5 ; Y \leq 0.50)$

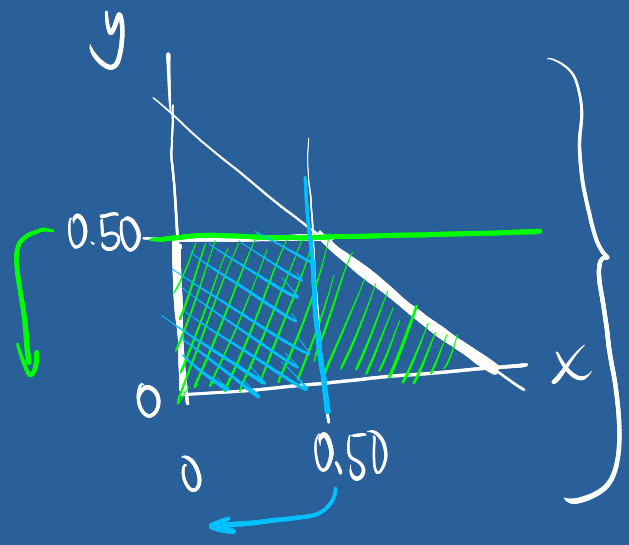




$$\int_0^{1/2}\int_0^{1/2} 2\,dx\,dy = \int_0^{1/2}(2x\Big|_0^{1/2})\,dy = \int_0^{1/2} 1\,dy = y\Big|_0^{1/2} = \frac{1}{2}$$

R/ $100 \times \frac{1}{2} = 50$. Aproximadamente 50 muestras contendrán menos del 50% de cada sustancia

c) $P(X \leq 0.50 \mid Y < 0.50) = \dfrac{P(X < 0.50 \; ; \; Y < 0.50)}{P(Y < 0.50)} = \dfrac{1/2}{3/4} = \dfrac{2}{3}$ ///



$P(X < 0.5 \; ; \; Y < 0.5) = \int_0^{1/2} \int_0^{1/2} 2 \, dx \, dy = \frac{1}{2}$

$h(y) = \int_0^{1-y} 2 \, dx = 2(1-y)$

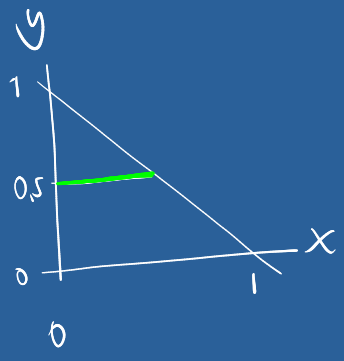
$\int_0^{0.50} 2(1-y) \, dy = 2\left(y - \frac{y^2}{2}\right)\Big|_0^{1/2}$

$= 2y - y^2 \Big|_0^{1/2} = 1 - \frac{1}{4} = \frac{3}{4}$

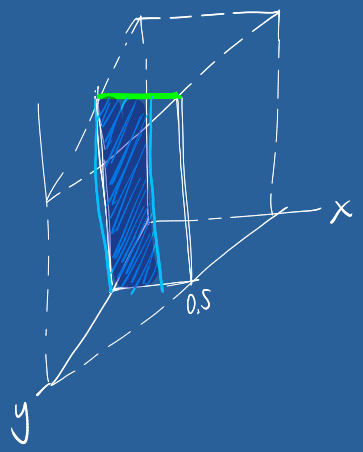
R/ Aproximadamente 67 mezclas de las 100 que contienen menos del 50% de la sustancia 2, tendrán menos del 50% de la sustancia 1.

d) $P(X < 0.40 \mid Y = 0.50)$

$$f_{X|Y=0.50} = \frac{f_{XY}(x, 0.50)}{h(0.50)} = \frac{2}{2(1-0.50)} = 2$$



$$f_{X|Y=0.50} = \begin{cases} 2 & \text{si } 0 \le x \le 1 \\ 0, & \text{en cualquier otro caso} \end{cases}$$

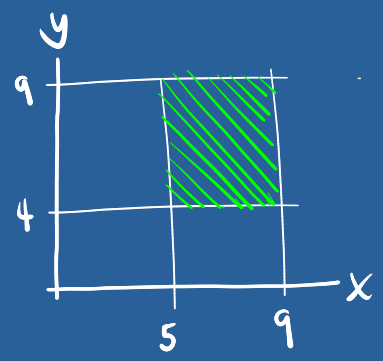


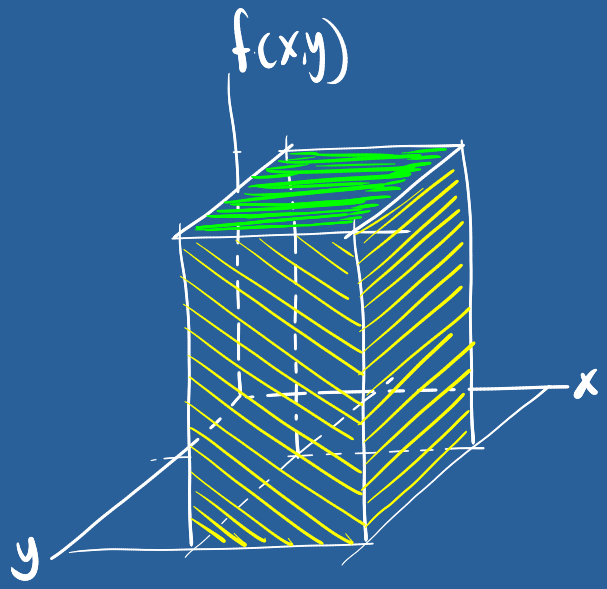
$$P(X < 0.40 \mid Y = 0.50)$$

$$= \int_0^{0.40} 2 \, dx = 0.80 \; //$$

PUNTO 2

$$f(x,y)=\begin{cases} k & 5\le x\le 9 \\ & 4\le y\le 9 \\ 0, & \text{en cualquier otro caso} \end{cases}$$





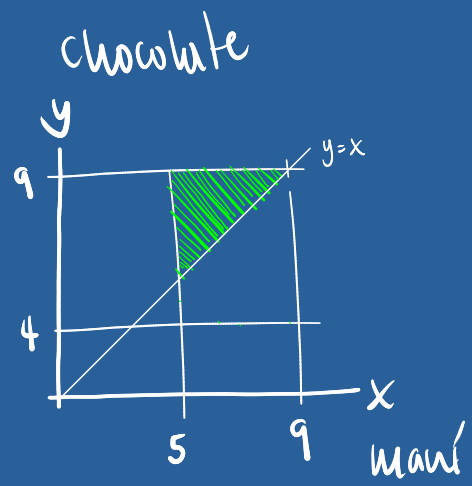
$$\int_4^9 \int_5^9 k\,dx\,dy = 1$$

$$= \int_4^9 \left[kx \Big|_5^9 \right] dy = 4ky \Big|_4^9 = 20k$$

$k = \frac{1}{20}$ //

$$f(x,y)=\begin{cases} 1/20 & 5\le x\le 9 \\ & 4\le y\le 9 \\ 0, & \text{en cualquier otro caso} \end{cases}$$

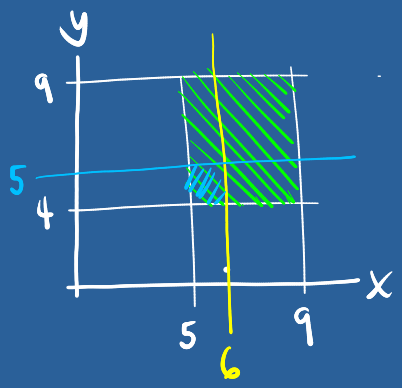
b) $P(X<Y)$



$$\int_5^9 \int_x^9 \frac{1}{20}\, dy\, dx = \int_5^9 (9-x)\, dx$$

$$= \left(\frac{9x}{20} - \frac{x^2}{40}\right)\Big|_5^9 = 0.40$$

c) $$P(Y<5 \mid X<6) = \frac{P(X<6;\ Y<5)}{P(X<6)} = \frac{1/20}{1/4} = \frac{1}{5}$$



$$P(X<6;\ Y<5) = \int_5^6 \int_4^5 \frac{1}{20}\, dy\, dx = \int_5^6 \frac{(5-4)}{20}\, dx = \frac{(6-5)}{20} = \frac{1}{20}$$

$$g(x) = \int_4^9 \frac{1}{20}\, dy = \left(\frac{9-4}{20}\right) = \frac{1}{4}$$

$$P(X<6) = \int_5^6 \frac{1}{4}\, dx = \frac{(6-5)}{4} = \frac{1}{4}$$

R/ De los 100 paquetes que contienen menos de 6 kg de maní aproximadamente 20 paquetes contienen menos de 5 kg de chocolate

d) $P(X > 8 \mid Y = 5)$

$$f_{X|Y}(x|y_0) = \frac{f(x, y_0)}{h(y_0)} = \frac{1/20}{1/50} = \frac{1}{4}$$

$$h(y) = \int_5^9 \frac{1}{20}\, dx = \frac{4}{20} = \frac{1}{5}$$

R/ $200 \times \frac{1}{4} = 50$

De los 200 paquetes que contienen 5 kg de chocolate, aproximadamente 50 paquetes contienen más de 8 kg de maní.