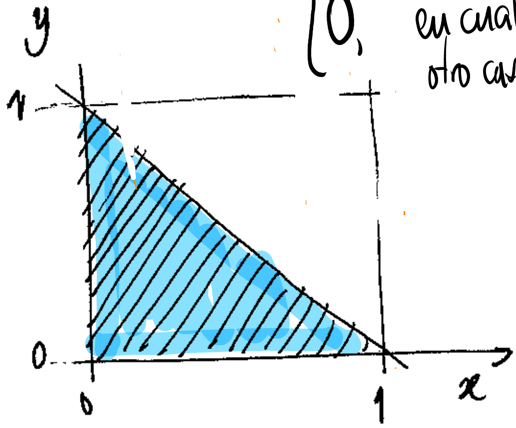
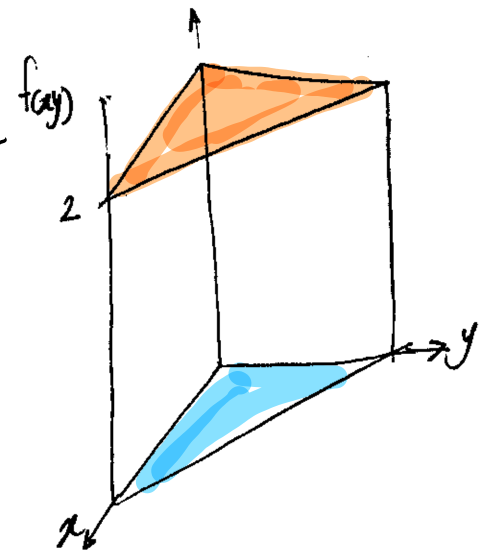
PUNTO 1

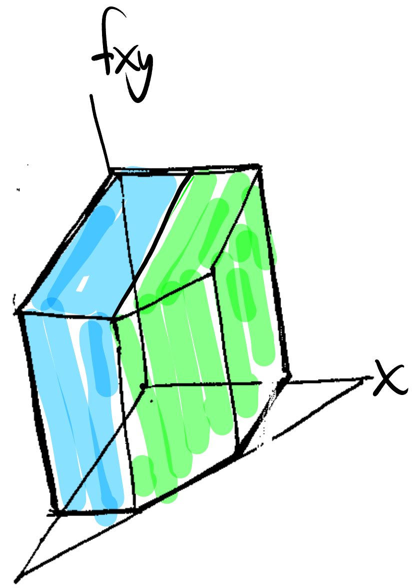
$$f(x,y)=\begin{cases}2, & 0\le x\le 1,\ 0\le y\le 1,\ x+y\le 1\\ 0, & \text{en cualquier otro caso}\end{cases}$$



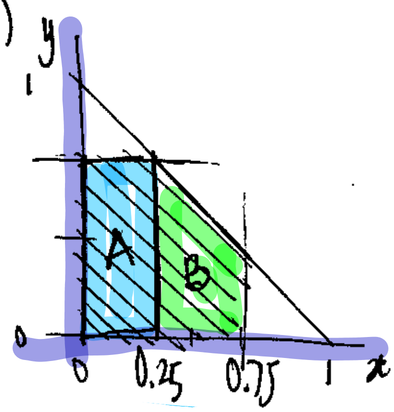


$x+y\le 1$

$y=1-x$



a)



$$P(X<0.75;\ Y<0.75)=\int_0^{0.75}\int_0^{0.25}2\,dx\,dy+\int_{0.25}^{0.75}\int_0^{1-x}2\,dy\,dx=0.375+0.50=0.875$$

$$A=\int_0^{0.75}2x\Big|_0^{0.25}dy$$

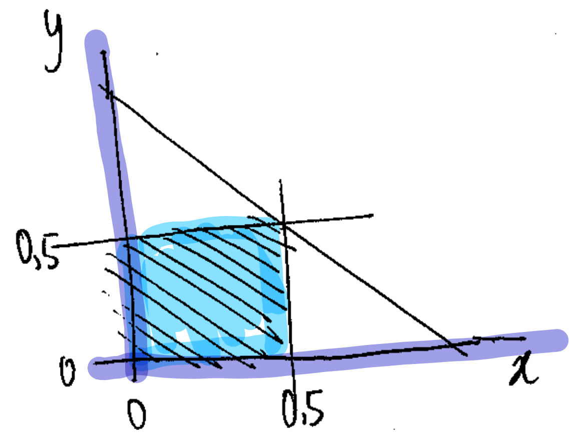
$$\int_0^{0.75}0.5\,dy$$

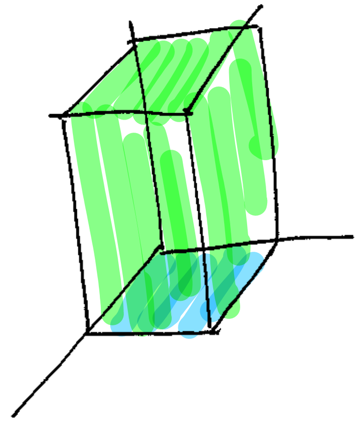
$$0.5\Big|_0^{0.75}=0.375$$

$$B=\int_{0.25}^{0.75}\left(2y\Big|_0^{1-x}\right)dx=\int_{0.25}^{0.75}2(1-x)\,dx=$$

$$2x-x^2\Big|_{0.25}^{0.75}=0.50$$

b)





$$\int_0^{0.5}\int_0^{0.5} 2\,dx\,dy = \int_0^{0.5}\left(2x\Big|_0^{0.5}\right)dy = \int_0^{0.5} 1\,dy = y\Big|_0^{0.5} = 0.50$$

$100 \times 0.50 = 50$ muestras contendran menos de 50% de cada sustancia

c) $$P(X<0.50 \mid Y<0.50) = \frac{P(X<0.50;\ Y<0.50)}{P(Y<0.50)} = \frac{0.50}{0.75} = 0.667$$

se requiere encontrar la distribucion MARGINAL de Y

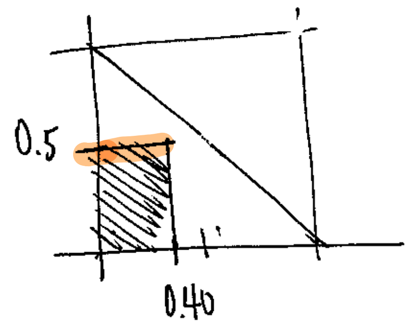
$$h(y) = \int_0^{1-y} 2\,dx = 2x\Big|_0^{1-y} = 2(1-y)$$

$$P(Y<0.50) = \int_0^{0.50} 2(1-y)\,dy = 2y - y^2\Big|_0^{0.5} = 0.75$$

R/ Aproximadamente 67 mezclas de las 100 que contienen menos de un 50% de la sustancia 2, tendran menos del 50% de la sustancia 1.

d) $$P(X<0.40 \mid Y=0.50) = \frac{P(X<0.40;\, Y=0.50)}{P(Y=0.50)}$$

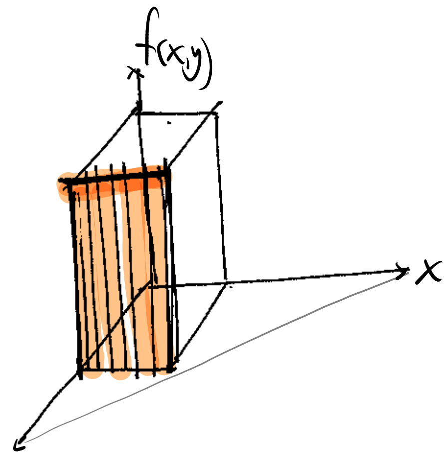
$$P(X<0.40;\, Y=0.50) =$$



$$= f_{X|Y}(x|y_0) = \frac{f_{XY}(x, y_0)}{2(1-y_0)}$$

$$\frac{\cancel{2}}{\cancel{2}(1-y_0)} = \frac{1}{1-y_0}$$

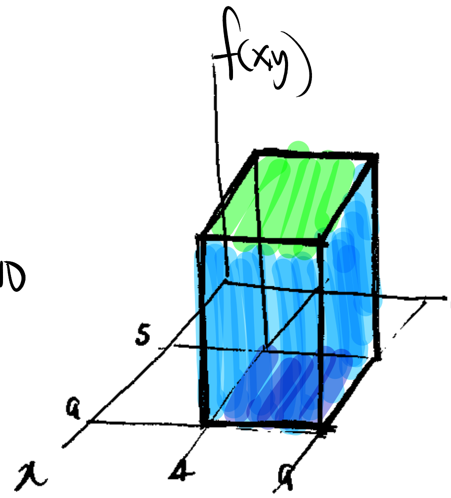
$$= \frac{1}{1-0.5} = 2$$

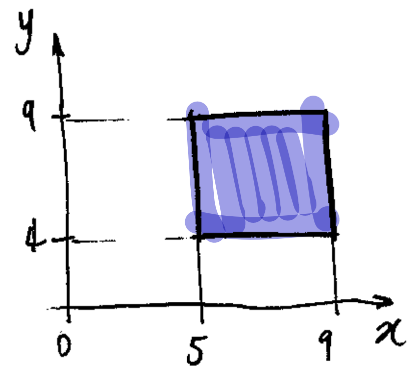


$$P(X<0.40 \mid Y=0.50) = \int_0^{0.40} 2\,dx = 0.80$$

PUNTO 2

$$f_{xy} = \begin{cases} K & 5 \leq x \leq 9 \\ & 4 \leq y \leq 9 \\ 0, \text{ en otro caso} \end{cases}$$





$$\int_4^9 \int_5^9 k \, dx \, dy = \int_4^9 \left(kx \Big|_5^9 \right) dy$$

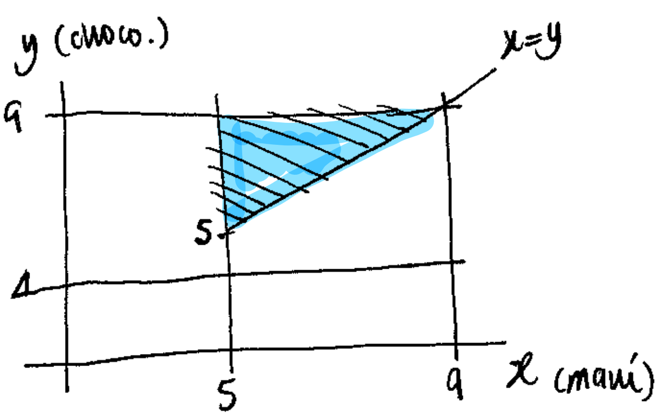
$$\int_4^9 k(9-5) \, dy = 4ky \Big|_4^9 = 4k(9-4)$$

$$20k = 1$$

$$k = \frac{1}{20}$$

$$f_{xy}(x,y) = \begin{cases} 1/20 \;, & 5 < x < 9 \\ & 4 < y < 9 \\ 0, \text{ en otro caso} \end{cases}$$

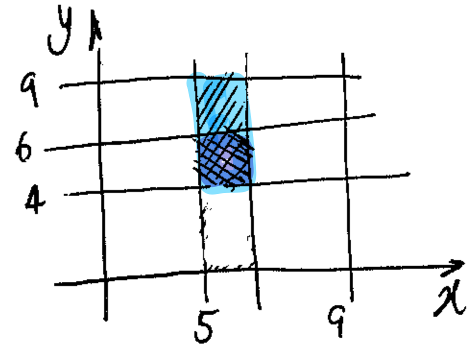
b)



$$\int_5^9 \int_x^9 1/20 \, dy \, dx = \int_5^9 \frac{(9-x)}{20} \, d = \frac{9x}{20} - \frac{x^2}{40} \Big|_5^9 = 0.40$$

R/ El 40% de las veces en que se selecciona un paquete de manera aleatoria contiene menos cantidad de MANI que de CHOCOLATE.

c) $P(Y<5 \mid X<6)$



$$= \frac{P(X<6;\, Y<5)}{P(X<6)} = \frac{1/20}{1/4} = 0.20 \qquad 100 \times 0.20 = 20 \text{ paquetes}$$

$$P(X<6;\, Y<5) = \int_5^6 \int_4^5 \frac{1}{20} \, dy \, dx = \int_5^6 \frac{(5-4)}{20} \, dx = \frac{(6-5)}{20}$$

$$= 1/20$$

R/ de 100 paquetes que contienen menos de 6kg de maní, 20 paquetes contienen menos de 5kg de chocolate.

$$g(x) = \int_4^9 \frac{1}{20} \, dy = \frac{(9-4)}{20} = \frac{1}{4}$$

$$P(X<6) = \int_5^6 \frac{1}{4} \, dx = \frac{(6-5)}{4} = \frac{1}{4}$$

d) $P(X>8 \mid Y=5) =$

$$f_{X|Y}(x \mid y=5) = \frac{f(x, y_0)}{h(y_0)} = \frac{1/20}{1/5} = 0{,}25$$

$$h(y) = \int_5^9 \frac{1}{20}\,dx = \frac{4}{20} = \frac{1}{5}$$ //

$200 \times 0.25 = 50$ //

R/ De los 200 paquetes que contienen 5 kg de CHOCOLATE, 50 paquetes contienen más de 8 kg de MANÍ.