PHYS304 HW3

Intouch Srijumnnong (Indy)
Haverford College

(Dated: February 15, 2023)

1. EXERCISE 3.6: DETERMINISTIC CHAOS AND THE FEIGENBAUM PLOT

In this problem, we are asked to plot the Feigenbaum plot from the logistic map, one of the most famous examples of the phenomenon of chaos, defined by the equation

$$x^{'} = rx(1-x) \tag{1}$$

We are asked to write a program that calculates and displays the behavior of the logistic map for a given values of r from 1 to 4 in steps of 0.01, starting with x = 1/2, and iterate the logistic map equation a thousand times. I start with stating the value of x = 1/2 from the first r, r = 1, then iterate the logistic map 1,000 times in a for loop using x = 1/2 and r = 1 for the calculation in $x^{'} = rx(1-x)$. After I adjust the plotting frame, I repeat the whole calculation for the values of r from 1 to 4 in a while loop with a created list of x values in a loop running from r = 1 to r = 4 using the append function. I also increment r from 1 to 4 in steps of 0.01. Finally, using scatter plot function, the program resulted in the plot shown below (1).

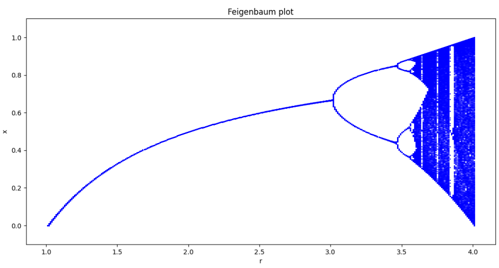


FIG. 1: Feigenbaum plot from the logistic map for a given values of r from 1 to 4 in steps of 0.01 starting with x = 1/2

a) For a given value of r, a fixed point on the Feigenbaum plot is at x = 0 where the value of x does not change, and settles down to a fixed number and stays there. For the period of fixed points, it lies between $1 \leq r \leq 3$. The limit cycle is a period between $3 \leq r \leq 3.6$ where the values of x does not settle down to a single value, and fluctuates between two or more values into a periodic pattern, rotating around a set of values. And, The deterministic chaos is the period between $3.6 \leq r \leq 4$ where it goes crazy and converges to a non-periodic value, generating a seemingly random sequence of numbers that appear to have no rhyme or reason to them at all.

b) Based on the plot, the "edge of chaos" is at around r = 3.6 where the system move from orderly behavior (fixed points or limit cycles) to chaotic behavior

2. EXERCISE 3.8: LEAST-SQUARES FITTING AND THE PHOTOELECTRIC EFFECT

In this problem, we are asked to use the method of least squares which is used to find the straight line that gives the best compromise fit to the data. The straight line can be represented in the familiar form y = mx + c with the values of the slope m and intercept c corresponded to the measured data.

We can calculate the vertical distances between the data points and the line, then calculate the sum of the squares of those distances, which we denote X^2. If we have N data points with coordinates (x_i, y_i), then X^2 is given by

$$X^2 = \sum_{i=1}^{N}(mx_i + c - y_i)^2. \tag{2}$$

The least-squares fit of the straight line to the data is the straight line that minimizes this total squared distance from data to line. We find the minimum by differentiating with respect to both m and c and setting the derivatives to zero, which gives

$$m\sum_{i=1}^{N} x_i^2 + c\sum_{i=1}^{N} x_i - \sum_{i=1}^{N} x_i y_i = 0 \tag{3}$$

Then we can define the following quantities as:

$$E_x = \frac{1}{N}\sum_{i=1}^{N} x_i,\ E_y = \frac{1}{N}\sum_{i=1}^{N} y_i,$$
$$E_{xx} = \frac{1}{N}\sum_{i=1}^{N} x_i^2,\ E_{xy} = \frac{1}{N}\sum_{i=1}^{N} x_i y_i \tag{4}$$

Solving these equations simultaneously for m and c in the equations now we have

$$m = \frac{E_{xy} - E_x E_y}{E_{xx} - E_x^2},\ c = \frac{E_{xx}E_y - E_x E_{xy}}{E_{xx} - E_x^2} \tag{5}$$

a) a) Using the data in millikan.txt with two columns of numbers, giving the x and y coordinates of a set of data points. We are asked to write a program to read these data points and make a graph with one dot for each point, resulted in the graph (2) below (the problem asks to give the x and y coordinates, so I label the axis as x and y not as frequency (Hz) and kinetic energy (J) for the actual experiment).

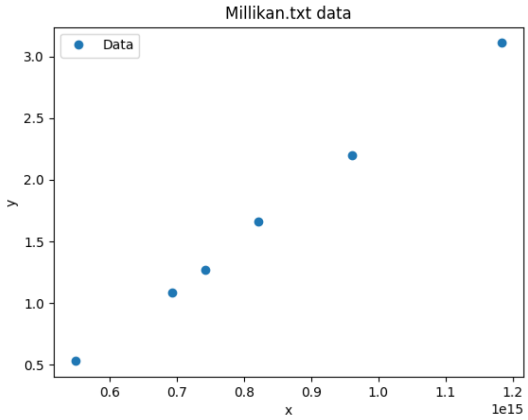


FIG. 2: The graph with the dotted data from millikan.txt

b) We are asked to add code to calculate the quantities E_x, E_y, E_{xx}, and E_{xy} defined above, and from them calculate and print out the slope m and intercept c of the best-fit line, resulted in The slope m of 4.0882273585175l6e-15 and the intercept c of -1.7312358039813558.

c) I write the code using the recursion function that goes through each of the data points in turn and evaluates the quantity $mx_i + c$ using the values of m and c calculated, and then graph as a solid line, on the same plot as the original data, resulted in the graph with a plot of the data points and a fit straight line (3) as shown below.

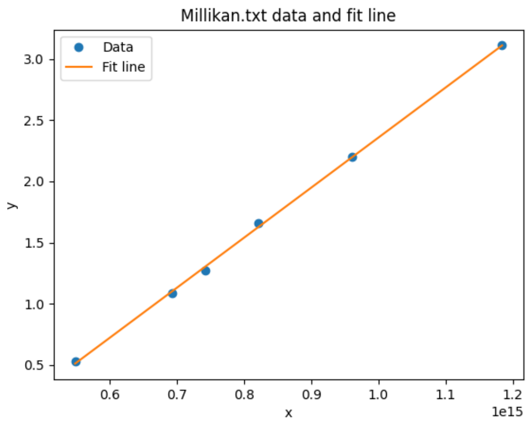


FIG. 3: The graph with the dotted data and the fit straight line from millikan.txt

d) The data in the file millikan.txt are taken from a historic experiment by Robert Millikan. It represents frequencies in hertz (first column) and voltages in volts (second column) from photoelectric measurements. Using the equation,

$$V = \frac{h}{e}v - \phi \tag{6}$$

where h is Planck's constant (Js), ν is the frequency of the light (Hz), V is the energy of an ejected electron (V), ϕ is the work function (J), and the charge on the electron e is 1.602e-19 C, the calculated Millikan's experimental value from the written program for Planck's constant is 6.549340228345061e-34 Js, and the work function is 1.7312358039813558 V. Comparing to the accepted value for Planck's constant of 6.62607015e-34 Js, our calculation is off by only 1.1716 %

Additionally, I add the plot of the photoelectric effect with the corrected experimental axis labels as shown below (4).

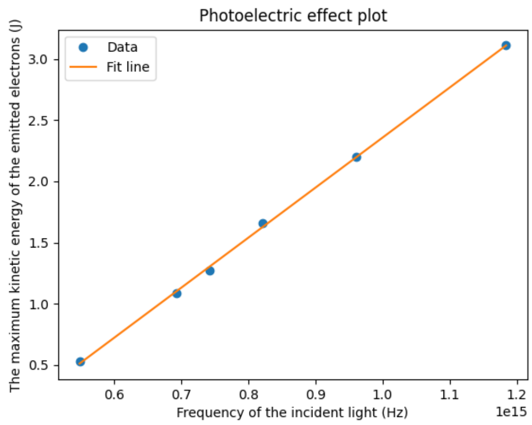


FIG. 4: The graph of the photoelectric effect plot with the dotted data and the fit straight line from millikan.txt

3. FEEDBACK

I spent about 9 hours on this homework including meeting with classmates and attending office hours. I enjoyed the problem set very much even though, I spent a lot of time learning the background information behind the problems, and writing the report in the Latex (many equations and explanations). These are really cool ideas and it is very interesting to see how they can be applied with the materials in the course that we learned so far.

3. 6)

a) → loadtex = data

→ $x = \text{data}[:, 0]$

$y = \text{data}[:, 1]$ # lodnel

→ plot → "o" for dot

b) Add $\Sigma x, \Sigma y, \Sigma xx, \Sigma xy$ → use np.sum

$$m = \frac{\Sigma xy - \Sigma x \Sigma y}{\Sigma xx - \Sigma x^2}$$

$$c = \frac{\Sigma xx \Sigma y - \Sigma x \Sigma xy}{\Sigma xx - \Sigma x^2}$$

c) def f(x):
return mx + c

$y' = f(x)$

→ combined plot a, b

d) $V = \frac{h}{e} \nu - \phi$

$e = 1.602 \times 10^{-19}$ C

$h = ?$ → # m e

$\phi = ?$ → # ν (work function)

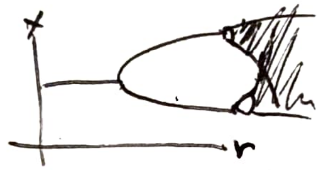
3.6)

$x' = rx(1-x)$

→ $x = \frac{1}{2}$ → iterate 1000 times

→ plot (r, x) "ko"

→ repeat 1 to 4 in steps of 0.01

→ 

a) fixed point? limit cycle? chaos?

b) r order → chaos (edge of chaos)

→ for i in range (N−1) N = 1000

$x' = rx(1-x)$

x_list.append(x)

→ for i in range (1000) logistic map 1000 times

$x' = rx(1-x)$

loop for x

→ repeat r from 1 to 4 in steps of 0.01

while r <= 4

r += 0.01

→ create list of x values

x_list.append(x)

→ plt.scatter

3.6) continue

→

for $x' = r \cdot x \cdot (1-x)$ 1000 loop

plt.figure

⇓

for r 1-4 in steps of 0.01

list of x in 1000 loop

for $x' = r\, x\, (1-x)$

r += 0.01

list of r

plt.scatter

⇓

plt.show