PHYS304 HW6

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1. EXERCISE 8.4A & 8.5A-B: NONLINEAR PENDULUM AND DRIVEN PENDULUM

1.1. Exercise 8.4a

In 8.4, the question asks us to calculate the motion of a nonlinear pendulum based on the example 8.6 previously in the book (p. 349), showing the approximation of the behavior of a pendulum by a linear differential equation.

a) We are asked to write a program to solve the two first-order equations, according to the equations shown in the example 8.6 below,

$$\frac{d\theta}{dt} = \omega \tag{1}$$

and

$$\frac{d\omega}{dt} = -\frac{g}{l} sin\theta \tag{2}$$

Using the fourth-order Runge–Kutta method for a pendulum with a 10 cm arm. For the case of the pendulum when it is released from a standstill at $\theta = 179$ degrees from the vertical, we can calculate the angle θ of displacement for several periods by combining the two variables, θ, and ω into a single vector, r = (θ, ω), then apply the fourth-order Runge–Kutta method in vector form to solve the two equations. We now can solve for θ which we can use to plot the graph of θ as a function of time. The result is shown below in 1

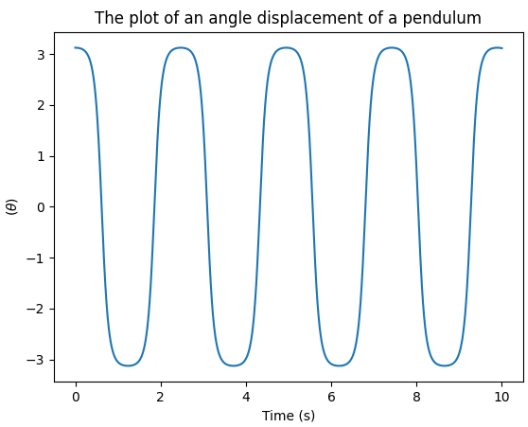

FIG. 1: The nonlinear pendulum plot of θ as a function of time for $l = 10$ cm, and released from a standstill at $\theta = 179$ degrees from the vertical.

1.2. 8.5a-b

A pendulum like the one in Exercise 8.4 can be driven by, for example, exerting a small oscillating force horizontally on the mass. Then the equation of motion for the pendulum becomes,

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l} sin\theta + C cos\theta \Omega t, \tag{3}$$

where C and Ω are constants.

a) Given values of l = 10cm, $C = 2s^{-2}$, and $\Omega = 5s^{-2}$, in the same approach in Exercise 8.4, we used the same function of the fourth-order Runge–Kutta method to solve, but instead of nonlinear pendulum equations, we use the driven pendulum equation for this problem. Then, we can make a plot of of θ as a function of time from t = 0s to t = 100s, with the added condition starting at rest with $\theta = 0$, and $\frac{d\theta}{dt} = 0$, resulting in the plot 2

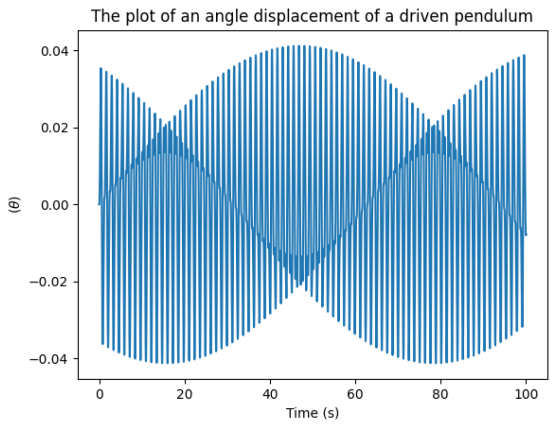

FIG. 2: The driven pendulum plot of θ as a function of time from t = 0s to t = 100s, starting the at rest with with $\theta = 0$, and $\frac{d\theta}{dt} = 0$.

b) As we keeping the value of C, we can vary the value of Ω to find a value for which the pendulum resonates with the driving force and swings widely from side to side. We find that the best value of Ω to represent the resonance is 9.48 s^{-2}, shown in the plot 3

2. EXERCISE 8.3: THE LORENZ EQUATIONS

Chaos theory is the concept describing how small changes in initial conditions in one state can escalate in

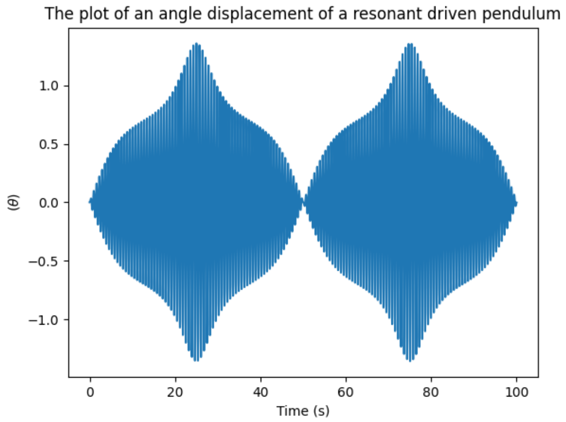

FIG. 3: The resonant driven pendulum plot of θ as a function of time from t = 0s to t = 100s, with Ω = 9.48 s^{-2}.

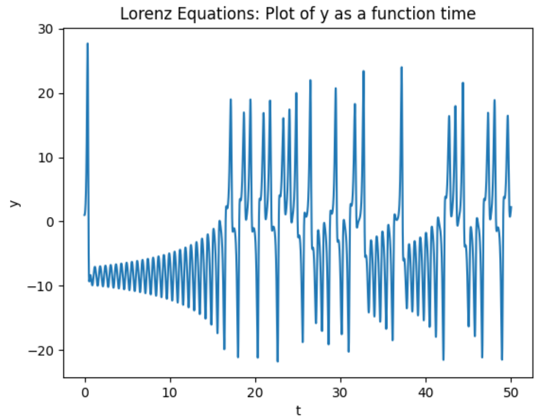

FIG. 4: The plot of Lorenz system for y as a function of time over the period of t = 0 to t = 50 with initial conditions of (x, y, z) = (0, 1, 0).

to large differences in a later state. The theory is studied by Edward Lorenz in 1963 which derived and simplified sets of differential equations from a model of weather patterns to represent the chaotic behavior, also known as the Lorenz system. Lorenz system is one of the first incontrovertible examples of deterministic chaos,the occurrence of apparently random motion even though there is no randomness built into the equations. The Lorenz equations are as follow

$$\frac{dx}{dt} = \sigma(y - x), \tag{4}$$

$$\frac{dy}{dt} = rx - y - xz, \tag{5}$$

$$\frac{dz}{dt} = xy - bz, \tag{6}$$

where σ, r, and b are constants a) For the case of σ = 10, r = 28, and $b = \frac{8}{3}$, in the range from t = 0 to t = 50 with (x, y, z) = (0, 1, 0) as initial conditions, we can solve the the Lorenz system by creating the array of values of x, y, and z. Then we can use the given values to solve the differential equations using the fourth order Runge-Kutta method which are calculated in a loop of the initial conditions. After solving the Lorenz equations, we can make the plot of y as a function of time, shown in 4.

The plot shows the Lorenz system as a function of time. The graph begins with a large swing starting at the fixed point of 0 then quickly jumps to peak and drops down immediately to around -10 on the x axis, then oscillates with a stabilizing and slowly increasing period up until around t = 16. After that point, the graph oscillates into a non-periodic behavior showing the unpredictable nature of the motion of the system.

b) We can modify the program to plot z points against x points which shows us the famous "strange attractor" of the Lorenz equations, a lop-sided butterfly-shaped plot that never repeats itself, resulted in the 5.

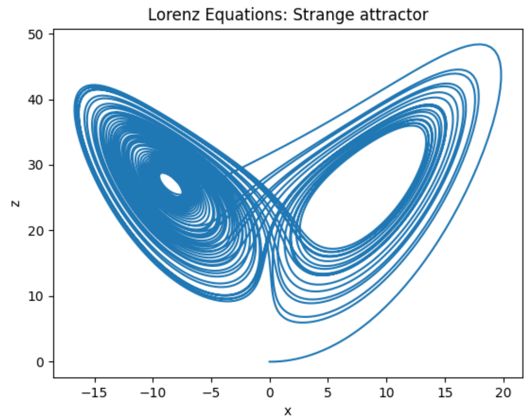

FIG. 5: The plot of Lorenz strange attractor in x-z phase plane over the period of t = 0 to t = 50 with initial conditions of (x, y, z) = (0, 1, 0).

According to the phase plane, no point in the plane is repeated by the same trajectory and no two trajectories intersect which is the chaotic behavior of a strange attractor.

3. FEEDBACK

I spent about 10 hours on this homework including meeting with classmates and attending office hours outside of classes. I enjoy the problems and appreciate the variety of options for us to choose according to our interests. I found the Lorenz attractor very fascinating which ended up being my final project.

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8.4 (a) $\ell = 10$ cm $= 0.1$ m

$\theta = 179° = 3.124$

→ Look at eq 8.45 & 8.46 p. 350

for def f(r,t) for pendulum

→ x = [θ, 0] (radians) for rk 4 loop

tpoints = arange(a, b, h)

xpoints = []

→ Plot (tpoints, xpoints) for θ as t function

8.5 a)

$$\frac{d^2\theta}{dt^2} = -\frac{g}{\ell}\sin\theta + C\cos\theta\sin\Omega t$$

$\ell = 10$ cm $= 0.1$ m

$C = 2.0\ s^{-2}$

$\Omega = 5\ s^{-1}$

→ Change def f(r,t) to rk 4 loop

→ Plot

b) $\Omega = 9.48\ s^{-1}$ Best ✓

→ Plot

8.3) $\frac{dx}{dt} = \sigma(y-x)$, $\frac{dy}{dt} = rx - y - xz$, $\frac{dz}{dt} = xy - bz$

a) $\sigma = 10$, $r = 28$, $b = 8/3$, $t = 0 \to 50$

$(x, y, z) = (0, 1, 0)$

→ def f(r, t)

x = r[0]
y = r[1]
z = [2]

$f_x = \frac{dx}{dt}$
$f_y = \frac{dy}{dt}$
$f_z = \frac{dz}{dt}$

return array x y z

given r = array [0, 1, 0]

loop rk4

→ Plot

→ tpoint, y

→ tpoint, z