PHYS304 HW4

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1. EXERCISE 5.4: THE DIFFRACTION LIMIT OF A TELESCOPE

Light from stars can be treated as coming from a point source at infinity. When such light, with wavelength λ, passes through the circular aperture of a telescope and is focused by the telescope in the focal plane, it generates a circular diffraction pattern consisting of central spot surrounded by a series of concentric rings. The intensity of the light in this diffraction pattern is given by

$$I(r) = (\frac{J_1(kr)}{kr})^2, \quad (1)$$

where r is the distance in the focal plane from the center of the diffraction pattern, $k = 2\pi/\lambda$, and $J_1(x)$ is a Bessel function. The Bessel functions $J_m(x)$ are given by

$$J_m(x) = \frac{1}{\pi}\int_0^{\pi} cos(m\theta - xsin\theta)d\theta, \quad (2)$$

a) In this problem, we are asked to write a function $J(m,x)$ that calculates the value of $J_m(x)$ using Simpson's rule with N = 1000 points, and make a plot, on a single graph, of the Bessel functions J_0, J_1, and J_2 as a function of x from x = 0 to x = 20. I start with defining the function $J(m,x)$ using the equation given in the problem with all the values given including r as $\sqrt{x^2+y^2}$ using Pythagorean theorem. Then using the Simpson's rule according to Newman, 5.1.2 to calculate the integration of the function over the interval of 0 and π. For the plot, I use the linspace function from numpy to return evenly spaced numbers from x = 0 to x = 20, making a single graph, of the Bessel functions J_0, J_1, and J_2 as a function of x from x = 0 to x = 20, resulting in the plot below (1).

b) We are asked to make a density plot of the intensity of the circular diffraction pattern of a point light source with $\lambda = 500$ nm, in a square region of the focal plane. I start by making a grid of values for the density plot using mgrid function from numpy. The mgrid function can make a multi-dimensional "meshgrid", which returns the same dimensions and number of the output arrays as to the number of indexing dimensions. In this case, I use to make a 2 dimensional array of x and y. Using the formula of the intensity of the light in this diffraction pattern with the given values from the problem, the density plot is resulted as shown below (2).

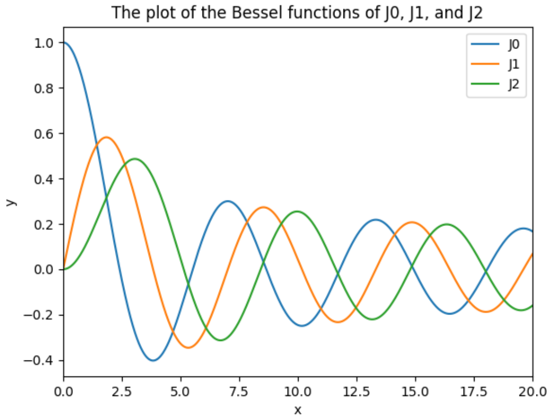


FIG. 1: The plot of the Bessel functions of J0, J1, and J2 as a function of x from x = 0 to x = 20

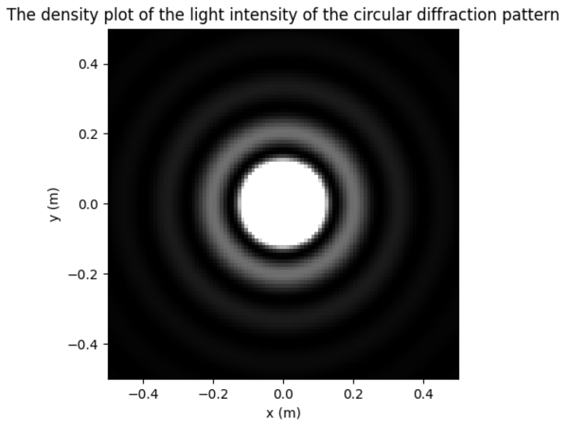


FIG. 2: The density plot of the light intensity of the circular diffraction pattern of a point light source with $\lambda = 500$ nm

2. EXERCISE 5.9: HEAT CAPACITY OF A SOLID

Debye's theory of solids gives the heat capacity of a solid at temperature T to be

$$C_V = 9V\rho k_B(\frac{T}{\theta_D})^3\int_0^{\theta_D/T}\frac{x^4e^x}{(e^x-1)^2}dx, \quad (3)$$

where V is the volume of the solid, ρ is the number density of atoms, kB is Boltzmann's constant, and θ_D is the so-called Debye temperature, a property of solids that depends on their density and speed of sound.

a) In this problem, we are asked to write function $cv(T)$ that calculates C_V for a given value of the temperature, for a sample consisting of 1,000 cubic centimeters of solid aluminum, which has a number density of $\rho = 6.022 \times 10^{28}\ m^{-3}$ and a Debye temperature of $\theta_D = 428$ K. Using, Gaussian quadrature to evaluate the integral, with N = 50 sample points, I call the function gaussxwab from gaussxw.py provided in Newman, 5.6.2. The function gaussxwab is an alternative function of Gaussian integration that calculates the positions and weights and then does the mapping. Then I import gaussxwab to create gaussint function for the calculation of the integral for the function in this problem, calculating C_V with N = 50 sample points for the given values and function.

b) I use the function above to make a graph of the heat capacity as a function of temperature from T = 5 K to T = 500 K by using the loop of T to calculates the integral in the function, and gives values for the plotting, resulted in the graph shown below (3).

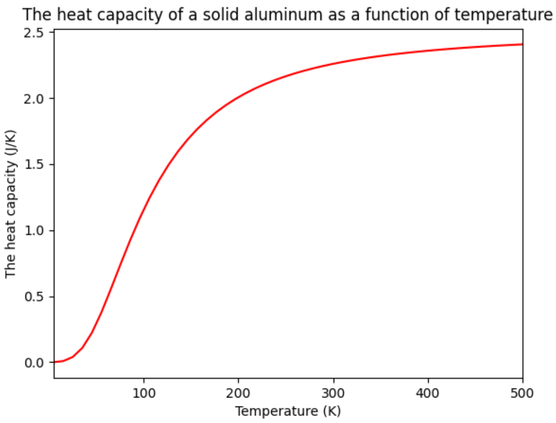


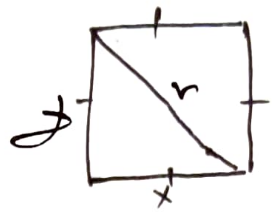
FIG. 3: The graph of heat capacity of a solid aluminum as a function of temperature from T = 5 K to T = 500 K

3. FEEDBACK

I spent about 12 hours on this homework including meeting with classmates and attending office hours outside of classes. I spent most of my time in this week homework trying to learn and use different kinds of integration methods. I spent about 3-4 hours writing the function of Simpson's rule and about 2-3 hours understanding and learn to use Gaussian integration. I think it is helpful for Prof. Dan to give us the option to use any integration methods in the homework this week, not limited to the problems. If I decide to use the integration function that we already learned and written in the class, I could have finished this homework around 8 hours. However, I learned to use Simpson's rule and Gaussian quadrature as asked in the problems out of my own interests which results in more hours spent. In conclusion, I think the homework is appropriate and corresponds well with the class materials at the time. I enjoyed doing this one.

5.4

$$I(r) = \left(\frac{J_1(kr)}{kr}\right)^2$$



$$r = \sqrt{x^2+y^2}$$

$$k = 2\pi/\lambda$$

$$J_m(x) = \frac{1}{\pi}\int_0^{\pi} \cos(m\theta - x\sin\theta)\,d\theta, \quad m, x \geq 0$$

a) $N = 1000$ points, J_0, J_1, J_2, $x = 0 \to 20$

→ Gaussian Quadrature or Simpson

↳ guassian see Newman 5.6 ✓

→ def Jm(x)

↳ f(theta)

return cos(m ...

for N = 1,000

~~#30~~

→ simpson → see Newman p.145 ✓

$s = f(a) + f(b) + \ldots f(a+kh)$

↳ use this for Jm(x)

↳ call in function

return I = h/3 s

for k in (1, N/2

→ Plotting x=0 to x=20

x = linspace(0, 20)

b) λ = 500 nm = 0.5 m

→ need a grid value for density plot

→ . array of x value

→ meshgrid? → see how to use?

↳ use ~~[illegible]~~

↳ np. mgrid [_:_ , _:_]

↳ array([ ])

→ Try mgrid [-1:1:100, -1:1:100]

for 2d square of arrays? need to be "100j"

→ For $I(r) = \left(\frac{J_1(kr)}{kr}\right)^2$

↳ plot . imshow (J, ...)

(5.9)

$C_V = \ldots$

→ θ_D = Debye temp

a) write cv(T)

$V = 1000\ cm^3 = 1\times10^{-6}\ m^3$

$\rho = 6.022\times10^{28}\ m^{-3}$

$\theta_D = 428\ K$

N = 500 sample point

→ Use gaussian quadrature "gaussxw" or "gaussxwab"

↳ creating integral function for Cv

↳ use for positions and weights

see on p.170

→ def f(x) - Cv

↳ return equation for integration

b) → temp = linspace (5, 500)

↳ for T = 5 K to 500 K

→ for T in temp

↳ append to integrate f(x) to the end of the loop

integrate the equation

→ plot (temp, integrated ...)