Phys 304: Assignment 1

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[Survey question: This homework took me 2.5 hours to complete. I learned how to plan my work ahead of time with a flowchart, build equations, then use those equations in a physical context. The problem set had a just right length, considering the high content of information.]

1. PROBLEM 1: A SATELLITE IN ORBIT

A satellite makes a complete rotation around the Earth with period T.

1.1. Part a: Deriving the altitude h

We know that the gravitational force F_G is given by:

$$F_G = \frac{GmM}{r^2}, \quad (1)$$

where the gravitational constant $G = 6.67 * 10^{-11} m^3 kg^{-1} s^{-2}$, m is the mass of the satellite, M is the mass of the Earth, and r is the radius of the satellite's orbit from the center of the Earth. We also know that the centripetal force is given by

$$F_C = \frac{mv^2}{r}, \quad (2)$$

where m is once again the mass of the satellite, v is the velocity of the satellite, and r is again the radius of the satellite's orbit from the center of the Earth.

When the satellite is in orbit, the gravitational force must equal the centripetal force. Otherwise, the satellite will either crash into Earth or fly off into space. Setting $F_G = F_C$, we can plug in the value of v to reduce it further. Plugging in $v = \frac{2\pi r}{T}$, where r is the same as above and T is the period of the orbit, we get:

$$\begin{aligned} F_G &= F_C \\ \frac{GmM}{r^2} &= \frac{mv^2}{r} \\ \frac{GM}{r} &= (\frac{2\pi r}{T})^2 \\ r^3 &= \frac{GMT^2}{4\pi^2} \\ r &= (\frac{GMT^2}{4\pi^2})^{1/3}. \end{aligned} \quad (3)$$

Remembering that r represents the orbit of the satellite from the center of the Earth, it can also be expressed as $r = R + h$, where R is the radius of the Earth and h is the altitude of the satellite. This means we can express Eq. 3 as:

$$h = (\frac{GMT^2}{4\pi^2})^{1/3} - R. \quad (4)$$

1.2. Part b: Code for desired T to h

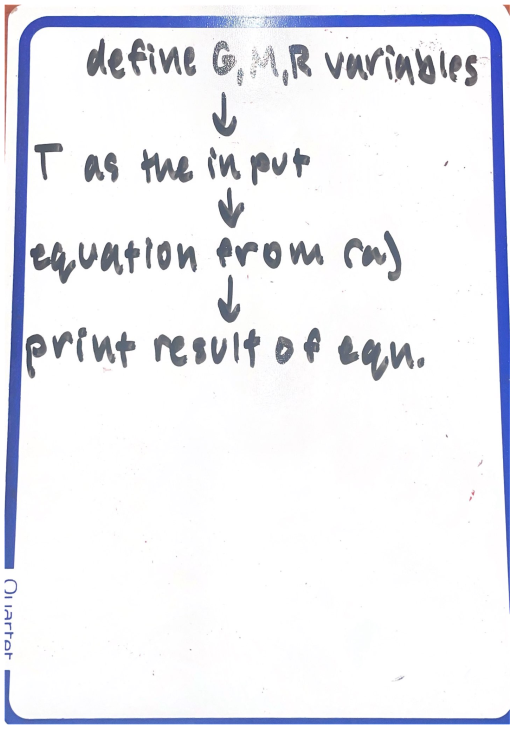

FIG. 1: [Flowchart of the code for Problem 1b. The code prints the altitude of the satellite in meters for a given period in seconds.]

1.3. Part c: Calculating orbits

Using the code created above, a satellite that orbits the Earth once a day does so at an altitude of $h = 35855910.18m$. A satellite that orbits the Earth every 90 minutes does so at an altitude of $h = 279321.63m$.

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Lastly, according to the code I developed, a satellite that orbits the Earth every 45 minutes does so at an altitude of $h = -2181559.9m$. This negative number is likely due to the fact that it is impossible for a satellite to make a complete rotation of the Earth in that small of a period.

1.4. Part d: The sidereal day

A true geosynchronous satellite orbits the Earth once every sidereal day, or 23.93 hours. This is because a sidereal day is the amount of time it takes to make a full rotation about its axis with respect to far away stars, versus with respect to the Sun. Therefore, a satellite rotating with a period of a sidereal day will have a more precisely geosynchronous orbit than if the satellite with a period of a solar day. A satellite with a period of a sidereal day will orbit at $h = 35773762.33m$. This means there is a difference of $h = 82147.85m$ between a sidereal orbit and a solar orbit, which presents quite a large difference.

2. PROBLEM 2: CATALAN NUMBERS

The Catalan numbers C_n can be found using recursion in the following code.

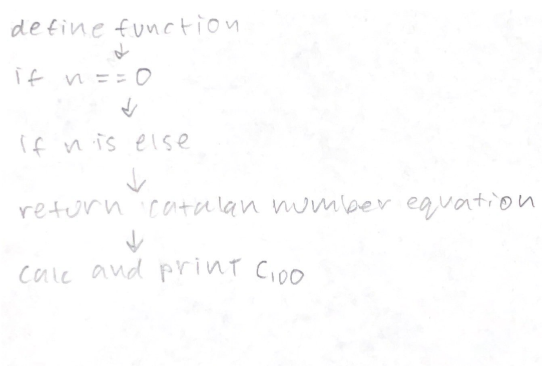

FIG. 2: [Flowchart of the code for Problem 2. The code finds the function for Catalan numbers C_n and prints C_100.]