# Phys 304: Assignment 4


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This problem set took me about 5 hours to complete. I learned a lot of interesting things about the different plots you can make with matplotlib and how to apply those plots. I also learned about when to use empty lists when plotting a function. This problem set was the perfect length. My collaborators for this problem set were Joey Carol and Luke Smithberg.


## 1. PROBLEM 1: DIFFRACTION LIMIT OF A TELESCOPE

The ability for precise measurements to be made with a telescope is limited by the diffraction of light. When observing far away stars, the light can essentially be treated as coming from a point source at infinity. Telescopes take in light with wavelength $\lambda$ through a central aperture, which is then focused in the focal plane. This produces a circular diffraction pattern caused by the bending of light as it comes in through the aperture. The intensity of light is given by

$$I(r) = (\frac{J_1(kr)}{kr})^2 \tag{1}$$

where r is the distance in the focal plane from the center of the diffraction pattern, $k = 2\pi/\lambda$, and $J_1(x)$ is a Bessel function. Bessel functions are given by

$$J_m(x) = \frac{1}{\pi}\int_0^{\pi} \cos m\theta - x\sin\theta d\theta \tag{2}$$

where m is a nonnegative integer and $x \geq$. The pseudocode for this section can be found in Figure 1.

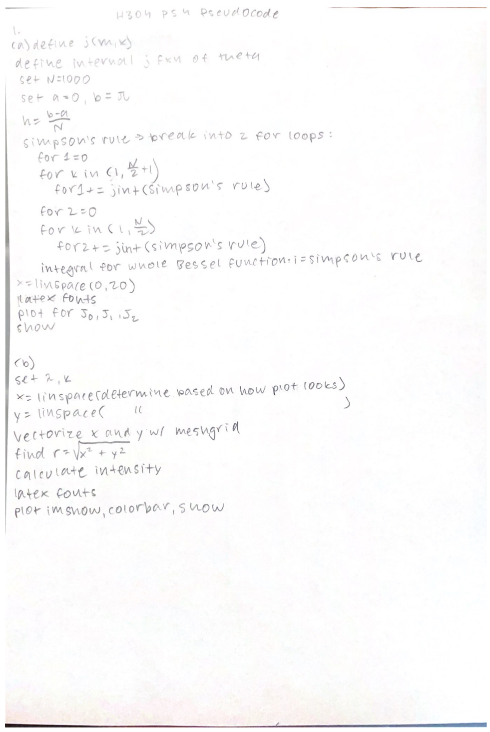


FIG. 1: [The pseudocode for Problem 1, the diffraction limit of a telescope.]

### 1.1. Part (a): Finding the Bessel function

The Bessel Function, shown in Eq. 2 was approximated using Simpson's rule. To do so, I created 2 for loops to sum over the summations given in the rule, then applied those sums back into Simpson's rule. This gives an approximation of an integral, which I applied to the Bessel Function. Then, I plotted the Bessel function for several values of m and plotted those in Figure 2.

### 1.2. Part(b): Intensity of the diffraction

To make a density plot of the intensity of the circular diffraction pattern of a point light source, the process highlighted in Figure 1 was followed. I created arrays for both x and y, then vectorized the x and y arrays. These were then used to calculate the distance from the center of the diffraction pattern $r$, and that was used to create the intensity function. Lastly, I plotted the intensity in Figure 3.

## 2. PROBLEM 2: HEAT CAPACITY OF A SOLID

Debye's theory of solids gives the heat capacity of a solid at temperature T as

$$C_V = 9V\rho k_B(\frac{T}{\theta_D})^3\int_0^{\frac{\theta_D}{T}} \frac{x^4e^x}{(e^x-1)^2}dx, \tag{3}$$

where V is the solid's volume, $\rho$ is the number density


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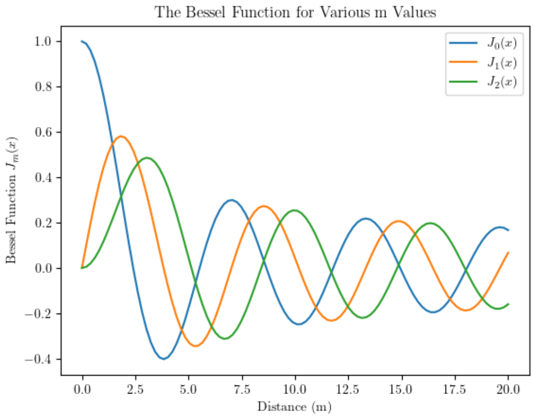



FIG. 2: [The Bessel functions for several values of m, plotted against distance.]

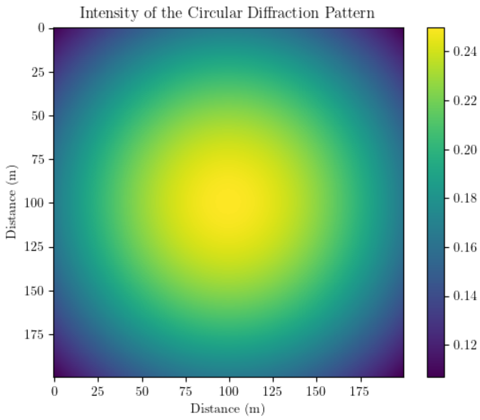
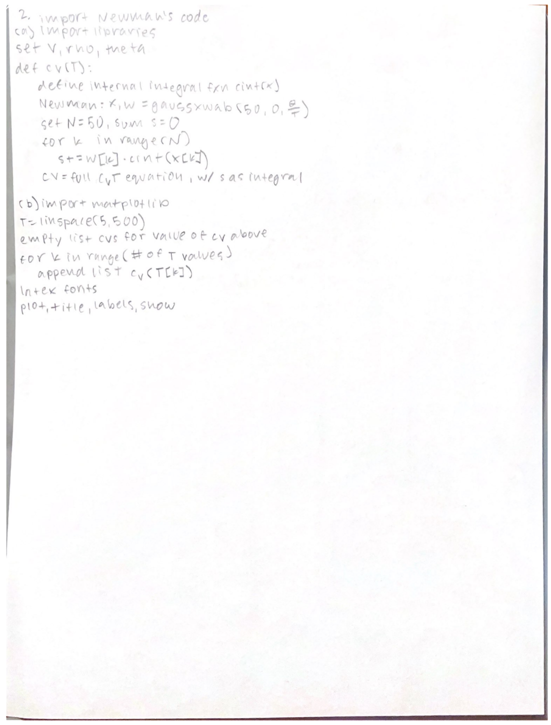



FIG. 3: [The intensity of the circular diffraction pattern of a point light source.]

of atoms, $k_B$ is the Boltzmann constant, and $\theta_D$ is the Debye temperature. This Debye temperature is a property of solids that depends on their density and the speed of sound. The pseudocode for this section can be found in Figure 4.

### 2.1. Part (a): Calculating heat capacity

To create a function for the heat capacity at constant volume $C_V$ for the given conditions provided, I followed the process highlighted in the pseudocode in Figure 4. I defined an internal integral function of x, then used the Guassian quadrature function provided by Mark Newman in the textbook to approximate a solution to that integral. I then plugged the result of this integral into Equation 3. This generates heat capacity values to be used in the next section.



2. import Newman's code
(a) import libraries
set V, rho, theta
def cv(T):
    define internal integral fxn cint(x)
    Newman: x, w = gaussxwab(50, 0, $\frac{\theta}{T}$)
    set N=50, sum s=0
    for k in range(N)
        s+=w[k]·cint(x[k])
    cv = full $C_V$T equation, w/ s as integral

(b) import matplotlib
T=linspace(5, 500)
empty list cvs for value of cv above
for k in range(# of T values)
    append list $c_V$(T[k])
latex fonts
plot, title, labels, show

FIG. 4: [The pseudocode for Problem 2, find the heat capacity of a solid.]

### 2.2. Part (b): Plotting heat capacity

To create a plot of the heat capacity as a function of temperature from $T = 5$K to $T = 500$K, I followed the processes outlined in Figure 4. I created an array of values for T and an empty list for the values of heat capacity. Then I appended this list in the range of the number of T values as the heat capacity at each temperature. From here, a plot of heat capacity values was generated in Figure 5.

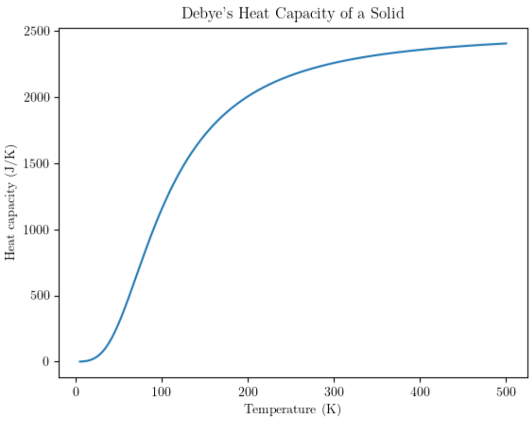



FIG. 5: [The heat capacity of a solid from temperatures 5-500K.]