Phys 304: Homework 6

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[Put abstract here]

1. PROBLEM 1: ASYMMETRIC QUANTUM WELL

Turning the section in late for partial credit - finished Problem 2 however and wanted credit for it.

The pseudocode for this section can be found in Figure 1.

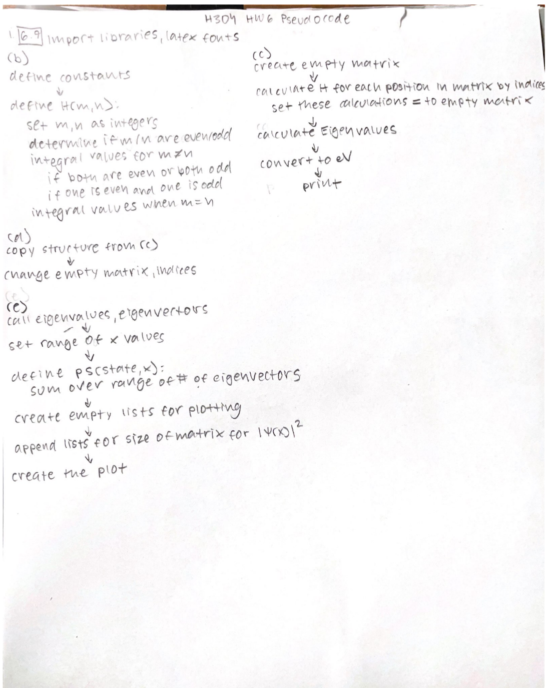

FIG. 1: [The pseudocode for the asymmetric quantum well.]

1.1. Part (a): Schrodinger's equation in matrix form

1.2. Part (b): Expression for H_{mn} for arbitrary m and n

1.3. Part (c): The 10x10 array for H

1.4. Part (d): The 100x100 array for H

1.5. Part (e): Plotting for different states

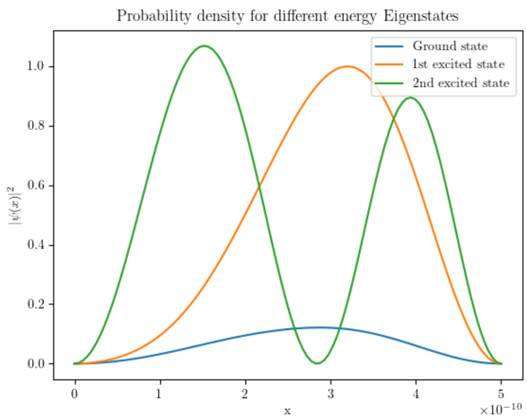

FIG. 2: [The probability density as a function of position for different energy Eigenstates. Note how the number of nodes change as the particle enters higher states. The asymmetric behavior of the graphs are because of the variance of the potential $V(x)$ inside the well.]

2. PROBLEM 2: THE RELAXATION METHOD

Consider the equation

$$x = 1 - e^{-cx}, \tag{1}$$

where c is a known parameter and x is unknown. This form of equation appears in a variety of processes, including the phsycis of contact processes, mathematical models of epidemics, and the theory of random graphs. The pseudocode for this section can be found in Figure 3.

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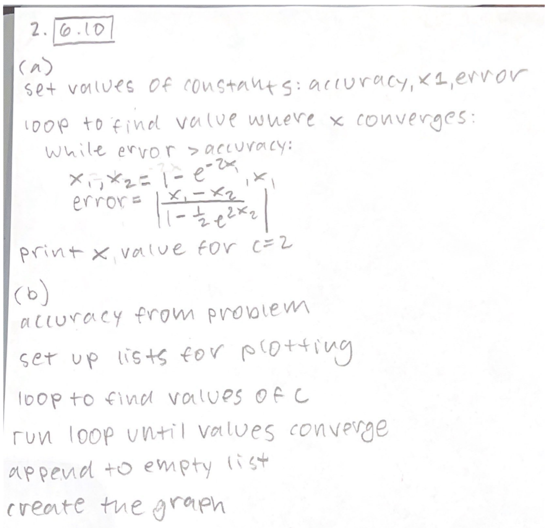

2. $\boxed{6.10}$
(a)
set values of constants: accuracy, x1, error
loop to find value where x converges:
while error > accuracy:
$x_1, x_2 = 1 - e^{-2x}, x_1$
error = $\left|\frac{x_1 - x_2}{1 - \frac{1}{2}e^{2x_2}}\right|$
print x_1 value for c=2
(b)
accuracy from problem
set up lists for plotting
loop to find values of c
run loop until values converge
append to empty list
create the graph

FIG. 3: [The pseudocode for using the relaxation method to solve Equation 1.]

2.1. Part (a): Solving for a specific case

A program was created to solve Equation 1 using the relaxation method for the case where c=2, to an accuracy of at least 10^{-6}. In order to do this, the method outlined in Figure 3 was used. A while loop to find the value where x converges for all values of error¿accuracy. The error was determined using the equation

$$\epsilon' = \frac{x - x'}{1 - 1/f'(x)}. \tag{2}$$

For the value c=2, the error can be determined to be $\epsilon' = \frac{1}{2}e^{2x}$. Using this process for when c=2, x=0.7968195968986895.

2.2. Part (b): Plotting x as a function of c

The program from above was modified to calculate the solution for values of c from 0 to 3 in steps of 0.01, in order to make a plot of x as a function of c. This is an example of a phase transition, known in physics as the percolation transition.

Using the method outlined in Figure 3, the plot in Figure 4 was generated. A loop was used to append an empty list for each value of x and c until the error in Equation 1 reaches the accuracy of 10^{-6}.

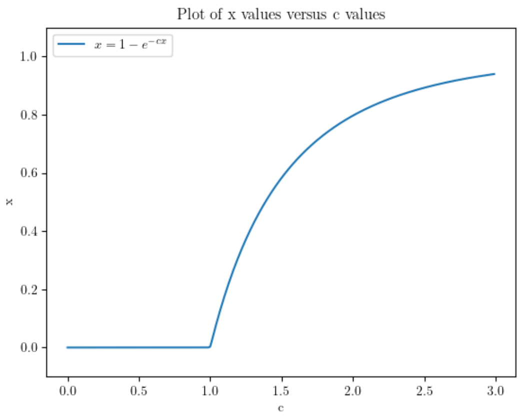

FIG. 4: [The plot of x as a function of c for the Equation 1. There is a clear transition from a regime in which x=0 to a regime of nonzero x.]