HW5 Question 2

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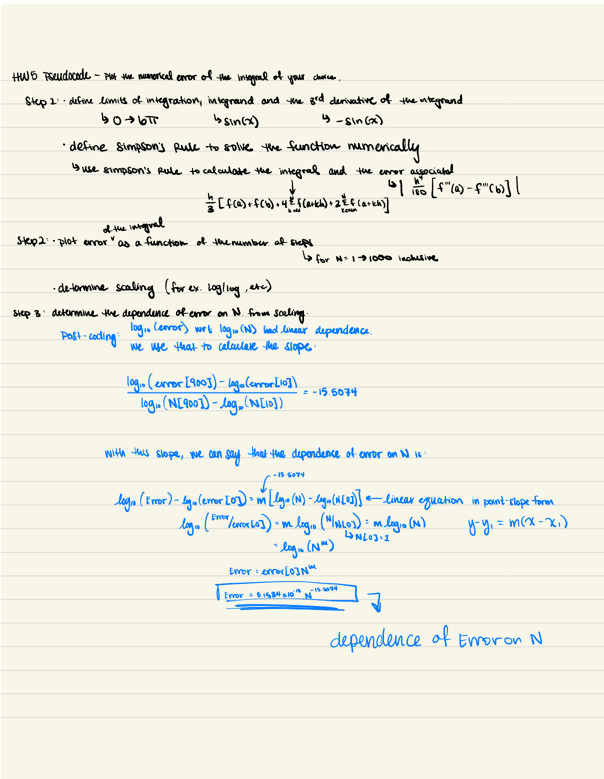
In this assignment, an algorithm was written to calculate the integral of $sin(x)$ using Simpson's method and plot the error as a function of the number of bins used.

1. INTRODUCTION

In this algorithm, we calculate the integral of $\sin(x)$ in order to evaluate how the error of the method of integration changes with the number of bins used to In Simpson's rule, the approximate error is calculated by:

$$\frac{h^4}{180}[f'''(a) - f'''(b)] \tag{1}$$

where h is the step-size calculated from $\frac{b-a}{N}$; a and b the limits of integration and N is the number of bins.



HW5 Pseudocode – Plot the numerical error of the integral of your choice.

Step 1: · define limits of integration, integrand and the 3rd derivative of the integrand

↳ $0 \to 6\pi$ ↳ $\sin(x)$ ↳ $-\sin(x)$

· define Simpson's Rule to solve the function numerically

↳ use Simpson's Rule to calculate the integral and the error associated

$\frac{h}{3}[f(a)+f(b)+4\sum_{k\,odd}^{N} f(a+kh)+2\sum_{k\,even}^{N} f(a+kh)]$ ↳ $\left|\frac{h^4}{180}[f'''(a)-f'''(b)]\right|$

Step 2: · plot error of the integral as a function of the number of steps

↳ for $N = 1 \to 1000$ inclusive

· determine scaling (for ex. log/log, etc)

Step 3: determine the dependence of error on N from scaling.

Post-coding: $\log_{10}$(error) wrt $\log_{10}$(N) had linear dependence. we use that to calculate the slope:

$$\frac{\log_{10}(error[900]) - \log_{10}(error[10])}{\log_{10}(N[900]) - \log_{10}(N[10])} = -15.5074$$

With this slope, we can say that the dependence of error on N is:

$\log_{10}(Error) - \log_{10}(error[0]) = \overset{-15.5074}{m}[\log_{10}(N) - \log_{10}(N[0])]$ ← linear equation in point-slope form

$\log_{10}(Error/error[0]) = m\log_{10}(N/N[0]) = m\log_{10}(N)$ ↳ $N[0]=1$ $y - y_1 = m(x - x_1)$

$= \log_{10}(N^m)$

$Error = error[0]N^m$

$\boxed{Error = 5.1534 \times 10^{-13} N^{-15.5074}}$

dependence of Error on N

FIG. 1: Pseudocode delineating the plan for coding the algorithm that used Simpson's rule to calculate the integral and error of integration ofsin(x). Calculations for determining the dependence of the error on the number of bins is also shown.

The integral is evaluated using:

$$\int_a^b f(x)\,dx = \frac{h}{3}\Bigg[f(a) + f(b) + 4\sum_{\substack{k_{odd}\\1\ldots N-1}} f(a+kh) + 2\sum_{\substack{k_{even}\\1\ldots N-2}} f(a+kh)\Bigg] \tag{2}$$

2. EXPERIMENT

In this algorithm, the Simpson's method was used to calculate the integral of $\sin(x)$ and the error in that calculation based on the number of bins (Fig. 1. Then, the error was plotted as a function of the number of bins to determine the dependence of that relationship.

3. RESULTS

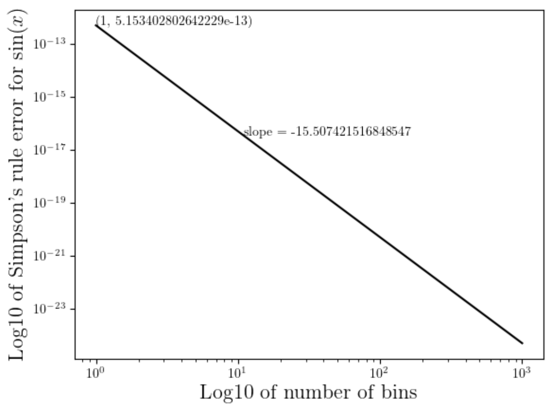


FIG. 2: The log-log dependence of the error of integration of $\sin(x)$ as a function of the number of bins.

Figure 2 shows the linear relationship between the base 10 logarithm of the error against the base 10 logarithm of the number of bins. The equation describing this linear relationship can be written in point-slope form:

$$\log(E) - \log(E_0) = m[\log(N) - \log(N_0)] \tag{3}$$

where m is the slope, E is the error and N is the number of bins used to calculate the integral of $sin(x)$. The slope

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was calculated from values of the error calculated using 10 and 900 bins. The values used for E_0 and N_0 were from the error of the integral when it was calculated using one bin. From this equation, the relationship between the error and the number of bins is found, as shown in Figure 1, to be described by:

$$E = 5.1534 \times 10^{-13} N^{-15.5074} \tag{4}$$

4. CONCLUSIONS

By plotting the error with respect to the number of bins when evaluating $\sin(x)$, we found that the relation had a log-log dependency and could be described by Equation 4.

Appendix A: Comprehension Questions

This assignment took about 5.5 hours. Writing the pseudocode took 1 hour, coding took 4 hours because of some issues debugging the log/log plot, and the write-up took 0.75 hours. I think it took me a reasonable amount of time to do this problem.

It was very useful to me to go through the steps to determine the dependence on N of the error. Although it is something that I have done before in math classes, I don't know why but I was initially struggling to apply it to the problem until I talked it through with Dan. The method is simple, but I needed the refresher.