Navier-Stokes Eq'ns

Conservation of momentum: $\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v} = -\frac{1}{\rho}\nabla P + \nu\nabla^2\vec{v} + \vec{F}$

velocity field; pressure field; external force on fluid / m

ρ: density → incompressibility → constant

Continuity: $\underbrace{\nabla\cdot\vec{v}}_{\text{divergence}} = 0$ → for an infinitesimally small volume (3D) or area (2D) element, all fluid flowing in has to also be flowing out simultaneously.

flow in = flow out

These equations model flow of incompressible fluids. (when density is constant)

In 2 dimensions, the continuity equation becomes these top 2 equations where u and v are the x- and y- components of the velocity field.

The Third equation, sort of unexpectedly actually comes out of the continuity equation. By taking the divergence of both sides of the momentum, we find this equation for pressure that constrains the momentum equations to abide by the continuity requirement. This equation assumes that the divergence of F is zero.

We discretize these equations in order to implement them in our algorithm.

talk about this in PDE slide

The discretized equations are really long and overwhelming so I won't show them but I'll explain how I got to them on the next slide.

So, now that I've explained these equations and where they come from, let me take a second to talk about what they mean for my model.

- Now, blood is compressible → it contains an abundance of different substances that can interact to form complex mechanical processes and form unique local behaviors such as clotting processes. We are ignoring these facts. however it is reasonable to approximate it as incompressible when

·2D means we are looking at an area element.

- The pressure field in the cardiovascular system is driven by a very specific pump, which is the heart. For my model, I decided to start by ignoring the heart and if I had time to add a pulsatile pressure, but did not end up having time.

The external forces described by

- F have physical counterparts in the bloodflow model, such as viscous friction of the blood and arterial resistance and the heartbeat. Arteries are lined with muscle that helps pump blood through them. I could not find a value for the sum of these forces that was anatomically relevant, so I chose an arbitrary value.

Solving partial differential equations

2 strategies for solving PDEs: Boundary problem vs initial value problem.

all finite differences:

- Forward differences for time derivatives
- Backward-differences (1st spatial derivatives) and central differences (2nd spatial derivatives) for spatial derivatives
  - ↳ works best for fully-defined initial conditions which is really impossible to obtain clinical data for, so we just make it up a little bit.

- Relaxation of pressure field at each time-step
  - ↳ helps us get a more realistic pressure field. Even though this is an initial-value problem, this gives the algorithm the character of a boundary-value problem.

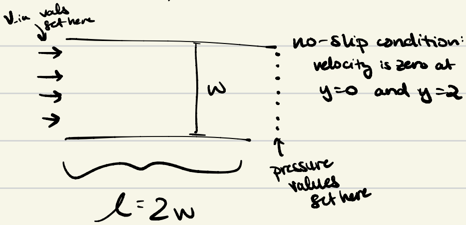
This is how we get our discretized equations that we implement into the algorithm.

in these equations,
i,j indicate a spot on the grid of our artery
and n indicates time.
Δt, Δx, Δy are set in the algorithm.

We are looking at a longitudinal section of an artery to approximate flow.

Draw on the board an artery & label

$u_{in} = \text{constant}$, $v_{in} = 0$



I set a constant pressure going out and a constant velocity field going in.

Because we start w/ laminar flow, this pressure is actually constant through artery @ start $P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \ldots$ (decide to use horz artery for simplicity)