Homework 1

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The altitude of a satellite and the probabilities of electron transmission and reflection across a quantum potential step were calculated using python algorithms.

1. EXERCISE 2.2: ALTITUDE OF A SATELLITE

1.1. Introduction

The altitude of the satellite h is derived using Newton's Second Law of Motion. Because the gravitational force acting on the object must equal the net centripetal force experienced by the object, the altitude can be related to the satellite's speed v:

$$m\frac{v^2}{R+h} = \frac{GMm}{(R+h)^2} \tag{1}$$

Because average speed is $\frac{distance}{time}$, in order to relate the speed to the orbital period we take the distance traveled during a single orbit, which is the circumference of the orbit:

$$v = \frac{2\pi(R+h)}{T} \tag{2}$$

Using the relations in Eq. 1 and Eq. 2, the altitude can be found as a function of the orbital period:

$$h = \left(\frac{GMT^2}{4\pi^2}\right)^{\frac{1}{3}} - R \tag{3}$$

The goal of this exercise is to generate an algorithm based on Eq. 3 that asks for the user to input the desired period of the orbit, converts the input to the required units to perform the calculation and subsequently prints the corresponding satellite altitude.

1.2. Experiment

To write the algorithm, the following pseudocode was used:

- ask for the period of the satellite in minutes using input command.
- convert the input to a float so that arithmetic operations can be performed using the input period.
- convert the period to seconds by multiplying the input number by 60.
- define constants G, M, and R that are used in the satellite altitude calculation.
- plug in constants and period in seconds into the satellite altitude formula to calculate the satellite altitude.
- print the altitude of a satellite with the desired period.

The calculation was performed using arithmetic operations in python.

1.3. Results

The calculated altitudes for different periods are given by Table I.

TABLE I: Resulting altitude of a satellite orbiting Earth from the desired period calculated using a python algorithm

T	24 hours	23.93 hours	90 minutes	45 minutes
h(m)	35855910	35773762	279321	-2181559

As anticipated, the altitude of the satellite will be higher when the period is longer. Additionally, because the altitude of a satellite with a period of 45 minutes is a negative number, it is not possible to have a satellite orbit with this period. This is because the required radius between the satellite and the center of the Earth is smaller than the radius of the Earth.

A geosynchronous satellite orbits the Earth once every sidereal day instead of every full day because of the Earth's rotation. The length of the day depends on both the rotation of the Earth and the distance it travels during that day to change the angle that the sun is hitting it.[1] The sidereal day is the time it takes for the earth to do a full rotation, while the length of the full day is slightly longer than that. For the satellite to be perfectly synchronized with the Earth's rotation, it would need to have a period of a sidereal day instead of a full day. If the satellite had a period of a full day, it would have a period slightly longer than the earth's rotation so it would orbit the earth slower than the earth rotates. The difference between the altitude of the satellite with a period of a day versus a sidereal day is 82,148 meters.

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1.4. Conclusions

Using this algorithm, we found that it is not possible to launch a satellite from the Earth with a period of 45 minutes and that the difference between the altitude of the satellite with a period of a day versus a sidereal day is 82,148 meters.

2. EXERCISE 2.5: QUANTUM POTENTIAL STEP

2.1. Introduction

In a one-dimensional quantum potential step, an electron starts by traveling with energy E and the wavevector k_1:[2]

$$k_1 = \frac{\sqrt{2mE}}{\hbar} \tag{4}$$

As it travels, the potential energy abruptly increases by V. When $V < E$, the electron either gets reflected by the potential step, keeping the same energy and the wavevector, or transmitted through the potential step, losing energy to have new wavevector, k_2:[2]

$$k_2 = \frac{\sqrt{2m(E-V)}}{\hbar} \tag{5}$$

The probabilities that the electron is reflected (R) or transmitted (T) across the potential step can be found as functions of the wavevectors before and after transmission over the potential step, k_1 and k_2:[2]

$$R = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2 \tag{6}$$

$$T = \frac{4k_1k_2}{(k_1 + k_2)^2} \tag{7}$$

2.2. Experiment

In order to perform the calculation, I used Below is the pseudocode used to design the algorithm:

- from math import sqrt
- set m equal to the mass of an electron, energy E to 10eV and the potential V of the potential step to 9eV as desired by the problem and define $\hbar$.
- using sqrt, multiplication and division operations, calculate wavevectors k_1 and k_2.
- using power, multiplication, division, addition and subtraction operations, calculate probabilities of electron reflection and transmission R and T according to Eqs. 6 and 7.
- print R and T.

The algorithm produced by this pseudocode worked effectively and the strategy did not have to be changed.

2.3. Results

The probability of transmission of an electron with 10eV across a 9eV potential step was calculated to be 0.73, and the probability of reflection was calculated to be 0.27.

2.4. Conclusions

An electron with 10eV of energy is nearly three times more likely to be transmitted across a 9eV one-dimensional potential step than it is to be reflected.

[1] C. Crockett, *Sidereal time: What is it?*, https://earthsky.org/astronomy-essentials/what-is-sidereal-time/ (2012).

[2] M. Newman, *Computational Physics* (2013), revised and expanded ed., ISBN 978-1-4801-4551-1.

Appendix A: Answers to Comprehension Questions

Figure 1 shows work done for Exercise 2.2(a), as well as the original pseudocode for Exercise 2.5.

Appendix B: Collaborators

I did not collaborate with anyone for this assignment.

Appendix C: Survey Questions

This assignment took me approximately 3 hours for the write-up and about 1 hour of coding per problem. I thought the homework set was the right length to learn the material and get a feel for how to set up calculations in Python, and start to interact with the text editors.

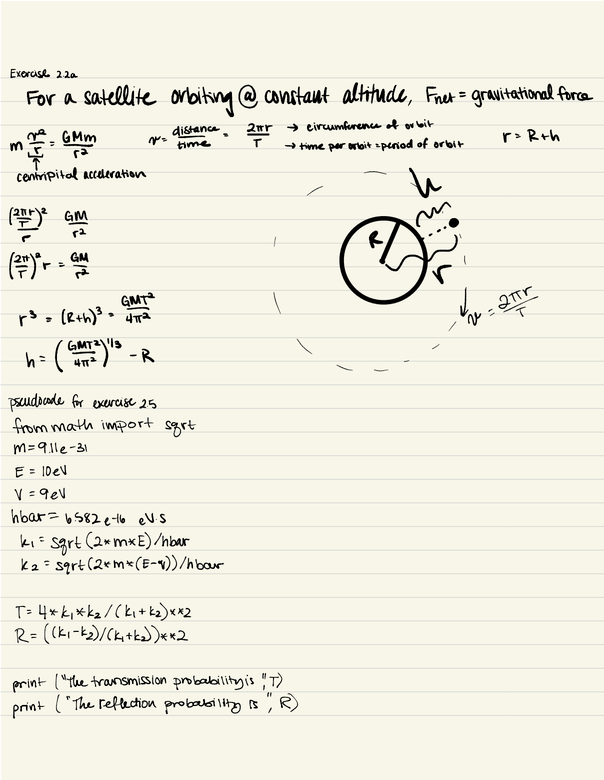

FIG. 1: Work for Exercise 2.2a and Pseudocode for Exercise 2.5

I learned what a sidereal day was, and how to use the math package to perform certain mathematical calculations such as taking the square root. I thought the most interesting problem was Exercise 2.2 because I hadn't thought about the possible time limits on a period and I hadn't known what a sidereal day was.