

# Higher Order Simpson's Rule Integration

- Trapezoidal is first order, Simpson's rule is quadratic, can go to cubic, etc.
- Replace weights with those shown in general form of Simpson's rule
- In a situation with N sample points, a Simpson's rule integration will be exact at the order N-1
- Leads to Gaussian quadrature

$$\int_a^b f(x) dx \simeq \sum_{k=1}^N w_k f(x_k)$$

Degree	Polynomial	Coefficients
1 (trapezoidal rule)	Straight line	$\frac{1}{2}, 1, 1, \dots, 1, \frac{1}{2}$
2 (Simpson's rule)	Quadratic	$\frac{1}{3}, \frac{4}{3}, \frac{2}{3}, \frac{4}{3}, \dots, \frac{4}{3}, \frac{1}{3}$
3	Cubic	$\frac{3}{8}, \frac{9}{8}, \frac{9}{8}, \frac{3}{4}, \frac{9}{8}, \frac{9}{8}, \frac{3}{4}, \dots, \frac{9}{8}, \frac{3}{8}$
4	Quartic	$\frac{14}{45}, \frac{64}{45}, \frac{8}{15}, \frac{64}{45}, \frac{28}{45}, \frac{64}{45}, \frac{8}{15}, \frac{64}{45}, \dots, \frac{64}{45}, \frac{14}{45}$