Romberg integration

A method of integration that uses Richardson

Extrapolation repeatedly on the trapezoidal rule.

- $I_{1} \equiv R_{1,1}$ $I_{2} \equiv R_{2,1} \rightarrow R_{2,2}$ $I_{3} \equiv R_{3,1} \rightarrow R_{3,2} \rightarrow R_{3,3}$ $I_{4} \equiv R_{4,1} \rightarrow R_{4,2} \rightarrow R_{4,3} \rightarrow R_{4,4}$
- Evaluates the integral at equally spaced points
- Works best with integrands that are not highly variable
- Process:
 - 1. Use trapezoidal integration over the first two intervals

$$R_{i,m+1} = R_{i,m} + \frac{1}{4^m - 1} (R_{i,m} - R_{i-1,m}),$$

- 2. Use for $R(2,2) \rightarrow$
- 3. Use trapezoidal rule for R(3,1) and the above for R(3,2) R(3,3)
- 4. Repeat step 3 for each additional level.
- 5. Use to calculate error

$$c_m h_i^{2m} = \frac{1}{4^m - 1} (R_{i,m} - R_{i-1,m}) + O(h_i^{2m+2}),$$