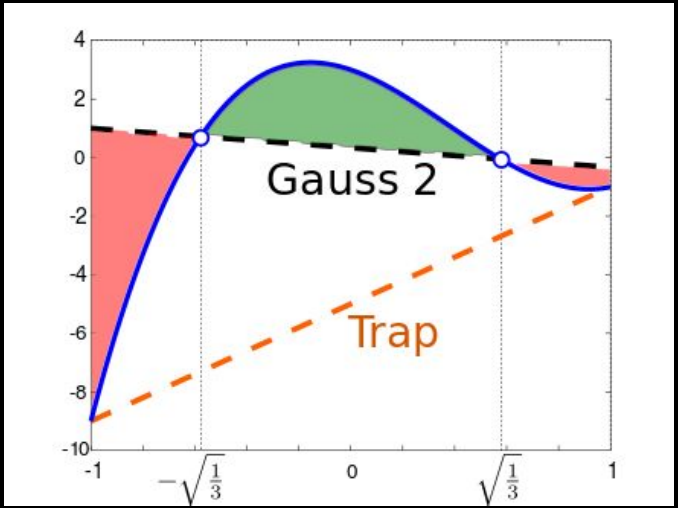
PHYS304 Computational Physics: Gaussian Quadrature Integration

Presented by Petra Mengistu, Seamus Flannery, Aaron Xu, Emma Martignoni, and Indy Srijumnong



What is Gaussian quadrature?

Named after Carl Friedrich Gauss, a German mathematician, the quadrature rule is a method to perform numerical integration over interval and even yield an exact result for polynomials of degree 2n - 1 or less

The Gauss quadrature integration has been implemented in almost every finite element analysis software due to its simplicity and computational efficiency.

Carl Friedrich Gauss

How does Gaussian quadrature work?

The goal is to use a non-uniform, small (N) set of sample points of the original function. From these points, an "interpolating polynomial" is created, order N-1, unique to the set of sample points.

Then, using the interpolating polynomial, the weights used for Simpson's Rule can be adjusted to calculate the integral of our original function with ease.

This is computationally expensive at first, but allows recalculation of different integrands within the same domain very quickly, using the wight and position transformations in eq. 5.61 and 5.62 (Newman 168)

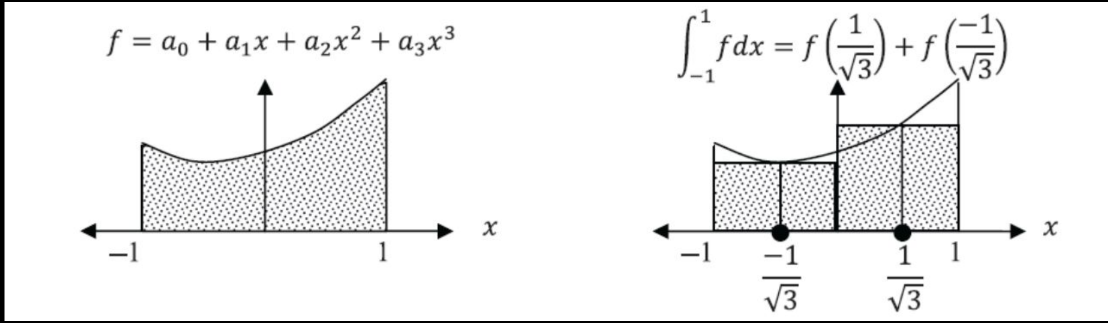


Figure showing Gauss integration for a third degree polynomial

Integration Equations

- To calculate the weights determined Simpson's rule, we use the following formula:

$$w_k = \int_a^b \phi_k(x)\,\mathrm{d}x. \tag{5.59}$$

- To evaluate the integral, we use equation 5.63 below:

$$\int_a^b f(x)\,\mathrm{d}x \simeq \sum_{k=1}^{n} w'_k f(x'_k). \tag{5.63}$$