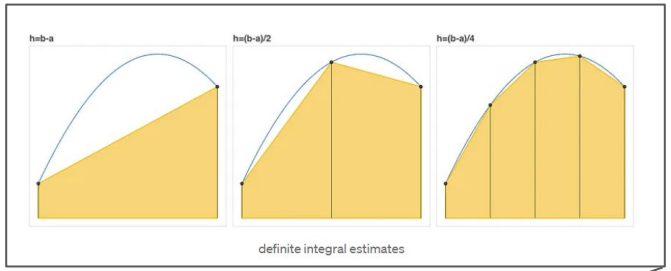
Romberg Integration



(Trapezoidal)

$$I_1 \equiv R_{1,1}$$
$$I_2 \equiv R_{2,1} \rightarrow R_{2,2}$$
$$I_3 \equiv R_{3,1} \rightarrow R_{3,2} \rightarrow R_{3,3}$$
$$I_4 \equiv R_{4,1} \rightarrow R_{4,2} \rightarrow R_{4,3} \rightarrow R_{4,4}$$

Higher Order Estimates

The leading order error between two step sizes for the trapezoidal rule is ⅓ (I_i-I_{i-1}).

We can improve our guess $R_{i,1}$=I_i by adding this error:

$R_{i,2}$ = I_i+⅓ (I_i-I_{i-1}) = $R_{i,1}$+ ⅓($R_{i,1}$- $R_{i-1,1}$).

We can continue to add higher order error terms, and we end up with…

$$R_{i,m+1} = R_{i,m} + \frac{1}{4^m - 1}(R_{i,m} - R_{i-1,m}), \qquad (5.51)$$

1. Calculate first two estimates using the trapezoidal rule $R_{1,1}$ and $R_{2,1}$
2. Calculate more accurate estimate $R_{2,2}$ using equation 5.51
3. Calculate next trapezoidal rule estimate $R_{3,1}$. With $R_{3,1}$ and equation 5.51, we can find $R_{3,2}$ and $R_{3,3}$.