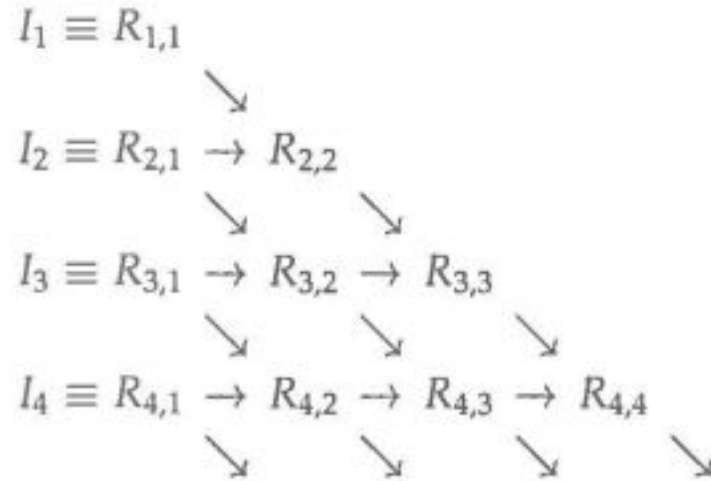


Romberg Integration

Rachel McQueen

Applying the trapezoidal rule successively to make the sum more refined and accurate.



I_i is each use of the trapezoidal rule for an estimate, and each R_{ij} is the refined (and even further more refined) version of that estimate

1. Make two initial estimates using the trapezoidal rule (R_{11} and R_{21})
2. Calculate a more accurate version (R_{22}) using the equation:

$$R_{i,m+1} = R_{i,m} + \frac{1}{4^m - 1} (R_{i,m} - R_{i-1,m}),$$

3. Use the trapezoidal rule again (R_{31}), and then use the above equation to calculate the next level (R_{32} and R_{33})
4. Repeat these steps for the i th term
5. BONUS: Calculate the error to

$$c_m h_i^{2m} = \frac{1}{4^m - 1} (R_{i,m} - R_{i-1,m}) + O(h_i^{2m+2}),$$