Romberg Integration

$$R_{i,1} = I_i, \qquad R_{i,2} = I_i + \tfrac{1}{3}(I_i - I_{i-1}) = R_{i,1} + \tfrac{1}{3}(R_{i,1} - R_{i-1,1}). \tag{5.42}$$

$$R_{i,m+1} = R_{i,m} + \frac{1}{4^m - 1}(R_{i,m} - R_{i-1,m}),$$

$$\begin{array}{ccccccc}
I_1 \equiv R_{1,1} & & & & & & \\
 & \searrow & & & & & \\
I_2 \equiv R_{2,1} & \rightarrow & R_{2,2} & & & & \\
 & \searrow & & \searrow & & & \\
I_3 \equiv R_{3,1} & \rightarrow & R_{3,2} & \rightarrow & R_{3,3} & & \\
 & \searrow & & \searrow & & \searrow & \\
I_4 \equiv R_{4,1} & \rightarrow & R_{4,2} & \rightarrow & R_{4,3} & \rightarrow & R_{4,4} \\
 & \searrow & & \searrow & & \searrow & & \searrow
\end{array}$$

Limitations:

- May not work efficiently for functions that have noise or wild fluctuations