Romberg Integration

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Romberg integration is a method that improves upon the trapezoidal rule with accuracy by iteratively refining estimates of integrals through successive calculations.It is best applied to smooth functions whose form can be determined accurately from only a small number of equally sample points.

Steps:

1. Use **initial estimates** of integral with the **trapezoidal rule.**
2. **Refine** using equation 5.51
3. **Iterate** the process as shown in the diagram for I3, I4 and so on
4. **Successively calculate** one more trapezoidal rule estimate
5. **Calculate the error** for each estimate using Eq. (5.49). Which will stop the calculation once the error on the estimate of the integral meets the desired target.

$$R_{i,m+1} = R_{i,m} + \frac{1}{4^m - 1}(R_{i,m} - R_{i-1,m}), \qquad (5.51)$$

$$\begin{array}{lllllll}
I_1 \equiv R_{1,1} & & & & & & \\
 & \searrow & & & & & \\
I_2 \equiv R_{2,1} & \rightarrow & R_{2,2} & & & & \\
 & \searrow & & \searrow & & & \\
I_3 \equiv R_{3,1} & \rightarrow & R_{3,2} & \rightarrow & R_{3,3} & & \\
 & \searrow & & \searrow & & \searrow & \\
I_4 \equiv R_{4,1} & \rightarrow & R_{4,2} & \rightarrow & R_{4,3} & \rightarrow & R_{4,4} \\
 & \searrow & & \searrow & & \searrow & & \searrow
\end{array}$$

$$c_m h_i^{2m} = \frac{1}{4^m - 1}(R_{i,m} - R_{i-1,m}) + \mathrm{O}(h_i^{2m+2}), \qquad (5.49)$$