non uniform sample points

$$\int_a^b f(x)\,\mathrm{d}x \simeq \sum_{k=1}^{N} w_k f(x_k),$$

method of interloping polynomials

- interpolating polynomial to find the weights, ω_k, and then integrate functions using the equation above
- do not have to recalculate weights for new domains can rescale weights

$$w_k = \int_a^b \phi_k(x)\,\mathrm{d}x. \longrightarrow w'_k = \tfrac{1}{2}(b-a)w_k.$$

errors

- Converge extremely quickly on true value of integral
- Function must be reasonably smooth
- No direct equivalent for $\epsilon_2 = ch_2^2 = \frac{1}{3}(I_2 - I_1)$. in practice
- I2N such better estimate than In, so we can treat it as the true value

$$\epsilon_N \simeq I_{2N} - I_N.$$

sample points for gaussian quadrature

- Need points that match zeros of Nth Legendre polynomial (no proof given)
- Exterior resources often need to find these
  - Gaussxw.py → gaussxw(N) returns arrays x and w
  - These return points in standard interval -1 to 1 → need to map to our domain
- These points have the below weights:

$$w_k = \left[\frac{2}{(1-x^2)}\left(\frac{\mathrm{d}P_N}{\mathrm{d}x}\right)^{-2}\right]_{x=x_k},$$