Romberg Integration

Rachel McQueen

Applying the trapezoidal rule successively to make the sum more refined and accurate.

$$\begin{array}{ccccccccc}
I_1 \equiv R_{1,1} & & & & & & & & \\
 & \searrow & & & & & & & \\
I_2 \equiv R_{2,1} & \rightarrow & R_{2,2} & & & & & & \\
 & \searrow & & \searrow & & & & & \\
I_3 \equiv R_{3,1} & \rightarrow & R_{3,2} & \rightarrow & R_{3,3} & & & & \\
 & \searrow & & \searrow & & \searrow & & & \\
I_4 \equiv R_{4,1} & \rightarrow & R_{4,2} & \rightarrow & R_{4,3} & \rightarrow & R_{4,4} & & \\
 & \searrow & & \searrow & & \searrow & & \searrow &
\end{array}$$

I_i is each use of the trapezoidal rule for an estimate, and each R_{ij} is the refined (and even further more refined) version of that estimate

1. Make two initial estimates using the trapezoidal rule (R_{11} and R_{21})
2. Calculate a more accurate version (R_{22}) using the equation:

$$R_{i,m+1} = R_{i,m} + \frac{1}{4^m - 1}(R_{i,m} - R_{i-1,m}),$$

3. Use the trapezoidal rule again (R_{31}), and then use the above equation to calculate the next level (R_{32} and R_{33})
4. Repeat these steps for the *i*th term
5. BONUS: Calculate the error to

$$c_m h_i^{2m} = \frac{1}{4^m - 1}(R_{i,m} - R_{i-1,m}) + O(h_i^{2m+2}),$$