

Integrals over Infinite Ranges

PHYS304 Slides

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Romberg Integration

① $I_1 = R_{1,1}$ and $I_2 = R_{2,1}$ (trapezoid rule).

② $R_{i,m+1} = R_{i,m} + \frac{1}{4^m-1}(R_{i,m} - R_{i-1,m})$

The first index i is the i^{th} step in the adaptive method (Sec 5.3).
The second index m tells us that the integral is accurate to order h^{2m-1} and has an error of order h^{2m} . (i.e. i specifies the level of subdivision, while higher value of m indicates higher-order extrapolations to improve accuracy)

We can then calculate $R_{2,2} \rightarrow R_{3,1} \rightarrow R_{3,2} \rightarrow R_{3,3} \dots$

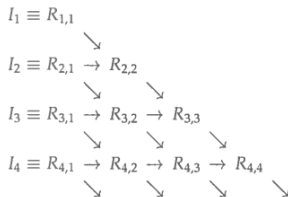


Figure: Romberg Integration

ERROR TERM: $c_m h_i^{2m} = \frac{1}{4^m-1}(R_{i,m} - R_{i-1,m}) + O(h_i^{2m+2})$