Week 6 Newman 5.4 → 5.7

Romberg Integration

Trapezoidal rule: $c\, h_i^2 = \frac{1}{3}(I_i - I_{i-1})$ → error calc.

some constant ↑ i step w/ stepsize h

↳ so $I = I_i + \frac{1}{3}(I_i - I_{i-1}) + \underbrace{O(h_i^4)}_{\text{next term in series}}$

$I_i \equiv R_{i,1}$ → $R_{n,m}$ matrix

↑ where I_2 is trapezoidal w/ $\frac{1}{2}$ step size of I_1

$$R_{i,m+1} = R_{i,m} + \frac{1}{4^m - 1}\left(R_{i,m} - R_{i-1,m}\right)$$

↑ next term in matrix

$$\begin{array}{cccc}
R_{1,1} & & & \\
 & \searrow & & \\
R_{2,1} & \rightarrow R_{2,2} & & \\
 & \searrow & \searrow & \\
R_{3,1} & \rightarrow R_{3,2} & \rightarrow R_{3,3} & \\
 & \searrow & \searrow & \searrow R_{n,n}
\end{array}$$

- Decrease # of steps needed by using error to estimate integral (much faster)

Gaussian Quadrature

Nonuniform points: (w/ N-1 degrees of freedom)

(5.53) $\phi_k(x) = \prod \frac{(x - x_m)}{(x_k - x_m)}$ w/ weights (w_k)

$$\int_a^b f(x)\,dx \simeq \int_a^b \phi(x)\,dx = \sum_{k=1}^{N} f(x_k) \underbrace{\int_a^b \phi_k(x)\,dx}$$

with weights $w_k = \int_a^b \phi_k(x)\,dx$

☆ Associated weights w/ N (# of steps)

"weighted sum of function values @ quadrature points" ↳ Physics Forum

faster at estimating integral!

☆ fits 1 polynomials to values in $f(x_k)$