Homework 7 Write-Up

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1. EXERCISE 8.2

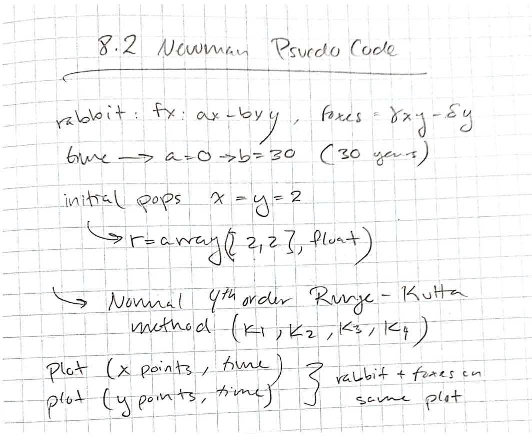

FIG. 1: Pseudo code for Exercise 8.2 Newman.

In this exercise, we plotted the changes in population of rabbits and foxes from a system of first order differential equations, described as the Lotka-Volterra predator-and-prey model. The rate of change in population for rabbits is given by Equation 1 and rate of change in population for foxes is given by Equation 2. Plugging these equations into the Runge-Kutta method code given by Newman, with a time interval of 30 years, and initial populations of 2 (thousand) for both species, the population over time can be seen in Figure 2.

$$\frac{dx}{dt} = \alpha x - \beta xy \tag{1}$$

$$\frac{dy}{dt} = \gamma xy - \delta y \tag{2}$$

In Figure 2, it can be seen how the predator and prey populations oscillate. This is because when the rabbit population is high, then the fox population will rapidly increase since there is abundant food. Until, there are too many foxes and the number of rabbits begins to decrease because they are being eaten faster than they can reproduce. Almost immediately, the fox population drops from lack of food allowing the bunnies to flourish again. Then the cycle repeats itself.

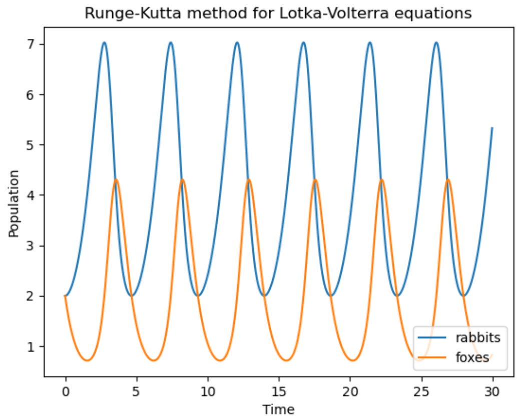

FIG. 2: Population of rabbits and foxes over time.

2. EXERCISE 8.4A

In this exercise, we use the Runge-Kutta method once again to calculate theta of a non-linear pendulum over time. While this is technically a second derivative problem, it can easily be turned into a system of first order differential equation by splitting it into Equation 3 and Equation 4.

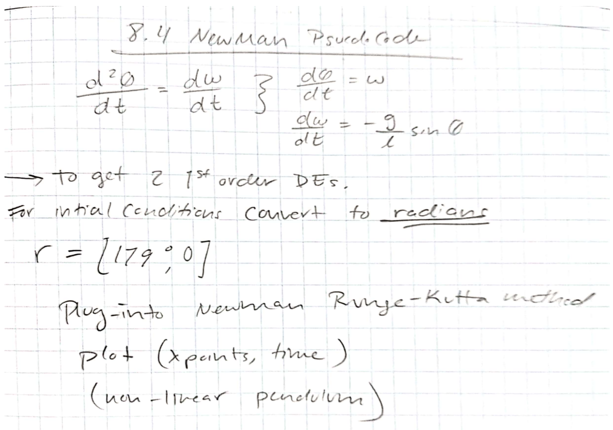

FIG. 3: Pseudo code for Exercise 8.4a Newman.

$$\frac{d\theta}{dt} = \omega \tag{3}$$

$$\frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = -\frac{g}{l} sin\theta \tag{4}$$

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Using the same method as in Exercise 8.2, I used 4th order Runge-Kutta method to get theta and time for the system. I then plotted theta over time, when the pendulum is released from a standstill at 179°, resulting in Figure 4. As you can see, it is not a perfect sine wave because it is a non linear pendulum.

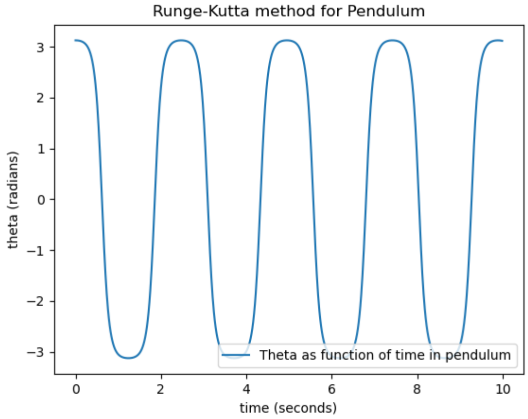

FIG. 4: Theta of swinging pendulum over time.

3. EXERCISE 8.5

This exercise is similar to 8.4, however, this is driven pendulum described in Equations 5 and 6 with constants C and Ω where Ω is the driving frequency.

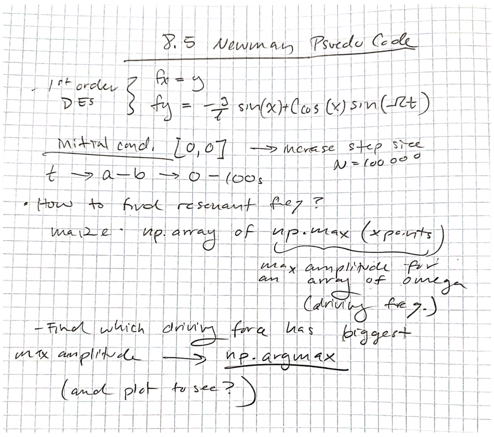

FIG. 5: Pseudo code for Exercise 8.5 Newman.

$$\frac{d\theta}{dt} = \omega \tag{5}$$

$$\frac{d\omega}{dt} = -\frac{g}{l} sin\theta + Ccos(\theta)sin(\Omega t) \tag{6}$$

First, we plot theta over time when C=2 and Ω=5, resulting in 6.

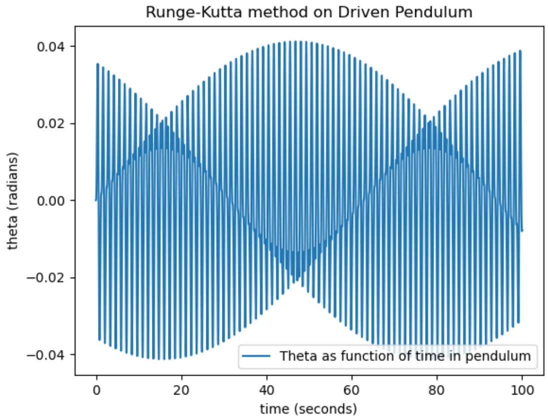

FIG. 6: Theta of driven pendulum over time.

However, we can figure out the resonant frequency of the system by finding at which driving force the maximum amplitude is largest because when driving frequency corresponds with the resonant frequency, the pendulum will swing violently. In a simple harmonic oscillator, the resonant frequency is at $\sqrt{\frac{g}{l}}$, and while this system is not simple, it is still a pretty good guess. So, I created a 'for loop' to loop through driving frequencies (Ωs) from approximately 1 to $2\sqrt{\frac{g}{l}}$. Then for each Ω I calculated the thetas like usual and got the maximum value using np.max. From this 'for loop', I eventually for an array of all of the max amplitudes for each Ω value. I then used np.argmax to get the Ω value with corresponded with the largest theta max. A plot of the maximum amplitude over Ω is seen in Figure 7.

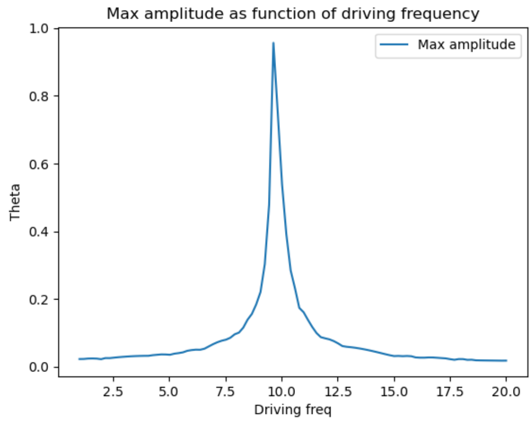

FIG. 7: Theta of driven pendulum over time.

I then used np.argmax to get the exact Ω value with corresponded with the largest theta max. We found that the amplitude is largest when $\Omega \approx 9.6363$ thus, finding the resonant frequency. This is pretty close to $\sqrt{\frac{g}{l}}$ but not exactly.

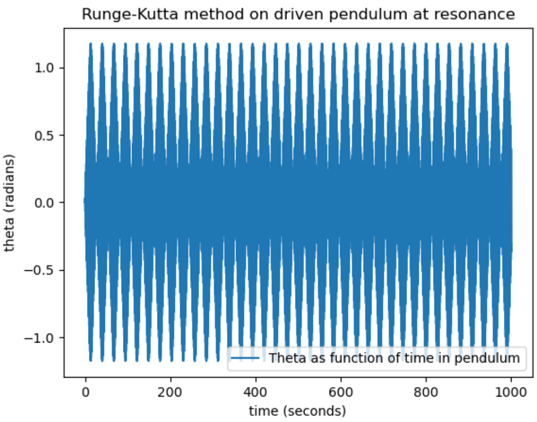

FIG. 8: Theta of driven pendulum over time.

You can see in Figure 8. that theta is around 1 radian when the pendulum is driven at resonant frequency. I also plotted it over along period of time, to show that it stays at the max amplitude and that the frequencies match up well.

4. SURVEY QUESTIONS

The homework this week took approximately 3 hours. While, they were a little repetitive, it is good review of the Runge-Kutta method. It was not ”too-easy’ - especially, after last weeks homework.