Problem Set 5

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1. ROMBERG FUNCTION

For this exercise, I created a function that uses Romberg Integration to calculate any integral. First I calculated the trapezoidal results when N, the number of steps, doubles between each run. I created an array where given some starting N, it doubles for each following value. Then using Equation(1), I calculated the integral and put results into an array.

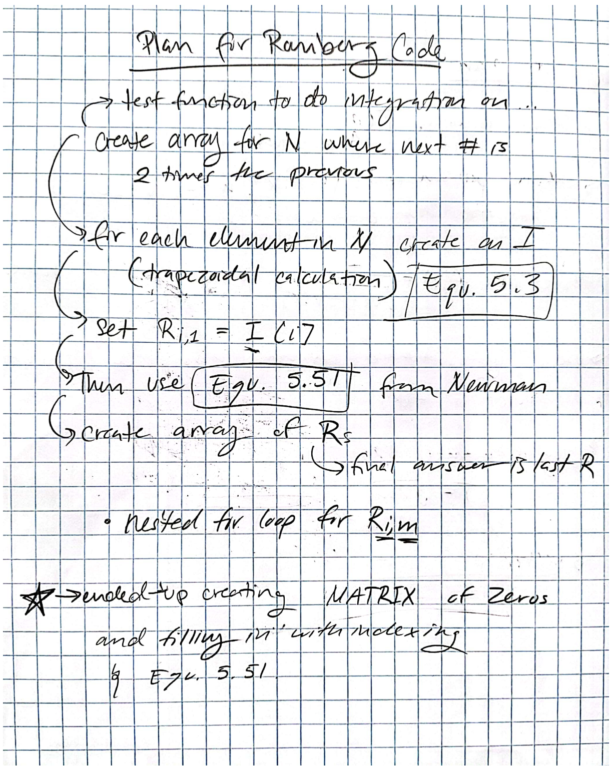

FIG. 1: Plan write-up for Romberg function.

$$I(a,b) \approx h\left(\frac{1}{2}f(a) + \frac{1}{2}f(b) + \sum_{k=1}^{N-1} f(a+kh)\right) \tag{1}$$

For calculating the Romberg value (R), I created a matrix of zeros with the expected dimensions. Then preceded to fill in the matrix such that the first column is the array of trapezoidal results (I) described above.

$$R_{i,1} = I_i \tag{2}$$

Then using a nested for loop I was able use indices to specify each value in the matrix. Using Equation(3) I looped through and placed new values in place of zeros.

$$R_{i,m+1} = R_{i,m} + \frac{1}{4^m - 1}\left(R_{i,m} - R_{i-1,m}\right) \tag{3}$$

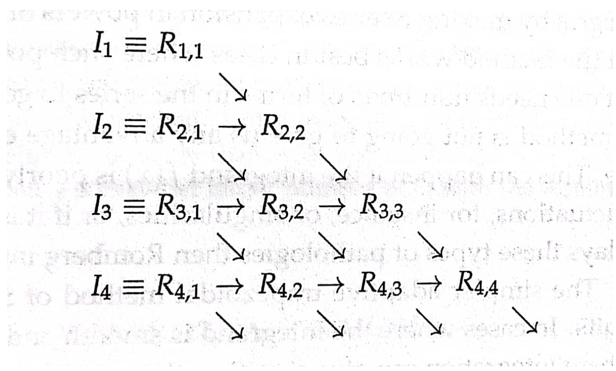

FIG. 2: Romberg Diagram.

Such that the diagram in Figure 2 looks like the matrix in Equation(4).

$$\begin{bmatrix} R_{1,1} & 0 & 0 & 0 \\ R_{2,1} & R_{2,2} & 0 & 0 \\ R_{3,1} & R_{3,2} & R_{3,3} & 0 \\ R_{4,1} & R_{4,2} & R_{4,3} & R_{4,4} \end{bmatrix} \tag{4}$$

I made the function return the final R value (right-most bottom corner in the matrix). I checked my function with a few input functions to which I already knew the answer to and found that it works!

2. EXERCISE 5.10

In this exercise we explore anharmonic oscillators through a combination of calculations and plots.

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2.0.1. Part (a)

For Part (a) of this exercise we are asked to show how Equation(5) can be rearranged into Equation(6). I used separation of variables to moved variables x and t to different sides of the equation and then integrated them to their respective limits. Because at one forth of the period the ball reaches the origin after being released from position a, we can integrate x from 0 to a, while we integrate t from 0 to T/4. You can see my work in Figure 3.

$$E(x) = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + V(x) \tag{5}$$

$$T = \sqrt{8m}\int_0^a \frac{dx}{\sqrt{V(a)-V(x)}} \tag{6}$$

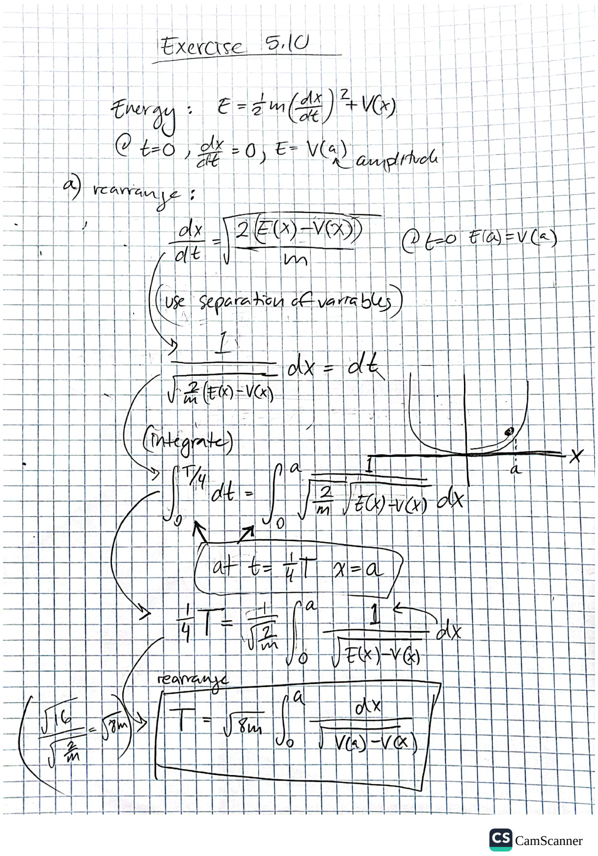

FIG. 3: Calculations for Part (a).

2.0.2. Part (b)

Next we are asked to create function that solves for the period (T) using Gaussian quadrature and then plot it over varying amplitudes. I used the built-in function for Gaussian quadrature in the Newman textbook, "gaussxwab(N,a,b)". In the case $V(x) = x^4$.My process can be seen in Figure 4.

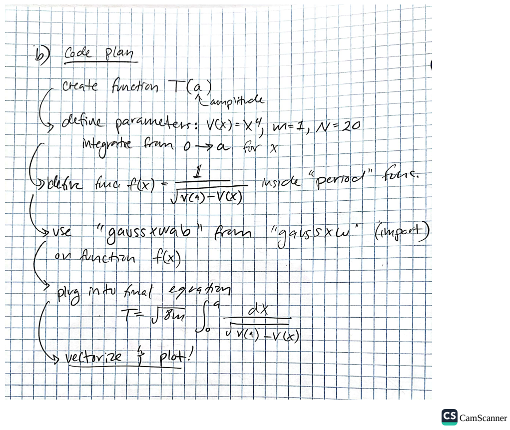

FIG. 4: Plan write-up for Exercise 5.10b code.

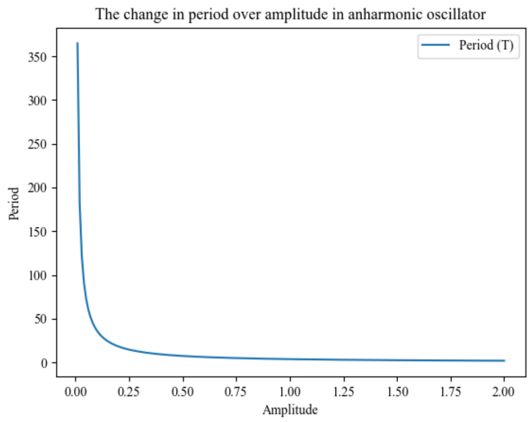

FIG. 5: Period as function of Amplitude in Anharmonic Oscillator.

2.0.3. Part (c)

After plotting the period of the oscillations as a function of amplitude, in Figure 5 it indeed can be seen that the oscillator gets faster as the amplitude increases. This is because this is an anharmonic oscillator which is not a perfect x^2 curve and therefore, the period is not independent of the amplitude or the position. In Equation(6), it can be seen that because $V(x) = x^4$, the period decreases as 'a' increases and increases as 'x' increases.

The code initially ran into some trouble plotting the period because the period diverges as the amplitude goes to zero; when the amplitude is zero then the denominator of the equation for the period is zero and therefore, has no solution. To avoid this problem, I graphed for amplitudes starting at slight above zero (0.01). We can assume it continues up infinitely.

3. SURVEY QUESTIONS

The homework this week took approximately 7 hours. I learned how to make a function for the Romberg method which gave me a deeper understanding of the method and allowed me to practice making matrices in Python. Thanks to Alec Wallach helped me fix my matrix indexing!