

## Week 6 Newman 5.4 → 5.7

### Romberg Integration

Trapezoidal rule:  $C h_i^2 = \frac{1}{3} (I_i - I_{i-1}) \rightarrow$  error calc.  
some constant  $\uparrow$   $i$  step w/ stepsize  $h$

$$\hookrightarrow \text{so } I = I_i + \frac{1}{3} (I_i - I_{i-1}) + O(h_i^4)$$

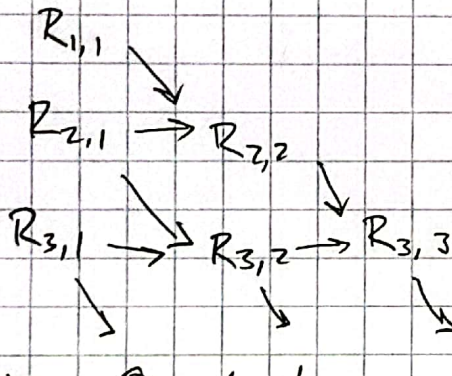
next term in series

$I_i \equiv R_{i,1} \rightarrow R_{n,m}$  matrix

$\uparrow$  where  $I_2$  is trapezoidal w/  $\frac{1}{2}$  step size of  $I_1$

$$R_{i,m+1} = R_{i,m} + \frac{1}{4^m - 1} (R_{i,m} - R_{i-2,m})$$

$\uparrow$  next term in matrix



• Decrease # of steps needed  
by using error to  
estimate integral (much faster)

### Gaussian Quadrature

Nonuniform points:  
(w/  $N-1$  degrees of freedom)

$$\phi_k(x) = \prod_{m \neq k} \frac{(x - x_m)}{(x_k - x_m)} \quad \text{w/ weights } (w_k) \quad (5.53)$$

$$\int_a^b f(x) dx \approx \int_a^b \phi(x) dx = \sum_{k=1}^N f(x_k) \underbrace{\int_a^b \phi_k(x) dx}_{w_k}$$

with weights  $w_k = \int_a^b \phi_k(x) dx$

★ Associated weights w/  $N$  (# of steps)

"weighted sum of function values @ quadrature points"  
faster at estimating integral  $\rightarrow$  Physics Forum

★ fits 1 polynomials to values in  $f(x_k)$