PHYS H304 Homework 3

Petra Mengistu, Collaborators: Emma Martignoni*
(Dated: February 18, 2023)

This is the lab report write-up of the approach and methods used for the third problem set in PHYS H304.

1. INTRODUCTION

This is a summary of the methods and major equations used for this problem set. In this report, we discuss iterations and the applications to various areas in Physics, namely chaos theory and popular fitting models for regression trends. We show the plots we produced representing the progression of a system from a steady state towards chaos and the demonstration of the photoelectric effect as observed with a linear least-squares fitting model to sample data. The major equations used are included where appropriate and a summary of the approach used to develop the code is included as well. All equations are taken from Computational Physics by Mark Newman [1].

2. METHODS

2.1. Exercise 3.6

Chaos refers to the state of random, disordered motion of particles. [1] provides an example of a system that appears to reflect chaos known as the Feigenbaum or logistic map. This is described by the expression given in Eq. 1, which shows the iterative nature of the value of x that is determined as a function of a variable constant “r”. In this system, the current value of x assumes the difference of the previous value and 1, scaled by the constant “r”. Three resulting patterns can emerge from this sort of iteration: the particle may remain in a steady state (i.e. the value of x remains fixed); the particle exhibits a certain level of periodicity (i.e. x cycles between a few values, known as the limit cycle); and the particle undergoes complete disorder (the x values change apparently randomly). This last case is termed as deterministic chaos, as the system appears disordered but is, in fact, defined by the iterative function.

$$x' = rx(1-x) \tag{1}$$

For a fixed value of r, we can determine a range of values for x. In Computational Physics, [1] indicates that the first thousand values of x produced for a fixed value of r=3 show an uneven behavior that is not representative of the actual behavior of the system. Therefore, to produce a logistic map that is reflective of the system’s true behavior, we use the next thousand values of x. To do so, we define a function in Python that generates the first two thousand values of x, but only outputs the last thousand values. We then determine the values for x for a range of values of “r” from 1 to 4 with increments of 0.01 (a total of 300 values of r). We then plot the resulting logistic map of the values of x as a function of “r”.

2.2. Exercise 3.8

In order to determine the behavior of electrons, Robert Millikan devised an experiment that comprised of light shining on a metal surface. [1] The observed effect was the ejection of electrons from the surface, which came to be known as the photoelectric effect. [2] In the field of quantum mechanics, Townsend [2] describes that this effect serves as evidence for the quantized nature of light, as there was a minimum threshold energy that light had to possess to eject an electron from the metal surface. By measuring the energies of the ejected electrons and frequencies, it was possible to determine how energy is related to frequency, and the limitations on the smallest energy a packet of light, known as a photon, can possess. [2]

Using the sample data provided by [1], we determine the quantitative relationship between the given voltages (that scale proportionally to the energies) and frequencies of the ejected electrons. We plot the voltages as a function of frequencies using the scatter plot function in the matplotlib library. As the data resembles a remarkably linear trend, we fit a linear best-fit trend to it. To do so, we use the least-squares method that determines the slope of the line by relating the square of the difference of each data point to the hypothetical trend line and selecting the slope and intercept that result in the minimum average difference. This method involves determining the average value of the frequencies, the voltages, the product of the voltages and frequencies, and the square of frequencies that we compute using the sums laid out in Equations 2, 3, 4, and 5.

$$E_x = \frac{1}{N} \cdot \sum_{i=1}^{N} x_i \tag{2}$$

*Electronic address: pmengistu@haverford.edu

$$E_y = \frac{1}{N} \cdot \sum_{i=1}^{N} y_i \tag{3}$$

$$E_{xy} = \frac{1}{N} \cdot \sum_{i=1}^{N} x_i \cdot y_i \tag{4}$$

$$E_{xx} = \frac{1}{N} \cdot \sum_{i=1}^{N} x_i^2 \tag{5}$$

To determine the slopes, we can set up a system of linear equations involving these average values. This system is given in Equation 7. From this system, we can express the slope "m" and the intercept "c" in terms of these averages as shown in Equations 8 and 9.

$$mE_{xx} + cE_x = E_{xy} \tag{6}$$

$$mE_x + c = E_y \tag{7}$$

$$m = \frac{E_{xy} - E_x E_y}{E_{xx} - Ex^2} \tag{8}$$

$$c = \frac{E_{xx} E_y - E_x Exy}{E_{xx} - Ex^2} \tag{9}$$

We implement these sums using loops in python and determine the resulting slopes and values. We then estimate the voltages using the determined relations from the linear trend $y = mx + c$ and over-plot the best fit line of original frequencies "x" against these values of voltage "y" on the scatter plot of the raw data. From the photoelectric experiment, Einstein had determined the equation relating the voltage to the frequency, as given in Eq. 10 where e is the charge of an electron, ϕ is the work function (threshold energy to excite an electron), and h is Planck's constant.

$$V = \frac{h}{e}\nu - \phi \tag{10}$$

This reflects a linear relationship of the voltage as a function of frequency where the work function "ϕ" corresponds to the intercept and the term $\frac{h}{e}$ corresponds to the slope, m. Therefore, we derive an estimate for the value of Planck's constant and compare it to other experimentally determined values using Eq. 11.

$$h = m \cdot e \tag{11}$$

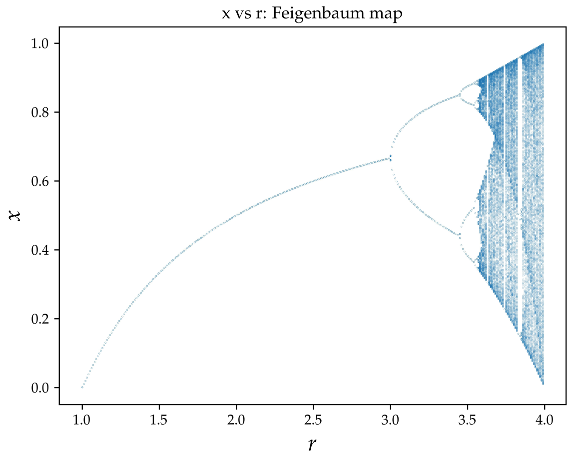

FIG. 1: The Feigenbum plot representing states of a fixed point, limit cycle, and deterministic chaos. All of these states are determined by Eq. 1

.

3. RESULTS AND CONCLUSION

For the Chaos series exercise, we obtained the following logistic map. As we observe, the system remains at a steady state for many iterations of x, reflecting the fixed point phase of "x". Beyond a certain value of "r", specifically r=3, it begins to show period doubling – that is, for certain value of r, there are roughly two values of x. Thus, the system enters the limit cycle phase of repeating only few points of x. At approximately r=3.5, the system shows further branches with period quadrupling, such that there are four values of x at any given value of r. The branches increase until it reaches a state of infinite branches, which appear disordered. This occurs at approximately r=3.6. Zooming in on these regions shows that there is distinct structure to the apparent disorder, showing the system exhibits deterministic chaos. These images are shown in Figure 1. The shape of the plot gives this system its characteristic name of "fig-tree", as [1] describes that the progression of the states of the system appear to be like a tree of many branches leaning forwards.

For Exercise 3.1, plotting the initial raw data of voltage as a function of frequency yields the scatterplot with an inherent linear trend shown in Figure 3. Using the methods outlined in Section 2.2, we determined that the slope and intercept of the least-squares fit are $m = 4.088x10{-}15\frac{\mathrm{J \cdot s}}{\mathrm{e}}$ and $c = -1.731V$, where "V" represents units of volts as c is the intercept with the y-axis. With these values, we overplot the best-fit line shown in Figure 4. Since we determine Planck's constant using the slope, from Eq. 11, we find that $h = 6.54934x10^{-34}\mathrm{J \cdot s}$ for Millikan's experimental sample data. According to the table of values on CODATA [3], the value of Planck's constant is 6.62607e-34 $\mathrm{J \cdot s}$.

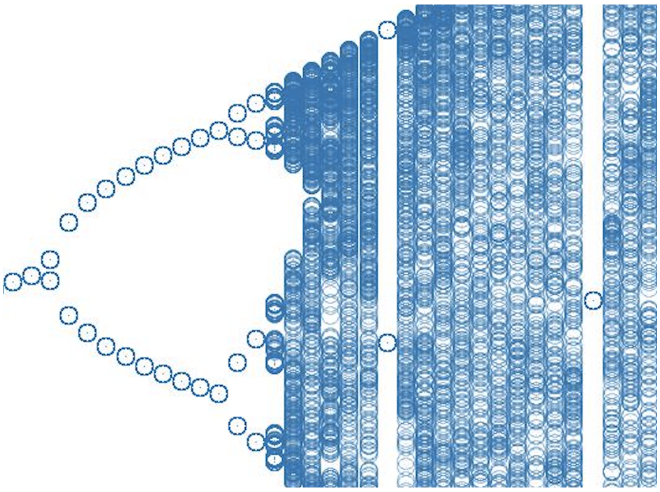

FIG. 2: A zoomed-in portion of the Feigenbum plot in the state of deterministic chaos. While there is seemingly no pattern on the broader level, there are patterns on the infinitely small scale

.

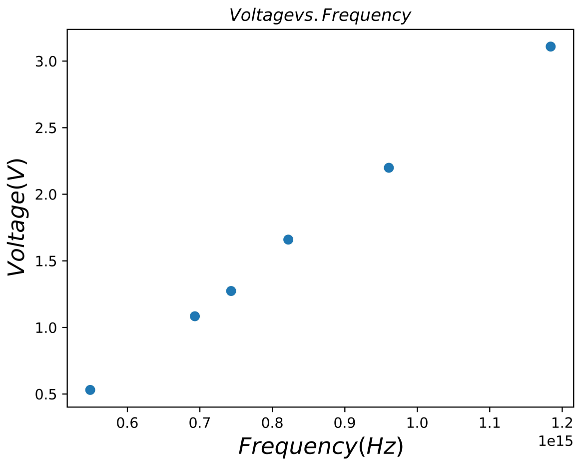

FIG. 3: A plot of the voltage against frequency for the sample data taken by Millikan. The individual data points correspond to single measurements.

Hence, our value determined from the least-squares fit is within 0.5% of the accepted value, confirming the validity of the trendline fitting method and Millikan's data.

For further description, see the codes attached in the folders. The outputs are printed in each code file.

4. SURVEY QUESTION

The most interesting problem was 3.6. The homework took me about 7 hours to complete. I learnt how to apply the coding we have been learning to practical concepts in Physics, and the results required more reflection than those we have done so far. I think the problem set length and difficulty were just right.

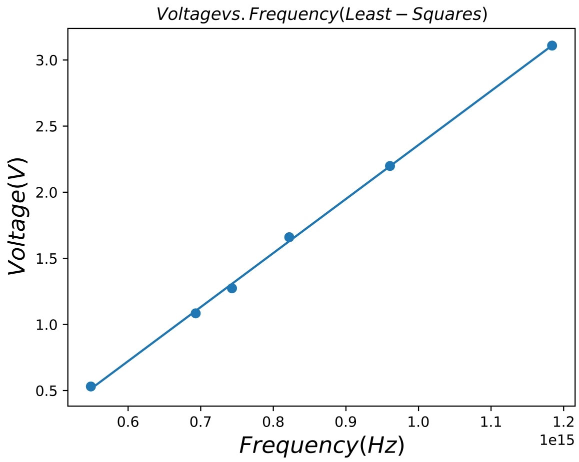

FIG. 4: A plot of Millikan's data of voltage against frequency with a least-squares fit overplotted.

[1] M. Newman, *Computational Physics* ([Createspace], 2012), URL http://www-personal.umich.edu/~mejn/cp/index.html.

[2] J. S. Townsend, *Quantum Physics: A Fundamental Approach to Modern Physics* ([University Science Books], 2010).

[3] P. J. Mohr, D. B. Newell, and B. N. Taylor, Reviews of Modern Physics **88** (2016), URL https://doi.org/10.1103%2Frevmodphys.88.035009.