PHYS H304 Homework 1

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This is a trial of the LateX tutorial for the first problem set in PHYS H304.

1. INTRODUCTION

This is a summary of the methods and major equations used for this problem set. The methods and proofs of major equations used are included where appropriate and a summary of the approach used to develop the code is included as well. All equations are taken from Computational Physics by Mark Newman.

2. METHODS

2.1. Exercise 2.6

According to the conservation of energy, the energy at the aphelion must equal that measured at the perihelion. Given that v_2 and v_1 represent the velocities at aphelion and perihelion respectively (with corresponding distances of l_2 and l_1, we can determine an expression for the velocity in terms of these other valuables. The expression for Energy is given by Eqn 1 where "m" and "M" are the masses of the satellite and sun respectively.

$$E = \frac{1}{2}mv^2 - \frac{GmM}{r} \tag{1}$$

At aphelion, $v = v_2$; hence, we can express Eq. 1 as Eq. 2:

$$E_2 = \frac{1}{2}mv_2^2 - \frac{GmM}{l_2} \tag{2}$$

Similarly, at perihelion, $v = v_1$ such that Eq. 1 becomes Eq. 3.

$$E_1 = \frac{1}{2}mv_1^2 - \frac{GmM}{l_1} \tag{3}$$

Equating Eq. 3 and Eq. 2, we can solve for v_2 as shown in Eq. 4, where 'm', the mass of the satellite, drops out of the equation one both sides.

$$v_2^2 = v1^2 - \frac{2GM}{l1} + \frac{2GM}{l_2} \tag{4}$$

Using the Keplerian relation in Eq. 5, we can subsitute an expression for l_2 in terms of l_1 and v_1. Hence, given the velcotiy and distance at perihelion as inputs from the user, the velocity at aphelion can be determined from the quadratic equation in Eq. 6.

$$v_2 l_2 = v_1 l_1 \tag{5}$$

$$v_2^2 - \frac{2GM}{l_1 v_1}v_2 - [v1^2 - \frac{2GM}{l1}] = 0 \tag{6}$$

To solve for v_2, we use the quadratic equation in Eq. 7, where we set $a = 1$, $b = \frac{-2GM}{l_1 v_1}$, and $c = -(v1^2 - \frac{2GM}{l1})$. As we require the velocity at aphelion, we use the smaller root as v_2, which we determine using an if statement comparison of the two possible roots in Python.

$$v_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{7}$$

Provided that the determinant was greater than zero, we set the value of the linear velocity at aphelion to the smaller root, as the velocity of an orbiting satellite reaches a minimum velocity at the farthest distance (in accordance with the conservation of energy). We then expressed determined the distance at aphelion from Eq. 5. The values of the orbital parameters were expressed using Eqs 8, 9, 10, 11 below.

$$a = \frac{1}{2}(l_1 + l_2) \tag{8}$$

$$b = \sqrt{(l_1 + l_2)} \tag{9}$$

$$T = \frac{2\pi ab}{l_1 v_1} \tag{10}$$

$$e = \frac{l_2 - l_1}{l_2 + l_1} \tag{11}$$

2.2. Exercise 2.2

From Kepler's third law, we have an established relationship between the period and the semi-major axis, as shown in Eq. 12.

$$T^2 = \frac{4\pi^2 a^3}{GM} \tag{12}$$

Assuming that the Earth's mass is concentrated at its center (that is, its center of mass), then the semi-major axis of the orbit is expressed as the sum of the satellite's

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altitude above the Earth's surface (the altitude h) and the Earth's radius, R, in meters. Hence, $a = h + R$. Substituting this expression for "a" into Eq. 12, we can rearrange to solve for the altitude in Eq. 13.

$$h = (\frac{GMT^2}{4\pi^2})^{1/3} - R \tag{13}$$

For part b, we determined the altitude for any value of period entered by the user. For part c, we determined the altitude for two values of the period - 90 minutes and 45 minutes. As the altitude for the latter was negative, we determined that a period of 45 minutes was too small for the satellite. For part d, we calculate the altitude for a 24 hour day period and 23.94 hour day, otherwise known as a sidereal day. Sidereal time exists due to the impact of reference frames when determining the period of rotation [1]. We determine the difference in altitude for these two versions of a day by calculating the altitude using each length of day as an estimate for the period and take the difference between the two.

2.3. Exercise 2.10

From the semi-empirical mass formula, we determine the binding energy as shown in Equation 14, where a_1, a_2, a_3, and a_4 are energies given in MeV (Mega electron volts) and A and Z represent the mass and atomic number respectively.

$$B = a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A} + \frac{a_5}{A^{1/2}} \tag{14}$$

We determine the value of a_5 based on the value of the mass number A and atomic number Z. When A is odd, a_5 is set to zero; however, when A is even, a_5 takes on the value of -12 or 12 when Z is odd or even respectively. We coded for this using if, elif and else statements. For part (a) and (b), we took in A and Z as inputs from the user and used Eq. 14. To determine the binding energy per nucleon (BPN), we used Eq. 15 below.

$$BPN = \frac{B}{A} \tag{15}$$

For part (c), we only take Z as an input and determine the value of A for the range of possible values between Z to 3Z. To do so, we used conditional statements that checked the maximum binding energy for each value of A tested. Similarly for part (d), we allow Z to range from 1 to 100, and the value of A to the range of possible values between Z to 3Z for each value of Z. To determine the choice of Z and A that produce a maximum binding energy, we used conditional statements that checked the maximum binding energy for each value of A tested. Then, we determined the maximum binding energy associated with each Z using conditional statements to compare the energy and mass number selected for each Z value. See Python code for full description.

3. RESULTS AND CONCLUSION

See the codes attached in the folders for the results. The outputs are printed in each code file.

[1] B. Ryden and B. M. Peterson, *Foundations of Astrophysics* (Cambridge University Press, 2020).