Hw 1

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[Solve exercises 2.2, 2.6, and 2.10]

1. EXERCISE 2.2

Part a)

A satellite is to be launched into a circular orbit around the earth so that it orbits the planet once every T seconds. Show that the altitude h above the earth's surface that the satellite must have is

$$h = (\frac{GMT^2}{4\pi^2})^{1/3} - R, \tag{1}$$

First we define the constants: $G = 6.67e^{-11}m^3kg^{-1}s^{-2}$ is Newton's gravitational constant M = the mass of the earth R = is the radius T = Time (in seconds) it takes for the satellite to complete one orbit. h = the altitude of the satellite above the Earth's surface

So first, we know that the gravitational force between the satellite and the earth is calculated using Newton's law of gravitation:

$$F = \frac{GMm}{R^2} \tag{2}$$

where in this case m is the mass of the satellite, and r is the distance between the center of the earth and the satellite.

We know that the satellite is in circular orbit, which means that the force of gravity is equal to the centripetal force that keeps it in its orbit:

$$F = \frac{mv^2}{r} \tag{3}$$

If we substitute the gravitational force F into the centripetal force equation, we have:

$$\frac{GMm}{R^2} = \frac{mv^2}{r} \tag{4}$$

Then if we solve for v we get:

$$v = (\frac{GM}{r})^{1/2} \tag{5}$$

and since we know that T can be found using the velocity and radius, we get:

$$T = \frac{2\pi r}{(\frac{GM}{r})^{1/2}} \tag{6}$$

Finally we solve for r,

$$r = (\frac{GMT^2}{4\pi^2})^{1/3} \tag{7}$$

since the altitude h can be found by subtracting the radius of the earth R (h=r-R) we get

$$h = (\frac{GMT^2}{4\pi^2})^{1/3} - R, \tag{8}$$

Part b)

The satellite that orbits the earth the less times a day has the highest the altitude. In other words, it means that the fastest satelite is closer to the earth (has lowest altitude)

Part c)

It doesn't make that much of a difference, but the satellite has a lower altitude for the 23.93 hours

24 hours: 35855910.17617497 meters

23.93 hours: 35773762.329895645 meters

2. EXERCISE 2.6

Part a)

Things we know:

perihelion - closest point to the sun:

$$l_1v_1 \tag{9}$$

aphelion - most distant point from the sun:

$$l_2v_2 \tag{10}$$

Kepler's second law tells us:

$$l_2v_2 = l_1v_1 \tag{11}$$

Total energy, kinetic plus gravitational is given by

$$E = 1/2mv^2 - G\frac{mM}{r} \tag{12}$$

where m is the planet's mass, $M = 1.9891x10^{30}kg$ is the mass of the Sun, and $G = 6.6738x10^{-11}kg^{-1}s^{-2}$

Now, we can substitute $l_2 = l_1$ and $v_2 = v_1$ into the total energy equation:

$$E = \frac{1}{2}mv_2^2 - G\frac{mM}{l_2} = \frac{1}{2}mv_1^2 - G\frac{mM}{l_1} \tag{13}$$

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If we substitute $l_2 = l_1$ into the equation, we are left with:

$$\frac{1}{2}mv_2^2 - G\frac{mM}{l_1} = \frac{1}{2}mv_1^2 - G\frac{mM}{l_1} \tag{14}$$

Finally after rearranging the equation, we are left with the equation that v_2 is the smaller root of the quadratic equation:

$$v_2^2 - \frac{2GM}{v_1 l_1}v_2 - [v_1^2 - \frac{2GM}{l_1}] = 0 \tag{15}$$

3. SURVEY QUESTIONS

The homework did take me some time, mainly because I kept getting stuck on small errors on python, like on question 2.10 a, when I kept trying to use an equation, I kept getting an error saying that the ”float” object was not callable. I finally figured out after office hours. I learned on how to put complex equations on latex, I had to google how to put subscripts and small stuff like that, but now I feel way more comfortable. The most interesting problem to me was 2.10, because I got to use the if statement, which I found to me kinda confusing sometimes. I do think the homework is kind of long,(I wasn't able to get 2.10.c :( ) even though there are only 3 exercises, the are multiple parts of each and we still have to do the Latex and pseudo code so i do think it might be a little too long.