Flow Chart

Exercise 5.3 - Computation of the error function through integral using Trapezoidal int & Gaussian quadrature.

Reminder

Trapezoidal rule

$$I(a,b) = \int_a^b f(x)\,dx$$

$$h = \frac{(b-a)}{N}$$

Euler-Maclaurin formula

$$\epsilon = \frac{1}{12} h^2 \left[f'(a) - f'(b) \right]$$

Gaussian quadrature

$$\omega_k = \left[\frac{2}{(1-x^2)} \left(\frac{dP_N}{dx} \right)^{-2} \right]_{x = x_k}$$

$$x'_k = \tfrac{1}{2}(b-a)x_k + \tfrac{1}{2}(b+a)$$

$$w'_k = \tfrac{1}{2}(b-a)\,w_k$$

Redo the problem 5.3 using Gaussian quadrature

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Define the Gaussian quadrature

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Calculate the sample points & weights, then map them to the required integration

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Perform integration

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Create the Error Gaussian method

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Create the Error Trapezoidal method

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Make a plot with both error curves (Gaussian vs Trapezoidal) in it.

Exercise 5.13: Quantum uncertainty in the harmonic oscillator

Equations: $\psi_n(x) = \frac{1}{\sqrt{2^n n! \sqrt{\pi}}} e^{-x^2/2} H_n(x)$ for $n = 0 \ldots \infty$

$$H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x)$$

First write the user-defined function $H(n,x)$ that calculates $H_n(x)$ for a given x and any int. $n>0$

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Make a plot that shows the harmonic oscillator wavefunction for $n=0, 1, \& 3$ with range $x=-4$ to $x=4$

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Make a second plot of the wavefunction for $n=30$ from $x=10$ to $x=10$

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Last, evaluate the integral $\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 |\psi_n(x)|^2 dx$ using Gaussian quadrature on 100 points, then calculate the uncertainty for $n=5$