PHYS 304 HW 6

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1. EXERCISE 8.3

The Lorenz equations are a set of differential equations that demonstrate the phenomenon of deterministic chaos. They are given by the following,

$$\frac{dx}{dt} = \sigma(y - x)$$
$$\frac{dy}{dt} = rx - y - xz$$
$$\frac{dz}{dt} = xy - bz$$

where r, b, and σ are constants.

To solve these equations, I utilized the Runge-Kutta 4th order integration method. I solved for the case where $\sigma = 10$, $r = 28$, and $b = \frac{8}{3}$, in the range of $t = 0tot = 50$. I first produced a plot of y values versus time 1, which demonstrates the equations' unpredictability. I then produced a plot of the z values against the x values 2, resulting in an oddly shaped plot known as the "strange attractor."

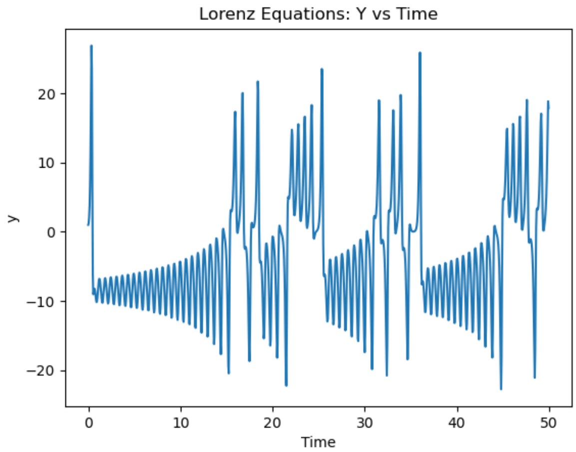

FIG. 1: Y as a function of time as given by the Lorenz equations.

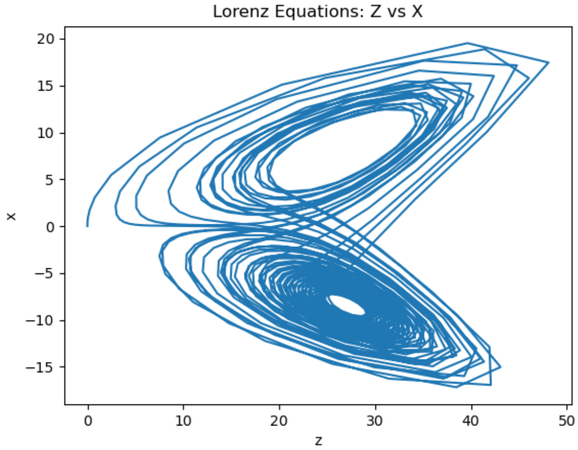

FIG. 2: The "Strange Attractor"

2. EXERCISE 8.4

The equation for the motion of a pendulum is given by 1, where ω is defined as 2. These two first order differential equations were derived from a single second order differential equation.

$$\frac{d\omega}{dt} = -\frac{g}{l}\sin\theta \tag{1}$$

$$\frac{d\theta}{dt} = \omega \tag{2}$$

These two equations were solved by using the 4th order Runge-Kutta method. For this exercise, the length of the pendulum arm was said to be 10 cm (0.1 m), and the angle of the pendulum's release was 179°. g is 9.81 m/s. I was able to create a graph of the angle, θ, as a function of time 3, which shows the clear oscillatory nature of the system.

3. EXERCISE 8.5

This exercise builds upon the previous by adding a driving force to the pendulum system. The equation of motion in this case is given by 3, where C and Ω are constants. This was once again solved using the Runge-Kutta 4th order method.

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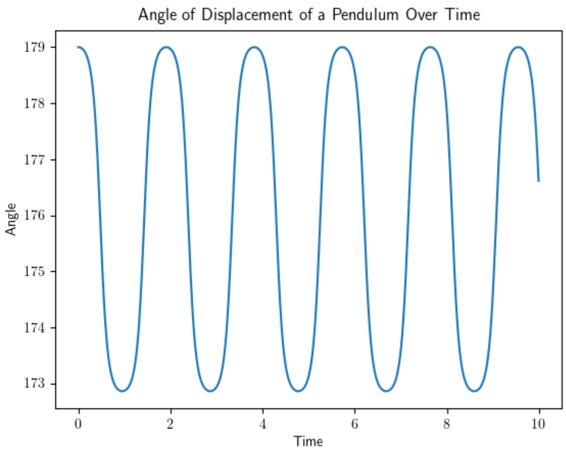

FIG. 3: The angle displacement of a pendulum over time.

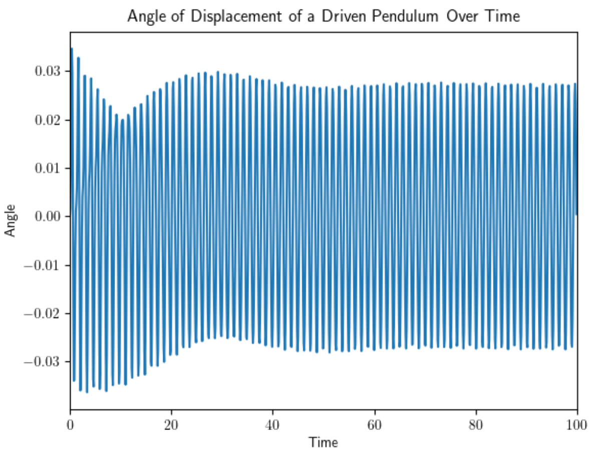

FIG. 4: Angle of displacement of a driven pendulum over time.

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin\theta + C\cos\theta\sin\Omega t \qquad (3)$$

For this exercise, l and g are the same as in exercise 8.4, but the pendulum starts at rest and at 0°. C is set to $2s^{-1}$ and $\Omega = 5s^{-1}$. The time interval is from 0 to 100 seconds. The resultant plot is shown in figure 4. To determine the value for which the pendulum resonates with the driving force, you vary the value of Ω. I found that a value of $10s^{-1}$ resonated 5.

4. SURVEY

I honestly do not remember exactly how long this homework took me. I would say it was probably around 4-5 hours, which is a perfect length for these problem

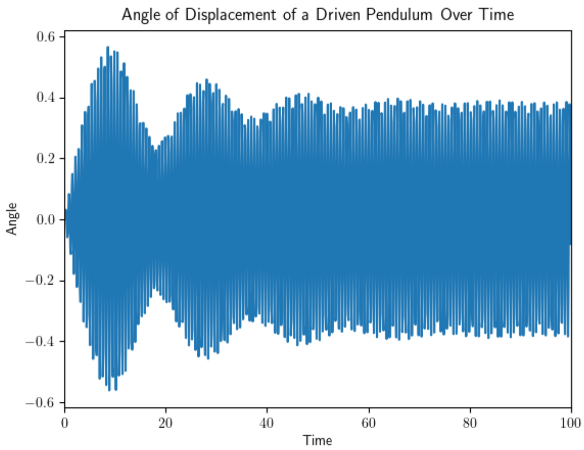

FIG. 5: Angle of displacement of a driven pendulum over time, with $\Omega = 10$

sets. I enjoyed the Lorenz equations problem, especially playing around with the values to create interesting plots.