PHYS 304 HW 5

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1. EXERCISE 5.4

The clarity and detail of objects seen through a telescope is limited by the diffraction of light. Light passing through the aperture of a telescope produces a diffraction pattern of many concentric rings. The intensity of the pattern is given by the following equation 1. Where r is the distance from the focal plane, $k = \frac{2\pi}{\lambda}$ (where λ is wavelength), and J_1 is a bessel function.

$$I(r) = \left(\frac{J_1(kr)}{kr}\right)^2 \tag{1}$$

The bessel functions are given by the following equation 2. Using Simpson's rule with N = 1000 points to evaluate the integral, I plotted the first three bessel functions, J_0, J_1, J_2, on a single graph 1.

$$J_m(x) = \frac{1}{\pi}\int_0^{\pi} \cos(m\theta - x\sin\theta)d\theta \tag{2}$$

To create the density plot of the intensity 2, I calculated the intensity using equation 1, assuming a wavelength of 500 nm and a radius of 0 to 1 micrometers. Rather than 1000 points, this program was run using only 100 points, as I struggled to optimize the time required to run the program.

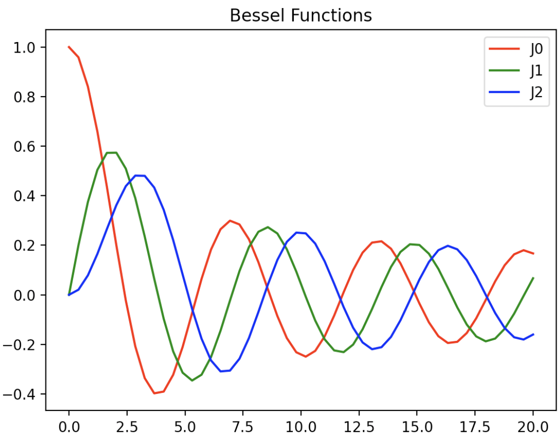

FIG. 1: Bessel functions.

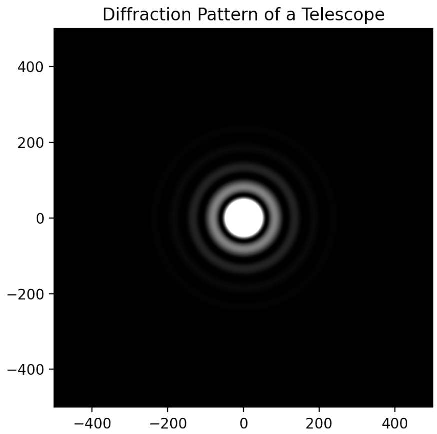

FIG. 2: The diffraction of a telescope.

In addition to these calculations, I created a program that would calculate and plot the fractional error between the values of the Bessel functions computed using Simpson's rule and values of the Bessel functions computed using recursion. I used equation 3 to compute the Bessel functions via recursion. I set the values of J_0 and J_1 using the previous method.

$$J_{n+1}(x) + J_{n-1}(x) = \frac{2n}{x}J_n(x) \tag{3}$$

I computed the values of the Bessel functions $J_2, J_3, and J_4$ using each method, and then calculated the fractional error by subtracting the Simpson's method values from the recursion values, and then dividing by the recursion values. I chose to plot these over the same range of x values in figure 1. The resulting plot

2. EXERCISE 5.13

The wave function of the nth level of a one-dimensional harmonic oscillator is given by equation 4, where H_n is the Hermite polynomial. The Hermite polynomials are given by equation 5, and the first two polynomials are defined as $H_0(x) = 1$ and $H_1(x) = 2x$.

$$\psi_n(x) = \frac{1}{\sqrt{2^n n!\sqrt{\pi}}}e^{-x^2/2}H_n(x) \tag{4}$$

$$H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x) \tag{5}$$

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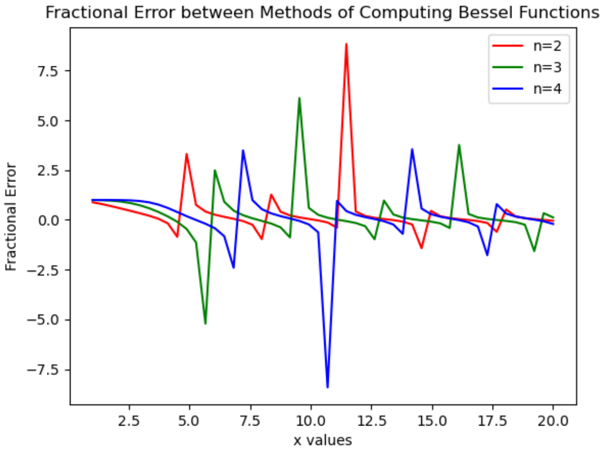

FIG. 3: The fractional error of Bessel functions J_2, J_3, J_4 evaluated using Simpson's rule vs recursion.

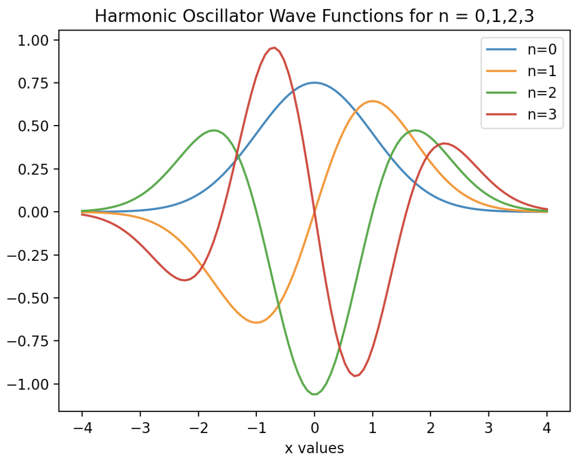

FIG. 4: Harmonic oscillator wave functions for n = 0,1,2,3

With these equations, I was able to write a program to calculate the Hermite polynomial for any given x, and then calculate the wave function. The wave functions of the first four states of the harmonic oscillator are plotted in figure 4.

From there, I was able to do the same process for a value of n = 30, shown in figure 5.

Finally for this exercise, I wrote a program that evaluates the quantum uncertainty in the position of a particle in any level n. This is quantified by its root-mean-square, the square root of its expectation value, given by 6. I evaluated the integral using Gaussian Quadrature on 100 points, using the program provided by Mark Newman. For n = 5, the quantum uncertainty I calculated is approximately 1.88.

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 |\psi_n(x)|^2 dx \tag{6}$$

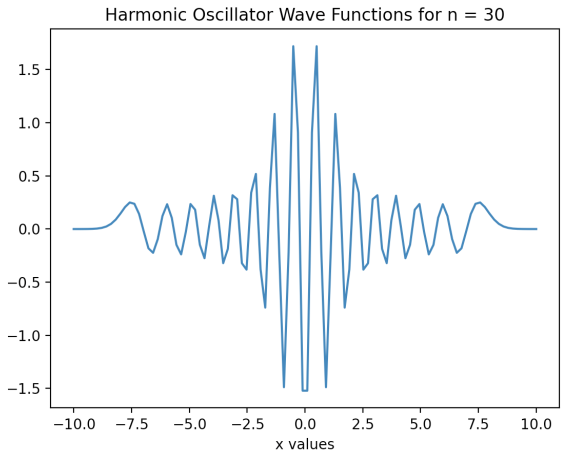

FIG. 5: Harmonic oscillator wave function for n = 30

3. SURVEY

I spent around 5 hours on this homework set. It was interesting to see how the Bessel functions differed with the different computation methods. I was having trouble understanding what exactly the first problem was asking me to do, but I realized that I was over-complicating a simple problem.

5.4

- Copy over Simpson's rule evaluation of the Bessel functions from last week
- Define the Bessel function
  - if m = 0,1 use Simpson's rule calculation
  - else, return the equation
- Define x values
- Calculate the fractional error between the Simpson's rule calculations and the recursion calculations
- Plot

5.13a

- Define the Hermite polynomials
  - if n = 0, return 1
  - if n = 1, return 2x
  - else, return the equation
- Define the wave function
- Define x values
- Define y values (empty array)
- Calculate y values for each wavefunction
- Plot

5.13b

- Repeat above
- Update x values
- Solve for only n = 30
- Plot

5.13

- Import gaussian quadrature
- Repeat above
- Define the integrand

- Define the number of points and the limits
- Use gaussian quadrature
- Take the square root
- Print