PHYS 304 HW 0

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1. EXERCISE 2.10

The semi-empirical mass formula 1 is a formula for calculating the nuclear binding energy (B) of an atomic nucleus. The nucleus' atomic number is given by Z, and its atomic mass number is given by A. The variables a_1, a_2, a_3, a_4, and a_5 are constants, with a_5 varying based on the even/odd nature of A and Z.

$$B = a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A} + \frac{a_5}{A^{1/2}} \quad (1)$$

To create a program to determine B for any given A and Z, I first had to define the inputs (A and Z) and define the constants a_1, a_2, a_3, and a_4. To define a_5, I first had to create a loop that would check the even/odd status of A and Z. From there, the appropriate a_5 value could be assigned, and the nuclear binding energy, B, could be calculated. For an atomic number of 28 and an atomic mass of 58, the nuclear binding energy is 493.936 MeV. To determine the nuclear binding energy of each individual nucleon, you divide the total energy, B, by the atomic mass A. This results in a binding energy of 8.516 MeV per nucleon. I modified my program to only take an input of Z, and use values of A between Z and 3Z. Before checking the even/odd status, I added a statement that defined A as the range of values between Z and 3Z. To find the most stable nucleus with the given atomic number, I had to find the value of A that maximized B. This turned out to be 58, and the binding energy per nucleon is 0.517 MeV. A similar process was used to restrict the values of Z to the range of 1 to 100.

2. EXERCISE 2.2

The altitude of a satellite (h) orbiting Earth is given by the shown equation 2. This equation is found by rearranging Kepler's law of periods, and subtracting the radius of the Earth. The radius is subtracted because Kepler's law of periods tells us the total distance from Earth's center. In order to find the altitude above the surface, you must subtract the Earth's radius.

$$h = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} - R \quad (2)$$

To create a program to determine altitude (h) based on a given period (T), I first had to define the constants M (mass of the Earth), G (gravitational constant), and R (radius of the Earth). Then, I defined the input T, period in seconds. Then, the program had to compute altitude using the given formula. Using this program, I was able to obtain the altitudes of satellites orbiting the Earth at different periods. A satellite with an orbital period of one day rests at an altitude of $3.586 \cdot 10^7$m. An orbit of one sidereal day has a $8.214 \cdot 10^4$m difference. An orbit of 90 minutes has an altitude of $2.793 \cdot 10^5$m. It is impossible for a satellite to orbit the Earth with a period of 45 minutes.

3. EXERCISE 2.6

The total energy of a planet with velocity v and distance r from the Sun is given by the following equation 3, where m is the mass of the planet, M is the mass of the Sun, and G is the gravitational constant. Kepler's second law tells us that $l_1 v_1 = l_2 v_2$, meaning that the distance (l_1) times the velocity (v_1) of a planet at its perihelion is equal to the distance (l_2) times the velocity (v_2) of that planet at its aphelion. Given that orbits sweep out equal area in equal time, and the perihelion is larger than the aphelion, v_2 must be smaller than v_1 4.

$$E = \frac{1}{2} m v^2 - G\frac{mM}{r} \quad (3)$$

$$v_2^2 - \frac{2GM}{v_1 l_1} v_2 - (v_1^2 - \frac{2GM}{l_1}) = 0 \quad (4)$$

The values of v_1 and l_1 can be used to calculate v_2 and l_2. From there, these values can be used to compute a variety of the planet's orbital parameters, such as its semi-major axis (a)5, semi-minor axis (b)6, orbital period (T)7, and orbital eccentricity (e)8.

$$a = \frac{1}{2}(l_1 + l_2) \quad (5)$$

$$b = \sqrt{l_1 l_2} \quad (6)$$

$$T = \frac{2\pi ab}{l_1 v_1} \quad (7)$$

$$e = \frac{l_2 - l_1}{l_2 + l_1} \quad (8)$$

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Creating a program to calculate these values based on an input of perihelion length (l_1) and velocity (v_1) began with defining the constants, M and G. Then, I had to define the inputs, l_1 and v_1. From there, I computed the values of the length of the aphelion, the velocity at the aphelion, the orbital period, and eccentricity. I checked that the program was functioning correctly by using the values for Earth and Halley's Comet.

define inputs A and Z

define constants

$a_1 = 15.8$

$a_2 = 18.3$

$a_3 = 0.174$

$a_4 = 23.2$

$$a_5: \begin{cases} 0 & \text{if } A = \text{odd} \\ 12 & \text{if } A, Z = \text{even} \\ -12 & \text{if } A = \text{even} \quad Z = \text{odd} \end{cases}$$

check even/odd status of A and Z

assign appropriate a_5 value

define B, $B = a_1 A - a_2 A^{2/3} - a_3 Z^2/A^{1/3} - a_4 (A-2Z)^2/A + a_5/A^{1/2}$

print B value

b) same as above, but print B/A instead

c) define input Z

define A as (Z, 3Z)

define constants

check even/odd status of A (in range (Z, 3Z)) and Z

assign appropriate a_5 value

define B

find value of A that maximizes B

print value of A that maximizes B

print max value of B/A

d) define Z as range of values (1, 100)

define A as (Z, 3Z)

define constants

check even/odd status of A (in range (Z, 3Z)) and Z

assign appropriate a_5 value

define B

find value of A that maximizes B

print value of A that maximizes B

print max value of B/A, and what Z it occurs at

2.2) a) kepler's law of periods

$T^2 = \frac{4\pi^2}{GM} h^2$ rearrange $\rightarrow$ $h = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3}$

this shows the total distance from Earth's center, in order to just find altitude above surface, you must subtract the Earth's radius, R.

b) define constants

$G = 6.67 \times 10^{-11}\ m^3 kg^{-1} s^{-2}$

$M = 5.97 \times 10^{24}\ kg$

$R = 6371\ km = 6.371 \times 10^6 m$

define input T

define $h = (GMT^2/4\pi^2)^{1/3} - R$

print value of h

c) once a day, 86400 sec $\rightarrow$ h = 35855910.176 m

once every 90 min, 5400 sec $\rightarrow$ h = 279321.625 m

once every 45 min, 2700 sec $\rightarrow$ h = -2181559.898 m

↳ it is impossible for something to orbit the Earth every 45 min

d) 1 sidereal day = 23.93 hr = 86148 sec $\rightarrow$ 35773762.330 m

35855910.176 m - 35773762.330 m = 82147.846 m difference

2.6) a) $\ell_2 v_2 = \ell_1 v_1$

total energy $E = \frac{1}{2}mv^2 - G\frac{mM}{r}$ $m = 5.97 \times 10^{24}$ kg $M = 1.989 \times 10^{30}$ kg

$v_2^2 - \frac{2GM}{v_1 \ell_1} v_2 - \left(v_1^2 - \frac{2GM}{\ell_1}\right) = 0$ $G = 6.67 \times 10^{-11}\ m^3\,kg^{-1}\,s^{-2}$

b) input constants M, G

define inputs ℓ_1 and v_1

compute v_2, ℓ_2, semi-major axis, semi-minor axis, period, and eccentricity

$v_2 = (2GM/\ell_1 v_1) - v_1$

$\ell_2 = (\ell_1 v_1)/v_2$

$a = (\ell_1 + \ell_2)/2$

$b = \sqrt{\ell_1 \ell_2}$

$T = (2\pi ab)/(\ell_1 v_1)$

$e = (\ell_2 - \ell_1)/(\ell_2 + \ell_1)$

print desired values

c) check values ✓