PHYS 304 HW 4

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1. EXERCISE 5.4

The clarity and detail of objects seen through a telescope is limited by the diffraction of light. Light passing through the aperture of a telescope produces a diffraction pattern of many concentric rings. The intensity of the pattern is given by the following equation 1. Where r is the distance from the focal plane, $k = \frac{2\pi}{\lambda}$ (where λ is wavelength), and J_1 is a bessel function.

$$I(r) = \left(\frac{J_1(kr)}{kr}\right)^2 \tag{1}$$

The bessel functions are given by the following equation 2. Using Simpson's rule with N = 1000 points to evaluate the integral, I plotted the first three bessel functions, J_0, J_1, J_2, on a single graph 1.

$$J_m(x) = \frac{1}{\pi}\int_0^{\pi} \cos(m\theta - x\sin\theta)d\theta \tag{2}$$

To create the density plot of the intensity 2, I calculated the intensity using equation 1, assuming a wavelength of 500 nm and a radius of 0 to 1 micrometers. Rather than 1000 points, this program was run using only 100 points, as I struggled to optimize the time required to run the program.

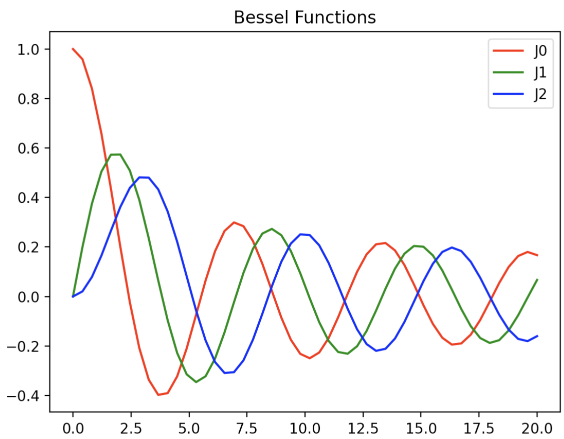

FIG. 1: Bessel functions.

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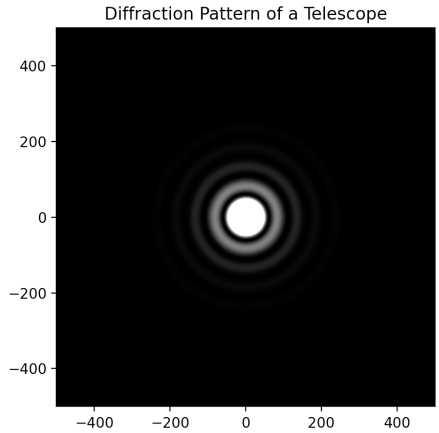

FIG. 2: The diffraction of a telescope.

2. EXERCISE 5.9

Debye's theory defines the heat capacity of a solid at temperature T as given by the following equation 3.

$$C_v = 9V\rho k_b \left(\frac{T}{\theta_D}\right)^3 \int_0^{\theta_D/T} \frac{x^4 e^x}{(e^x - 1)^2} dx \tag{3}$$

V is the volume of the solid, ρ is the number density of atoms, k_B is the Boltzmann constant, and θ_D is the Debye temperature, which depends on the properties of the material. I wrote a program that would calculate the value of C_v for any given temperature, T, using gaussian quadrature to evaluate the integral. The gaussian quadrature program that I implemented, gaussxw.py, was provided by Mark Newman as supplemental material to the textbook. For this problem, the integral was evaluated over 50 points, and the constants were defined as the following:

$$V = 10^{-6}m^3$$
$$\rho = 6.022 \cdot 10^{28}m^{-3}$$
$$\theta_D = 428K$$
$$k_B = 1.381 \cdot 10^{-23}m^2kgs^{-2}K^{-1}$$
$$5K \leq T \leq 500K$$

I plotted the resulting C_v values against the temperature, which resulted in the following plot 3

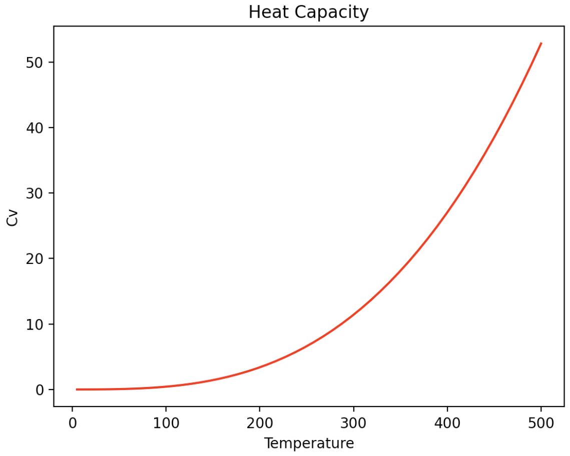

FIG. 3: Heat capacity as a function of temperature.

3. SURVEY

I believe I spent about 9-10 hours on this homework set. I spent a lot of time creating a working Simpson's rule program, but once I accomplished this I had a much easier time adapting it to the other problems. I spent a long time working on the plots for this assignment, particularly for exercise 5.9. I attempted to complete this problem using Simpson's rule for multiple hours, but was unable to produce results, despite my Simpson's rule code working normally in problem 5.4. I think this homework was a good length, and once again I appreciate the ability to choose which problems I'd like to do.

5.4a
define bessel function
set N
define simpson’s rule
calculate simpson’s rule
define y values for each bessel function
plot

5.4b
same as above
define intensity function
define parameters
calculate intensity values
plot

5.9
import gaussian quadrature
define constants
define function Cv
define limits and N
evaluate the integral
plot