PHYS 304 HW 3

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1. EXERCISE 3.6

In mathematics, chaos describes a situation in which the typical solutions of a system do not converge, but continue to display unpredictable behavior. A famous example of chaos is the logistic map, described by equation 1.

$$x' = rx(1 - x) \tag{1}$$

This equation is an iterative map, repeating the calculation with a value of x results in one of three outcomes. The value could settle down to a single number, becoming a *fixed point*. It could settle into a periodic pattern rotating over a set of values, a *limit cycle*. Or, it could generate a seemingly random series of numbers, which is *deterministic chaos*. The goal of this exercise was to create a program that would calculate and display the behavior of the logistic map. First, I defined the equation 1, the values of r and x, and the number of iterations. r is the range of values between 1 and 4, with a step of 0.01. In the problem, x is defined as 1/2. The number of iterations is 2000, but only the second 1000 were plotted, as the first 1000 were used to account for any limits or convergences that may appear. I used these to calculate all the values I needed, and then plotted them, showing the graph known as the Feigenbaum Plot 1.

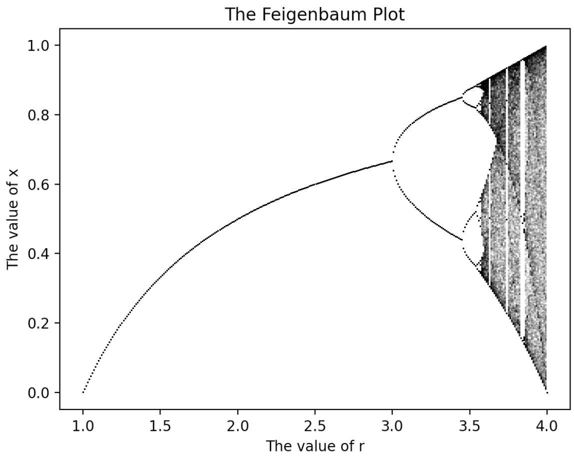

FIG. 1: The Feigenbaum Plot.

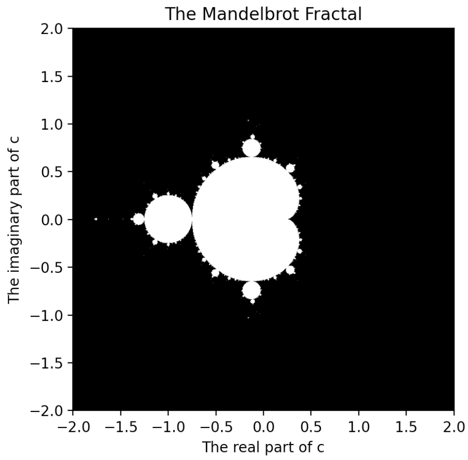

FIG. 2: The Mandelbrot Fractal.

On the Feigenbaum plot, a fixed point would simply look like a point in space. A limit cycle would appear to be something similar to the points ranging from 1.0 to 3.0 on the x-axis. And chaos would appear like the area between 3.5 and 4.0 on the x-axis. The r value at which the system seems to move into chaos is around 3.5.

2. EXERCISE 3.7

The Mandelbrot set is a fractal defined by the iterative equation shown below 2, where c is a complex number $x + iy$. The Mandelbrot set is the set of points in the complex plane that satisfy:

For a given complex value of c, start with $z = 0$ and iterate repeatedly. If the magnitude $|z|$ of the resulting value is ever greater than 2, then the point in the complex plane at position c is not in the Mandelbrot set, otherwise it is in the set. (Newman, pg 122)

$$z' = z^2 + c \tag{2}$$

The goal of this exercise was to create a program that would produce an image of the Mandelbrot set. To do this, I first defined a function that would calculate the Mandelbrot set given a particular number of iterations, the value $|z|$ cannot exceed, and the ranges of x and y, which were both between -2 and 2 for this exercise. Then, I input the values and produced the plot 2.

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3. SURVEY

This homework took me about 5 hours in total. This problem set felt a bit more manageable than the previous ones have, and I appreciated the ability to choose which two exercises I wanted to complete. I definitely learned more about iteration and its applications. I also feel like I've strengthened my plotting skills, and hope to bring some of them to my own research.

3.6)
import packages
define the equation
define the values of variables
    list of r values, x
calculate
    use for loop
create arrays of x and r values
plot

3.7)
import packages
define function to compute set
    define variables
    return set
plug in variables to function
plot