PHYS 304 HW 7

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1. EXERCISE 8.14

The one-dimensional time-independent Schrodinger equation is given by equation 1. The harmonic potential is given by 2, where V_0 and a are constants.

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi}{dx^2} + V(x)\Psi(x) = E\Psi(x) \tag{1}$$

$$V(x) = V_0 x^2/a^2 \tag{2}$$

For this exercise, $V_0 = 50eV$, a = $10^{-11}m$, and m is the mass of an electron. Breaking up the Schrodinger equation into two first order differential equations and using Runge-Kutta fourth order to evaluate them yields a ground state energy value of 178.38 eV, within the predicted range of 100-200 eV. Conducting the same steps with an anharmonic oscillator potential, 3, yields a ground state energy value of 205.31 eV.

$$V(x) = V_0 x^4/a^4 \tag{3}$$

2. EXERCISE 8.15

The double pendulum is a chaotic system with two masses of the same mass m, and two arms of the same length l. The equations of motion for the double pendulum are derived from the Lagrangian. The potential energy of the system is given by 4, and the kinetic energy is given by 5. Combining these two equations gives an equation for the entire energy of the system, 6, where ω has replaced $\dot{\theta}$.

$$V = -mgl(2\cos\theta_1 + \cos\theta_2) \tag{4}$$

$$T = ml^2[\dot{\theta}_1^2 + \frac{1}{2}\dot{\theta}_2^2 + \dot{\theta}_1\dot{\theta}_2\cos(\theta_1 - \theta_2)] \tag{5}$$

$$E = ml^2[\omega_1^2 + \frac{1}{2}\omega_2^2 + \omega_1\omega_2\cos(\theta_1 - \theta_2)] - mgl(2\cos\theta_1 + \cos\theta_2) \tag{6}$$

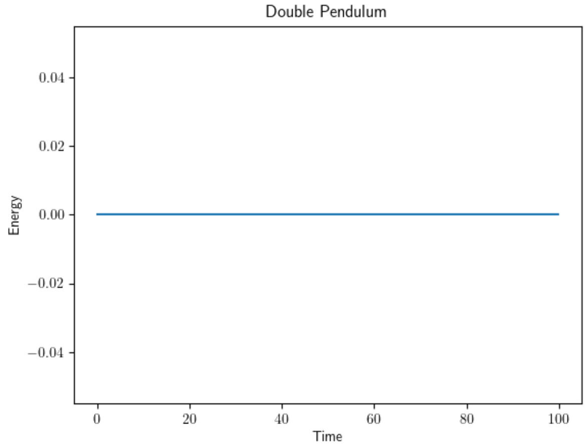

FIG. 1: Energy of a double pendulum over time.

Using the fourth order Runge-Kutta method, I was able to solve the equations of motion for the case where l is 40 cm, m is 1 kg, both θ start at 90°, and both ω equal 0. Plotting the calculated energy values against time, for a period of 100 seconds, yields a straight line, due to the conservation of energy.

3. SURVEY

I really struggled with this problem set. I believe it took me around 6-7 hours. I am not totally satisfied with my work here, but I have run out of time. For 8.14, I struggled for a long time to get a numerical answer rather than an error message, and I would like to thank George and Prof. Grin for their advice in the class slack channel. My issue with problem 8.15 came when calculating the energy. It took me a very long time to figure out the correct combination of arrays and numpy commands to be able to plug my values into the equation for E. I am still unsure if I did either of these problems correctly.