53.5/55

PHYS 304 HW 6

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Bryn Mawr College
(Dated: March 22, 2023)

1. EXERCISE 8.3

+24.5

The Lorenz equations are a set of differential equations that demonstrate the phenomenon of deterministic chaos. They are given by the following,

$$\frac{dx}{dt} = \sigma(y - x)$$
$$\frac{dy}{dt} = rx - y - xz$$
$$\frac{dz}{dt} = xy - bz$$

where r, b, and σ are constants.

To solve these equations, I utilized the Runge-Kutta 4th order integration method. I solved for the case where $\sigma = 10$, $r = 28$, and $b = \frac{8}{3}$, in the range of $t = 0tot = 50$. I first produced a plot of y values versus time 1, which demonstrates the equations' unpredictability. I then produced a plot of the z values against the x values 2, resulting in an oddly shaped plot known as the "strange attractor."

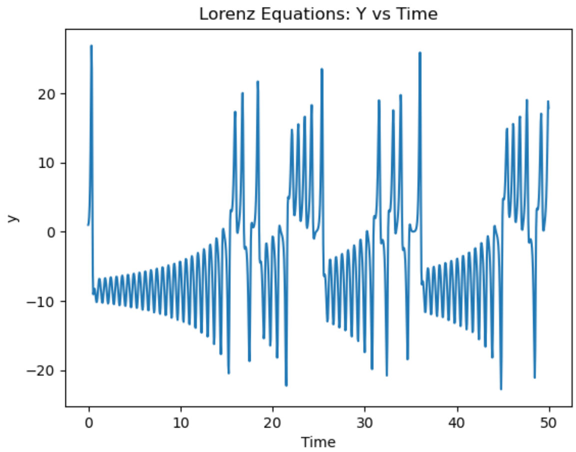

FIG. 1: Y as a function of time as given by the Lorenz equations.

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This plot could use more points to be smoother

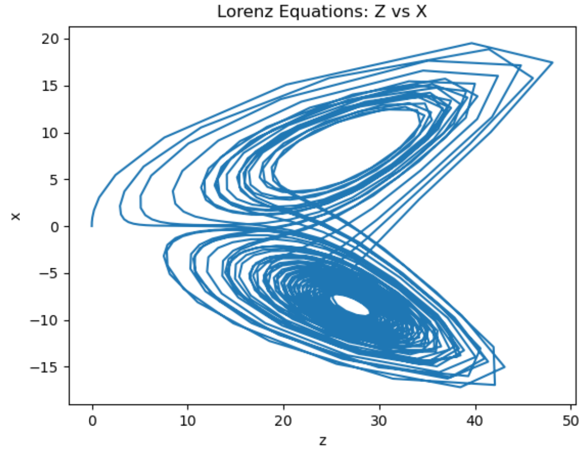

FIG. 2: The "Strange Attractor"

2. EXERCISE 8.4

+24

The equation for the motion of a pendulum is given by 1, where ω is defined as 2. These two first order differential equations were derived from a single second order differential equation.

$$\frac{d\omega}{dt} = -\frac{g}{l}\sin\theta \tag{1}$$

$$\frac{d\theta}{dt} = \omega \tag{2}$$

These two equations were solved by using the 4th order Runge-Kutta method. For this exercise, the length of the pendulum arm was said to be 10 cm (0.1 m), and the angle of the pendulum's release was 179°. g is 9.81 m/s. I was able to create a graph of the angle, θ, as a function of time 3, which shows the clear oscillatory nature of the system.

3. EXERCISE 8.5

This exercise builds upon the previous by adding a driving force to the pendulum system. The equation of motion in this case is given by 3, where C and Ω are constants. This was once again solved using the Runge-Kutta 4th order method.

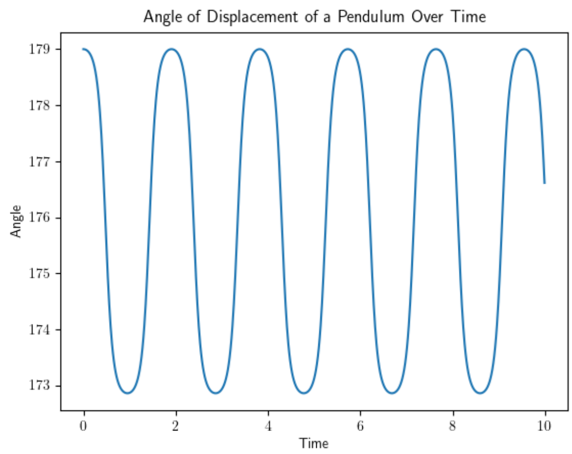

FIG. 3: The angle displacement of a pendulum over time.

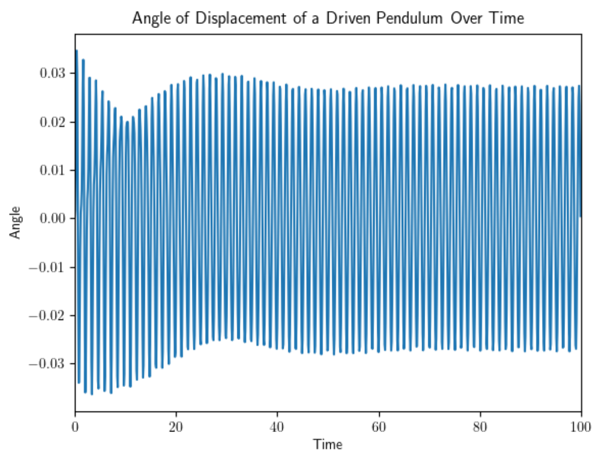

FIG. 4: Angle of displacement of a driven pendulum over time.

For this exercise, l and g are the same as in exercise 8.4, but the pendulum starts at rest and at 0°. C is set to $2s^{-1}$ and $\Omega = 5s^{-1}$. The time interval is from 0 to 100 seconds. The resultant plot is shown in figure 4. To determine the value for which the pendulum resonates with the driving force, you vary the value of Ω. I found that a value of $10s^{-1}$ resonated 5.

+5

4. SURVEY

I honestly do not remember exactly how long this homework took me. I would say it was probably around 4-5 hours, which is a perfect length for these problem sets. I enjoyed the Lorenz equations problem, especially playing around with the values to create interesting plots.

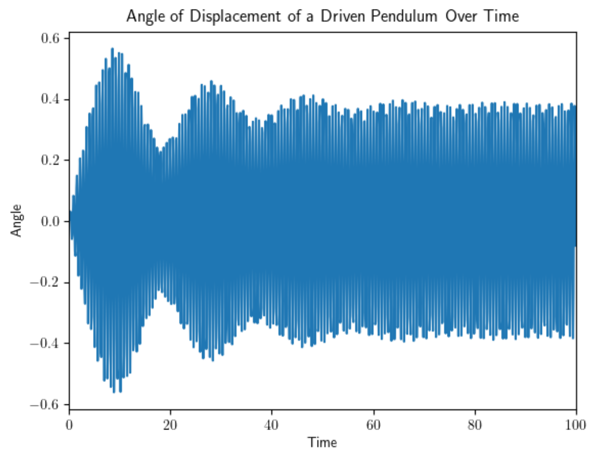

FIG. 5: Angle of displacement of a driven pendulum over time, with $\Omega = 10$

The beginning of these plots looks good but then they get muddled.

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin\theta + C\cos\theta\sin\Omega t \tag{3}$$

Problem 8.4/5

Lanava +24

Computational Physics/Astrophysics, Winter 2023:

Grading Rubrics [1]

Haverford College, Prof. Daniel Grin

For coding assignments, roughly 25 points will be available per problem.

+3 1. Does the program complete without crashing in a reasonable time frame? If yes, up to +3 points.

+1 2. Does the program use the exact program files given (if given), and produce an answer in the specified format? If yes, +1 points

+2

3. Does the code follow the problem specifications (i.e numerical method; output requested etc.) Up to +2 points

It looks like you did not use a high enough N in the Runge Kutta method. This causes your plots to diverge from what they should be after about t=20. When I changed N from 1000 to 10000 it worked great.

+3.5 4. Is the answer correct? Up to+4points

5. Is the code readable? Up to+2points

+2

. 5.1. Are variables named reasonably?

. 5.2. Are the user-functions and imports used?

. 5.3. Are units explained (if necessary)?

. 5.4. Are algorithms found on the internet/book/etc. properly attributed?

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[1] Inspired by rubric of D. Narayanan, U. Florida, and C. Cooksey, U. Hawaii

+3 6. Is the code well documented? +3points

. 6.1. Is the code author named?

. 6.2. Are the functions described and ambiguous variables defined?

. 6.3. Is the code functionality (i.e. can I run it easily enough?) documented?

+9.5 7. LaTeX writeup (up to 10 points)

. Are key figures and numbers from the problem given? (3 points)
. Is a brief explanation of physical context given? (2 points)
. If relevant, are helpful analytic scalings or known solutions given? (1 point)
. Are 3-4 key equations listed (preferably the ones solved in the programming assignment) and algorithms named? (2 points)
. Are collaborators clearly acknowledged? (1 point)
. Are any outside references appropriately cited? (1 point)

You should include more detail on the small angle approximation that is normally used for pendula and how this method is different.

Note, even if (1), (2), (3), or (4) are not correct, one can still obtain many points via (5), (6), and (7).

Problem 8.3

Lanava +24.5

Computational Physics/Astrophysics, Winter 2023:

Grading Rubrics [1]

Haverford College, Prof. Daniel Grin

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+3 1. Does the program complete without crashing in a reasonable time frame? If yes, up to +3 points.

+1 2. Does the program use the exact program files given (if given), and produce an answer in the specified format? If yes, +1 points

+2 3. Does the code follow the problem specifications (i.e numerical method; output requested etc.) Up to +2 points

You should use more points on the strange attractor plot so it is smoother

+3.5 4. Is the answer correct? Up to+4points

+2 5. Is the code readable? Up to+2points

. 5.1. Are variables named reasonably?

. 5.2. Are the user-functions and imports used?

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7. LaTeX writeup (up to 10 points)

+10
- Are key figures and numbers from the problem given? (3 points)
- Is a brief explanation of physical context given? (2 points)
- If relevant, are helpful analytic scalings or known solutions given? (1 point)
- Are 3-4 key equations listed (preferably the ones solved in the programming assignment) and algorithms named? (2 points)
- Are collaborators clearly acknowledged? (1 point)
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Note, even if (1), (2), (3), or (4) are not correct, one can still obtain many points via (5), (6), and (7).