```
sigz = {{1, 0}, {0, -1}}
```

```
{{1, 0}, {0, -1}}
```

```
externalfield = -KroneckerProduct[KroneckerProduct[sigz, id2], id2] -
  KroneckerProduct[KroneckerProduct[id2, sigz], id2] -
  2 KroneckerProduct[KroneckerProduct[id2, id2], sigz]
```

```
{{-4, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, -2, 0, 0, 0, 0, 0}, {0, 0, 0, 2, 0, 0, 0, 0}, {0, 0, 0, 0, -2, 0, 0, 0},
 {0, 0, 0, 0, 0, 2, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 4}}
```

```
id2 = IdentityMatrix[2]
```

```
{{1, 0}, {0, 1}}
```

```
spincoupling = KroneckerProduct[KroneckerProduct[sigz, sigz], id2] +
  2 KroneckerProduct[KroneckerProduct[sigz, id2], sigz] +
  2 KroneckerProduct[KroneckerProduct[id2, sigz], sigz]
```

```
{{5, 0, 0, 0, 0, 0, 0, 0}, {0, -3, 0, 0, 0, 0, 0, 0},
 {0, 0, -1, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0},
 {0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, 0, -3, 0}, {0, 0, 0, 0, 0, 0, 0, 5}}
```

```
hf = externalfield + spincoupling
```

```
{{1, 0, 0, 0, 0, 0, 0, 0}, {0, -3, 0, 0, 0, 0, 0, 0},
 {0, 0, -3, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, -3, 0, 0, 0},
 {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, -3, 0}, {0, 0, 0, 0, 0, 0, 0, 9}}
```

```
MatrixForm[hf]
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 \end{pmatrix}$$

```
u = MatrixExp[I hf t] // MatrixForm
```

$$\begin{pmatrix} e^{i t} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{-3 i t} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{-3 i t} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{i t} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{-3 i t} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{i t} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & e^{-3 i t} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{9 i t} \end{pmatrix}$$

```
Eigenvalues[hf]
```

```
{9, -3, -3, -3, -3, 1, 1, 1}
```

```
Eigenvectors[hf] // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
htotal[s_] := (1 - s) hinitial + s hf
```

```
hinitial = b (KroneckerProduct[KroneckerProduct[sigx, id2], id2] +
     KroneckerProduct[KroneckerProduct[id2, sigx], id2] +
     KroneckerProduct[KroneckerProduct[id2, id2], sigx])
```

```
{{0, b, b, 0, b, 0, 0, 0}, {b, 0, 0, b, 0, b, 0, 0},
 {b, 0, 0, b, 0, 0, b, 0}, {0, b, b, 0, 0, 0, 0, b}, {b, 0, 0, 0, 0, b, b, 0},
 {0, b, 0, 0, b, 0, 0, b}, {0, 0, b, 0, b, 0, 0, b}, {0, 0, 0, b, 0, b, b, 0}}
```

```
sigx = {{0, 1}, {1, 0}}
```

```
{{0, 1}, {1, 0}}
```

```
MatrixForm[hinitial]
```

$$\begin{pmatrix} 0 & b & b & 0 & b & 0 & 0 & 0 \\ b & 0 & 0 & b & 0 & b & 0 & 0 \\ b & 0 & 0 & b & 0 & 0 & b & 0 \\ 0 & b & b & 0 & 0 & 0 & 0 & b \\ b & 0 & 0 & 0 & 0 & b & b & 0 \\ 0 & b & 0 & 0 & b & 0 & 0 & b \\ 0 & 0 & b & 0 & b & 0 & 0 & b \\ 0 & 0 & 0 & b & 0 & b & b & 0 \end{pmatrix}$$

```
(1 - s) hinitial + s hf
```

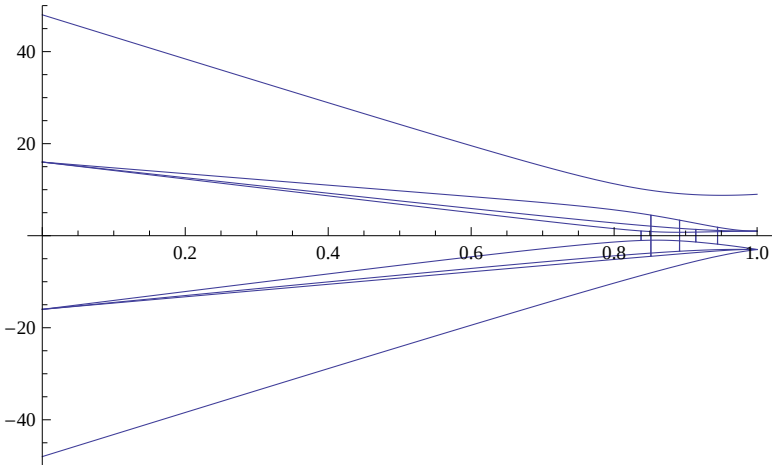
```
{{s, b (1 - s), b (1 - s), 0, b (1 - s), 0, 0, 0},
 {b (1 - s), -3 s, 0, b (1 - s), 0, b (1 - s), 0, 0},
 {b (1 - s), 0, -3 s, b (1 - s), 0, 0, b (1 - s), 0},
 {0, b (1 - s), b (1 - s), s, 0, 0, 0, b (1 - s)},
 {b (1 - s), 0, 0, 0, -3 s, b (1 - s), b (1 - s), 0},
 {0, b (1 - s), 0, 0, b (1 - s), s, 0, b (1 - s)},
 {0, 0, b (1 - s), 0, b (1 - s), 0, -3 s, b (1 - s)},
 {0, 0, 0, b (1 - s), 0, b (1 - s), b (1 - s), 9 s}}
```

```
Clear[htotal]
```

htotal[s]

```
{{s, b (1 - s), b (1 - s), 0, b (1 - s), 0, 0, 0},
 {b (1 - s), -3 s, 0, b (1 - s), 0, b (1 - s), 0, 0},
 {b (1 - s), 0, -3 s, b (1 - s), 0, 0, b (1 - s), 0},
 {0, b (1 - s), b (1 - s), s, 0, 0, 0, b (1 - s)},
 {b (1 - s), 0, 0, 0, -3 s, b (1 - s), b (1 - s), 0},
 {0, b (1 - s), 0, 0, b (1 - s), s, 0, b (1 - s)},
 {0, 0, b (1 - s), 0, b (1 - s), 0, -3 s, b (1 - s)},
 {0, 0, 0, b (1 - s), 0, b (1 - s), b (1 - s), 9 s}}
```

Plot[Eigenvalues[htotal[s]], {s, 0, 1}]



b = 16

```
16
```

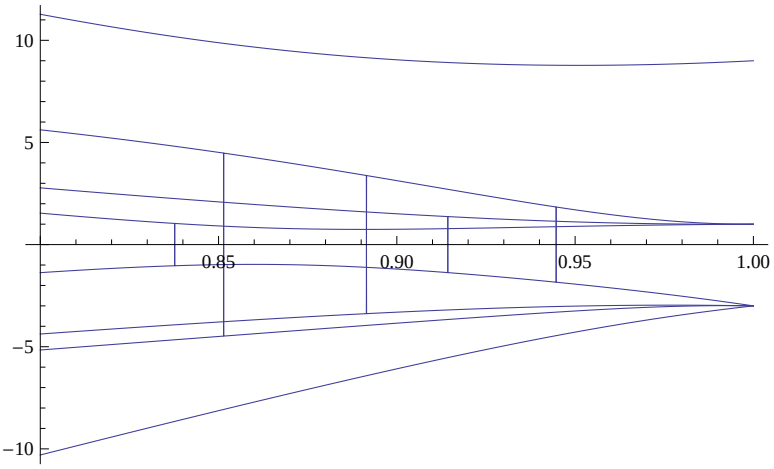
Eigenvalues[htotal[1]]

```
{9, -3, -3, -3, -3, 1, 1, 1}
```

Eigenvalues[htotal[0]]

```
{-48, 48, -16, -16, -16, 16, 16, 16}
```

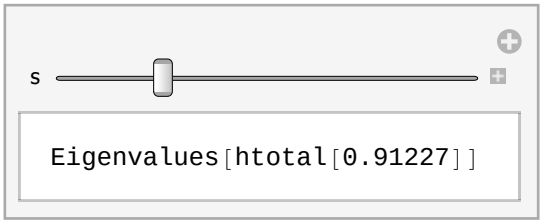
```
Plot[Eigenvalues[htotal[s]], {s, 0.80, 1}]
```




```
htotal[0.914] // MatrixForm
```

$$\begin{pmatrix} 0.914 & 1.376 & 1.376 & 0. & 1.376 & 0. & 0. & 0. \\ 1.376 & -2.742 & 0. & 1.376 & 0. & 1.376 & 0. & 0. \\ 1.376 & 0. & -2.742 & 1.376 & 0. & 0. & 1.376 & 0. \\ 0. & 1.376 & 1.376 & 0.914 & 0. & 0. & 0. & 1.376 \\ 1.376 & 0. & 0. & 0. & -2.742 & 1.376 & 1.376 & 0. \\ 0. & 1.376 & 0. & 0. & 1.376 & 0.914 & 0. & 1.376 \\ 0. & 0. & 1.376 & 0. & 1.376 & 0. & -2.742 & 1.376 \\ 0. & 0. & 0. & 1.376 & 0. & 1.376 & 1.376 & 8.226 \end{pmatrix}$$

```
Manipulate[Eigenvalues[htotal[s]], {s, .91, .92}]
```




```
(* cnot with 4 qubits *)
```

```
externalfieldc =
 KroneckerProduct[KroneckerProduct[KroneckerProduct[sigz, id2], id2], id2] +
  KroneckerProduct[KroneckerProduct[KroneckerProduct[id2, sigz], id2], id2] +
  KroneckerProduct[KroneckerProduct[KroneckerProduct[id2, id2], sigz], id2] +
  4 KroneckerProduct[KroneckerProduct[KroneckerProduct[id2, id2], id2], sigz]
```

```
{{7, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 5, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, -3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 5, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, -3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, -5, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 5, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, -3, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -5, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -5, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -7}}
```

```
spincouplingc =
 2 KroneckerProduct[KroneckerProduct[KroneckerProduct[sigz, sigz], id2], id2] -
  2 KroneckerProduct[KroneckerProduct[KroneckerProduct[sigz, id2], sigz], id2] -
  4 KroneckerProduct[KroneckerProduct[KroneckerProduct[sigz, id2], id2], sigz] -
  2 KroneckerProduct[KroneckerProduct[KroneckerProduct[id2, sigz], sigz], id2] -
  4 KroneckerProduct[KroneckerProduct[KroneckerProduct[id2, sigz], id2], sigz] +
  4 KroneckerProduct[KroneckerProduct[KroneckerProduct[id2, id2], sigz], sigz]
```

```
{{-6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, -6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 18, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, -6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, -6, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, -6, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -6, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 18, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -6, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -6}}
```

```
hfc = externalfieldc + spincouplingc
```

```
{{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 15, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 7, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, -9, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, -3, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, -3, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 7, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, -9, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -3, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -3, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 21, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -11, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -13}}
```

```
Eigenvalues[hfc]
```

```
{21, 15, -13, -11, -9, -9, 7, 7, -3, -3, -3, -3, 3, -1, 1, 1}
```

```
Eigenvectors[hfc] // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$