Environmental Modelling & Software 25 (2010) 129–145

Contents lists available at ScienceDirect

Environmental Modelling & Software

journal homepage: www.elsevier.com/locate/envsoft

Elicitator: An expert elicitation tool for regression in ecology

Allan James[a,b,*], Samantha Low Choy[b], Kerrie Mengersen[b]

[a] High Performance Computing and Research Support, Queensland University of Technology (QUT), Brisbane, Australia
[b] School of Mathematical Sciences, QUT, Brisbane, Australia

ARTICLE INFO

Article history:
Received 23 December 2008
Received in revised form
3 July 2009
Accepted 4 July 2009
Available online 13 August 2009

Keywords:
Elicitation software
Expert elicitation
Expert opinion
Bayesian statistics
Regression
Ecology
Java

ABSTRACT

Expert elicitation is the process of retrieving and quantifying expert knowledge in a particular domain. Such information is of particular value when the empirical data is expensive, limited or unreliable. This paper describes a new software tool, called Elicitator, which assists in quantifying expert knowledge in a form suitable for use as a prior model in Bayesian regression. Potential environmental domains for applying this elicitation tool include habitat modelling, assessing detectability or eradication, ecological condition assessments, risk analysis and quantifying inputs to complex models of ecological processes. The tool has been developed to be user-friendly, extensible and facilitate consistent and repeatable elicitation of expert knowledge across these various domains. We demonstrate its application to elicitation for logistic regression in a geographically based ecological context. The underlying statistical methodology is also novel, utilizing an indirect elicitation approach to target expert knowledge on a case-by-case basis. For several elicitation sites (or cases), experts are asked simply to quantify their estimated ecological response (e.g. probability of presence), and its range of plausible values, after inspecting (habitat) covariates via GIS.

Software availability

Title: Elicitator, a software tool for expert elicitation for regression
Developers: Allan James (main developer) and Samantha Low Choy (statistical computing)
Contact Address: High Performance Computing and Research Support and School of Mathematical Sciences, Queensland University of Technology, GPO Box 2434, Brisbane QLD 4001, Australia
Tel.: +61 7 31382308 (School of Mathematical Sciences, QUT)
Fax: +61 7 31382310
E-mail: elicitator@qut.edu.au
Software availability: above e-mail or contact authors
Software Requirements: Java JRE 1.6, MySQL 5
Program Size: 8.9Mb
Format for importing data: Text-readable (ASCII) files in a custom file format
Format for exporting output prior distributions: Text appropriate for input into WinBUGS 1.4

1. Introduction

Developing accurate models can often require large datasets. This can be a problem in environmental contexts where observational data are not yet available or are otherwise limited, costly to obtain or subject to design and quality concerns. Developing a model from such data can bias predictions (Manel et al., 2001), and in these cases, expert knowledge can provide a valuable contribution by addressing information gaps. Within the environmental context, expert knowledge has successfully contributed to models using a range of methodologies: estimating study bias in meta-analysis of demographic models of spotted owls in the US (Boyce et al., 2005); using dominance-based rough-set approach to classify brownfields in US cities to support environmental management (Chen et al., 2009); collating consensus opinion via the Delphi technique for assessing status of US wildlife species (Clark et al., 2006); using a fuzzy logic decision support system to inform wildlife relocation in Namibia (Paterson et al., 2008), assessing aquatic habitat suitability in Belgium (Mouton et al., 2009), or assessing soil condition in Argentina (Ferraro, 2009); and informing Bayesian model-based classification to delineate bioregional boundaries in Australia (Accad et al., 2005). Expert elicitation may also act as a crucial modelling tool: for uncertainty assessment of model simulations (Refsgaard et al., 2007), generating scenarios for assessing and predicting from environmental models (Mahmoud et al., 2009), and for assessing reliability to complement

* Corresponding author. School of Mathematical Sciences, QUT, Brisbane, Australia. Tel.: +61 7 31382308.
E-mail address: ar.james@qut.edu.au (A. James).

1364-8152/$ – see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.envsoft.2009.07.003

quantitative analysis and risk assessment (Van der Sluijs et al., 2005; Refsgaard et al., 2007).

Knowledge elicited from experts can be formally incorporated into statistical models, where expert opinion can be used to augment and complement observational data. Bayesian statistical modelling provides a useful framework for achieving this, by formulating elicited knowledge as informative priors. Communicating with experts to elicit regression parameters has been found useful in several contexts relevant to environmental applications, ranging from ecology to socio-economics. Ecological examples include quantifying impacts of grazing on species (Martin et al., 2005), assessment of vegetation condition (Low Choy et al., 2005, 2009a), prediction of the distribution of rare or threatened species (Denham and Mengersen, 2007; O'Leary et al., 2009; Murray et al., 2009), and water infrastructure management (Garthwaite and O'Hagan, 2000). Other examples include comparing medical treatments (Chaloner et al., 1993) or examining factors impacting on real estate prices (Denham and Mengersen, 2007), designing highway pavements (Kadane et al., 1980) and defining characteristics of university students (Winkler, 1967). In this paper we describe software devised to assist with elicitation of information suitable for use in regression models, for broad application across these contexts, environmental and otherwise.

From the modeller's perspective the easiest elicitation is the *direct* approach (Winkler, 1967), also called structural (Kadane et al., 1980). However, it requires experts to directly express their beliefs about parameters of prior distributions. In regression, a direct elicitation approach would require experts to quantify the impact of a change in a covariate value on the response variable, given the other covariates under consideration in the model. This requires that experts not only have a good understanding of the relative impact of covariates, but can also interpret regression parameters (possibly in the context of a response transformed via a link function). In practice however, many experts often find it difficult to express, or indeed conceptualise, their knowledge directly in this way. Despite its simplicity from the statistical modeller's perspective, a direct approach to elicitation can sometimes produce less accurate results compared to an indirect approach, especially from experts where the notion of probability is a somewhat foreign concept (Ayyub, 2001, pp. 113; Winkler, 1967). Instead many experts are more comfortable estimating observables, which in regression is equivalent to estimating responses, given specific values of covariates (Low Choy et al., 2009a; Recommendation iv). Expert preferences for supplying exemplar decisions rather than "developing rigid model-based preference rules" have been noted in other environmental modelling contexts (Greco et al., 2001; Chen et al., 2009). This motivates the focus of this paper on an *indirect* approach to elicitation (Winkler, 1967), which is easier for experts, but often requires more effort by the modeller in designing the elicitation and encoding method to transform expert responses into the required form.

Designing effective elicitation to suit available expert knowledge within a modelling framework that also embraces observable data can be challenging (Low Choy et al., 2009a). When eliciting a probability, depending on the exact information elicited, dozens of different methods have been proposed to encode the prior distribution (Hughes and Madden, 2002; O'Hagan et al., 2006). These include the fractile approach of eliciting quantiles, its reverse the interval approach of eliciting cumulative probabilities, or a hybrid of these (Winkler, 1967; Spetzler and Staël von Holstein, 1975). Elicitation software could potentially support several encoding methods to help communicate with a wide range of experts (e.g. Denham and Mengersen, 2007).

In addition to flexible communication, a number of immediate benefits have been identified for using software tools to assist in the expert elicitation process (Low Choy et al., 2009b). Compared to more traditional hard-copy materials, software-based tools provide more flexibility and immediate feedback (e.g. visualisation using graphs). When designed for general application, such tools streamline elicitation in a range of different contexts, and by incorporating a number of encoding methods and options, tools allow elicitation to be tailored to meet the specific needs of experts and researchers. They also support a consistent, repeatable, and structured method of elicitation, with more robust results when complex calculations are required *in situ*. Moreover, software tools may be more accessible to geographically dispersed experts because the modeller need not be present to undertake the calculations. Finally, digital media may be easily integrated into the elicitation process, such as GIS maps, photography, video and audio.

A few software tools have been developed specifically for elicitation of expert opinion suitable for input into Bayesian regression. Du Mouchel (1988) reports a graphical tool for directly eliciting multiple comparisons of regression coefficients. Stand-alone tools have been developed to train experts, elicit particular quantiles and provide feedback; see for example O'Hagan (1997) for asset management in the water industry and Leal et al. (2007) for describing patient risks. Chaloner and Duncan (1983) developed a graphical tool for indirect elicitation of the probability of disease under different treatments, in order to estimate the regression coefficients for new treatments in a survival model. One indirect approach requires experts to describe univariate response curves by specifying fractiles of the response conditional on one covariate. These tools have been implemented as a toolbox for WinBUGS (Kynn, 2006; Spiegelhalter et al., 2003) or in Java (Al-Awadhi and Garthwaite, 2006), and were inspired by the conditional mean prior approach of Bedrick et al. (1996). More recently, software was developed by Denham and Mengersen (2007) to provide an interactive, site-by-site approach to elicitation, based on the posterior predictive distribution (Kadane et al., 1980, Kadane and Wolfson, 1998). It was tailored to a specific environmental problem and was embedded within a commercial Geographic Information System (GIS).

This paper describes a software tool, *Elicitator*, which extends considerably the work of Denham and Mengersen (2007), both methodologically and computationally. In particular, it comprises a new underlying encoding method, is stand-alone in nature, uses open-source components, can be used for a wide range of applications, and is designed to be generally extensible to a range of regression models. It thus arguably fills a much-needed niche in the environmental modelling software arena for elicitation (Leal et al., 2007). The tool is demonstrated via a case study that aims to develop a logistic regression model to describe the habitat suitability and predict the geographic distribution of a rare species, the Australian brush-tailed rock-wallaby (*Petrogale penicillata*) (Murray et al., 2008, 2009). As is typical in these and similar situations, data are sparse, with observed presences and absences at sites barely representative of complex environmental gradients across a given geographical region (Murray et al., 2008). In these situations, more comprehensive information (both spatially and ecologically) may be available from ecologists who are expert in the field of interest (O'Leary et al., 2009).

The specification of the Elicitator software tool is detailed in Section 2, commencing with an outline of the motivation for its development and a summary of the benefits of using the software (Section 2.1), then proceeding to a specification of the statistical methods and technical details of using the system to perform elicitations (Section 2.2). In Section 3, an insight into the system architecture and the design strategies employed during development is provided. Finally, the discussion in Section 4 addresses broader issues including potential further extensions to the software.

2. Specification

2.1. Overview

Packaging well-designed elicitation methods has several benefits, since it promotes the use of a designed approach to elicitation, by providing carefully considered, default and flexible choices for specifying the steps involved, as described by Low Choy et al. (2009a): (E1) motivation and (E2) goal of elicitation, (E3) the statistical model, (E4) underlying computations to translate elicited information into statistical distributions via an encoding method, (E5) management of ever-present uncertainty, and (E6) practical elements within an elicitation protocol. Elicitator has been designed explicitly to meet these aims, as detailed below.

Elicitator may generally be applied (E1) to a wide range of regression problems, such as habitat modelling (O'Leary et al., 2009), environmental niche modelling (Austin, 2002), species distribution mapping and detectability (Guisan and Zimmermann, 2000) or other ecological responses to environmental change (Martin et al., 2005). The tool supports a variety of applications and users, whose main motivation for eliciting information may be to derive a stand-alone expert-defined regression (Alho et al., 1996) or to formulate informative priors to supplement observational data within a full Bayesian regression (see case studies in Low Choy et al., 2009a). Open-source libraries have been used, which not only widens the applicability of the tool, but also prolongs its lifetime, by removing dependence on commercial products (and thus reducing development and maintenance overheads).

The tool provides several graphical interfaces to help experts communicate their knowledge (E2). Elicitator assists experts to estimate the response in a regression for a set of cases, which may for example be geographically defined sites, given covariates corresponding to each case. This elicitation approach suits experts who are more comfortable estimating the observable response for known covariates (Denham and Mengersen, 2007), compared with other elicitation approaches available for regression (O'Leary et al., 2009). The tool may be tailored to various applications or types of experts using the various options provided for elicitation.

The tool provides a basis for a standard elicitation approach generally suitable for a wide range of geographically based case studies (E1). To cater for experts whose knowledge is largely location-based (Leal et al., 2007), a map-based elicitation approach can be adopted, where experts explore and query the spatial data relating to elicitation sites by viewing a GIS alongside Elicitator (Denham and Mengersen, 2007).

A multi-phase approach has been employed in the development of Elicitator. The first phase (v1.0) is described in this paper, and implements a constrained statistical and elicitation model whilst focussing on developing the extensible application framework. However, the tool has wider potential for extension to elicitation for generalized linear models (GLMs) (McCullagh and Nelder, 1989) due to its underlying modular design (E3). For specificity, this first development phase focuses on logistic regression, providing a test-bed for later extension to GLMs.

The elicitation interface and protocol are highly similar to the prototype (Denham and Mengersen, 2007). However the underlying encoding method (E4) used to translate elicited information into statistical distributions is new, and is described in more detail in Section 2.2. An indirect elicitation method is used since it is well suited to the types of experts encountered in environmental contexts (see case study; Denham and Mengersen, 2007; Low Choy et al., 2009a). Elicitator adopts an indirect case-by-case approach to elicitation that extends the conditional means approach of Bedrick et al. (1996), which is more tractable (E3) and more applicable in general (E1) compared to the posterior predictive approach of the prototype. Essentially the expert is asked, for a number of cases, to estimate the value, and plausible range, of conditional means for known values of covariates. In logistic regression the conditional mean is the probability of success. These elicitations are compiled across several cases and analysed to provide prior distributions of regression coefficients. We modify the approach of Bedrick et al. (1996), where estimation relies on the rather severe assumption that the number of elicitations equals the number of covariates. To facilitate estimation for the more typical situation where the number of elicited values may exceed the number of covariates, we reformulate the elicitation model as a measurement error model (*in sensu* Lindley, 1983).

To facilitate accurate elicitation (E5), Elicitator provides immediate informative graphical feedback, which improves the quality of elicitations (Kadane and Wolfson, 1998). Several interactive feedback and diagnostic graphs are provided to achieve this. Univariate response graphs highlight the main effects of each covariate associated with each of the elicitation sites, where the elicited probability (y-axis) is plotted against the values of the site covariates (x-axis), and a standard regression plot is then overlaid to assess the fit of the encoded prior model. A fortunate by-product of the new elicitation model formulation (E3) is its expression as a regression model, so that various diagnostic graphs are also available that enable the modeller and experts to evaluate the elicited values aggregated across all elicitation sites (see Section 2.3.3 for more details). The complexity of the highly multivariate relationships underlying regression is hidden from the expert, and tedious computations undertaken by the tool free the elicitor to focus on guiding the expert.

Modern Graphical User Interface (GUI) functionality is exploited to improve communication with the expert, thereby improving accuracy (E5) and practicality (E6). The tool allows experts to explore and query the covariate values representing environmental characteristics for each case. Where cases correspond to geographic locations, a case-by-case covariate database is easily obtained using a spatial intersection operation performed within a Geographic Information System (GIS). Hence the tool and a GIS are loosely coupled, providing a link to support further spatial analysis and visualisation, including mapping of contextual information and covariates. This relaxes the tight coupling of the software produced by Denham and Mengersen (2007), which was embedded within a commercial GIS package. The advantages of loose coupling are reduced dependence on complex GIS data structures and on version changes, and more flexibility to consider cases without geographical information.

To facilitate a transparent and repeatable elicitation, the tool provides several features (E6). Automation of several elicitation steps contributes to standardisation and reduces time required by both experts and elicitors, and therefore reduces overall cost and resource requirements of elicitation projects. Overall, streamlined logistical management of the elicitation process reduces potential for administrative and computational errors by elicitors. The software also provides a platform for transparent and repeatable elicitation that helps maintain standards and consistency in elicitation methodology, and information across individual elicitations and elicitation projects. Output from the model is in the form of probability distributions that are immediately interpretable in their own right, and may also act as priors in subsequent Bayesian analyses.

Elicitator has been developed to be flexible by allowing for the addition of sites at any time during the elicitation process, since the elicitation method does not explicitly condition on sites, enabling a staged elicitation approach. However since the elicitation method does explicitly condition on covariates, Elicitator allows for addition of new covariates at the outset of the elicitation process. This first version of the tool does allow for the addition of sites without restarting the elicitation, and currently has capacity to add covariates or restrict encoding to a subset of covariates selected by the modeller. Another benefit of the elicitation method used in Elicitator is that the conditioning on covariate values is quite flexible. The modeller may use the same elicited information to investigate different combinations and functional forms of covariates used to encode the priors (Bedrick et al., 1996; Denham and Mengersen, 2007). In the current version of Elicitator, the modeller may import a previous set of elicitations. The columns in the covariate dataset determine which variables are to be used when encoding the prior; thus the modeller can edit this covariate table before importing to change the variables that are used. In future development phases, the flexibility of the tool will be improved, both by allowing the modeller to include additional covariates, and also allowing the modeller to encode the prior model based on different sets of covariates. This contrasts with some other elicitation methods for regression, which explicitly condition on the covariates and their functional form, so that elicitation must be performed for *each covariate set* of potential interest to modellers (Kynn, 2005, 2006; Al-Awadhi and Garthwaite, 2006; Kuhnert et al., 2005; O'Leary et al., 2009).

2.2. Statistical elicitation method

The aim of elicitation is to derive an expert-defined model, for potential use as a "prior" model within the Bayesian framework (Section 2.2.1). To capture the expert's conceptual model, we propose a new measurement error formulation using Beta regression, which extends the ideas of Bedrick et al. (1996) (Section 2.2.2). This framework for aggregating expert opinions across sites, by necessity, defines what information is required from experts at each site. We describe the method for encoding expert opinions at each site into individual Beta distributions (Section 2.2.3). The elicitation model provides a basis for interpretation, especially uncertainty (Section 2.2.4). Implementation in Elicitator means addressing several computational issues (Section 2.2.5) as well as exploiting the regression framework (Section 2.2.6). This elicitation method is presented in a different format for a statistical audience in Low Choy et al. (in press).

2.2.1. Aim of elicitation: a Bayesian prior distribution

Statistical inference for regression within a classical context focuses on the likelihood function $p(Y|X,\beta)$, which specifies a sampling model that relates the observed data Y to model parameters β and covariates X. Typically, inference proceeds via numerical optimisation of the likelihood over the parameter space, such as maximising the likelihood or minimising prediction errors. In a Bayesian context a prior distribution $p(\beta)$ is specified, describing the statistical distribution (or plausible values) for model parameters before observing the empirical data, and may itself depend on hyper-parameters ϕ, so that $p(\beta) = p(\beta|\phi)$. Inference is then based on the posterior distribution of the parameters, given the observed data and the prior, via Bayes Theorem:

$$p(\beta|X,Y,\phi) \propto p(Y|X,\beta)p(\beta|\phi) \tag{1}$$

This assumes that the priors and likelihood have been independently specified, so they cannot, for example, rely on the same information set. In the case study, this meant that elicited information used to define the prior on β was obtained from experts who had not seen observed data Y. See Ellison (2004) for a readable introduction to Bayesian statistics in an ecological setting.

For specificity, we describe the elicitation method in the context of a logistic regression model that falls within the broader class of generalized linear models (McCullagh and Nelder, 1989). Suppose observations on a binary response Y_i (such as presence/absence data) are available for several independent cases, labelled $i = 1,\ldots,N$, which may represent geographic sites for example. A standard logistic regression model assumes a Bernoulli distribution for observations Y_i indexed by a probability of success (such as presence) θ_i with a logit link to a linear combination of the J covariate values $X_{i1},\ldots,X_{iJ}$ for that case. For logistic regression the conditional mean is the probability of success $E[Y_i|X_i] = \theta_i$. Thus, the model and likelihood within Equation are:

$$\begin{aligned} &Y_i = \mathrm{Bern}(\theta_i) \text{ with } \mathrm{logit}(\theta_i) = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \cdots + \beta_J X_{iJ} \\ &\text{So that } p(y_i|X_i,\beta) = \frac{\exp\left(y_i X_i^T \beta\right)}{1 + \exp\left(X_i^T \beta\right)} \end{aligned} \tag{2}$$

In the Bayesian paradigm, expert knowledge can be introduced to supplement limited observational datasets by specifying informative prior distributions for the regression parameters β (O'Hagan et al., 2006; Low Choy et al., 2009a). Typically, normal prior distributions are adopted, with zero covariances to reflect the assumption that covariates are essentially orthogonal:

$$\beta_j \sim N\left(\mu_j, \sigma_j^2\right), \quad j = 1,\ldots,J \tag{3}$$

If collinearity amongst covariates is non-negligible, then a multivariate Normal distribution can instead be specified (Denham and Mengersen, 2007). A weakly informative prior sets the prior mean to zero to favour neither negative nor positive effects ($\mu_j = 0$) with a wide variance to reflect vague knowledge ($\sigma^2 \to 0$).

The purpose of this paper is to describe how Elicitator works with experts to estimate the hyper-parameters, being the prior expected values μ_j and the prior standard deviations σ_j of the regression coefficients β_j in Equation (3). This expert-defined prior can then be input to a Bayesian analysis together with *observed* data X to provide updated posterior estimates using Bayes Theorem (Equation (1)). In the next sections we describe how *expert* "data" is used to define the prior.

2.2.2. Combining elicitations via beta regression, a measurement error model

A *direct* elicitation approach would simply require eliciting μ_j and σ_j^2 from experts (e.g. Fleishman et al., 2001). Importantly, this requires an explicit understanding of the potential nature and magnitude of the effect of each covariate on the response Y, accounting for all the other covariates in the model as well as the transformation via the link function. However, often the expert's knowledge cannot be easily expressed in this form, especially when there is more than a single covariate. In these cases we may apply an *indirect* approach, such as the conditional means approach (Bedrick et al., 1996), which focuses on eliciting the distribution of the conditional mean response $E[Y_i|X_i,\beta]$. For a logistic regression, this amounts to seeking information from the expert on the probability of success θ_k, for a number of cases $k = 1,\ldots,K$ with known combinations of covariate values $X_{k1},\ldots X_{kJ}$. For example, in habitat modelling this means asking the expert about the probability of presence at a number of sites with known habitat and environmental predictors. Specifically, Bedrick et al. (1996) assign a Beta prior distribution to θ_k:

$$\theta_k \sim \mathrm{Beta}(a_k, b_k) \tag{4}$$

Under the condition that $K = J$, that is the number of elicitations equals the number of covariates, the prior (Equation (4)) on the conditional mean $p(\theta_k|a_k,b_k)$ induces a prior distribution on the regression coefficients (Bedrick et al., 1996):

$$p(\beta|a,b) = \prod_{k=1}^{K} \frac{\exp\left\{a_k X_k^T \beta\right\}}{\left(1+\exp\{X_k^T\beta\}\right)^{a_k+b_k}} \tag{5}$$

However, in practical situations such as the case study, it is likely that the number of elicitations exceeds the number of covariates, so that Equation (5) does not apply. To address this we may reformulate the elicitation model as a measurement error model (Equation (6)), similar in spirit to Lindley (1983). Then the expert's opinion on the probability of presence Z_k is a measurement of its expected value, being the true probability of presence θ_k. This true probability of presence is a latent (unobserved) variable, and it is this true value rather than the expert's opinion, which is related to the linear predictor involving covariates X and parameters β as in Equation (2). This measurement error model constitutes a Beta regression (e.g. Branscum et al., 2007) although we have reparameterized shape and scale parameters a_k and b_k in terms of the effective prior sample size γ_k and expected value θ_k:

$$\begin{aligned} &Z_k|\theta_k \sim \mathrm{Beta}(a_k,b_k), \quad E[Z_k|\theta_k] = \theta_k, \\ &\mathrm{logit}(\theta_k) = X_k^T\beta \quad \gamma_k = a_k + b_k \end{aligned} \tag{6}$$

In Elicitator, for each site the expert "datum" Z_k is elicted, together with sufficient information to estimate a_k, b_k and therefore γ_k (see Section 2.2.3). There are then several options for estimating regression coefficients β. Typically in encoding a simple Beta distribution, without the complication of the relationship to regression covariates, a deterministic approach is used (e.g. O'Hagan et al., 2006; pp. 124–132). This involves eliciting the minimal amount of information required to solve equations relating elicited quantities (e.g. mode, mean or median and quantiles or cumulative probabilities). As described above, Bedrick et al. (1996) took a deterministic approach to encoding regression coefficients when conditional means are elicited. However, when more than minimal information can be comfortably elicited (here $K > J$), then a statistical rather than deterministic approach to encoding can be taken, potentially providing more accurate encoding as well as providing an estimate of elicitation error (Low Choy et al., 2008). By eliciting expert "data" $\{Z_k, k = 1,\ldots,K\}$, with expected value set to θ_k (Equation (2)), we enable a statistical rather than deterministic approach to encoding (Bedrick et al., 1996).

2.2.3. Encoding probability of presence at each site

The challenge is now to elicit sufficient information from the expert to encode the probability distribution, or range of plausible values, for the probability of success, by specifying shape and scale parameters a_k and b_k in Equation (6). Estimation of these two parameters minimally requires elicitation of two summary statistics about the required prior distribution of the expert's assessment of the probability of presence (Low Choy et al., 2008). The most obvious starting point is to ask the expert for their *best estimate* of the probability of presence at a site. Typically measurement error models are structured so that the arithmetic mean of the measurement (here the elicited probability of presence) is set equal to the true value. However, conceptually the arithmetic average is difficult to elicit, particularly for skewed distributions such as the Beta. The median is often elicited (O'Hagan et al., 2006; Kynn, 2005; Denham and Mengersen, 2007), but again requires cognitive effort from the expert to determine a value, such that it is equally likely that the probability of presence falls above or below it. Instead, using Elicitator the expert is asked for the mode, which relates to the more intuitive concept of their most likely estimate of the probability of presence for a particular scenario, or set of covariates X_k. Mathematically, the mode m_k of Z_k is related simply to the Beta parameters via $m_k = (a_k - 1)/(a_k + b_k - 2)$. Hence the elicitation (measurement error) model is centered on the easily conceptualised mode.

Full characterisation of the Beta distribution requires additional information. In Elicitator, we follow the well-established practice (e.g. Kynn, 2005; O'Hagan et al., 2006) of eliciting several quantiles, which is particularly important with skewed distributions (Low Choy et al., 2008). Asking the expert at the outset to specify the upper and lower bounds (corresponding to a 99% or 95% credible interval) on the plausible values for the probability of presence helps broaden their thinking initially, and thus helps to avoid over-conservatism arising from anchoring biases (Kynn, 2008; Low Choy et al., 2009a).

Experts are also asked to estimate another interval, such as the 50% credible interval (CrI), so that the probability of presence has a 25% chance of falling either below or above these bounds. In practice, we have found that asking for the 95% and 50% CrIs before seeking the mode is also crucial for avoiding representation bias, where experts may confuse the 50% CrI for Z_k with the precision of their estimate of its mean or mode. Elicitator uses a simple numerical procedure to determine the closest fitting Beta distribution corresponding to the mode and either two or four of these quantiles, fixing b as a function of the specified mode λ. If only two quantiles are used for fitting, then the remaining two can be used for feedback (Kynn, 2005). Alternatively, instead of estimating the mode and four quantiles, the mode and just one quantile could be estimated by the expert (e.g. median and upper quantile), allowing algebraic solutions to be used (Denham and Mengersen, 2007). However, the latter approach tends not to be as accurate, unless the expert has substantially greater difficulty assessing the additional three quantiles (Low Choy et al., 2008). The modular structure of Elicitator allows for later addition of alternative methods of encoding distributions such as these.

2.2.4. Interpretation

This formulation of the elicitation model introduces a separation between the expert's stated opinions Z_k and their underlying conceptual model θ_k. Instead of assuming that the expert statements precisely reflect their underlying beliefs, the measurement error formulation allows for an expert to inaccurately communicate their beliefs. In addition it provides a simple regression-based framework for aggregating expert beliefs across sites, whilst adjusting for expert "error". Thus asking experts to assess probability of presence Z at an increasing number of sites, K, will lead to increasingly more accurate estimation of the expert's underlying "conceptual model" for how probability of presence θ relates to habitat covariates X. This approach differs subtly from that proposed by Bedrick et al. (1996) which implicitly assumes that expert stated beliefs at J sites precisely (and are therefore sufficient to) reflect their underlying conceptual model.

Due to the formulation as a regression problem, it is straightforward to extend to a weighted regression by incorporating weights for each site. These weights may, for instance, be used to reflect the expert's varying uncertainty in their estimates of probability of presence at each site. For example, they may find it easier to identify habitats that clearly correspond to absence or to presence, but greater difficulty estimating probability of presence for intermediate habitats.

Thus the elicitation method based on Beta regression captures two major sources of uncertainty.

1. The measurement error model addresses the discrepancy between the expert's stated opinions (at individual sites) and the underlying conceptual model (across sites), and provides

a basis for using regression to aggregate expert opinions at individual sites.
2. A weighted regression provides a basis for including expert uncertainty about their assessment at each site. A Beta distribution is used to represent expert opinion on the range of plausible values for the probability of presence for each covariate set (e.g. habitat profiles). This captures two additional sources of uncertainty.
3. The expert's assessment of the probability of presence at the site Z_k with known covariates X_k, is interpreted as the most likely value for this probability at similar sites with similar covariate values.
4. The effective sample size γ_k can be interpreted to reflect the expert's assessment of the range of possible values for the probability of presence at similar sites, having similar covariate values.

2.2.5. Computation in Elicitator

To perform the Beta regression of expert "data" Z on covariates X (Equation (6)), which will provide expert-defined estimates of β, either a Bayesian or classical approach may be taken. Modern Bayesian approaches to inference for logistic regression would require implementation of Markov Chain Monte Carlo (MCMC) algorithms (Marin and Robert, 2007), which are difficult to automate (for inclusion in a tool like Elicitator) since convergence of the MCMC algorithm will depend on the problem's expert "data" as well as covariates. In many cases, before eliciting expert knowledge no prior information is typically available, so a Bayesian analysis with non-informative priors will provide similar estimates of model parameters β compared to a classical (Frequentist) approach (Marin and Robert, 2007). For both these reasons, and in order to simplify encoding, we therefore chose to implement a classical approach to encoding the Beta regression prior model using Elicitator.

There are few packages available that implement Beta regression, even within a classical framework. Unfortunately, it was not possible to utilise the additional information on expert uncertainty provided by γ_k in *betareg* (de Bustamante Simas and Rocha, 2008), which is the only library on a suitable platform (Sections 2.2.5 and 3.6) that performs maximum likelihood estimation for Beta regression (Ferrari and Cribari-Neto, 2004). However we may approximate the Beta regression problem $Z_k \sim \text{Beta}(a_k, b_k)$, with linear predictor defined in Equation (2), as a Binomial regression problem, which is essentially a discrete version with the same linear predictor. There are two different ways of formulating a binomial regression with the probability of success θ_k and the same linear predictor. One approach would simplify the expert's opinion into whether they consider that a presence or absence would occur at these types of sites (with covariates X), thereby assigning the expert response Z^* a Bernoulli distribution. This Bernoulli distribution has a single parameter θ_k, that determines both the expected value and the variance. However this single parameter is insufficient to distinguish between habitat profiles with the same probability of presence θ_k but different variability; sites in one habitat may have wider range of probabilities of presence than in the other habitat. Therefore we choose an alternative approach that retains the information elicited on γ_k, which captures the varying probabilities of presence across sites with similar covariates (Item 4, Section 2.2.4). This is achieved by defining $Z_k^* \sim \text{Bin}(\theta_k, \gamma_k^*)$ with $\gamma_k^* = \text{Ceiling}[\gamma_k]$, a discretized version of the effective sample size γ_k. To fit the binomial regression the response is rescaled as $Z_k^* = Z_k \gamma_k^*$ which represents the expert's assessment of how many sites out of γ_k^* sites, with similar covariate values, that they consider will have a presence.

2.2.6. Benefits

A key benefit of the measurement error formulation as a Beta or Binomial regression is that the standard regression diagnostics provide feedback to the expert on goodness-of-fit of the encoded prior model, or how well the prior model fits their assessments at each elicitation site. Elicitator utilizes R (R Development Core Team, 2008) to estimate β using Binomial regression with two parameters via a classical approach, essentially equivalent to a non-informative Bayesian approach (Marin and Robert, 2007). This provides a point estimate $\widehat{\beta}_j$ and standard error $\text{se}(\widehat{\beta}_j)$ which can be interpreted as the prior estimate of the mean and standard deviation of the regression coefficient used to define the prior in (1):

$$\beta_j \sim N\left(\widehat{\beta}_j, \text{se}\left(\widehat{\beta}_j\right)\right) \tag{7}$$

2.2.7. Implementation specifications

In summary, the main calculations required are: implementation of a numerical algorithm to encode the Beta distribution representing the expert site-by-site assessments of the probability of presence (Equation (6); Section 2.2.3); and application of iteratively reweighted least squares to fit a binomial regression to aggregate expert opinions across sites and encode the prior distribution for the regression coefficients (Equation (7); Section 2.2.2).

In addition some exploratory data analysis is also required: calculation of "pretty" histograms to represent the range of covariate values represented within the elicitation sites; calculation of summary statistics from Beta distributions to provide feedback to the expert on site-by-site encoding; and computation of regression diagnostics to help the experts review their aggregated assessments.

These statistical calculations are implemented in Elicitator by utilizing existing and well-tested functions within the freely available R statistical package (R Development Core Team, 2008; Venables and Ripley, 2002). The tool communicates with R via a TCP/IP server application called RServe (Section 3.6).

2.3. Software organisation

A flowchart of the basic elicitation process is shown in Fig. 1. There are five main stages to consider when using the software for an expert elicitation session: initial preparation, the elicitation of expert knowledge, encoding expert knowledge into statistical distributions, feedback and review of the elicited values (with re-elicitation if necessary), and export of the generated expert model, which may be used as Bayesian priors to combine expert knowledge with observed data in (1). In all of these stages, except the computational encoding stage, users may interact graphically with the system. We now discuss each stage in more detail.

2.3.1. Project preparation

After loading the software, users are presented with the main application window. From this window the user is able to create a new elicitation project, or load an existing project. The software is based around a project model where all data related to a common set of elicitation sessions are encapsulated within a project. When a project is created or loaded successfully, a properties panel appears within a tab in the main window (Fig. 2), providing information about the currently active project.

Each project must contain one or more project *phases*. All elicitation sites and experts added to the project are linked to all phases of a project. Having multiple phases for a project allows numerous elicitation sessions to be performed with the same set of experts across the same collection of sites. This is useful for ensuring consistency when, for example comparing elicitations performed at

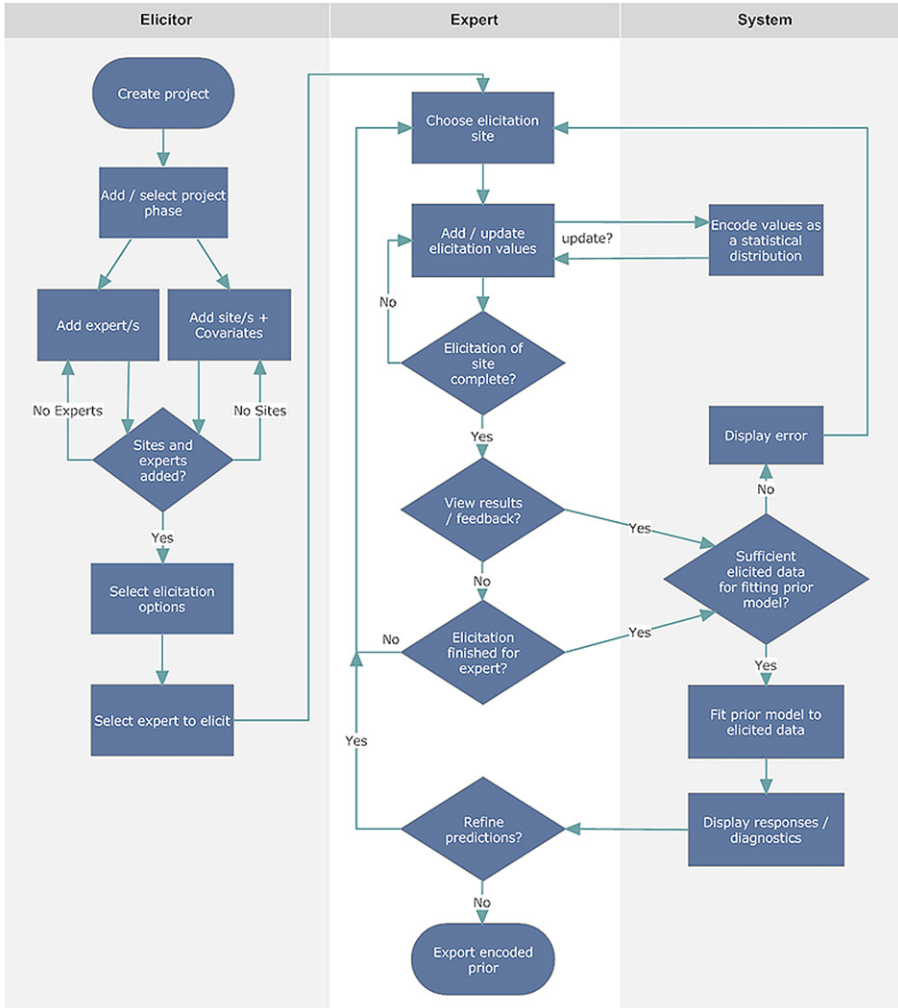

Fig. 1. Flowchart depicting the elicitation process when using the Elicitator tool, highlighting interactions between the elicitor, expert and the system.

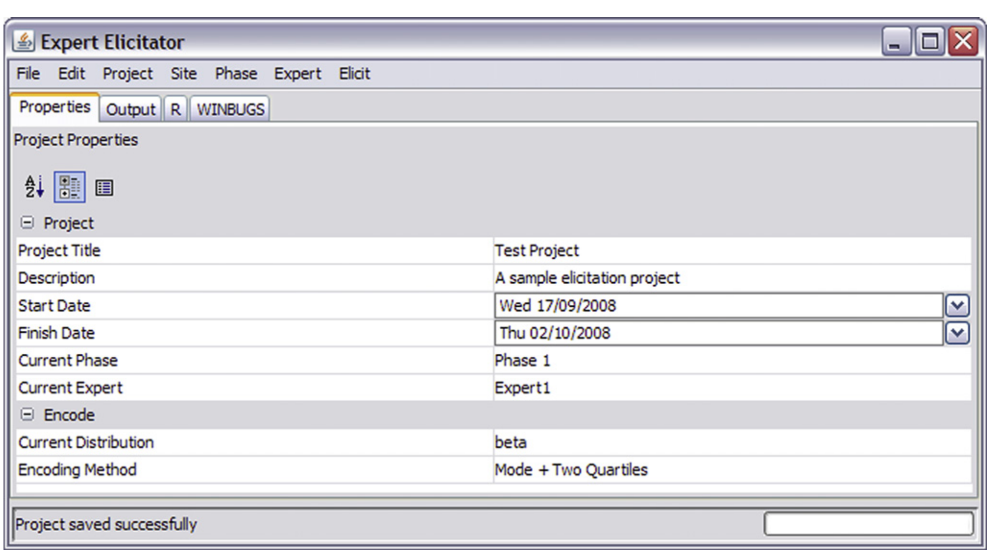

Fig. 2. Main application window showing properties of an example project.

Table 1
The contents of an example site file showing sites and associated covariate values.

Site	Easting	Northing	*Geology	*REMVEG	*Landcover	Elevation	Slope	Aspect
1	463,368	6,997,329	3	1	1	360	20	287
2	472,155	7,004,332	1	2	1	426	14	74
3	454,330	6,999,610	1	1	1	320	19	120
…								
…								
29	413,351	7,101,492	1	3	1	313	31	56
30	464,371	7,001,690	2	2	1	329	34	39

different times, using different encoding and elicitation methods, or interview techniques.

Elicitation is undertaken for several cases or sites, which in Elicitator are defined so that they may correspond to geographic locations (as in Denham and Mengersen, 2007) but in fact need not be indexed spatially. In the following, we refer to cases as *sites* to help with conceptualisation. Each site has a set of specific covariate values. Sites and associated covariates are added to the project by importing them from a site file following the example structure shown in Table 1, in comma-separated value (CSV) file format. The decision of how sites are selected for the elicitation is problem-specific and is therefore left up to the user. Adding sites to a project simply requires that each site has a unique site number, and additional sites can be added at any time during the elicitation process. If an imported site has the same site number as an existing site, then the newly imported site is ignored. The first line of the site file must begin with the three mandatory headings: site, easting and northing. These represent the unique site identification number, and geographical location coordinates in the Universal Transverse Mercator (UTM) format (Universal Transverse Mercator Coordinate System, 2009). Following these headings is a list of the site covariate headings, and headings prefixed by an asterisk (*) represent categorical covariates. Each site is then listed on its own line, with data values separated by commas and in the same order as the headings. This information is easy to export from most GIS packages after a spatial intersection of the site layer with appropriate environmental variable layers. All covariate values in the site file must be numeric (integer or floating point values). Categorical covariate values are specified as integers, but are mapped to the appropriate category names stored in the database to encourage more accurate interpretation of tables and graphs by users. If these categorical mappings do not already exist in the database for a particular categorical covariate, then they can also be imported into the application from a simple CSV file.

After sites and covariates have been added to the project, the main elicitation site window (Fig. 3) can be accessed from the *Elicit* menu by selecting the *Perform Elicitation* option. This window tabulates all elicitation sites added to the project, together with their associated elicited and encoded responses. This window can also be used to select the current site for elicitation, by clicking on the row in the upper table to highlight the desired site. The lower table then displays the list of covariates and values associated with the selected site. When other viewer windows that display the collection of sites are open, selecting a site in one will automatically select the same site in all open viewers. Histograms of covariates (Fig. 4) can also be displayed via the *Site* menu, which the expert

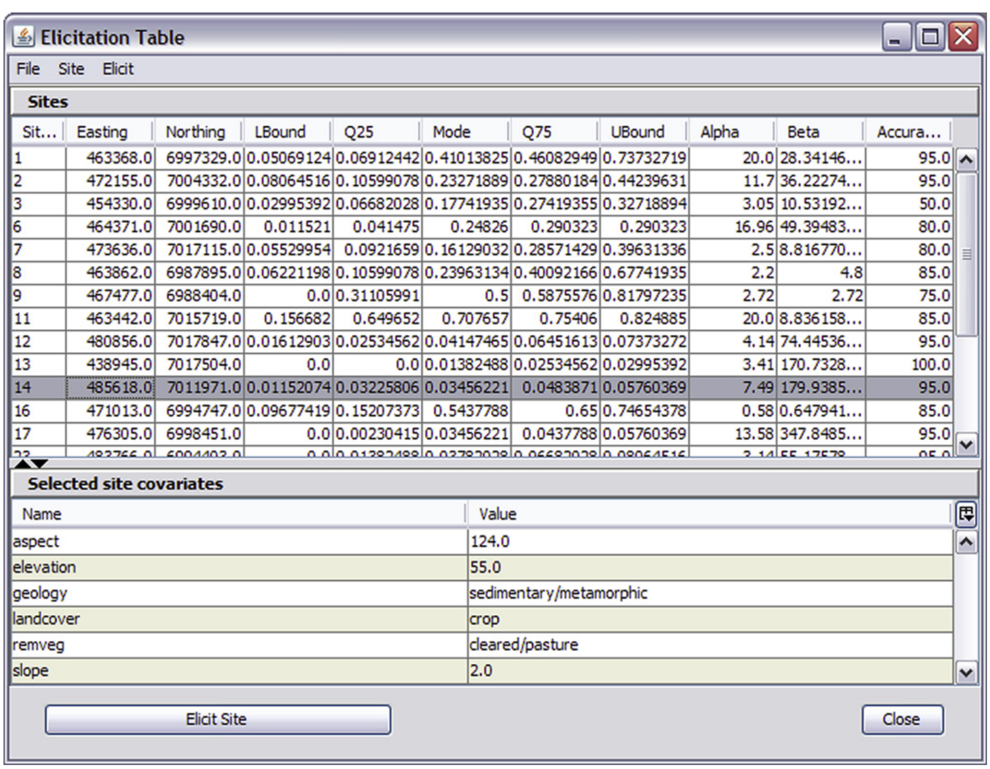

Fig. 3. A list of sample site elicitations showing the covariates for a selected site.

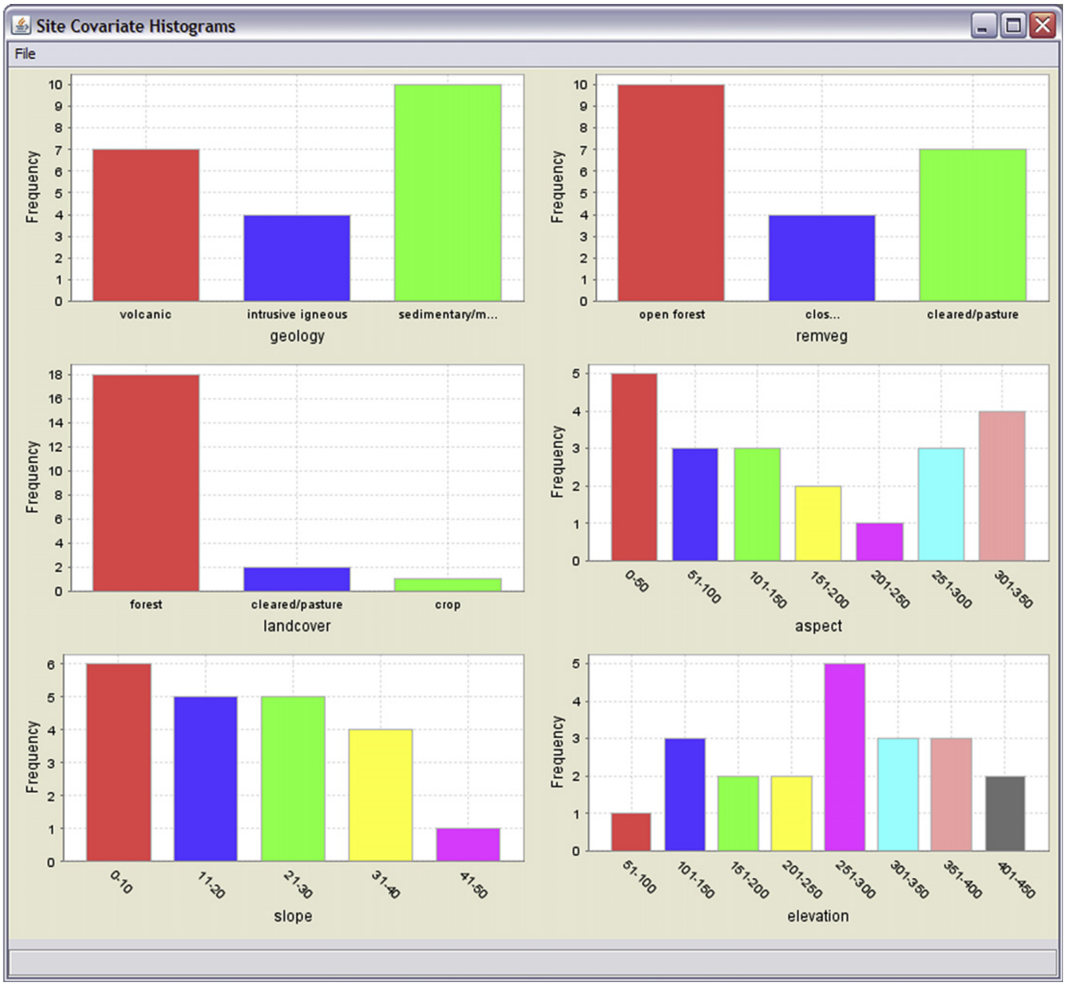

Fig. 4. The covariate histogram viewer showing the spread of covariate values across all elicitation sites.

will find useful for showing the frequencies of the different covariate values over the entire collection of elicitation sites.

2.3.2. Elicitation

The elicitation process begins by firstly ensuring that the required expert has been selected as the current active expert. Elicitation can then commence for a site (selected in any viewer) by clicking the *Elicit Site* button. The elicitation dialog box (Fig. 5) tabulates covariates and values for the selected elicitation site and also provides a number of interactive panels for helping the expert to specify their assessments about the probability of success at each site. Information elicited from experts using this dialog include their best estimate (encoded as the mode), upper and lower quantiles (currently as assumed to be quartiles), estimates of the upper and lower bounds, and a confidence rating for each probability.[1] While the expert is specifying or modifying values, the encoding module works to immediately encode the new assessments as a statistical distribution (Equation (6)) and the graphs within the dialog are updated accordingly.

The elicitation dialog provides a number of different interfaces to help the expert specify the required assessments $p(Z_k|a_k, b_k)$. The box and whisker plot can be modified by clicking and dragging the various vertical line segments, representing mode, quartiles and outer quantiles. The probability density curve can be modified directly by clicking and dragging the square nodes with the left mouse button. Alternatively, some experts may prefer to specify assessments numerically, entering them directly into text fields at the top of the dialog.

After the expert has finished providing predictions for the site and the *Save* button is clicked, this information is recorded within the project. The expert may then continue to the next elicitation site, or if a sufficient number of sites have been elicited, the data can then be encoded mathematically into an informative prior model via Equation (7). As noted in Section 2.2, this encoding step corresponds to a regression, and is achieved in R (Venables and Ripley, 2002). This statistical computing package provides regression diagnostics for GLMs, which are then processed in Elicitator for communication to the user in two forms: as text information summarizing goodness-of-fit of the regression, and as graphical feedback for evaluating regression assumptions (Section 2.3.3). After a successful fit, the results can be exported (Section 2.3.4) as the desired prior distributions for regression coefficients β (Equation (7)) in a form suitable for a Bayesian analysis on combination with observed field data (Equations (1) and (2)). Generating the encoded prior is achieved by selecting the *Fit Prior Model* option under the *Elicit* menu.

[1] The confidence rating, based on the scale used in Kynn (2005) and O'Leary et al. (2009), is to be implemented and utilised in future development phases of the tool.

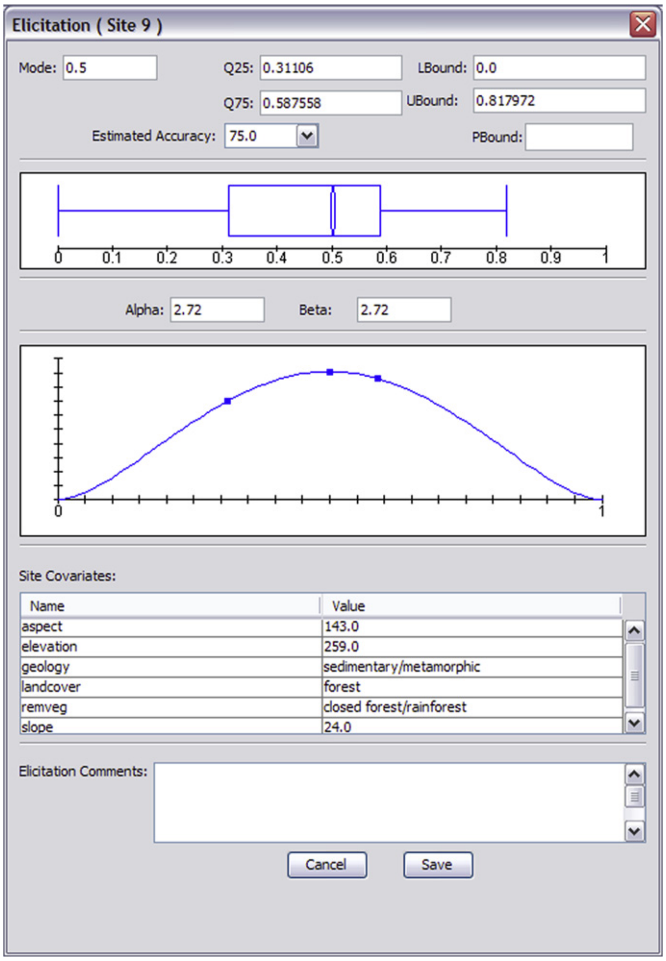

Fig. 5. Elicitation dialog showing the various interface types for specifying probabilities.

2.3.3. Feedback

Provided that sufficient information has been elicited and the application has successfully generated the prior model for regression coefficients, various plots can be displayed to help the expert visualise and evaluate their site-by-site assessments. Univariate response curves and prediction intervals generated from the elicitation (Fig. 6) show the main effects of each environmental covariate on the conditional mean, which for logistic regression is the probability of success. The y-axis depicts the probability of success and the x-axis depicts the values of one covariate. For categorical covariates, box and whisker plots show the range of responses for each value of the covariate, with particular elicited responses shown as bar charts. The response viewer can be displayed by choosing *Show Responses* under the *Elicit* menu.

Various other informative diagnostic plots can also be viewed (Fig. 7) by selecting the *Show Diagnostics* option, also under the *Elicit* menu. The diagnostic plots indicate how well the prior model (encoded by combining all elicitations) fits the individual elicitations (most plausible values of the assessed probabilities of presence) obtained at each site. Note that the *residual* is the deviance residual resulting from applying the iteratively reweighted least squares algorithm to estimating maximum likelihood estimates for Generalized Linear Models (Venables and Ripley, 2002). These indicate the item's contribution to the change in deviance, standardized so that the scale is comparable to a standard normal distribution. Thus these residuals simply indicate the scaled difference between the encoded and the elicited probability of presence at each site, with values beyond ± 2 indicating large residuals. Standard diagnostic plots are provided: a residual *vs* prior estimates plot (top left), to help highlight elicitations at sites that badly fit the encoded prior model (i.e. have large residuals); a plot of the encoded prior estimate against the elicited probability of presence for each site (top right), with a straight diagonal line indicating the result if all elicitations consistently fit the encoded prior model; a quantile–quantile plot to examine the distribution of the residuals (bottom left), with points lying on a straight diagonal line indicating desired normality of the residuals; a plot of Cook's distance for each elicitation site (bottom right), with unusually large values indicating elicitations at these sites have great influence on the encoded prior model. In this context the diagnostic plots enable both the elicitor/modeller and the expert to evaluate the elicited quantities from a perspective that aggregates across sites.

To encourage a cycle of improving elicitations, the software provides a feedback loop that allows experts to easily select sites (by clicking directly on the various graphs) to modify elicited assessments if desired. This provides experts with the opportunity to refine site-by-site assessments after viewing the feedback generated from encoding their elicitations for all sites.

2.3.4. Exporting encoded priors

After the expert is satisfied with their assessments and encoding has successfully provided an informative prior, this prior can then be exported from the tool. This prior can for example, be combined with field data within a Bayesian logistic regression model (Low Choy et al., 2009a; O'Leary et al., 2009). This first version of Elicitator currently provides the encoded prior information in the format required by popular Bayesian software WinBUGS (Spiegelhalter et al., 2003) (Fig. 8).

3. System architecture and design

Elicitator was motivated by the need to redesign a prototype elicitation tool developed by Denham and Mengersen (2007). The intention was to improve and expand on the functionality provided by the prototype software and develop the new tool using the Java programming language, object-oriented design methods and free, open-source libraries. The tool was designed with an intuitive and user-friendly graphical interface that allows users to easily navigate between the key steps in the elicitation process. Java Swing libraries were used to implement all of the graphical user interfaces and JFreeChart (Gilbert, 2008) was used for the implementation of the range of feedback graphs. Data persistence has been achieved through the use of a database implemented using MySQL (MySQL AB, 2008). All statistical calculations are performed through communication with the R statistical package (R Development Core Team, 2008) via a Java-based server package called RServe (Urbanek, 2006).

The tool consists of a number of primary components that promote an object-oriented, modular design. It is intended that the software will continue to be developed to provide further functionality and additional modelling options, and the design of the software facilitates this through the use of standard design patterns and the relatively loose coupling of the system components. Abstraction techniques have been employed where appropriate to ensure the software can be extended in the future with a minimum amount of effort and disruption to existing source code. The components of the software framework will now be discussed in more detail.

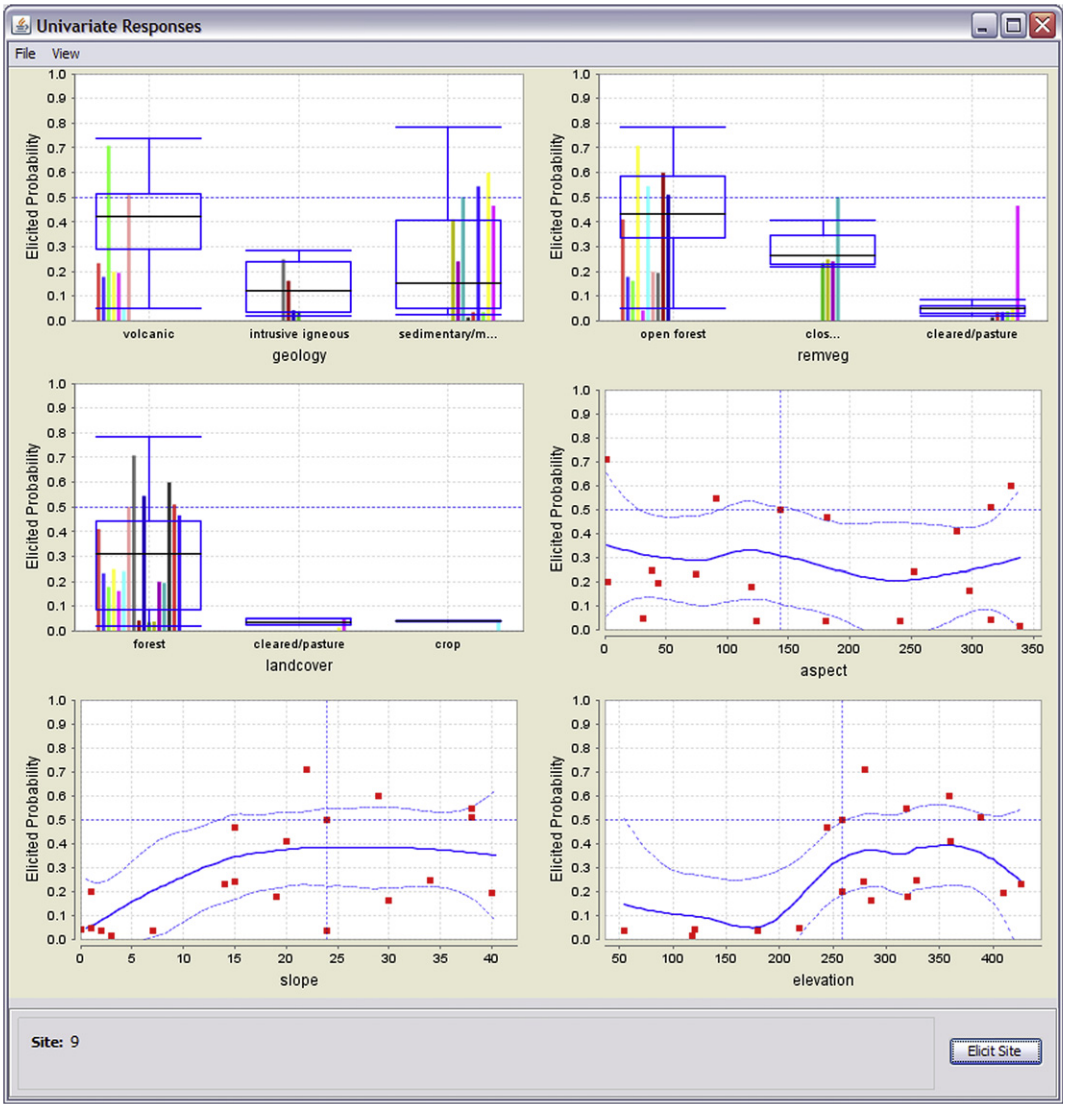

Fig. 6. Response viewer showing site predictions and associated regression for each covariate. Categorical covariates are represented as bars to emphasise the discrete nature of the variables.

3.1. Project data model

The project model used in the software serves to encapsulate all data and data relationships for an elicitation project. A conceptual class diagram of the data model is shown in Fig. 9. These objects contain persistent data that is saved to a backend database when a project is saved. Projects and all associated data can then be retrieved from the database when required.

The *Project* is the central object in the data model. A project instance has attributes such as a project name, a brief description, and expected start/finish dates, specified by the user when the project is created. Each project consists of a number of independent project phases used for performing multiple elicitations using the same set of experts and elicitation sites, for example using different encoding methods and/or distribution types.

Expert objects encapsulate information representing elicitation experts. In addition to some personal identifying information, this includes a collection of the expert's elicitations, with one elicitation per site assigned to a project. An expert is required to provide an assessment for each of these sites, for each project phase, and these values are used to encode a prior distribution of the conditional means for each site using the expert's assessments, based on the distributional assumptions chosen by the modeller (Equation (6)). This information is stored in associated *Encoded Response* objects, one for each distribution type.

Distribution objects encapsulate the attributes and methods that represent statistical distributions and common operations. Abstraction techniques have been used to ensure additional distribution types can be added to the system at a later date with minimal effort. An abstract parent class contains the common attributes and methods required to represent all distributions. This provides the framework for specifying particular distributional forms, such as Beta or Gamma distributions, which are then implemented as child classes that inherit and override the features of the parent class. *Site* objects represent the elicitation sites, and the environmental characteristics of each site are represented by a collection of *Covariate* objects. A covariate represents a single environmental variable whose value can either be continuous or categorical. A sample site showing an associated list of covariates has been selected in the elicitation site dialog shown in Fig. 3. Values for categorical covariates are mapped to appropriate category names via a *CovCategories* object. Additional category

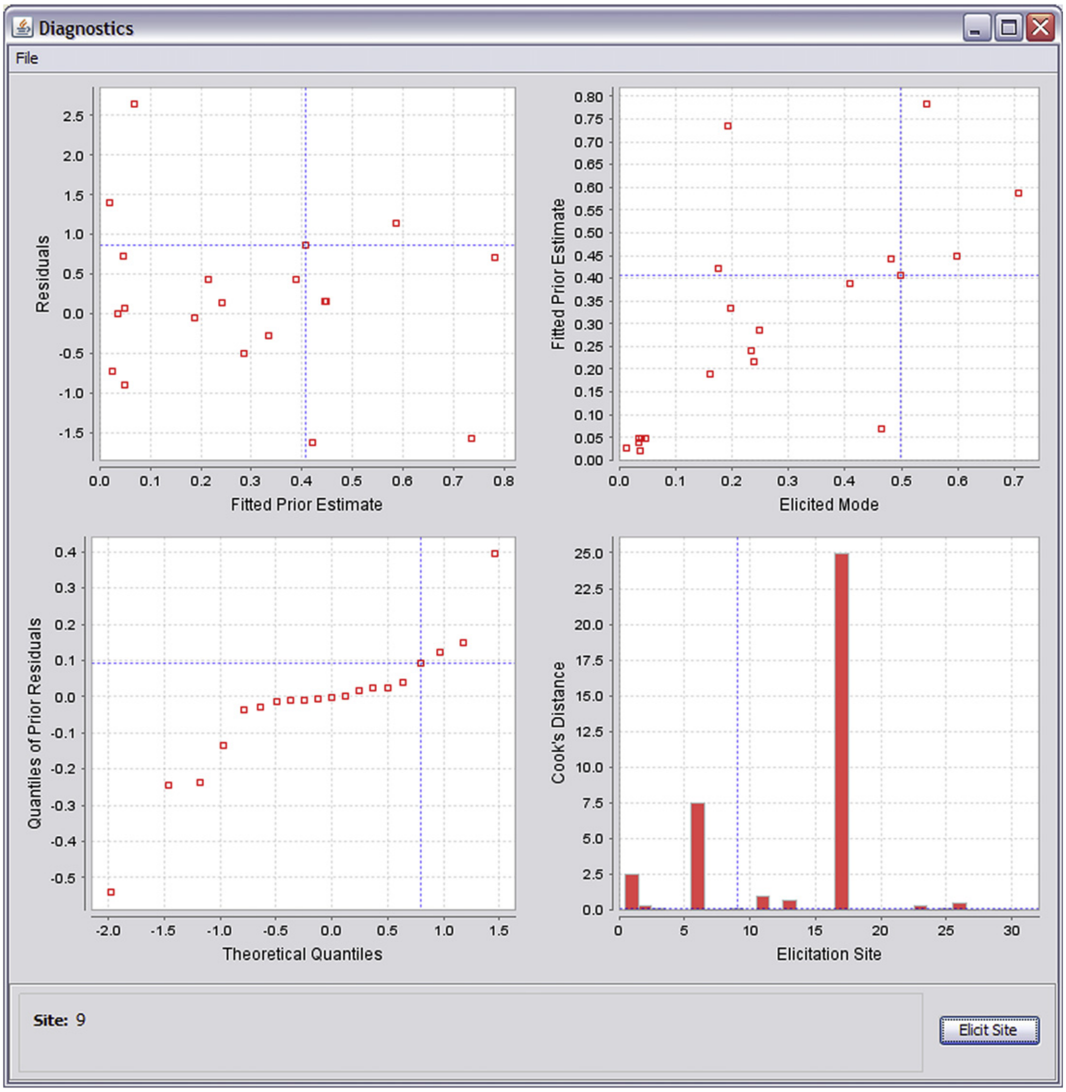

Fig. 7. Diagnostics viewer displaying various informative feedback plots resulting from a successful calculation of a prior model, fit to the elicitation data.

mappings can be added to the database by importing a file which is saved to the database along with the project.

3.2. Graphical user interface (GUI)

The graphical user interface (GUI) provided by Elicitator had to be versatile enough to handle a wide range of graphical objects (widgets), including standard textboxes, drop-down menus and windows, as well as more complex objects, such as interactive and dynamic graphs in the elicitation and feedback windows. In addition the elicitation process was designed to be highly interactive, allowing the user to provide information and determine the flow of control at any time via the graphical interface. For these reasons the GUI in Elicitator has been implemented using the Java Swing library, which provides both a large range of common GUI components that are cross-platform and easy to use, as well as an extensive event handling model for capturing user interaction events.

The core GUI is based on a variation of the well-known Model-View-Controller (MVC) software design pattern. The model holds data, and the views display the data contained within the model. The controller component is usually responsible for handling user interaction events, but in the case of Elicitator this functionality has been combined within each view. More generally, the Java variation of the MVC pattern implemented in the software is called the *Observer* pattern.

The Observer pattern is a commonly used and proven design pattern that allows one or more objects (called *observers*) to conveniently observe another object (the *subject* or observable) for changes. The pattern works by providing a publish-subscribe relationship between the subject and its observers. Observers can register to receive events from the subject, and when the subject needs to inform observers of an event (when the internal state has changed), it simply sends an update notification event to each observer. Elicitator uses this observer pattern to inform viewer windows of project updates. In this case, the project is the subject or observable, and the various viewer windows are registered as observers. When a project is modified, it dispatches update events to all registered observers, and provides an indication of the type of update and what data has changed. This ensures observers only update what is required, depending on the type of change that has occurred in the project.

Each of the registered project observers holds a reference to the project (observable), and changes made to data via user interaction

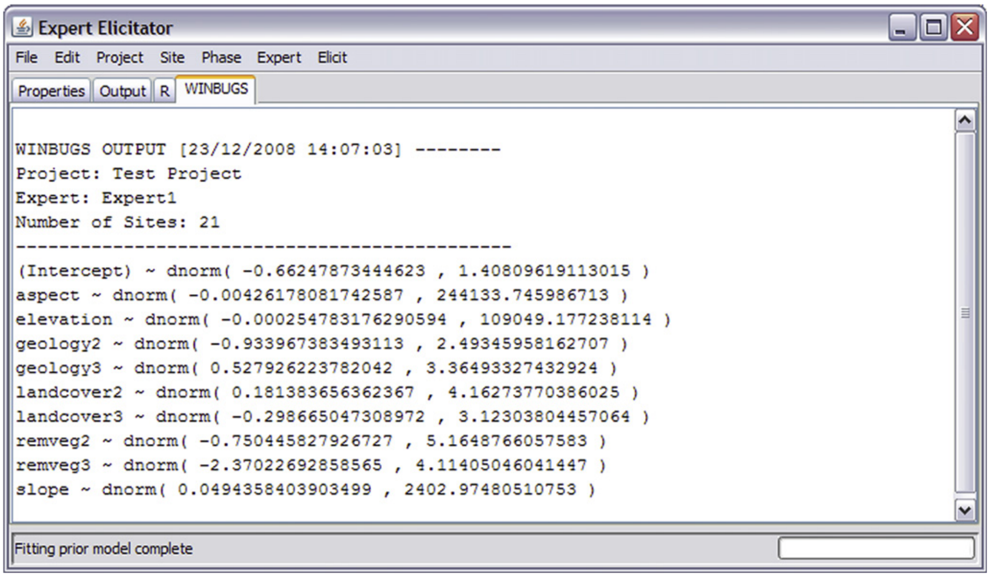

Fig. 8. The output of an encoded informative prior in WinBUGS format.

with the viewer windows can also affect the state of the project, which will in turn notify all other viewers of the change. This provides a two-way communication between the project model and the observer views, which for example, allows users to select a site in one of the views, and automatically have the same site selected in all other views.

Providing a separation of the various viewers from the project data model ensures the model has no direct dependence on actual viewer implementations. The loose coupling of these components also promotes a clean and extensible design for the software, where additional viewers can be developed to provide different views of the data, and added to the system simply by registering it as another observer of the project. The new viewer will then receive project update events and can respond to the relevant events accordingly.

3.3. *Visual feedback*

The *Feedback* viewers are a selection of windows that provide various interactive feedback graphs, generated from the elicitation data and successfully encoded prior models. The graphs have been implemented using the open-source JFreeChart package (Gilbert, 2008), a flexible library that supports a wide range of different graph types.

JFreeChart has been utilised to provide attractive and interactive graphs for Elicitator. Users have the ability to resize the graph panels in order to access a more detailed view of the data. The user can also zoom in on specific portions of the graphs to gain a closer look at the displayed values; this is especially necessary when a number of values occupy positions that lie closely together. Various other options are also available, such as changing colours and line thicknesses, changing the orientation of the plots, and the ability to export the graphs as images. These options can be accessed via a context menu (accessed by right clicking on any of the graphs).

The feedback viewers utilise a number of different graph types. For example, the univariate response viewer window (Fig. 6) contains graphs that display the univariate response curves resulting from the encoded prior model that aggregates the expert's opinion across all elicitation sites, showing the species response to each environmental attribute, as estimated from the individual site-by-site elicitations. Responses predicted under the encoded prior model are compared to elicited values for each site and represented by selectable red squares. For categorical covariates, the expert's best estimate of the probability of presence at each site is represented by a bar positioned at the site's covariate value. Results from encoding are summarised and overlaid as box and whisker plots. Bars have been used for the categorical variables to emphasise their discrete nature, and to provide a clearer representation for experts such as ecologists. For continuous covariates, these best estimates are represented by dots positioned at the site's covariate value, with results from encoding overlaid as a curve.

The diagnostics viewer (Fig. 7) displays the diagnostic graphs, described in Section 2.2, that help the expert evaluate how well the prior model encoded from their elicited information represents their knowledge overall. For consistency, the diagnostic plots also represent points corresponding to elicitation sites as selectable red squares. The exception is Cook's distance plot, where selectable bars have instead been used to ensure accurate selection of elicitation sites amongst the dense vertical lines.

Individual sites on feedback graphs can also be selected by clicking the red square nodes, and the expert may then choose to revisit elicitations for any site if desired. After elicited assessments for that site are updated and saved, the positions of the red nodes on the response graphs and feedback diagnostics are updated to reflect the new elicited and encoded prior values for that site. Previous values are retained as ghosted (gray) site nodes. This allows the modeller and expert to compare the new and previous elicited assessments, as well as their respective impacts on the encoded prior model.

3.4. *Capturing expert assessments of probabilities*

The elicitation dialog (Fig. 5) is a fundamental component of Elicitator. Its main task is to support the elicitation of the expert's assessments of the conditional mean (in the case study, probability of presence) for each of the sites in the project. This dialog has been designed to allow experts to interactively specify their assessments on the probability of success $p(Z_k|a_k,b_k)$, including: upper and lower limits, upper and lower quantiles, and the expert's best estimate (mode). To achieve this a number of different graphs provide options for different styles of interaction with the expert, either in

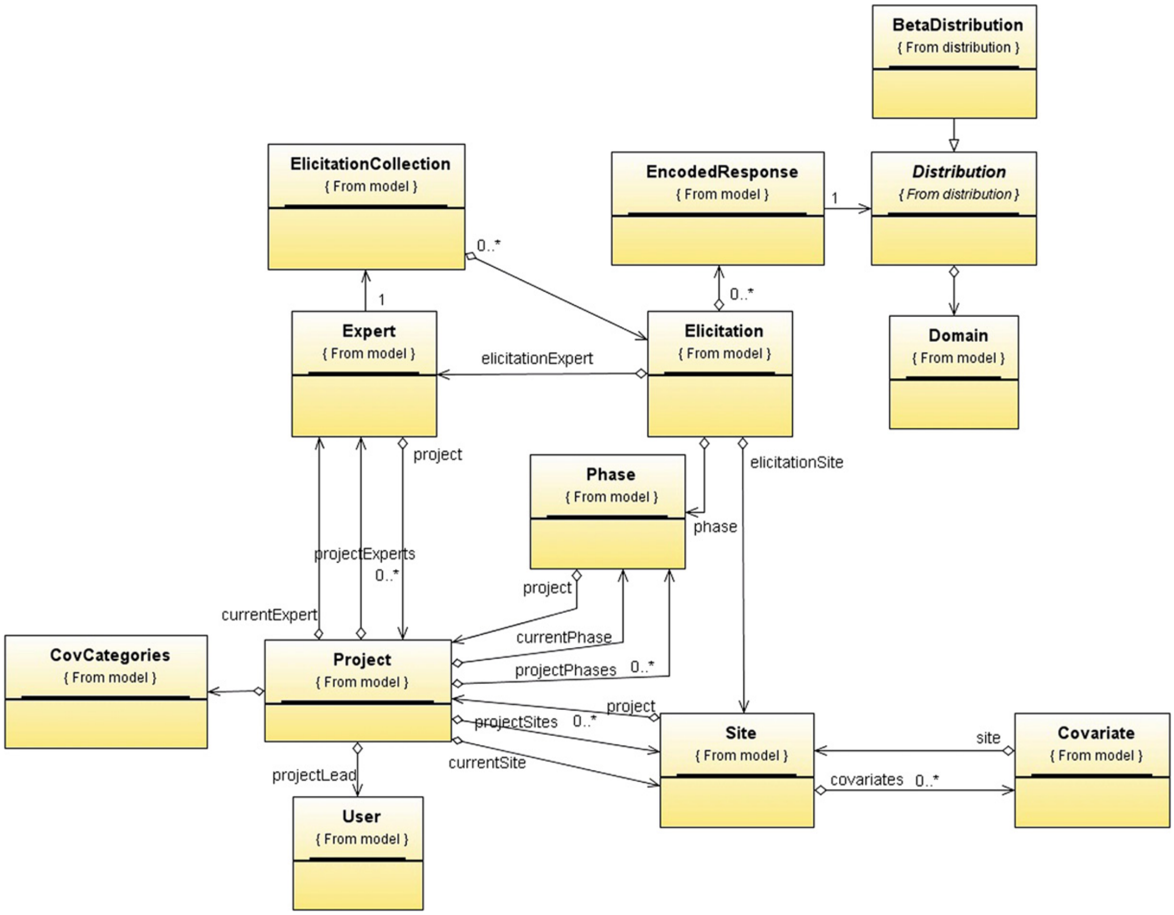

Fig. 9. Conceptual project data model for the Elicitator.

a graphical environment via a box plot or a probability density function plot, or via simple numerical specification of assessments. The dialog also provides a list box for the expert to specify their confidence in their assessment at each site. Such values could be drawn on to weight sites in the regression, used to encode elicited information into a prior.

Updates made by the user during interaction with any of the interaction panels will automatically update the other panels accordingly. When the expert interacts with these panels to update elicited values, this initiates a call to the Encoder module. Here the application communicates with the statistical package to update encoding of the estimated Beta distribution (Equation (6)). The probability density plot in the elicitation dialog is then updated to reflect the newly encoded distribution, providing the user with a dynamic view of the range of plausible values for the conditional mean.

3.5. Data persistence

The database backend provides a persistent storage space for project data, and the functionality to save, load and update project data. The design of the database was inspired directly by the conceptual data model diagram (Fig. 9) and implemented using the MySQL relational database package.

Each object template (class) within the data model has a corresponding table implemented in the database. Logical relationships between the various tables have been defined in the database. These enforced relationships provide a higher level of data integrity, by ensuring data is valid and contains valid references. The database will reject requests to insert data that do not conform to these constraints.

A Database Manager class functions as a proxy to the database connection and provides methods to create, retrieve and close database connections. If the database connection has closed or is invalid when the software requests a connection, the manager attempts to create a new database connection automatically.

3.6. R Communication via RServe

All statistical calculations required by Elicitator are performed via code written in the R statistical language (Venables and Ripley, 2002). The R package (R Development Core Team, 2008) has been used to avoid having to re-implement existing, well-accepted statistical algorithms, such as elementary probability distribution calculus used when encoding the prior model from the elicitation data. Feedback relies on R object-oriented computations for GLMs, and is therefore easily extensible to other GLM families. Elicitator communicates with the R libraries via an open-source component called RServe (Urbanek, 2006). RServe is a TCP/IP server that allows communication with R from within various programming environments, without the requirement of running an actual instance of the R software.

An *RManager* class has been implemented to manage RServe instances and is responsible for starting the RServe server and initialising connections. Calls to various functions are sent to R via the RManager and the results are returned in appropriate Java objects. Any exceptions (errors) thrown by the RServe client can also be caught to provide informative feedback and error messages to the user.

While the RServe server provides a convenient method of communication with R from Java, one of the disadvantages of using the server as a communication proxy is that it does not handle certain errors gracefully, and can terminate the connection and server instance when an error is encountered. This occurs for example, when the user attempts to fit a prior model when there is insufficient data for R to perform the calculation successfully. However, the RServe Java client library provides various exception types that are thrown when errors occur in R code, and these are used to provide informative error messages to users. The software catches the R exceptions and reconnects to a new RServe instance when it determines that the server connection has been lost.

3.7. Encoding expert assessments

The *Encoder* object handles the statistical encoding and the bulk of the statistical calculations required by the software, and is responsible for communicating with R through the RManager to encode expert elicitations into a distribution selected by the modeller.

When encoding expert assessments into a distribution, the currently active distribution type is used, and different distributions result in different encoded response values. See Section 2.2 for details regarding the statistical methods employed by the application. The extensible design of the software provides a foundation for defining additional distributions and encoding methods.

The Encoder is also responsible for calculating the prior model for an expert's set of assessments. It is possible for the user to select various methods for encoding the site-by-site elicitation distribution (Equation (6)), such as using the mode and two quartiles or the median and upper quartile (Denham and Mengersen, 2007). In later versions of the tool, it will also be possible for the user to select other encoding methods (e.g. Bedrick's exact method based on Equation (5)). The Encoder object holds references to the results generated from the last successful prior fit, which includes the encoded response data and diagnostic information for each elicitation site.

3.8. Application settings

Elicitator provides a settings dialog to allow modification of various application settings, such as default data file paths and database connection information, as well as encoding options. This window is accessible from the *Edit→Application Settings* menu. Available settings currently include connection settings for the database and the default file path for data files. The settings dialog is divided into related categories that make specific settings easier to find. This will become increasingly beneficial as additional settings are included in later versions, to improve the configurability of the system.

Application settings are loaded from a properties text file when the application is executed, and any changes made to properties through the application settings dialog are saved back to the properties file. In addition to storing application settings persistently, the use of an external properties file allows for modification of the values directly with a text editor, without the need to run the software itself, due to the format being a simple text file containing a list of properties and associated values. This is especially necessary when the software is first installed, for ensuring the database user and connection details are correct (as the software requires a valid connection to the elicitation database in order to run).

4. Discussion

A major benefit of the Elicitator software is that it implements an approach to elicitation that is easily interpreted, and suitable for a wide range of ecological regressions. This indirect approach to elicitation aims to elicit accurate information by asking experts about concrete observable quantities (Kadane et al., 1980; Kynn, 2008; Low Choy et al., 2009a). To enable this indirect approach, Elicitator conceals the mathematical complexity, undertaking behind-the-scenes two stages of encoding expert opinion into statistical distributions. The expert's opinion on probability of success given different covariate values is first encoded site-by-site as a conditional mean prior, which for logistic regression takes the form of a Beta distribution (Bedrick et al., 1996). In this paper we propose a new measurement error model approach to combine this information across sites to encode prior distributions for the desired regression coefficients (Section 2.2).

One of the primary benefits of using Elicitator to support elicitation is that it provides a consistent basis for gathering expert knowledge in a structured and repeatable way, and thus helps reduce undesirable elicitation biases. The tool helps ensure elicitations at each site are performed consistently. When multiple experts are interviewed, the tool helps ensure elicitations are comparable. A consistent environment, together with implicit definitions for elicitation, both reduce measurement error. For inexperienced elicitors, the software naturally imposes a statistically well-designed structure on the elicitation process, thereby improving representativeness and reducing other biases arising from the misapplication of elicitation. However, it is important to note that for repeatable and transparent results, the tool should be used in conjunction with training (on statistical concepts, and conditioning to alert experts to potential biases), and an interview proforma, with wording of questions carefully chosen to minimize biases (O'Hagan et al., 2006; Kynn, 2008; Low Choy et al., 2009a)

The software also provides immediate feedback in the form of graphical visualisations to help experts understand the overall implications of their site-by-site elicitations. This feature helps elicitors and experts to reflect and monitor expert assessments, an effective means of improving accuracy (Kynn, 2008, Recom. 9). Many of the feedback graphs are also interactive and dynamic, and elicitation sites can be selected for re-elicitation by clicking directly on graphs. This encourages a cycle of feedback (elicitation > feedback > re-elicitation), which helps ensure that elicited information more closely reflects the expert's opinion. The interactive nature of this feedback loop in Elicitator accommodates the *iterative* nature of refining a set of probability assessments.

There are many other benefits of using Elicitator to manage the elicitation process. The tool streamlines elicitations and enables the researcher to gather more expert opinions in a shorter time period. The time savings can be significant, especially when compared to traditional surveys and questionnaires, which make it difficult to provide instant (even hand-drawn) graphs of results. The use of the tool also allows more refined and accurate opinions to be elicited from experts, as less time and effort is required to complete the assessment. This is important since elicitation sessions should be limited, with appropriate breaks (Ayyub, 2001, pp. 111), since expert fatigue affects memory recall and accuracy. The tool also supports breaks, by allowing the user to save a project at any point during elicitation for later reloading. This provides the expert with greater flexibility on the order in which individual assessments are supplied, and the elicitor with more flexibility regarding the way in which experts are interviewed.

This first version of Elicitator is limited to a single method for encoding each site's assessment (mode with two quartiles) based on a particular distribution (Beta) for the conditional mean, for logistic regression for cases that are geographic locations. However, the software has been developed within an extensible framework that can be extended in various ways. Extensions planned for future development of the tool will initially focus on: providing the user with a greater range of modelling options, encoding distributions and encoding estimation methods; improvements to the user interface; and improved error handling in the interface to the statistical package. Future development will investigate the JRI package (JRI – Java/R Interface, 2006), which provides a direct Java to R interface without the need for the client-server model provided by RServe.

The addition of more methods for encoding distributions, such as that based on a median and quantiles, will also allow the elicitor to tailor elicitation to the information most easily and accurately obtained from an expert, increasing the generality of the software. Permitting alternative estimation methods for encoding the prior distribution (Exact, Frequentist or Bayesian) would also be an advantage, as would providing implementations for a wider range of generalized linear models, beyond logistic regression, would also be a practical addition to the software. Another valuable addition would be the ability to combine all expert opinions gathered within an elicitation project, to produce a final combined elicitation model.

The elicitation dialog presents a selection of several numerical or graphical interfaces for specifying expert assessments. This ensures the elicitation process is more accessible to a wide range of experts with different ways of thinking, areas of knowledge and levels of expertise. Elicitator currently provides *three* different interfaces within the elicitation dialog, more than other software (e.g. Kynn, 2006; Denham and Mengersen, 2007; Leal et al., 2007): an interactive box plot, probability density plot and numerical specification of key distributional quantities. To avoid overloading or confusing the expert, default settings could ensure that only one or two of these representations are visible.

In ecology, expert knowledge is commonly site or location-based. Thus a map-based elicitation approach, achieved via GIS software, allows experts to explore and query the spatial and covariate data relating to elicitation sites, within the context of surrounding areas (Denham and Mengersen, 2007). In this first phase of development, we decided to keep GIS tools separate from the elicitation tool in order to maintain independence from commercial software, reduce complexity and hence development overheads. However, it would certainly be very convenient for the modeller to have maps and a range of GIS tools available within the tool. Currently a large range of commercial GIS software is already available that offer extensive features and tools for manipulating spatial datasets. Re-implementing these features within the tool would be a substantial development task. Recent availability of higher level operations within open-source GIS libraries may lead to reassessment of this decision.

In addition to convenience, extending Elicitator with GIS functionality could also provide several other practical benefits. Many ecologists are already familiar with traditional GIS software, and incorporating a GIS view of imported sites and associated covariates to complement the table-based view of the elicitation sites would be a valuable addition to the tool. This would provide the ability to import GIS map layers representing location covariates, and overlay elicitation site locations on top of the layer stack, allowing the stack to be queried at site locations in order to retrieve the covariate values for each site. The tool could be extended so that the user could select elicitation sites directly by clicking site nodes using a map-based interface, and new sites could also be added by clicking on the map display directly.

Other possible improvements to the tool include providing a web-based interface for interacting with the elicitation tool via a webpage. This would centralise tool maintenance and avoid licensing issues, since secure access, backend computations and project data would be managed by a central server. In this case, the web interface would not be used for resource intensive operations such as those involving maps and other large datasets; these files and operations could instead remain resident on the client machine.

Elicitator has been designed to deliver a robust software framework for expert elicitation that provides many benefits for modellers and researchers. Here we report on the first development of the software, providing "proof-of-concept" of: extensions to the interactive dynamic style of elicitation (based on Denham and Mengersen, 2007); the use of a database to enable data persistence; a new encoding method based on Bedrick et al. (1996); encouraging a well-designed, transparent and repeatable elicitation process; a modular software design to permit extensibility; and the use of a modern computing platform linking to open-source libraries to provide generality and "future-proofing". As discussed, there are many possibilities for further extensions and improvements to the tool. Development will continue to further evolve the software by adding a greater range of features, improving modelling options, and providing support for a wider variety of elicitation-based ecological research applications.

Acknowledgements

This tool is inspired by case-specific prototypes implemented within ArcGIS by Robert Denham, whom we thank for provision of code. The authors would like to thank the Australian Research Council for funding provided through the ARC Discovery scheme and also to the Australian Research Centre of Excellence in Complex Dynamic Systems and Control.

Special thanks are given to Rebecca O'Leary and Justine Murray for testing and providing feedback on the tool. Acknowledgments also go to Mark Dwyer for initial efforts on the elicitation dialog.

References

Accad, A., Low Choy, S.J., Pullar, D., Rochester, W., 2005. Bioregional classification via model-based clustering. In: Zerger, A., Argent, R.M. (Eds.), MODSIM 2005 International Congress on Modelling and Simulation. Modelling and Simulation Society of Australia and New Zealand, pp. 1326–1332. December 2005.

Al-Awadhi, A.S., Garthwaite, P.H., 2006. Quantifying expert opinion for modelling fauna and habitat distributions. Computational Statistics 21 (1), 121–140.

Alho, J.M., Kangras, J., Kolehmainen, O., 1996. Uncertainty in expert predictions of the ecological consequences of forest plans. Applied Statistics 45, 1–14.

Austin, M., 2002. Spatial prediction of species distribution: an interface between ecological theory and statistical modelling. Ecological Modelling 157 (2–3), 101–118.

Ayyub, B.M., 2001. Elicitation of Expert Opinions for Uncertainty and Risks. CRC Press, Florida, USA.

Bedrick, E.J., Christensen, R., Johnson, W., 1996. A new perspective on priors for generalized linear models. Journal of the American Statistical Association 91 (436), 1450–1460.

Boyce, M.S., Irwin, L.L., Barker, R., 2005. Demographic meta-analysis: synthesizing vital rates for Spotted Owls. Journal of Applied Ecology 42, 38–49.

Branscum, A.J., Johnson, W.O., Thurmond, M.C., 2007. Bayesian beta regression: applications to household expenditure data and genetic distance between foot-and-mouth disease viruses. Australian and New Zealand Journal of Statistics 49 (3), 287–301.

de Bustamante Simas, A., Rocha, A.V., 2008. The Betareg Package. Version 1.2. http://medipe.psu.ac.th/cran-r/web/packages/betareg/index.html.

Chaloner, K., Church, T., Louis, T.A., Matts, J.P., 1993. Graphical elicitation of a prior distribution for a clinical trial. The Statistician, Special Issue: Conference on Practical Bayesian Statistics, 1992, 42 (4), 341–353.

Chaloner, K.M., Duncan, G.T., 1983. Assessment of a Beta prior distribution: PM elicitation. The Statistician, Proceedings of the 1982 I.O.S. Annual Conference on Practical Bayesian Statistics (Mar.-Jun., 1983) 32 (1/2), 174–180.

Chen, Y., Hipel, K.W., Kilgour, D.M., Zhu, Y., 2009. A strategic classification support system for brownfield redevelopment. Environmental Modelling & Software 24, 647–654.

Clark, K.E., Applegate, J.E., Niles, L.J., Dobkin, D.S., 2006. An objective means of species status assessment: adapting the Delphi technique. Wildlife Society Bulletin 34, 419–425.

Du Mouchel, W., 1988. A Bayesian model and a graphical elicitation procedure for multiple comparisons. In: Bernardo, J.M., de Groot, M.H., Lindley, D.V., Smith, A.F.M. (Eds.), Bayesian Statistics 3. Oxford University Press, Oxford, UK, pp. 127–145.

Denham, R., Mengersen, K., 2007. Geographically assisted elicitation of expert opinion for regression models. Bayesian Analysis 2 (1), 99–136.

Ellison, A.M., 2004. Bayesian inference in ecology. Ecology Letters 7, 509–520.

Ferrari, S., Cribari-Neto, F., 2004. Beta regression for modelling rates and proportions. Journal of Applied Statistics 31 (7), 799–815.

Ferraro, D.O., 2009. Fuzzy knowledge-based model for soil condition assessment in Argentinean cropping systems. Environmental Modelling & Software 24, 359–370.

Fleishman, E., Nally, R.M., Fay, J.P., Murphy, D.D., 2001. Modeling and predicting species occurrence using broad-scale environmental variables: an example with butterflies of the great basin. Conservation Biology 15 (6), 1674–1685.

Garthwaite, P.H., O'Hagan, A., 2000. Quantifying expert opinion in the UK water industry: an experimental study. The Statistician 49, 455–477.

Greco, S., Matarazzo, B., Slowinski, R., 2001. Rough set theory for multicriteria decision analysis. European Journal of Operational Research 129, 1–47.

Gilbert, D., 2008. JFreeChart API Documentation Available from: http://www.jfree.org/jfreechart/api/javadoc/index.html.

Guisan, A., Zimmermann, N.E., 2000. Predictive habitat distribution models in ecology. Ecological Modelling 135 (2–3), 147–186.

Hughes, G., Madden, L.V., 2002. Some methods for eliciting expert knowledge of plant disease epidemics and their application in cluster sampling for disease incidence. Crop Protection 21, 203–215.

JRI – Java/R Interface, 2006. JRI API Documentation Available from: http://www.rosuda.org/R/nightly/JavaDoc/index.html Project website: http://www.rforge.net/JRI/index.html.

Kadane, J.B., Dickey, J.M., Winkler, R.L., Smith, W.S., Peters, S.C., 1980. Interactive elicitation of opinion for a normal linear model. Journal of the American Statistical Association 75, 845–854.

Kadane, J.B., Wolfson, L.J., 1998. Experiences in elicitation. The Statistician 47, 3–19.

Kuhnert, P.M., Martin, T.G., Mengersen, K., Possingham, H.P., 2005. Assessing the impacts of grazing levels on bird density in woodland habitat: a Bayesian approach using expert opinion. Environmetrics 16, 717–747.

Kynn, M., 2005. Eliciting expert knowledge for Bayesian logistic regression in species habitat modelling in natural resources. PhD thesis, Queensland University of Technology.

Kynn, M., 2006. Designing elicitor: Software to graphically elicit expert priors for logistic regression models in ecology. http://www.winbugs-development.org.uk/elicitor/files/designing.elicitor.pdf.

Kynn, M., 2008. The "heuristics and biases" bias in expert elicitation. Journal of the Royal Statistical Society, Series A 171 (1), 239–264.

Leal, J., Wordsworth, S., Legood, R., Blair, E., 2007. Eliciting expert opinion for economic models: an applied example. Value in Health 10 (3), 195–203.

Lindley, D., 1983. Reconciliation of probability distributions. Operations Research 31 (5), 866–880.

Low Choy, S.J., Mengersen, K., Rousseau, J., 2008. Encoding expert opinion on skewed nonnegative distributions. Journal of Applied Probability and Statistics 3 (1), 1–21.

Low Choy, S., James, A., Mengersen, K., 2009b. Expert elicitation and its interface with technology: a review with a view to designing Elicitator. In: Braddock, R., et al. (Eds.), 18th IMACS World Congress – MODSIM09 International Congress on Modelling and Simulation, December 2009. ISBN: 978-0-9758400-7-8, pp. 4269–4275.

Low Choy, S., Murray, J., James, A., Mengersen, K. Indirect elicitation from ecological experts: from methods and software to habitat modelling and rock-wallabies. In: O'Hagan, A., West, M. (Eds.), Handbook of Applied Bayesian Analysis. Oxford University Press, Oxford, UK, in press.

Low Choy, S.J., O'Leary, R., Mengersen, K., 2009a. Elicitation by design for ecology: using expert opinion to inform priors for Bayesian statistical models. Ecology 90 (1), 265–277.

Low Choy, S., Stewart-Koster, B., Kelly, A., Eyre, T., Mengersen, K. M., 2005. Identifying good condition in native vegetation: a Bayesian regression and decision theoretic approach. Pages 435-441 in A. Zerger and R.M. Argent, editors. *MODSIM 2005 International Congress on Modelling and Simulation.* Modelling and Simulation Society of Australia and New Zealand, Canberra, Australia.

Mahmoud, M., Liu, Y., Hartmann, H., Stewart, S., Wagener, T., Semmens, D., Stewart, R., Gupta, H., Dominguez, D., Dominguez, F., Hulse, D., Letcher, R., Rashleigh, B., Smith, C., Street, R., Ticehurst, J., Twery, M., van Delden, H., Waldick, R., White, D., Winter, L., 2009. A formal framework for scenario development in support of environmental decision-making. Environmental Modelling & Software 24, 798–808.

Manel, S., Williams, H., Ormerod, S., 2001. Evaluating presence–absence models in ecology: the need to account for prevalence. Journal of Applied Ecology 38, 921–931.

Marin, J.M., Robert, C., 2007. Bayesian Core: A Practical Approach to Computational Bayesian Statistics. Springer, New York.

Martin, T.G., Kuhnert, P.M., Mengersen, K., Possingham, H.P., 2005. The power of expert opinion in ecological models: a Bayesian approach examining the impact of livestock grazing on birds. Ecological Applications 15 (1), 266–280.

McCullagh, P., Nelder, J.A., 1989. Generalized Linear Models, second ed. Chapman & Hall, London.

Mouton, A.M., De Baets, B., Goethals, P.L.M., 2009. Knowledge-based versus data-driven fuzzy habitat suitability models for river management. Environmental Modelling & Software 24, 982–993.

Murray, J., Low Choy, S.J., Possingham, H., Goldizen., A., 2008. The importance of ecological scale for wildlife conservation in naturally fragmented environment: a case study of the brush-tailed rock-wallaby (*Petrogale penicillata*). Biological Conservation 141, 7–22.

Murray, J.V., Goldizen, A.W., O'Leary, R.A., McAlpine, C.A., Possingham, H.P., Low Choy, S.J., 2009. How useful is expert opinion for predicting the distribution of a species within and between regions? A case study using brush-tailed rock-wallabies (*Petrogale penicillata*. Journal of Applied Ecology 46, 842–851.

MySQL AB, 2008. MySQL Reference Manual. Sun Microsystems, Inc. Manual available from: http://dev.mysql.com/doc/index.html.

O'Hagan, A., 1997. The ABLE story: Bayesian asset management in the water industry. In: French, S., Smith, J.Q. (Eds.), The Practice of Bayesian Analysis. Arnold, London, pp. 173–198.

O'Hagan, A., Buck, C.E., Daneshkhah, A., Eiser, J.R., Garthwaite, P.H., Jenkinson, D.J., Oakley, J.E., Rakow, T., 2006. Uncertain Judgements: Eliciting Expert Probabilities. Wiley, United Kingdom.

O'Leary, R.A., Low Choy, S.J., Murray, J.V., Kynn, M., Denham, R., Martin, T.G., Mengersen, K., 2009. Comparison of three expert elicitation methods for logistic regression on predicting the presence of the threatened brush-tailed rock-wallaby *Petrogale penicillata*. Environmetrics 20, 379–398.

Paterson, B., Stuart-Hill, G., Underhill, L.G., Dunne, T.T., Schinzel, B., Brown, C., Beytell, B., Demas, F., Lindeque, P., Tagg, J., Chris Weaver, C., 2008. A fuzzy decision support tool for wildlife translocations into communal conservancies in Namibia. Environmental Modelling & Software 23, 521–534.

R Development Core Team, 2008. R: A Language and Environment for Statistical Computing. R Foundation for Statistical Computing, Vienna, Austria. http://www.r-project.org/ Software manual available from: http://cran.r-project.org/manuals.html.

Refsgaard, J.C., van der Sluijs, J.P., Højberg, A.L., Vanrolleghem, P.A., 2007. Uncertainty in the environmental modelling process: a framework and guidance. Environmental Modelling & Software 22, 1543–1556.

Spetzler, C.S., Staël von Holstein, C.-A.S., 1975. Probability encoding in decision analysis. Management Science 22 (3), 340–358.

Spiegelhalter, D.J., Thomas, A., Best, N.G., Lunn, D., 2003. WinBUGS Version 1.4 User Manual. MRC Biostatistics Unit, Cambridge.

Universal Transverse Mercator Coordinate System, 2009, June 1. Wikipedia, the Free Encyclopedia Retrieved June 9, 2009, from: http://en.wikipedia.org/wiki/Universal_Transverse_Mercator_coordinate_system.

Urbanek, S., 2006. RServe Documentation Available from: http://rosuda.org/Rserve/doc.shtml.

Van der Sluijs, J.P., Risbey, J.S., Ravetz, J., 2005. Uncertainty assessment of VOC emissions from paint in the Netherlands using the NUSAP system. Environmental Monitoring and Assessment 105, 229–259.

Venables, W.N., Ripley, B.R., 2002. Modern Applied Statistics Using S-Plus, fourth ed. Springer-Verlag.

Winkler, R., 1967. The assessment of prior distributions in Bayesian analysis. Journal of the American Statistical Association 62, 776–800.