# Calculus 3 Study Guide

## Coordinate Systems

### Polar Coordinates

**Note:**
Polar coordinates describe a point in 2D space using two values:

- $r$: the distance from the origin
- $\theta$: the angle from the positive x-axis

This system is ideal for problems with circular or rotational symmetry, such as spirals or radial fields.

#### Polar to Cartesian

- $x = r\cos\theta$
- $y = r\sin\theta$

#### Cartesian to Polar

- $r = \sqrt{x^2 + y^2}$
- $\tan\theta = \frac{x}{y} \to \theta = \tan^{-1}\left(\frac{x}{y}\right)$

**Example 1:** Convert the Cartesian point (3,3) into Polar coordiantes.

$r = \sqrt{x^2 + y^2} = \sqrt{3^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$
$\theta = \tan^{-1}\left(\frac{x}{y}\right) = \tan^{-1}\left(\frac{3}{3}\right) = \tan^{-1}(1) = \frac{\pi}{4}$
**Answer:** $(3\sqrt{2}, \frac{\pi}{4})$

**Example 2:** Convert the Polar point $(2, \frac{\pi}{3})$ to Cartesian coordiantes.

$x = r\cos\theta = 2\cos\left(\frac{\pi}{3}\right) = 2 \cdot \frac{1}{2} = 1$
$y = r\sin\theta = 2\sin\left(\frac{\pi}{3}\right) = 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}$
**Answer:** $(1, \sqrt{3})$

### Cylindrical Coordinates

**Note:**
Polar coordinates describe a point in 3D space using three values:

- $r$: The distance from the origin in the xy-plane
- $\theta$: The angle from the positive x-axis in the xy-plane (same as polar coordiantes)
- $z$: The height above (or below) the xy-plane

This system is useful for objects with circular symmetry around the z-axis, like cylinders and spirals.

### Cylindrical to Cartesian

- $x = r\cos\theta$
- $y = r\sin\theta$
- $z = z$

### Cartesian to Cylindrical

- $r = \sqrt{x^2 + y^2}$
- $\tan\theta = \frac{x}{y} \to \theta = \tan^{-1}\left(\frac{x}{y}\right)$
- $z = z$

## Spherical Coordinates

**Note:**
Spherical coordinates describe a point in 3D space using three values:

- $\rho$: the ditance from the origin
- $\theta$: the angle from the positive x-axis in the xy-plane (same as polar coordiantes)
- $\phi$: the angle from the postive z-axis down to the point

This system is useful for problems with radial symmetry, like spheres and cones.

### Spherical to Cartesian

- $x = \rho\sin\phi\cos\theta$
- $y = \rho\sin\phi\sin\theta$
- $z = \rho\cos\phi$

### Cartesian to Spherical

- $\rho = \sqrt{x^2 + y^2 + z^2}$
- $\tan\theta = \frac{x}{y} \to \theta = \tan^{-1}\left(\frac{x}{y}\right)$
- $\phi = \arccos\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$