

Calculus 3 Study Guide

Coordinate Systems

Polar Coordinates

Note:

Polar coordinates describe a point in 2D space using two values:

- r : the distance from the origin
- θ : the angle from the positive x-axis

This system is ideal for problems with circular or rotational symmetry, such as spirals or radial fields.

Polar to Cartesian

- $x = r \cos \theta$
- $y = r \sin \theta$

Cartesian to Polar

- $r = \sqrt{x^2 + y^2}$
- $\tan \theta = \frac{y}{x} \rightarrow \theta = \tan^{-1} \left(\frac{y}{x} \right)$

Example 1: Convert the Cartesian point (3,3) into Polar coordinates.

$$r = \sqrt{x^2 + y^2} = \sqrt{3^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$$
$$\theta = \tan^{-1} \left(\frac{y}{x} \right) = \tan^{-1} \left(\frac{3}{3} \right) = \tan^{-1}(1) = \frac{\pi}{4}$$

Answer: $(3\sqrt{2}, \frac{\pi}{4})$

Example 2: Convert the Polar point $(2, \frac{\pi}{3})$ to Cartesian coordinates.

$$x = r \cos \theta = 2 \cos \left(\frac{\pi}{3} \right) = 2 \cdot \frac{1}{2} = 1$$
$$y = r \sin \theta = 2 \sin \left(\frac{\pi}{3} \right) = 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}$$

Answer: $(1, \sqrt{3})$

Cylindrical Coordinates

Note:

Polar coordinates describe a point in 3D space using three values:

- r : The distance from the origin in the xy-plane
- θ : The angle from the positive x-axis in the xy-plane (same as polar coordinates)
- z : The height above (or below) the xy-plane

This system is useful for objects with circular symmetry around the z-axis, like cylinders and spirals.

Cylindrical to Cartesian

- $x = r \cos \theta$
- $y = r \sin \theta$
- $z = z$

Cartesian to Cylindrical

- $r = \sqrt{x^2 + y^2}$
- $\tan \theta = \frac{x}{y} \rightarrow \theta = \tan^{-1} \left(\frac{x}{y} \right)$
- $z = z$

Spherical Coordinates

Note:

Spherical coordinates describe a point in 3D space using three values:

- ρ : the distance from the origin
- θ : the angle from the positive x-axis in the xy-plane (same as polar coordinates)
- ϕ : the angle from the positive z-axis down to the point

This system is useful for problems with radial symmetry, like spheres and cones.

Spherical to Cartesian

- $x = \rho \sin \phi \cos \theta$
- $y = \rho \sin \phi \sin \theta$
- $z = \rho \cos \phi$

Cartesian to Spherical

- $\rho = \sqrt{x^2 + y^2 + z^2}$
- $\tan \theta = \frac{x}{y} \rightarrow \theta = \tan^{-1} \left(\frac{x}{y} \right)$
- $\phi = \arccos \left(\frac{z}{\sqrt{x^2 + y^2 + z^2}} \right)$