Contents

List of Figures

List of Tables

1 Potential Scattering

Scattering calculations have been performed for a projectile, with charge z_{proj}, scattering off a structure-less potential (equivalently - a one-state target) of the form

$$V(r) = z_{\text{proj}}\left(1 + \frac{1}{r}\right)e^{-2r}. \tag{1}$$

In these scattering calculations, the following parameters were constant: $r_{\max} = 200$, dr $= 0.001$ and $\ell_{\min} = 0$. Two sets of calculations were performed:

1. With $\ell_{\max} = 5$; for $z_{\text{proj}} \in \{-1, +1\}$, for $E_{\text{proj}} \in \{E_k = \alpha + \beta k^2\}_{k=1}^{20}$ with α, β such that $E_1 = 0.1\,\text{eV}$ and $E_{20} = 50.0\,\text{eV}$, the calculation was performed, and the ICS and DCS curves extracted.

2. With $z_{\text{proj}} = -1$, and $E_{\text{proj}} = 25.0\,\text{eV}$; for $\ell \in \{0, \ldots, 9\}$, the calculation was performed, and the ICS and DCS curves extracted.

1.1 ICS Curves

The total and partial Integrated-Cross-Section (ICS) curves, extracted from the first set of calculations, are shown for an electron and positron projectile in Figure 1 and Figure 2 respectively.

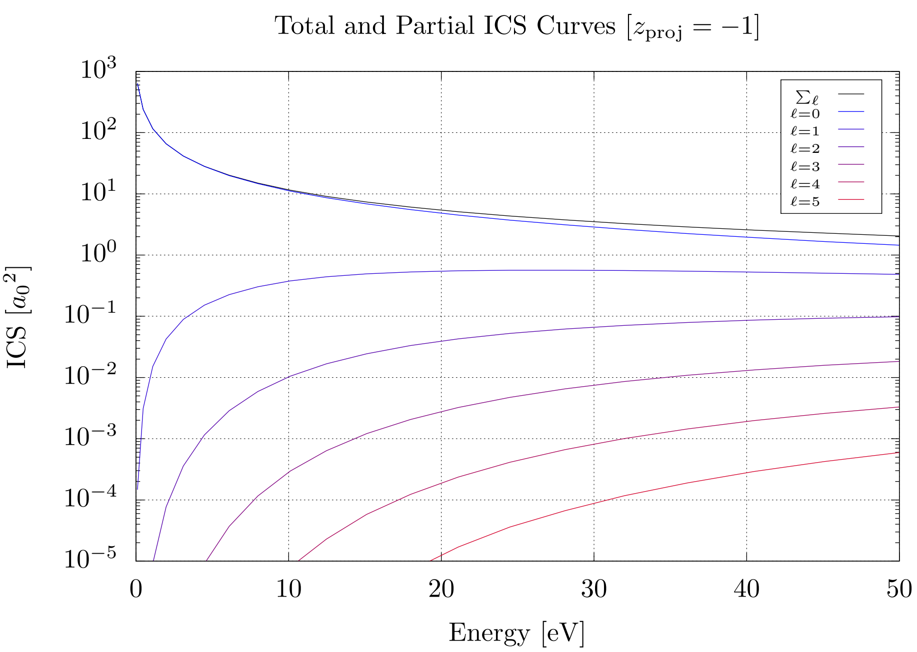

Figure 1: The total ICS curve (shown in black) and the partial ICS curves (shown in blue-to-red) are presented, across projectile energies $0.1\,\text{eV}$ *to* $50\,\text{eV}$*, for an electron projectile, with* $\ell_{\min} = 0$ *and* $\ell_{\max} = 5$*. Note that the y-axis is presented in log-scale.*

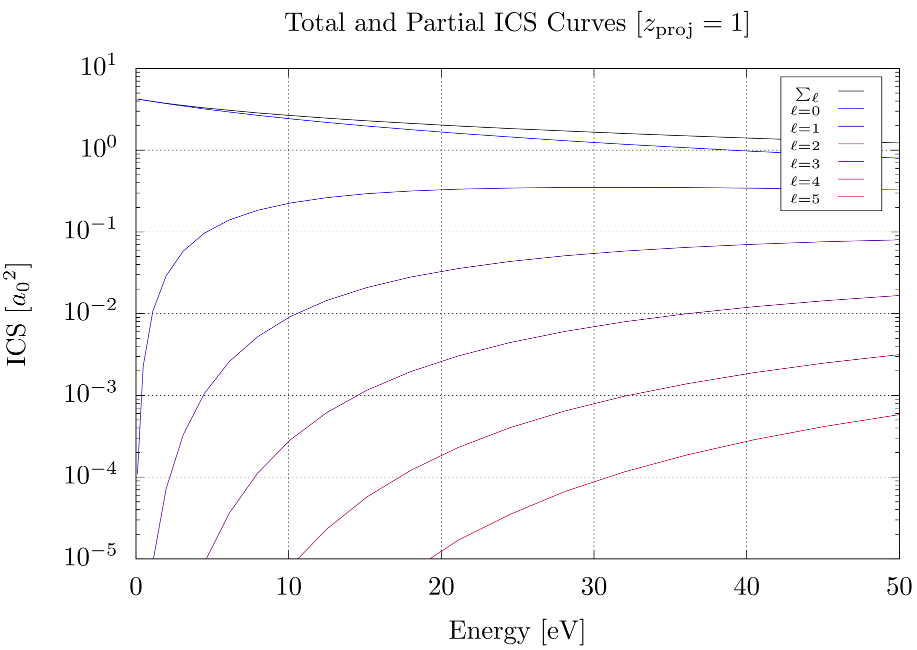

Figure 2: The total ICS curve (shown in black) and the partial ICS curves (shown in blue-to-red) are presented, across projectile energies $0.1\,\text{eV}$ *to* $50\,\text{eV}$*, for a positron projectile, with* $\ell_{\min} = 0$ *and* $\ell_{\max} = 5$*. Note that the y-axis is presented in log-scale.*

1.2 DCS Curves

The Differential-Cross-Section (DCS) curves, extracted from the first set of calculations, are shown for an electron and positron projectile in Figure 3 and Figure 4 respectively.

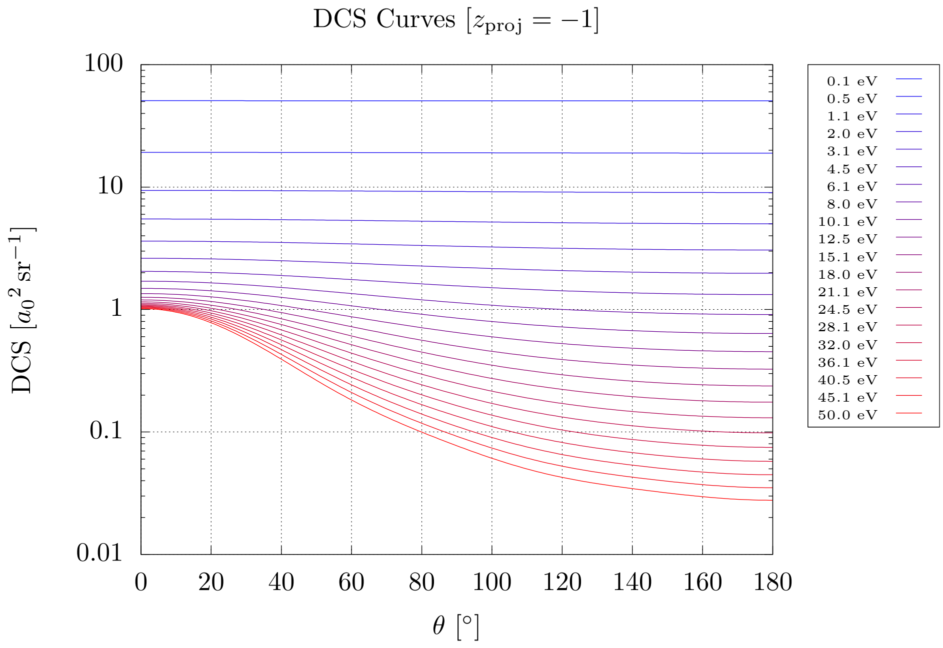

Figure 3: The DCS curves (shown in blue-to-red) are presented, across scattering angles 0° to 180°, for an electron projectile, with projectile energies ranging across $0.1\,\text{eV}$ to $50\,\text{eV}$, and with $\ell_{\text{min}} = 0$ and $\ell_{\text{max}} = 5$. Note that the y-axis is presented in log-scale.

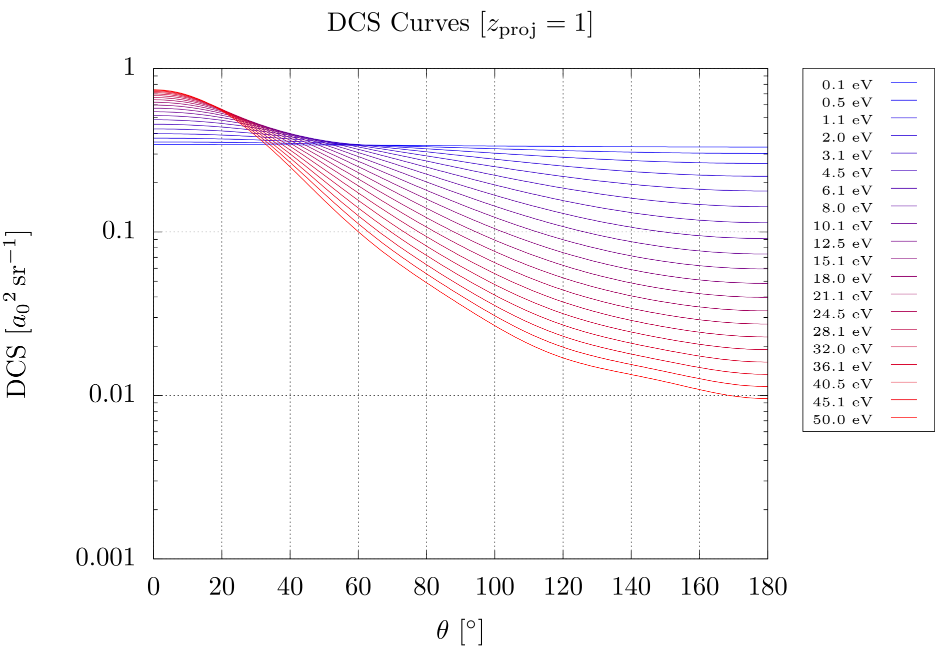

Figure 4: The DCS curves (shown in blue-to-red) are presented, across scattering angles $0°$ *to* $180°$*, for a positron projectile, with projectile energies ranging across* $0.1\,\text{eV}$ *to* $50\,\text{eV}$*, and with* $\ell_{\min} = 0$ *and* $\ell_{\max} = 5$*. Note that the y-axis is presented in log-scale.*

1.3 DCS Curve Convergence

The Differential-Cross-Section (DCS) curves, extracted from the second set of calculations, are shown in Figure 5.

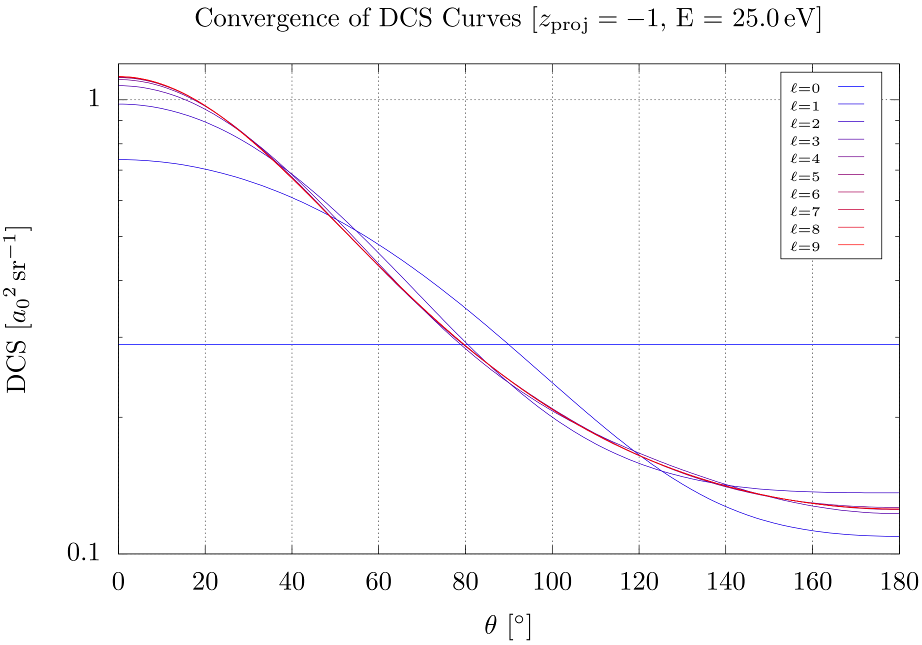

Figure 5: The DCS curves (shown in blue-to-red) are presented, across scattering angles $0°$ *to* $180°$*, for an electron projectile, with projectile energy* $E = 25.0\,\text{eV}$*, and* $\ell_{\min} = 0$*, with* $\ell_{\max}$ *ranging across* 0 *to* 9*. Note that the y-axis is presented in log-scale.*

It can be seen that the DCS converges rather quickly for this projectile energy of 25.0 eV. A point of interest is that the DCS curve, for $\ell_{\max} = 0$, is constant. This is a consequence of the behaviour of the zeroth-order Legendre polynomials $P_\ell(\cos\theta)$, for which $P_0(\cos\theta) = 1$. To see this, note that the differential cross section, for this scattering calculation, is of the form

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\theta) = |f(\mathbf{k}_f, \mathbf{k}_i)|^2$$

where $\mathbf{k}_f$ is such that $k_f = k_i$, and where $\cos\theta = \hat{\mathbf{k}}_f \cdot \hat{\mathbf{k}}_i$, with the scattering amplitude being of the form

$$f(\mathbf{k}_f, \mathbf{k}_i) = -\frac{\pi}{k_i^2} \sum_{\ell=\ell_{\min}}^{\ell_{\max}} (2\ell+1) T_\ell(k_i, k_i) P_\ell(\cos\theta).$$

Hence, where $\ell_{\min} = \ell_{\max} = 0$, we have that

$$f(\mathbf{k}_f, \mathbf{k}_i) = -\frac{\pi}{k_i^2} \sum_{\ell=0}^{0} T_0(k_i, k_i) P_0(\cos\theta) = -\frac{\pi}{k_i^2} T_0(k_i, k_i)$$

whence

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\theta) = \frac{\pi^2}{k_i^4} |T_0(k_i, k_i)|^2$$

demonstrating the constant behaviour of the DCS curve for $\ell_{\max} = 0$.

2 Derivation

3 Dimensional Analysis