Audio Engineering Society

Convention Paper 8590

Presented at the 132nd Convention
2012 April 26–29 Budapest, Hungary

Physical Model of the Slide Guitar: an Approach Based on Contact Forces

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ABSTRACT

In this paper we approach the synthesis of the slide guitar, which is a particular play mode of the guitar where continuous tuning of the tones is achieved by sliding a metal or glass piece, the bottleneck, along the strings on the guitar neck side. The bottleneck constitues a unilateral constraint for the string vibration. Dynamics is subject to friction, scraping, textured displacement and collisions. The presented model is physically inspired and is based on a dynamic model of friction, together with a geometrical model of the textured displacements and a model for collisions of the string with the bottlenck. These models are suitable for implementation in a digital waveguide computational scheme for the 3D vibration of the string, where continuous pitch bending is achieved by allpass filters to approximate fractional delays, friction is captured by nonlinear state-space systems in the slide junction and textured displacements by signal injection at a variable point in the waveguide.

1. INTRODUCTION

The slide guitar is a special play mode in which the player displaces a cylindrically shaped piece of metal or glass, the bottleneck, over the strings to produce continuously varying pitching, as opposed to discrete pitching constrained by the frets in a fretted instrument. In Figure 1 the coordinate system employed in this paper is shown, together with a bottleneck sliding on the string with velocity v_b.

Plucked by the player, the string oscillates in three dimensions. 3D string oscillations can be decomposed into two orthogonal transversal polarizations, which describe oscillations in planes orthogonal to the string rest line, plus longitudinal oscillations along the direction of the string rest line.

In our coordinate system, the transversal oscillations occur in the yz parallel planes, where for convention y is the horizontal direction, i.e. parallel to the fret

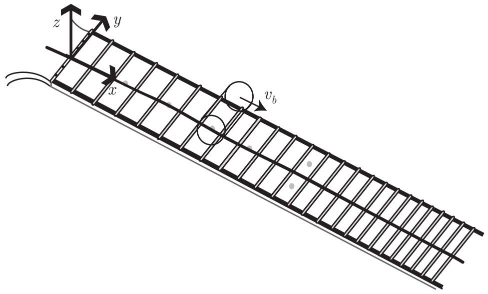

Figure 1: *Coordinate system and bottleneck sliding on a string.*

lines, and z is the vertical direction, i.e. orthogonal to the neck surface. Vertical and horizontal attributes refer to the natural directions when a guitar is lying flat on a table. The longitudinal oscillations occur along the x direction.

The main effect of slide guitar is the continuous variation of pitch resulting from the motion of the bottleneck. However, the sliding of the bottleneck along the string is accompanied by other side effects such as friction, scraping, textured displacement and collisions.

In normal play modes the player applies a slight force which reliably presses the bottleneck on the string, causing a local string deformation that travels with the bottleneck when this is displaced. The bottleneck is only supported by the string, it does not press or clamp the string against the neck or frets. Therefore, the string is only constrained not to trespass the bottleneck surface and that of the neck or fret. Thus, the string-bottleneck contact is unilateral, the string being able to vibrate underneath the bottleneck and to collide with it, accordingly, at locations near the contact point.

The movement of the bottleneck is subject to velocity dependent friction on the string surface. Depending on the type of string, different friction characteristics are observed: over a flat surface for plain strings or over regularly spaced bumps for wound strings. Friction and collisions also depend on the material of the strings.

An approach to the synthesis of the slide guitar has been previously proposed in [1]. However, their approach is empirical rather than physical since the effect of contact forces is obtained by the injection of filtered noise at the input of a comb filter generating string vibrations. In this paper, we attempt to model the complex of contact phenomena by taking a physical approach.

Friction is modeled as a dynamical system using a generalized Lu-Gre model [2, 3, 4]. In this model, the dynamics of friction is captured by the deflection of elastic or elasto-plastic bristles at contact area. Friction depends on velocity, resulting in stick and slip motion. The original model is able to capture dynamics at macroscopic level between two "flat" rigid surfaces. However, the type of string surface perturbs the deflection of the bristles in a way relevant to sound synthesis. In order to capture surface roughness needed for the synthesis of scraping sounds, an additive noise term was introduced in the Lu-Gre friction model in [5]. Experimental evidence of the fluctuation of the friction force over textured surfaces has been reported in [6].

Besides surface scraping of the bottleneck over the string, the profile and roughness of string and bottleneck also contribute to string displacement in the vertical z direction, orthogonal to the contact surface. Wound strings present a textured profile composed by the regularly spaced windings. The profile of the wound strings is modeled as a periodic array of connected "bumps". The bumps are semicircles in the case of round wound strings or rounded trapezoidal in the case of flat wound strings. The movement of the bottleneck gives rise to a perturbation of the z polarization mode injected at a variable point along the string according to the position of the bottleneck. At a constant bottleneck velocity the perturbation is periodic, where the frequency is directly proportional to the velocity of the bottleneck and inversely proportional to the winding spacing of the string. Due to friction and to the presence of the bumps, the movement of the bottleneck is also able to excite longitudinal modes, as in player's finger movements along the string [7].

The string and the bristles constitute two nonlinearly coupled dynamical systems. The resulting delay-free loops in the computation are resolved by applying the K-Method [8]. A nonlinear state-space system is obtained, in which the nonlinearity is reduced to a single equation relating current values of the state variables to past values of the state and the

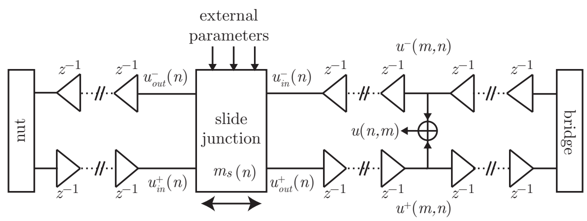

Figure 2: *Inclusion of a moving slide junction in a digital waveguides for string displacement waves. The required junction differs according to the mode of oscillation. The junction is displaced at integer string samples $m_s(n)$ and includes fractional delays.*

inputs. The solution amounts to root finding, which is handled by means of Newton-Raphson iteration.

The unilateral contact of the bottleneck on the string is enforced using methods previously developed by the author for the collisions of the strings with the neck [9]. Collisions are handled by a time-varying scattering junction to set proper post-collision initial conditions. The material of the string and that of the bottleneck determines the restitution coefficient of the collision. A single parameter controls the type of collision from completely inelastic to completely elastic.

2. DIGITAL WAVEGUIDES AND SLIDE JUNCTION

In a digital waveguide synthesis scheme [10] the string is spatially sampled. The spatial sampling interval is chosen as $X = cT$, where T is the time sampling interval and c is the propagation velocity. In principle, three digital waveguides are required in order to simulate the 3D vibration of the string. Due to different propagation speeds, the spatial sampling interval of the waveguide simulating longitudinal oscillations generally differs from that of the transversal modes. Forward and backward wave propagation, for each transversal and longitudinal mode, is simulated by means of two chains of elementary delays, one for each direction of propagation, constituting the two rails of the waveguide, shown in Figure 2. The waveguides corresponding to transversal and longitudinal modes are nonlinearly coupled at each point. However, coupling is weak and essentially occurs at points of strong amplitude of oscillation [11].

In sampled systems, the continuous variations of the active string length – the portion ranging from the bottleneck-string contact to the bridge – obtained by the positioning of the bottleneck requires interpolation for the simulation of the string-bottleneck contact dynamics. In digital waveguides, efficient simulation can be achieved by inserting a slide junction at a variable point along the waveguide, as shown in Figure 2. Displacing the junction has the effect of modulating the discrete length of waveguide portion from bottleneck to bridge.

Continuous length modulation is achieved by further introducing a variable fractional delays within the junction. The fractional delays are in charge of simulating the propagation delay in a small string portion of length proportional to the remainder of the integer division of the active string length versus the spatial sampling interval. They can be implemented by means of two IIR allpass filters or by means of two nearly allpass FIR filter, one for each rail of the waveguide. The number of needed allpass filters can be reduced to one by consolidating the total forward and backward fractional delays at the bridge, provided that no nonlinear processing with memory occurs at other portions of the string and within the bridge.

3. FRICTION MODEL

In order to simulate the stick-slip motion of the string in contact with the bottleneck, the development of a suitable computational model for friction is necessary.

In the horizontal y direction the string is free to oscillate, subject to friction against the bottleneck surface. The type of friction is independent of the string type, whether plain, round or flat wound, the motion occurs in a direction transversal to the string so that the string texture does not affect friction except for random roughness of the two surfaces. Generally, both string and bottleneck have quite smooth surfaces in this direction although some minor scraping can occur due to irregular surfaces and oxidation. Friction is also involved in the longitudinal direction x, where string texture critically influences the dynamics. Scraping does occur, especially noticeable for wound strings, which can excite longitudinal oscillation modes of the string.

3.1. Bristle-Based Friction Models

In this section we introduce a parametric friction model that is suitable for simulating friction in both horizontal and longitudinal directions. A sufficiently general scheme is derived from [2, 3], where the effect of friction is modeled as a dynamic system, know as the Lu-Gre model, which generalizes the Coulomb model. The contacting surfaces are thought of as being randomly coated by elastic bristles, which deflect as two surfaces are set in relative motion. Elastic bonds are created and destroyed along the motion.

Further generalization of the friction model has been introduced in [4]. The bristle based model has been previously used in sound synthesis to capture the dynamics of the violin bow [12], to model general friction interactions among rigid bodies [13] with modal synthesis and to model strig-fret interaction in digital waveguide simulation of the string [14].

The friction force f_f can be written in terms of bristle displacement and relative velocity as follows [2, 3]:

$$f_f(\xi, \dot{\xi}, v_{rel}) = \sigma_0 \xi + \sigma_1 \dot{\xi} + \sigma_2 v_{rel} + \sigma_3 w \qquad (1)$$

where v_{rel} denotes the relative velocity of the string-bottleneck contacting surfaces, ξ is the average deflection of elasto-plastic bristles and w is additive noise incldued here as in [5] to model scraping. The symbol $\dot{\xi} = \frac{d\xi}{dt}$ denotes first time derivative. The coefficients in (1) represent the parameters of friction and depend on the material and type of surfaces, where σ_0 represents the stiffness of the bristles' spring, σ_1 is a damping coefficient, σ_2 is the viscous friction coefficient and σ_3 is the scraping noise coefficient.

According to [3], the average deflection ξ of elasto-plastic bristles can be modeled by the following first order differential equation:

$$\dot{\xi} = v_{rel} \left(1 - \alpha(v_{rel}, \xi) \frac{\xi}{\xi_{ss}(v_{rel})} \right). \qquad (2)$$

The function $\alpha(v, \xi)$ allows us to capture the elasto-plastic behavior of the bristles for large displacement. In a simplified model (strictly Lu-Gre) one can let $\alpha(v_{rel}, \xi) = 1$.

The function $\xi_{ss}(v)$ provides the limit value for the deflection in steady state where the relative velocity v, instanciated by v_{rel} in (2), and the average bristle deflection are constant.

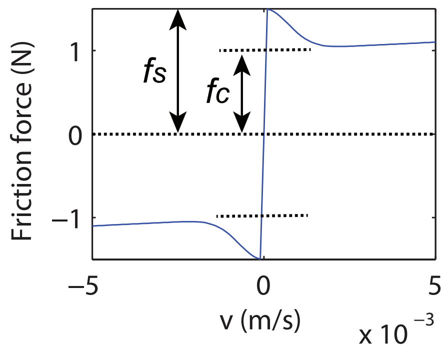

Figure 3: *Typical steady state friction force versus velocity.*

In the Lu-Gre parametrization [3] we have:

$$\xi_{ss}(v) = \frac{\mathrm{sign}(v)}{\sigma_0} \left(f_c + (f_s - f_c)\, e^{-(v/v_s)^2} \right) \qquad (3)$$

where f_c is the magnitude of Coulomb friction force, f_s is the magnitude of the static friction (*stiction*) force and v_s is the Stribeck velocity, which controls the characteristics of the Stribeck effect where friction continuously decreases as relative velocity increases in the low velocity regime. A typical plot of the steady state friction force versus velocity is shown in Figure 3.

Since the friction force depends on the relative velocity of the contact surfaces, the bristle model is able to capture stick-slip motion, continuously switching from static to kinetic friction according to velocity regimes.

4. DIGITAL WAVEGUIDE SIMULATION OF BOTTLENECK-STRING INTERACTION

In this section we consider the friction model reviewed in Section 3.1 to simulate the behavior of the string pressed by the bottleneck in the vertical z direction but free to move in the horizontal y and longitudinal x direction, subject to friction and scraping. We will first derive the continuous time systems describing the bottleneck-string interaction and then provide a discrete version of the model based on bilinear transform. Furthermore, we provide a scheme to compute the solution of the nonlinear difference equation describing the bottleneck-string node, which is based on the K-method [8].

4.1. Continuous Time Bottleneck-String Node

Let us denote by $u_y(x,t)$ and $u_z(x,t)$, respectively, the value of the string displacement at time t and position x along the string for the y and z polarization modes. Also, denote by $u_x(x,t)$ the longitudinal displacement of the string.

Disregarding nonlinear (coupling) [15] and dispersive effects [16, 17], the wave equation holds for segments of the string not in contact with other objects such as the plectrum or the player's finger and the bottleneck. Assume that the only object in contact with the string is the bottleneck, touching the string on a segment of width Δ and centered at coordinate x_b, then for a string of length L_s we have

$$c^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}; \qquad x \in \left]0, x_b - \tfrac{\Delta}{2}\right[\cup \left]x_b + \tfrac{\Delta}{2}, L_s\right[, \tag{4}$$

where $u(x,t)$ denotes any of the x, y or z string displacements and c is the propagation velocity. We have $c = \sqrt{K_0/\mu}$, where K_0 is the tension of the string, and μ is the linear mass density, for the y and z polarization modes. For the longitudinal propagation we have $c = c_L = \sqrt{ES/\mu}$ where E is the Young's modulus and S is the cross-section area of the string. We have disregarded all propagation losses along the string, as these can be consolidated at one of the extremities and embedded in the bridge model [10].

The solution of (4) can be written in D'Alembert form as a superposition of a left-going u^- and a right-going u^+ wave:

$$u(x,t) = u^-(x,t) + u^+(x,t) = u_l(t+x/c) + u_r(t-x/c), \tag{5}$$

where $u_l(x/c) = u_r(x/c) = u(x,0)/2$ for a static initial displacement condition.

4.1.1. Vertical polarization

For the vertical polarization mode u_z the portion of the string is subject to the force exterted by the player over the bottleneck. The contact is unilateral: the string can be assumed to be clamped unless a sufficient downward blow propagating to the contact zone detatches the string from the bottleneck. This detatchement is followed by subsequent collisions of the string with the bottleneck until stable contact is achieved again. This node therefore requires a time varying scattering junction as the one

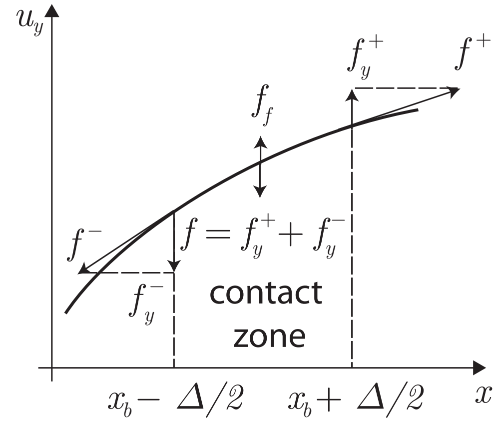

Figure 4: *String segment subject to tensile forces f^- and f^+ and friction f_f against the slide. For the y polarization, the tensile forces are projected along the y direction, obtaining f_y^- and f_y^+. The resultant $f = f_y^- + f_y^+$ of the projected tensile forces is considered as acting at the point x_b.*

introduced in [9] for collisions of the strings against the frets. In a simplified version, one can assume the string to be clamped, i.e., $u_z^+(x_b,t) \approx -u_z^-(x_b,t)$, at contact point with the bottleneck in the vertical direction and detect and handle collisions at nearby points towards the bridge. This is justified by the fact that the player normally exerts enough force for the bottleneck never to lose contact with the string.

4.1.2. Horizontal polarization

For the horizontal polarization mode u_y the string is free to oscillate subject to friction against the bottleneck surface and possible scraping noise. On the bottleneck-string contact segment, which we will also refer to as the contact zone shown in Figure 4, the equilibrium equation of the string with the bristle system (2) is enforced:

$$\mu\Delta \frac{\partial^2 u_y}{\partial t^2} = f(t) - f_f(\xi, \dot{\xi}, v_y) \qquad x \in \left]x_b - \tfrac{\Delta}{2}, x_b + \tfrac{\Delta}{2}\right[, \tag{6}$$

where the force $f(t)$ is the resultant of the transversal component of the tensile force of the string acting at the extreme points of the contact segment and $f_f(\xi, \dot{\xi}, v_y)$ is the friction force (1). The velocity v_y is the relative velocity of the string and the bottleneck. Since the player usually does not move the bottleneck in the y direction, v_y coincides with string displacement velocity in the y-polarization mode. Equation (6) can be rewritten in terms of $v_y(x,t) = \frac{\partial u_y}{\partial t}$:

$$\mu\Delta \frac{\partial v_y}{\partial t} = f(t) - f_f(\xi, \dot{\xi}, v_y) \qquad x \in \left]x_b - \tfrac{\Delta}{2}, x_b + \tfrac{\Delta}{2}\right[. \tag{7}$$

At small string displacements, for the tensile force we have:

$$f(t) = K_0 \left(\left.\frac{\partial u_y}{\partial x}\right|_{x=x_b+\frac{\Delta}{2}} - \left.\frac{\partial u_y}{\partial x}\right|_{x=x_b-\frac{\Delta}{2}} \right). \tag{8}$$

Since from (5) we have

$$\frac{\partial u}{\partial x} = \frac{1}{c}\left(v^-(x,t) - v^+(x,t)\right) \tag{9}$$

where

$$\begin{array}{l} v^-(x,t) = \frac{\partial u^-}{\partial t} \\ v^+(x,t) = \frac{\partial u^+}{\partial t} \end{array} \tag{10}$$

then (8) can be rewritten as follows:

$$f(t) = \frac{K_0}{c}\left(v_y^{in}(t) - v_y^{out}(t)\right), \tag{11}$$

where we have defined v_y^{in} as the velocity wave entering the contact zone and v_y^{out} as the velocity wave leaving the contact zone, i.e,

$$\begin{array}{l} v_y^{in}(t) = v_y^-(x_b + \frac{\Delta}{2}, t) + v_y^+(x_b - \frac{\Delta}{2}, t) \\ v_y^{out}(t) = v_y^+(x_b + \frac{\Delta}{2}, t) + v_y^-(x_b - \frac{\Delta}{2}, t). \end{array} \tag{12}$$

Assimilating $v_y(x_b,t)$ to $v_y^{out}(t)$, i.e., shrinking the system (7) to a point, while retaining the finite mass $\mu\Delta$, we obtain the bottleneck-string node equation for the horizontal mode:

$$\begin{array}{l} \mu\Delta \frac{dv_y^{out}}{dt} = \\ \frac{K_0}{c}\left(v_y^{in} - v_y^{out}\right) - \sigma_0\xi - \sigma_1\dot{\xi} - \sigma_2 v_y^{out} - \sigma_3 w, \end{array} \tag{13}$$

where we have substituted (1) and (11) in (6) after establishing that $v_{rel} = v_y^{out}$.

The bristle displacement function ξ in (13) must satisfy equation (2). Defining a state vector

$$\mathbf{x} = \begin{bmatrix} v_y^{out} \\ \xi \end{bmatrix}, \tag{14}$$

equations (13) and (2) can be put in the form of a nonlinear state space system:

$$\left\{ \begin{array}{l} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}v_y^{in} + \mathbf{d}w + \mathbf{e}\phi \\ \phi = \rho(\mathbf{x}) \end{array} \right., \tag{15}$$

where

$$\begin{array}{l} \mathbf{A} = \frac{-1}{\mu\Delta}\begin{bmatrix} \sigma_2 + \frac{K_0}{c} & \sigma_0 \\ 0 & 0 \end{bmatrix} \\ \mathbf{b} = \frac{K_0}{c\mu\Delta}\begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ \mathbf{d} = \begin{bmatrix} -\frac{\sigma_3}{\mu\Delta} \\ 0 \end{bmatrix} \\ \mathbf{e} = \begin{bmatrix} -\frac{\sigma_1}{\mu\Delta} \\ 1 \end{bmatrix} \end{array} \tag{16}$$

and

$$\rho\left(\begin{bmatrix} v_y^{out} \\ \xi \end{bmatrix}\right) = v_y^{out}\left(1 - \alpha(v_y^{out}, \xi)\frac{\xi}{\xi_{ss}(v_y^{out})}\right) \tag{17}$$

is a scalar function of the state vector.

In the form (15) the system describing the bottleneck-string node for the horizontal polarization is ready for suitable discretization required in digital simulations of strings.

4.1.3. Longitudinal Modes

In the longitudinal modes the string is assimilated to a thin bar with uniform section area S. The bottleneck-string node for the simulation of friction and scraping of the longitudinal modes is similar to that derived in the previous section for the horizontal modes. Indeed, equation (8) is replaced by the following

$$f(t) = ES\left(\left.\frac{\partial u_x}{\partial x}\right|_{x=x_b+\frac{\Delta}{2}} - \left.\frac{\partial u_x}{\partial x}\right|_{x=x_b-\frac{\Delta}{2}} \right), \tag{18}$$

which describes the force at the contact point x_b as the difference of pressure at the edges of the small contact zone.

The relative velocity of string and bottleneck in the longitudinal direction is $v_{rel} = v_x^{out} - v_b$, where v_x^{out} is the longitudinal velocity wave leaving the contact zone and v_b is the time-varying velocity of the bottleneck counted as positive in the nut to bridge direction. Thus, in the bottleneck-string node for the longitudinal oscillations, equation (13) is replaced by

$$\mu\Delta \frac{dv_x^{out}}{dt} = \frac{ES}{c_L}\left(v_x^{in} - v_x^{out}\right) - \sigma_0\xi - \sigma_1\dot{\xi} - \sigma_2(v_x^{out} - v_b) - \sigma_3 w, \tag{19}$$

where $c_L = \sqrt{ES/\mu}$. Consequently, system (15) becomes:

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}v_x^{in} + \mathbf{c}v_b + \mathbf{d}w + \mathbf{e}\phi \\ \phi = \rho(\mathbf{x}) \end{cases}, \tag{20}$$

where

$$\begin{aligned} \mathbf{A} &= \frac{-1}{\mu\Delta}\begin{bmatrix} \sigma_2 + \frac{ES}{c_L} & \sigma_0 \\ 0 & 0 \end{bmatrix} \\ \mathbf{b} &= \frac{ES}{c_L\mu\Delta}\begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ \mathbf{c} &= \begin{bmatrix} -\frac{\sigma_2}{\mu\Delta} \\ 0 \end{bmatrix} \\ \mathbf{d} &= \begin{bmatrix} -\frac{\sigma_3}{\mu\Delta} \\ 0 \end{bmatrix} \\ \mathbf{e} &= \begin{bmatrix} -\frac{\sigma_1}{\mu\Delta} \\ 1 \end{bmatrix} \end{aligned} \tag{21}$$

and

$$\rho\left(\begin{bmatrix} v_x^{out} \\ \xi \end{bmatrix}\right) = (v_x^{out} - v_b)\left(1 - \alpha(v_x^{out} - v_b, \xi)\frac{\xi}{\xi_{ss}(v_x^{out} - v_b)}\right) \tag{22}$$

is a scalar function of the state vector

$$\begin{bmatrix} v_x^{out} \\ \xi \end{bmatrix}. \tag{23}$$

4.2. Discrete Time Computation of the Bottleneck-String Node

In this section we carry out the discretization of the system (15) using the bilinear transformation and show how to handle the delay-free loops in the computation.

The system in (15) is characterized by first order derivatives. In Laplace transform a differentiator is equivalent to multiplication by the Laplace variable s. By bilinear transformation, s is replaced by $2(1-z^{-1})/T(1+z^{-1})$, where T is the sampling time interval. Accordingly, a first order differential equation of the type

$$\dot{\eta}(t) = f\left(\eta(t), t\right) \tag{24}$$

is led by bilinear transformation to the recurrence

$$\eta(n) = \eta(n-1) + \frac{T}{2}\left[f\left(\eta(n), n\right) + f\left(\eta(n-1), n-1\right)\right], \tag{25}$$

where we dropped the factor T in the arguments of the functions.

Using this rule, it is easy to discretize the system (15). The discrete version of equation for the first state component expresses the current value of the output velocity $v_y^{out}(n)$ in terms of past values of v_y^{out}, present and past values of the input velocity v_y^{in}, present and past values of w, present and past values of ϕ and present and past values of bristle deflection ξ. The discrete version of the equation for the second component of the state becomes the recurrence:

$$\xi(n) = \xi(n-1) + \frac{T}{2}(\phi(n) + \phi(n-1)). \tag{26}$$

This recurrence can be substituted in the first state component recurrence in order to remove the dependency on the present value of ξ, obtaining

$$\begin{aligned} &v_y^{out}(n) = c_1 v_y^{out}(n-1) + c_2\xi\,(n-1) \\ &+c_3\left[v_y^{in}(n) + v_y^{in}(n-1)\right] + c_4\left[w(n) + w(n-1)\right] \\ &+c_5\left[\phi(n) + \phi(n-1)\right] \end{aligned} \tag{27}$$

where

$$\begin{aligned} c_1 &= \frac{2\,\mu\,\Delta c - Tc\sigma_2\, - TK_0}{2\,\mu\,\Delta c + Tc\sigma_2\, + TK_0} \\ c_2 &= -\frac{2Tc\sigma_0}{2\,\mu\,\Delta c + Tc\sigma_2\, + TK_0} \\ c_3 &= \frac{TK_0}{2\,\mu\,\Delta c + Tc\sigma_2\, + TK_0} \\ c_4 &= -\frac{Tc\sigma_3}{2\,\mu\,\Delta c + Tc\sigma_2\, + TK_0} \\ c_5 &= -\frac{Tc\left(\sigma_1\, + \frac{T}{2}\sigma_0\right)}{2\,\mu\,\Delta c + Tc\sigma_2\, + TK_0}. \end{aligned} \tag{28}$$

We are then left with a recurrence for v_y^{out} that depends on known values, except for that of $\phi(n)$. Yet, the equation for ϕ requires the value of $v_y^{out}(n)$ in order to be computed, which is a delay-free loop of the system. This delay-free loop must be properly handled in order to be able to find the solution, as described next.

Substituting the recurrence for $v_y^{out}(n)$ and that for $\xi(n)$ in the vector argument of the function ρ in (15), one obtains an equation of the type

$$\phi(n) = g(\phi(n), n), \tag{29}$$

where g is a known function, which is built from ρ by isolating the dependency on ϕ and reducing all other dependencies to an explicit dependency on time index n. This equation can be solved by finding, at any sample index n, a local zero of the function

$$\zeta - g(\zeta, n), \tag{30}$$

which can be achieved by means of Newton-Raphson root finding method. Look-up tables for the roots can be precalculated in order to ease real-time computation [8]. The root ζ of (30) is assigned to $\phi(n)$ and all other quantities are known in order to compute $v_y^{out}(n)$ and $\xi(n)$, which describes how to handle the delay-free loop in the computation.

A very similar derivation of the discrete system simulating friction and scraping in the longitudinal mode can be performed, which is here omitted.

4.3. Slide Junction in Digital Waveguides

The discrete time realization of the bottleneck-string interaction blocks illustrated in the previous sections are directly usable as blocks in digital waveguides for the synthesis of strings based on velocity waves. The blocks are included in the waveguide simulating the horizontal and longitudinal modes. In order to fix our ideas we provide the details for the horizontal modes.

The input velocity for the horizontal mode block v_y^{in} is obtained by summing the input velocities v_{in}^{+} and v_{in}^{-} from the two rails of the waveguide. The output velocity v_y^{out} obtained from the fret-string system is equally fed to the two rails of the waveguide. In order to force the output velocity at the fret contact point, a scattering junction of the type

$$\begin{bmatrix} v_{out}^{-} \\ v_{out}^{+} \end{bmatrix} = \mathbf{S_c} \begin{bmatrix} v_{in}^{-} \\ v_{in}^{+} \end{bmatrix} + \frac{v_y^{out}}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{31}$$

where

$$\mathbf{S_c} = \frac{1}{2} \begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix} \tag{32}$$

is included, similar to what described in [9] in order to force after-collision displacement.

In the displacement variables, differentiator and integrator blocks have to be introduced in order to convert back and forth to velocity variables. These blocks can be derived by applying the bilinear transformation to the analog differentiator and integrator, similar to Section 4.2. The bilinear differentiator is given by the following recurrence:

$$v(n) = -v(n-1) + \frac{2}{T}\left(u(n) - u(n-1)\right) \tag{33}$$

while the bilinear integrator is

$$u(n) = u(n-1) + \frac{T}{2}\left(u(n) + u(n-1)\right) \tag{34}$$

as in (25). Bilinear integrator and differentiator also have the advantage of being inverse of each other.

The insertion of the slide junction in a displacement wave based digital waveguide is shown in Figure 2.

5. SCRAPING

In order to feed the slide junction illustrated in the previous section, one needs to specify the signal w representing scraping noise. We will address this problem in this section.

In the horizontal y polarization, the cross section of the string oscillates under the bottleneck. Therefore, the type of string, wound or plain, is immaterial. In this case, the signal w can be equated to 0 (no scraping as surfaces are smooth enough) or to a noise source of small amplitude depending on relative velocity. A $1/f$ noise source provides good acoustic results in this case [5], which can be justified by a small-scale fractal model of the contacting surfaces.

In the vertical z polarization mode, no friction junction is present as the motion of the bottleneck is orthogonal to this direction. However, on wound strings, the pressure of the bottleneck pushes the string down in a space-dependent way, due to the presence of the wound thread. The string texture in this case presents small ridges and valleys, as shown in Figure 5 for a round wound string.

The characteristics of the perturbation caused by the movement of the bottleneck can be analyzed with the help of the geometry sketched in Figure 6. There, the bottleneck (large circle of radius R) meets the winding of a round wound string, whose section is represented by small connected circles of radius r, which are enlarged in the figure for illustration purposes. The repetitive motion pattern of the bottleneck pushing down the windings of the string while sliding can be captured by an angle α such that $0 \leq \alpha < \theta$ where

$$\theta = 2 \arcsin \frac{r}{r+R}. \quad (35)$$

When α increases from 0 to θ, the corresponding linear displacement of the bottleneck along x is $2r$, which corresponds to the ridge to ridge distance of the winding. The linear displacement corresponding to the angle α is

$$x = 2(r+R) \sin \frac{\alpha}{2}. \quad (36)$$

The point of maximum displacement of the string occurs when $\alpha = 0$, where the bottleneck lies precisely on a ridge. The point of minimum displacement occurs when $\alpha = \theta/2$, where the bottleneck lies precisely on a valley in between two ridges. The distance z_P of the center O of the bottleneck section to the point P lying at intersection of the perpendicular line from O with the line connecting the centers of two adjacent thread sections O_1 and O_2 measures the local amplitude of the perturbation up to a constant value. From the figure we have:

$$z_P = (R+r) \left[\cos\alpha - \sin\alpha \tan\left(\theta/2 - \alpha\right)\right] \quad (37)$$

Using (36) one can write (37) as a periodic function of x. An average value can be subtracted from (37) so that the function is zero mean.

The linear displacement x_b of the bottleneck is obtained by integrating the velocity v_b in time. For a constant velocity, the perturbation of the oscillation in z caused by the movement of the bottleneck is a periodic function whose frequency is inversely proportional to the winding distance. As the bottleneck accelerates or decelarates the frequency of the perturbation increases or decreases, compatibly with observations. This perturbation is simply injected at the bottleneck contact point to the waveguide simulating the vertical z oscillation of the string.

In a similar way one can proceed in order to derive a form for the scraping noise w to be fed to the longitudinal modes slide junction. In this case, one can observe from Figure 6 that the displacement of the centers of the winding sections follows the law

$$w = 2r(1 - \cos\left(\theta/2 - \alpha\right)), \quad (38)$$

from which a periodic function of the bottleneck position can be derived via (36).

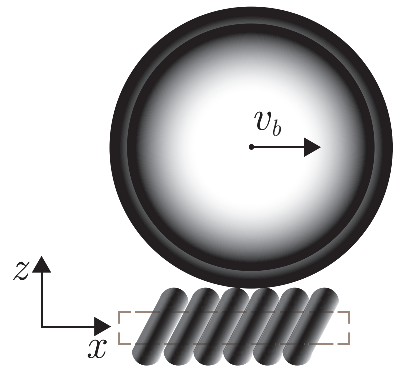

Figure 5: *The sliding of the bottleneck with velocity v_b over a round wound string.*

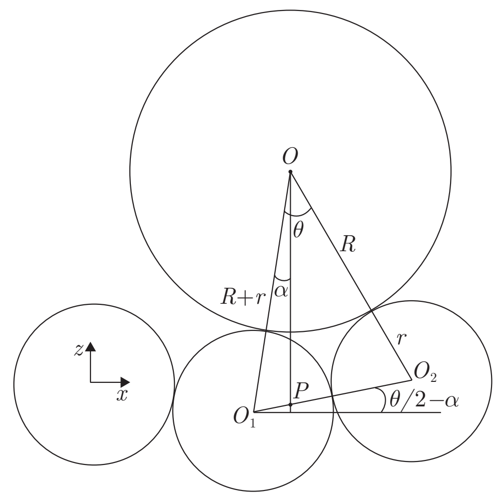

Figure 6: *Geometry of the displacement of a round wound string under bottlenck pressure.*

6. CONCLUSIONS

In this paper we proposed a model of the slide guitar based on a physical model. At an increased computational cost, which is still suitable for real time synthesis, the proposed approach provides a parametric model, which is easily adapted to various string materials and configurations and produces very realistic sound. The proposed model is a piece of a larger project for the high quality synthesis of the guitar and of the interactions of the player.

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