Digital Synthesis of Complex Spectra by Means of Multiplication of Nonlinear Distorted Sine Waves*

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A technique for the synthesis of complex dynamic spectra is described. The first step in the synthesis method is to distort sine waves with nonlinear transfer functions. The resulting spectra depend upon the input amplitudes and the nature of the distortions. By multiplying one such distorted source by another one obtains a spectrum that is the convolution of the spectra of the distorted sine waves. By varying the amplitudes of the input sine waves one can produce complex spectral evolutions. Harmonic as well as inharmonic spectra can be produced, and control over the formant structure is provided. With one distorted and one pure wave, results similar to those of Chowning's frequency-modulated technique are produced. With several distorted signals very complex spectral evolutions are possible. Various implementations will be discussed using a computer or special-purpose digital device.

0 INTRODUCTION

Synthesizing a sound by digital methods, for example, using the Music V program [1] or special digital circuits, implies the definition of the mathematical procedure to be followed for constructing the sound and for specifying its parameters. For example, additive synthesis consists of adding sine waves, the amplitudes of which are functions of time. Subtractive synthesis corresponds to the digital filtering of a complex waveform where one changes the filter characteristics with time. However, with these two methods one must be painstaking to obtain sounds with rich musical quality and various playing possibilities. Recent research has focused on global methods of synthesis, which can be called modulation methods: a sound is defined by the evolution in time of one amplitude, one or more frequencies determining a line spectrum, and one or more indexes of timbre determining the amplitude of these partials. This class of methods includes the frequency-modulation synthesis technique described by Chowning [2] where the time variables are the amplitude, the carrier and modulation frequencies, the modulation index(es), and the discrete-summation-formula method described by Moorer [3], [4] where the variables are the amplitude, the starting frequency, the frequency difference between partials, and one index controlling the ratio of the successive partials. The power of such methods depends upon their ability to generate a palette of related but distinct sounds. Similarly, in the syntax of the Music V program there are two levels: the instrument (INS definition) which can define a class of sounds, and the note parameters (NOT) which can define one specific sound inside this class. In this sense a global method is powerful if one can define differentiated instruments with enough possible articulations in notes. Nonlinear distortion of sine waves (also called wave shaping) is a global method which can be introduced as follows: The distortion of a sine wave by a "bad" amplifier produces a number of harmonics, the amplitude of which depends upon the amplitude of the input and the nonlinearity of the amplifier. This technique has already been used in computer music. (Risset [5] generated clarinetlike sounds by using a simulation of a saturated amplifier.) A first im-

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provement is to use a transfer function[1] in terms of a limited-degree polynomial so that the input is band-limited, eliminating foldover [1]. This method uses the fact that it is possible to calculate a transfer function of this form transforming a given input value into a given output spectrum. Conversely it is possible to calculate the output spectrum from every transfer function and input value (Shaefer [6], Suen [7]). For a given transfer function the variation of the input amplitude (which we will now call the index function) leads to a variation of the spectral balance of the output. The knowledge of a transfer function allows us to determine an instrument, that is, a class of sounds. Notes of different qualities are produced by choosing different index functions.

The first part of this paper deals with the standard use of nonlinear distortion. Although self-sufficient, this method becomes more interesting when we add amplitude modulation (multiplication of the output by a sine wave), thereby shifting the spectrum and folding it through zero (as in frequency modulation). As we shall explain, this allows us to produce spectra with formant structures, missing harmonics, and inharmonic sounds.

By multiplying more signals (which can be distorted or not), one can obtain more complex sounds. For example, an equivalent to the double frequency modulation [8] is presented whereby little additional computation yields complex sounds. Some simple examples are explicitly given, and some indications for producing more elaborate sounds are mentioned.

1 THE THEORY

The starting point of calculation has been clearly exposed by Shaefer [6] for a fixed-output spectrum and by Suen [7] for an evolving input. Here we give a presentation of their results in matrix form, which may be easier to understand and which is certainly easier to program.

Given in a nonlinear amplifier, the transfer function of which is $F(I)$, and an input $I(t)$, the output is $F(I(t))$. If now the input is a cosine wave of amplitude x and the transfer function an nth-order polynomial, we can write:

$$I(t) = x \cos(\omega t)$$

$$F(x) = d_0 + d_1 x + d_2 x^2 + d_3 x^3 + \cdots + d_n x^n \qquad (1)$$

$$O(t) = d_0 + d_1 x \cos \omega t + \cdots + d_n x^n \cos^n \omega t .$$

Developing $\cos^k(\omega t)$ in terms of $\cos(k\omega t)$ yields the following relations where h_i is the amplitude of the ith harmonic:

$$O(t) = (1/2)H_0 + H_1 \cos \omega t + \cdots + H_n \cos n\omega t \qquad (2)$$

with:

$$\begin{bmatrix} h_0 \\ h_1 \\ h_2 \\ \cdot \\ \cdot \\ \cdot \\ h_n \end{bmatrix} = 2 \cdot A \cdot \begin{bmatrix} d_0 \cdot (x/2)^0 \\ d_1 \cdot (x/2)^1 \\ d_2 \cdot (x/2)^2 \\ \cdot \\ \cdot \\ \cdot \\ d_n \cdot (x/2)^n \end{bmatrix} . \qquad (3)$$

A is an $(n+1, n+1)$ matrix which is abstracted from a general two-dimensional array. Fig. 1 gives the generative relations and presents the first ten lines and columns of this matrix. A practical example is also given which shows how to calculate the output spectrum partials for a given distortion and one index value.

A transfer function of degree n gives a spectrum composed of n harmonics (which is very important in digital synthesis because this can avoid foldover, provided the sampling frequency is greater than twice the nth-harmonic frequency value). There is complete independence between the odd harmonics (depending upon the values of the odd-order coefficients of the polynomial) and the even ones. This can be seen in the A matrix, for a_{ij} equals 0 if i and j do not have the same parity. This feature in particular permits an easy control of the balance between odd and even harmonics in a spectrum.

Conversely, the expression of $\cos(k\omega t)$ in terms of $\cos^k(\omega t)$ gives the relations between an output spectrum for a given value of the index x and the resulting d_i coefficients:

$$\begin{pmatrix} d_0 \\ d_1 \\ d_2 \\ \cdot \\ \cdot \\ \cdot \\ d_n \end{pmatrix} = ½ \begin{pmatrix} (2/x)^0 \\ (2/x)^1 \\ (2/x)^2 \\ \cdot \\ \cdot \\ \cdot \\ (2/x)^n \end{pmatrix} \cdot B \cdot \begin{pmatrix} h_0 \\ h_1 \\ h_2 \\ \cdot \\ \cdot \\ \cdot \\ h_n \end{pmatrix} . \qquad (4)$$

B is an $(n+1)$ matrix, the generative procedure and a (10,10) subset of which are given in Fig. 2. A practical example indicates how to calculate the polynomial coef-

[1] The term "transfer function" must be understood as meaning: waveshaping function (out = f(in)) and not at all as in substractive synthesis (linear filtering) the s or z transform.

The recursion formula for calculating A (where i is the line and j is the column) is

$$a(1,1) = 1$$

$$a(i,j) = a(i-1, j-1) + a(i+1, j-1), \quad i \neq 1$$

$$a(1,j) = 2 \times a(2, j-1) \qquad i = 1$$

An easier way to compute these relations is to set the first column and to calculate successive columns:

$$A = \begin{bmatrix} 1 & 0 & 2 & 0 & 6 & 0 & 20 & 0 & 70 & 0 \\ & 1 & 0 & 3 & 0 & 10 & 0 & 35 & 0 & 126 \\ & & 1 & 0 & 4 & 0 & 15 & 0 & 56 & 0 \\ & & & 1 & 0 & 5 & 0 & 21 & 0 & 84 \\ & & & & 1 & 0 & 6 & 0 & 28 & 0 \\ & & & & & 1 & 0 & 7 & 0 & 36 \\ & & & & & & 1 & 0 & 8 & 0 \\ & & & & & & & 1 & 0 & 9 \\ & & & & & & & & 1 & 0 \\ & & & & & & & & & 1 \end{bmatrix}$$

$P(x) = x + 2x^2 + 3x^3$. Calculation of H for $x = 0.3$:

$$\begin{pmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \end{pmatrix} = 2 \times \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \times 1 \\ 1 \times 0.3 \\ 2 \times 0.09 \\ 3 \times 0.027 \end{pmatrix}$$

Fig. 1. Computing and using the A matrix.

ficients to obtain a given output spectrum for one value of the index.

2 STANDARD USE OF NONLINEAR DISTORTION

The preceding section has shown how to calculate a transfer function $F(I)$. This allows us to produce for a given input of amplitude x an output spectrum with given values of the first n harmonics, using a polynomial of nth order. Our concern is now the evolution of the output spectrum in terms of the evolution of the x input.

With the same transfer function we can calculate every spectrum corresponding to other values of the input amplitude x (which is in fact a timbre index). Each sound produced this way has no more than n harmonics, and the evolution of the spectrum with x presents no discontinuity, though it can present individual large variations of the harmonics, including zeroing and changing of sign. As indicated earlier, the evolution of odd and even harmonics is absolutely unrelated (as a limit example, setting to zero the even-order coefficients of the polynomial ensures every spectrum to comprise only odd harmonics). For any transfer function we can draw the evolution of a spectrum as a function of the input amplitude. A possible representation is shown in Fig. 3 where the evolution of the spectrum is computed for different values of the index.

3 STANDARD USE OF NONLINEAR DISTORTION IN DIGITAL SYNTHESIS

To perform nonlinear distortion, a sound synthesis program or a digital device requires a polynomial function generator in order to get a table corresponding to the transfer function, and the simulation of a distorting amplifier through a table look-up device.

For matrix B the first line is as follows:

$$1 \quad 0 \quad -2 \quad 0 \quad +2 \quad 0 \quad -2 \quad 0 \quad +2 \quad \cdots.$$

The other lines can be computed according to the recursion formula

$$b(i, j) = b(i - 1, j - 1) - b(i, j - 2).$$

$$B = \begin{bmatrix} 1 & 0 & -2 & 0 & 2 & 0 & -2 & 0 & 2 & 0 \\ & 1 & 0 & -3 & 0 & 5 & 0 & -7 & 0 & 9 \\ & & 1 & 0 & -4 & 0 & 9 & 0 & -26 & 0 \\ & & & 1 & 0 & -5 & 0 & 14 & 0 & -30 \\ & & & & 1 & 0 & -6 & 0 & 20 & \\ & & & & & 1 & 0 & -7 & 0 & 27 \\ & & & & & & 1 & 0 & -8 & \\ & & & & & & & 1 & 0 & -9 \\ & & & & & & & & 1 & 0 \\ & & & & & & & & & 1 \end{bmatrix}$$

The output spectrum is

$$\begin{pmatrix} h_0 = 0 \\ h_1 = 11 \\ h_2 = 12 \\ h_3 = 13 \end{pmatrix}$$

For $x = 0.5$:

$$\begin{pmatrix} d_0 \\ d_1 \\ d_2 \\ d_3 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 \\ 4 \\ 16 \\ 64 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 11 \\ 12 \\ 13 \end{pmatrix}$$

$$\left(\frac{2}{x}\right)^i$$

Fig. 2. Computing and using the B matrix.

3.1 Polynomial Function Generator

A function generator has to compute a table of values corresponding to the transfer function, the input sequence being the coefficients $d_0, d_1, d_2, \cdots, d_n$ of the polynomial. Due to the fact that the function is digitally stored in a limited area and its absolute value must be bound, the generated function must be centered and normalized. Hence we have chosen to store the function in a $(2m + 1)$ area and to compute

$$F(j) = \left(\sum_{i=0}^{n} d_i \left(\frac{j}{m} \right)^i \right) / T.$$

T is a normalization value such that the maximum value of this function is 1. It must be clear that with this definition we can relate x and j by

$$j = m \cdot x \qquad \text{and} \qquad x = j/m$$

and so the excursion of the index of timbre x is limited to $(-1, 1)$ as stated in Fig. 4. This has proved not to be a drastic limitation, because one can transform the original mathematical polynomial and input by the change of variables $x' = x/x_{\max}$ and $d_i' = d_i x_{\max}{}^i$.

3.2 Table Look-up Unit Generator

We now need a table look-up device (or program). It must calculate an output from two inputs corresponding to: OUT = I1·F(I2). If it is calculated without interpolation (by truncation), the table must be chosen large enough to avoid a kind of quantification noise. Another feature is that the origin of the scanning is not the first location but the $(m + 1)$th one, so the input can be negative.

3.3 An Instrument for Standard Use of Nonlinear Distortion

We give here more specific details in the framework of a Music V implementation. A function generator has been written which calculates a polynomial function the values of which are stored in a 512-location area. Since it is not an odd value, they are effectively stored in 511 locations (Fig.

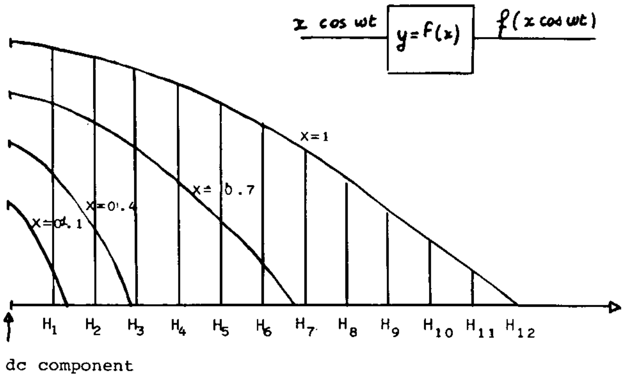


Fig. 3. Evolution of the output spectrum as a function of the index.

4), so the preceding value of m is 255. A new unit generator has been written, which is a table look-up with interpolation. The standard Music V oscillator could have been used in a degenerated way, with a null increment; however, this is cumbersome. An instrument following Mathews [1] definitions can now be defined in Fig. 5.

The index function F_1 is intended primarily to describe the variation of timbre with time. However, the loudness is also a function of the index, and care must be taken in the appreciation of the amplitude value P_5 that will often have to be variable and not fixed as in Fig. 5. Some automatic correction could be calculated by means of a normalization function, as discussed by Moorer [3]. However, it requires storage for a new function and computation time, it complicates the instrument, and it is only approximate. Moreover this variation of loudness often sounds natural, and the simplest way to use this instrument is to use the P_5 input if necessary as a postcompensation of loudness.

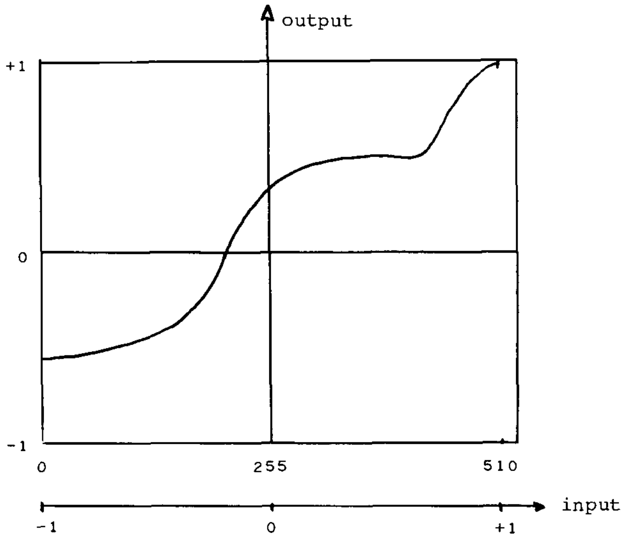


Fig. 4. Table containing the transfer function.

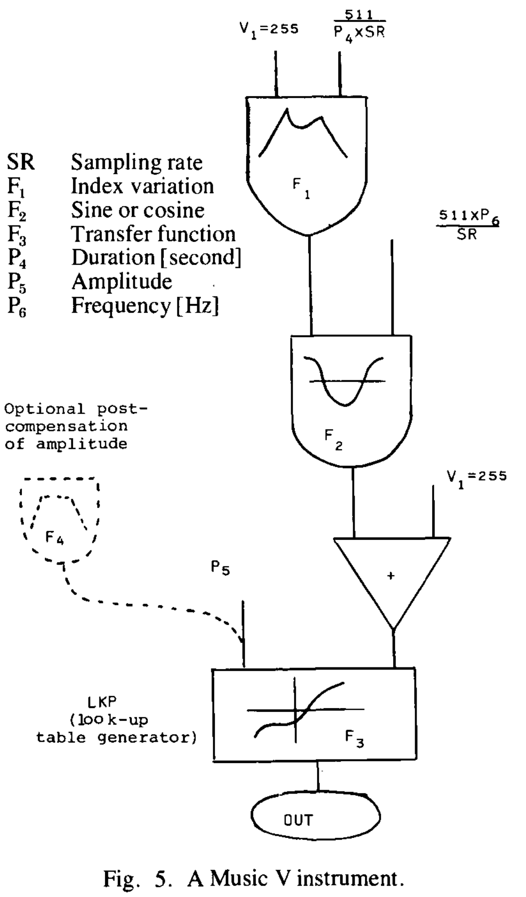


Fig. 5. A Music V instrument.

The transfer function F_3 generates the distortion and consequently determines the harmonic content of the output as a function of the input. In general the timbre becomes richer as the index increases, but this is highly dependent on the choice of this transfer function. However, some general remarks can be made: In the standard use of the nonlinear distortion the d_0 term (the constant of the polynomial) should be set to zero to avoid a click at the beginning and at the end of a note, especially with softer (small index value) notes. The odd and even harmonics are independent, and the maximum number of harmonics is equal to the degree of the polynomial (but the higher harmonics may be insignificant). When the order is small, one can use the direct equalities (3) to predict this evolution. With larger values one needs the aid of a computer program displaying the evolution of the spectrum as a function of the index value.

3.4 Choice of a Transfer Function

The characteristics of the nonlinear distortion are entirely determined by the coefficients of the polynomial. But there are many ways to calculate these coefficients.

3.4.1 Evaluation from a Steady-State Spectrum

Relation (4) allows us to calculate the d coefficients (the polynomial) starting from the h ones (the spectrum), given an index value x_0. We have seen that the distortion of a cosine wave gives only cosine components, the amplitudes of which are real but can be positive or negative. So different distortions, hence different timbre variations, can be used, which verify the initial conditions only by changing the signs of some components. An abruptly limited spectrum or a rich spectrum (more than 20 harmonics) often gives rise to irregularities in the evolution of the sound with the index, which is akin to the Gibbs phenomenon.

3.4.2 Evaluation from a Continuous Transfer Function

In this case the general form of the transfer function is known, and the problem is to get a polynomial approximation of it in order to obtain a band-limited spectrum and avoid foldover. A limited development of the function approximates it very well around the origin (small values of the index), but usually not in other areas. One can also take the complete transfer function, calculate the spectrum for a specific index value, and then use the first procedure with the first n harmonics. This approximates well the spectrum for that value of the index, but abrupt bandwidth limitation causes the ripple effect previously described, and it may prove useful to attenuate little by little the last partials to get smoother evolutions with almost identical spectra. (This is in effect a kind of windowing.) Other classical algorithms can be used, such as the least-mean-square approximation.

3.4.3 Direct Evaluation

An experimental and/or intuitive choice of the coefficients of the polynomial is also a good strategy. There is a strong but subtle connection between the regularity of variation of the coefficients (and their sign) and the homogeneity of the resulting timbre. Choosing to affect the same sign to all odd-order coefficients, and doing so for even-order coefficients (though this sign may be different from the other one), produces a very steep transfer function which may produce overly brassy sounds. A good practice can be to alternate the sign of successive odd-order coefficients (say, $a1$, $a5$, $a9$ positive and $a3$, $a7$, $a11$ negative) and do the same for even-order coefficients.

3.5 Some Examples

Examples 1 to 4 use the previously described Music V instrument, except that "amplitude" input (for correction of loudness) comes from an oscillator scanning once the compensation function F_4.

3.5.1 Example 1: Brilliant Sounds

Many nonlinear distortions create impulselike waveforms (Fig. 6). In this case the timbre becomes richer as the index increases.

3.5.2 Example 2: Clarinetlike Sounds

We have determined a distortion producing a sound with only odd harmonics, becoming richer as the index increases and able to produce 25 harmonics without ripple phenomenon (Fig. 7).

3.5.3 Example 3: Spectra Merging into one Harmonic

A Chebyshev function of nth order produces a distortion where the spectrum is restricted to harmonic number n when the index is 1. Let us assume that the index evolves as shown in Fig. 8. If n is odd, only odd harmonics are produced in the intermediate state; if it is even, the sound is an octave higher. With respect to this higher fundamental, harmonic $n/2$ is produced at the end, the intermediate state comprising both odd and even harmonics (Fig. 8).

3.5.4 Example 4: Sounds Merging into a Group of Harmonics

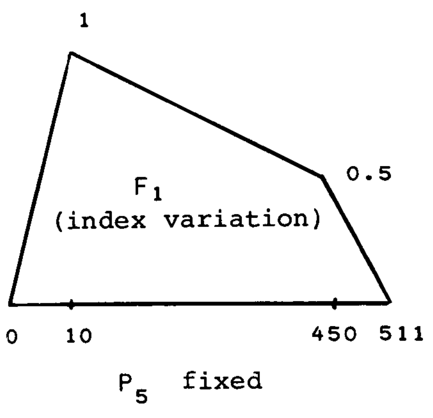
A formantlike spectrum can be achieved, for example, with the function described in Fig. 9 which has been calculated to give a formant spectrum for an x value of 0.8. The sound described in this figure evolves from a simple sine wave to this formantlike spectrum. The sound of Fig. 10 yields a percussive spectrum with a complex resonance by keeping the same distortion and by choosing a different index evolution.

4 SHIFTING AND FOLDING THE SPECTRUM

Through amplitude modulation the spectrum can be shifted to any desired center, thus folding around zero some components of the sound. In the Music V format this corresponds to the multiplication of the output of our preceding instrument by a sine wave at frequency f_{AM}; it can be represented as in Fig. 11. A Music V instrument may be the one shown in Fig. 12 in which the additional parameters are the amplitude-modulation frequency and the initial phase P_8.

4.1 How Amplitude Modulation Works

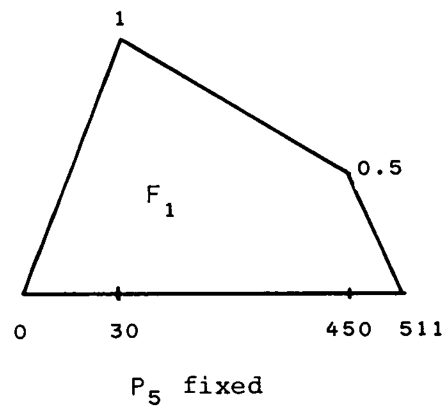
What happens to the components of the original sound because of amplitude modulation can be explained in the following way. The multiplication of time functions is equivalent to the convolution of their spectra. In our case the resulting spectrum is the complex sum of the spectrum



$400\text{ Hz} < P_6 < 1000\text{ Hz}$
$P_4 = 1$ second

$$\begin{aligned} P(x) = {} & x + 0.9x^2 + 0.8x^3 + 0.72x^4 \\ & + 0.63x^5 + 0.55x^6 + 0.50x^7 + 0.46x^8 \\ & + 0.43x^9 + 0.40x^{10} + 0.38x^{11} + 0.36x^{12} \\ & + 0.34x^{13} + 0.32x^{14} + 0.30x^{15} + 0.28x^{16} \\ & + 0.26x^{17} + 0.24x^{18} + 0.22x^{19} + 0.20x^{20} \\ & + 0.18x^{21} + 0.16x^{22} + 0.14x^{23} + 0.12x^{24} \\ & + 0.08x^{25} + 0.06x^{26} + 0.04x^{27} + 0.02x^{28} \end{aligned}$$

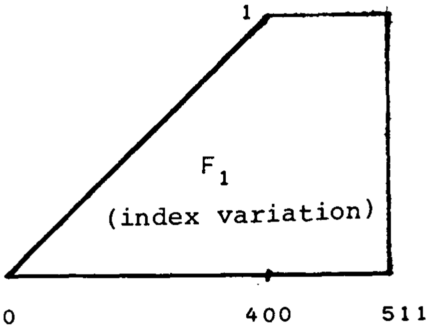
Fig. 6. Brilliant sounds.



$125\text{ Hz} < P_6 < 500\text{ Hz}$
$P_4 = 1$ second

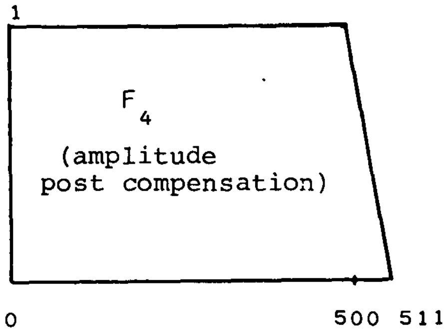
$$\begin{aligned} P(x) = {} & 5.141x - 33.246x^3 + 176.02x^5 \\ & - 668.494x^7 + 1857.797x^9 - 3848.826x^{11} \\ & + 5996.932x^{13} - 7017.083x^{15} \\ & + 6085.882x^{17} - 3801.572x^{19} \\ & + 1619.364x^{21} - 421.588x^{23} + 50.66x^{25} \end{aligned}$$

Fig. 7. Clarinetlike sounds.



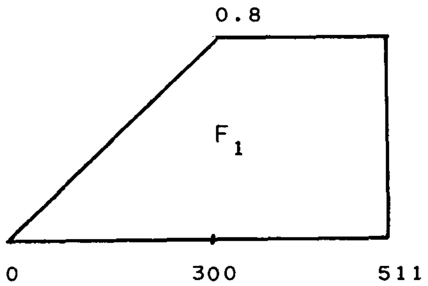
P_4 = 10 seconds
P_6 = 261 Hz (C_4)

$P_9(x) = 9x - 120x^3 + 432x^5 - 576x^7 + 256x^9$



$P_{10}(x) = 0 + 50x^2 - 400x^4 + 1120x^6 - 1280x^8 + 512x^{10}$

Fig. 8. Use of Chebyshev-like polynomials.



P_4 = 8 seconds
P_6 = 261 Hz (C_4)

$$P(x) = x + 0.71x^2 - 1.64x^3 - 6.17x^4 + 7.77x^5 + 19.3x^6 - 14.19x^7 - 24.84x^8 + 8.87x^9 + 11.08x^{10}$$

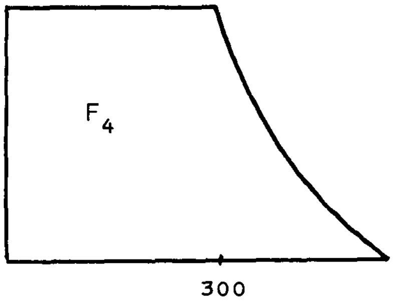


Fig. 9. Sound merging in a formant structure.

produced by the nonlinear distortion shifted to the right by the quantity f_{AM} and also shifted in phase, and of the same spectrum shifted to the left and reverse shifted in phase (Fig. 13). Taking, as we did, the nonlinear distortion of a cosine wave rather than a sine wave permits us to understand more easily the importance of the phase of the modulation (P_8 in our instrument).

1) If the amplitude modulation uses a cosine wave,

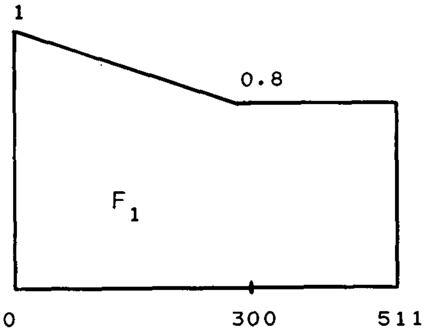
$$\cos(\omega t) = \frac{1}{2} (e^{j\omega t} + e^{-j\omega t})$$

the resulting spectrum is half the algebraic sum of the two oppositely shifted spectra because the initial distortion produces only cosine terms.

2) If the amplitude modulation uses a sine wave,

$$i \cdot \sin(\omega t) = \frac{1}{2} (e^{j\omega t} - e^{-j\omega t})$$

the resulting spectrum is half the algebraic difference of the two oppositely shifted spectra, and afterwards phase shifted by $\pi/2$. The phase has importance only if folded compo-



P_4 = 8 seconds
P_6 = 261 Hz (C_4)

$$P(x) = x + 0.71x^2 - 1.64x^3 - 6.17x^4 + 7.77x^5 + 19.3x^6 - 14.19x^7 - 24.84x^8 + 8.87x^9 + 11.08x^{10}$$

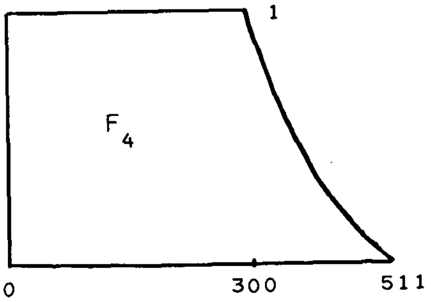


Fig. 10. Percussive sound with complex resonance.

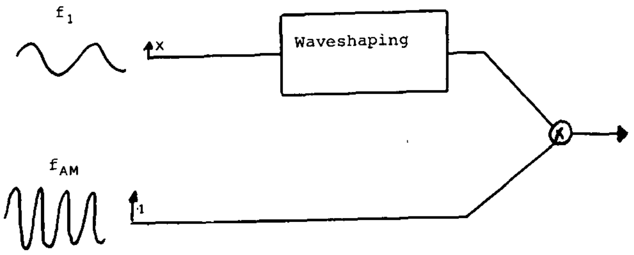


Fig. 11. Amplitude modulation.

nents are covering one another. A dc component can produce clicks, and in this case it is important to note that using a sine modulation wave eliminates this dc component to avoid clicks at the beginning and at the end of a note.

4.2 Harmonic and Inharmonic Spectra

We have seen that the spectrum thus shifted has components distributed around the amplitude modulation frequency f_{AM}, spaced at intervals f_1 (frequency of the undistorted sine wave). The components folded through zero may be thought of as originating from parts of the left spectrum (Fig. 13).

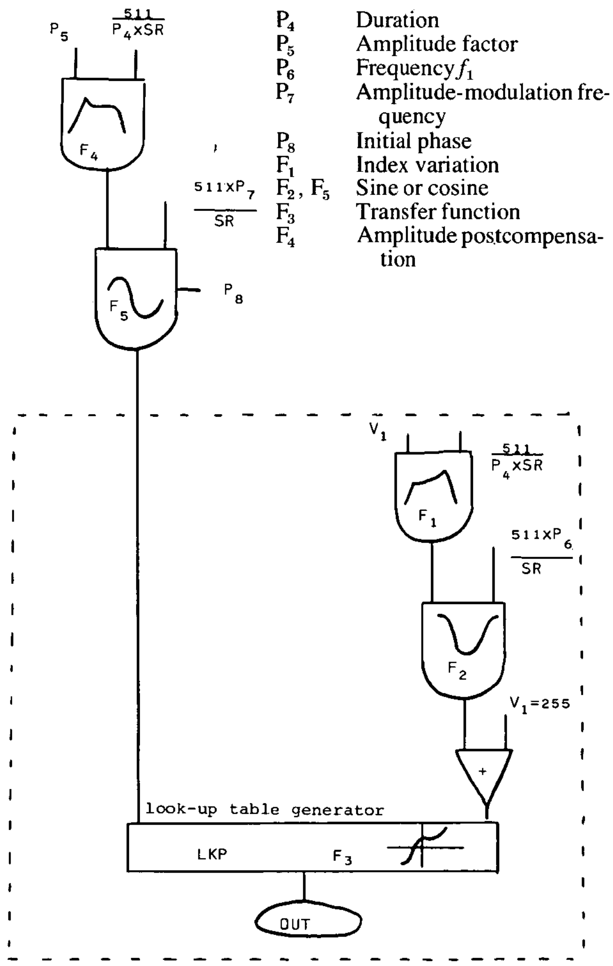


Fig. 12. A Music V instrument for amplitude modulation.

The use of amplitude modulation is very close to the frequency-modulation synthesis technique. Producing a harmonic spectrum needs:

$$\frac{f_1}{f_{AM}} = \frac{N_1}{N_2} \tag{5}$$

(where N_1 and N_2 are mutually prime), or written in another way,

$$f_1 = N_1 f_0$$

$$f_{AM} = N_2 f_0$$

where f_0 is the resulting fundamental frequency.

N_1 and N_2 being mutually prime, there is an exact superposition (and thus the phase becomes important) only if $N_1 = 1$ or 2. Fig. 14 shows the aspect of the spectrum for different values of N_1 and N_2 without any consideration of amplitude and phase values. (These depend on the nature of the distortion used.)

Inharmonic spectra are produced if f_1/f_{AM} cannot be expressed as a ratio of integers, that is, if folded components are not in simple harmonic relation with direct ones (Fig. 15). If f_1/f_{AM} approaches a simple fraction, some roughness or even beats can be heard, but the main impression is one of harmonicity (though no exact rational fraction can be written). If f_1/f_{AM} is represented by a nonsimple fraction (the limit being dictated by perception), inharmonicity can be heard, though the spectrum is quite harmonic (but with a sparse spectrum), and even noisy sounds can be produced this way (for example, by keeping $N_1 = 1$ and taking $N_2 = 10$ or 20, or even more).

The effect of the constant term of the polynomial is transferred to the carrier component. For a zero value of the index, if d_0 is nonzero, it produces a sine wave at the amplitude-modulation frequency. The presence of clicks also depends on the amplitude envelope. If the amplitude (postcompensation) rises and falls slowly, there will be no clicks.

4.3 Choosing the Amplitude Modulation

Choosing the carrier frequency (amplitude-modulation

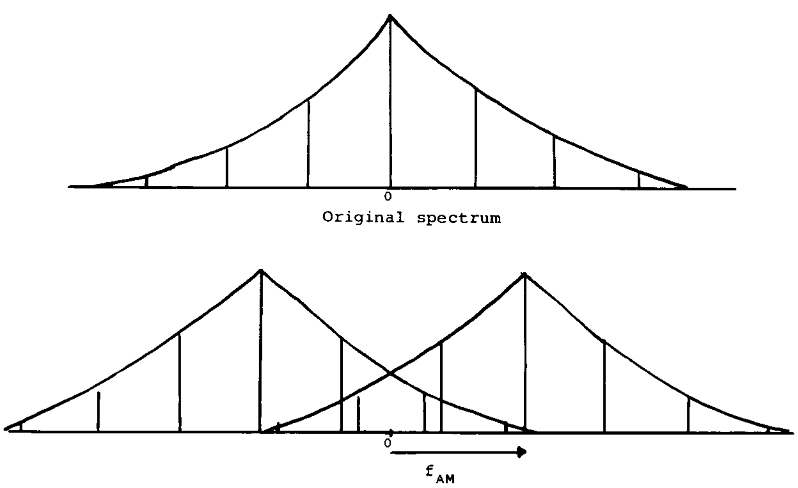


Fig. 13. Shifting and folding the spectrum.

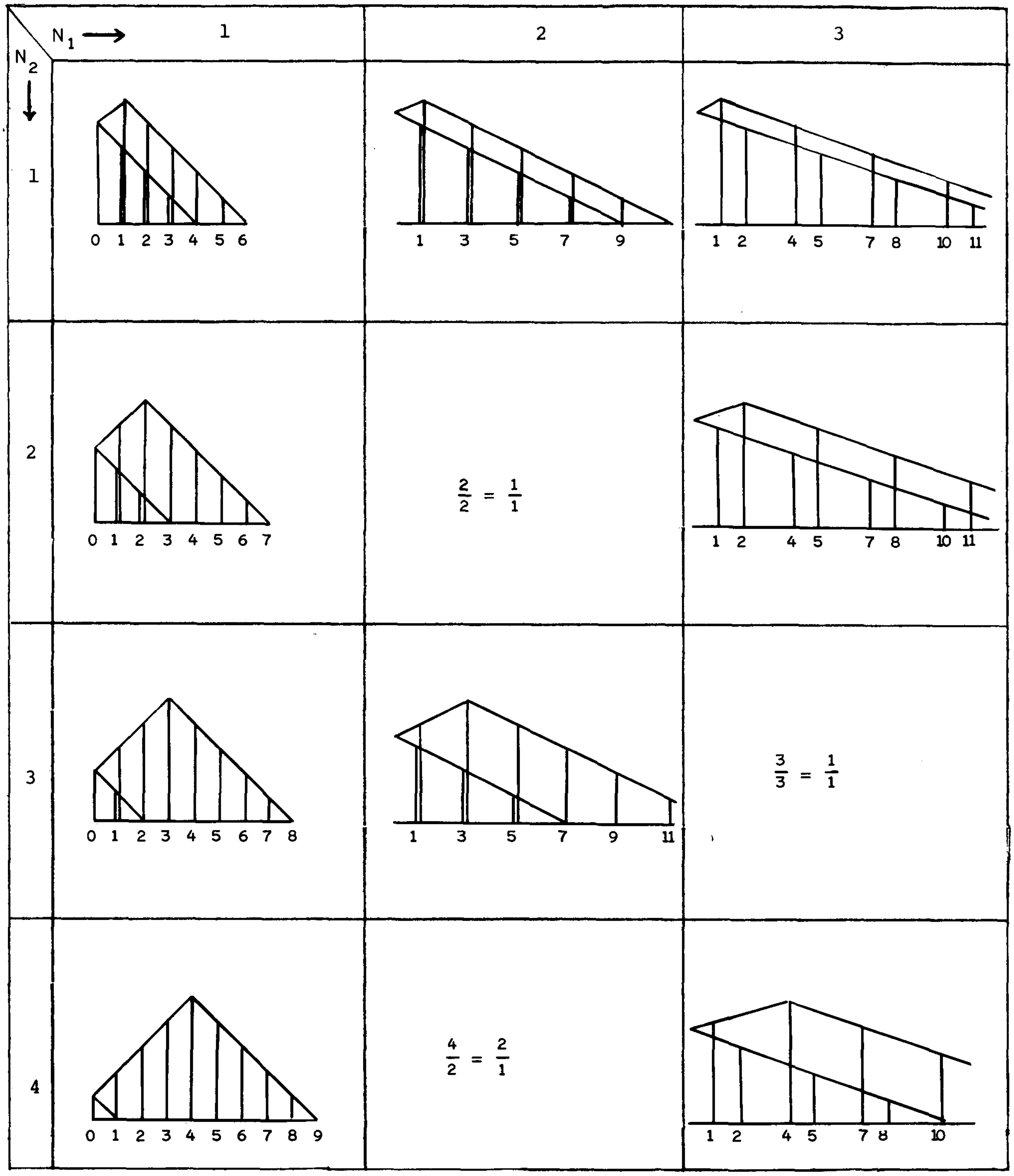


Fig. 14. Aspects of spectra with $f_{AM} = N_2 f_0$ and $f_1 = N_1 f_0$.

frequency f_{AM}) and the modulation frequency (input frequency f_1) gives the pattern on which the nonlinear distortion spectrum will be inscribed, shifted, and folded through zero. As was suggested for frequency modulation by Maillard [9], one can obtain a good image of what happens by drawing a frequency axis on transparent paper, marking a point at the carrier frequency, and distributing n equidistant points to the right and left of this point. (n is the degree of the polynomial.) These points may fall in the negative frequency domain. Then draw a spectrum, reporting the

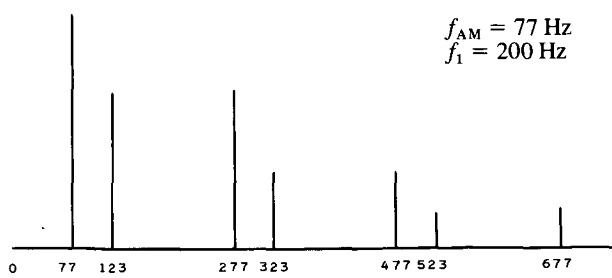


Fig. 15. Inharmonic spectrum.

harmonic lines of the distorted cosine wave symmetrically about the amplitude-modulation frequency. (If the output expression is, as stated, $(1/2)h_0 + h_1 \cos \omega t + \cdots h_n \cos n\omega t$, draw an h_0 component at the carrier frequency, h_1 at the immediate left and right, and so on.) If the modulation wave is a cosine, all you have to do is to fold your transparent paper at the frequency origin, thus obtaining the folded components to be added. If the modulation uses a sine wave, these folded components must be subtracted. If there is no coincidence between the two parts of the paper, the phase is of no real acoustical importance. If there is a component at the origin, take it without modification (cosine modulation) or not at all (sine modulation). The latter case avoids the formation of possible clicks, for the dc component is always zero. However, even with cosine modulation, clicks due to the dc component are rarely audible, for this component does not exist for small values of the index. Now if we use a modulation wave between sine and cosine, components are to be added in the complex frequency domain, one shifted and the other reverse shifted.

4.4 Some Examples

As for standard use, these are crude examples, that is, they use only nonlinear distortion and amplitude modulation. These processes must be made more complex to get musical sounds, as will be developed further.

4.4.1 Example 5: Sounds with Resonances

By shifting a spectrum up to a high harmonic we can simulate the equivalent of a filter of decreasing bandwidth through the tone (Fig. 16).

4.4.2 Example 6: Percussive Sounds

An inharmonic (and somewhat noisy) percussive sound occurs if the partial around which the formantlike structure lies is higher than before. This is obtained by changing the modulation frequency (Fig. 17).

4.4.3 Example 7: Plucked Sounds

If the formant structure is lower situated, the sound has the dual property of becoming thin but centered higher in frequency (Fig. 18) when the index decreases.

4.4.4 Example 8: Clarinetlike Sounds

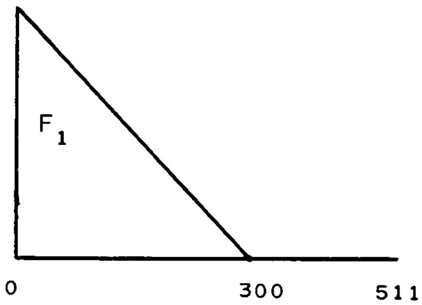
Odd harmonics can only be produced by an ordinary distortion (with odd and even terms in the polynomial) by choosing $f_1 = 2f_{AM}$ (Fig. 19) or $f_1 = 4f_{AM}$.

4.4.5 Example 9: Bell or Chime Imitation

Many inharmonic sounds can be obtained by using a nonrational value for f_1/f_{AM}, for example, $\sqrt{2}$ (Fig. 20). (This is a useful number because the folded components do not lie too close to zero or the initial fundamental frequency f_0.) With a fixed choice of carrier and initial fundamental frequency, one can choose different distortion, which makes it possible to play timbre melodies.

5 DOUBLE MODULATION AND MISCELLANEOUS

Chowning [8] uses a double frequency modulation so that the frequencies of the partials are now $\pm f_c \pm f_{m1} \pm f_{m2}$. A good description of the use of double frequency modulation for the simulation of piano and violin sounds is found in Schottstaedt [11]. This way procedure is a real improvement over classical frequency modulation, and one can ask if it is possible to find an equivalent with nonlinear distor-



P_4 = 4 seconds
P_6 = 80 Hz
P_7 = 800 Hz

$P(x) = 1 + x + x^2 + x^3 + x^4 + x^5 + x^6$

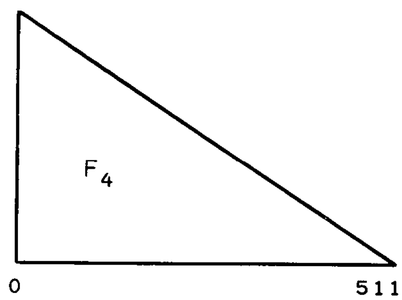
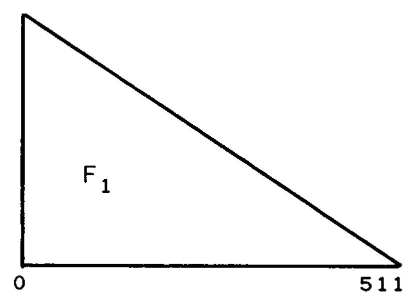


Fig. 16. Sound with a resonance on the 10th harmonic.



P_4 = 5 seconds
P_6 = 45 Hz
P_7 = 900 Hz

$P(x) = 1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10}$

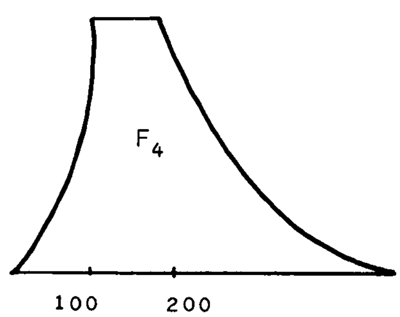


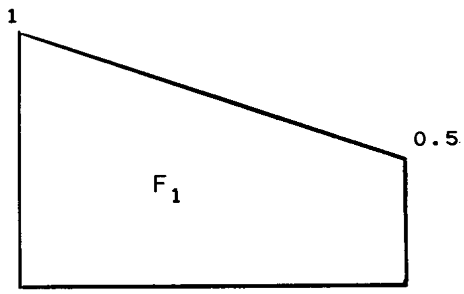
Fig. 17. Percussive sound.

tion. This result can be obtained by introducing amplitude modulation by an evolving complex wave. It consists (Fig. 21) in multiplying the output of two standard instruments. (Partial frequencies are $\pm \alpha f_1 \pm \beta f_2$ if the frequencies of the undistorted signals are f_1 and f_2.) To get the exact equivalent of the double frequency modulation, one has to multiply the output of two distorted waves and a pure sine wave so that partial frequencies are now $\pm f_{AM} \pm \alpha f_1 \pm \beta f_2$.

The interest of such a technique is twofold:

1) Using f_{AM}, f_1, and f_2 values such that they provide a harmonic spectrum (all multiples of a fundamental frequency) produces an extension of the harmonic content, keeping the order of the two polynomials low, hence more easily predictable. It is indeed hard to find musically good distortion polynomials with a rich spectrum because they tend to be brassy or noisy or to have ripple in the evolution. This problem is entirely solved by double modulation.

2) Using irrational ratios for f_{AM}, f_1, and f_2 provides an incredible richness of inharmonic sounds. Here the problem lies in distinguishing which of the three frequencies, two indexes, and two distortion functions is the actual predominant parameter, and how to give the values of all those parameters.



$P_4 = 2$ seconds
$P_6 = 261$ Hz
$P_7 = 261 \times 3$ Hz

$P(x) = 1 + x - 0.25x^2 - 0.25x^3 + 0.05x^4 + 0.05x^5$

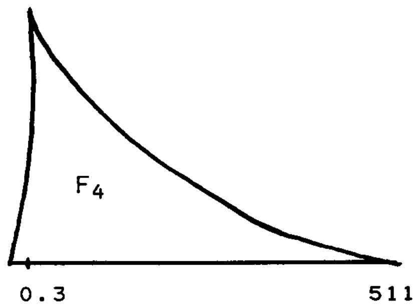
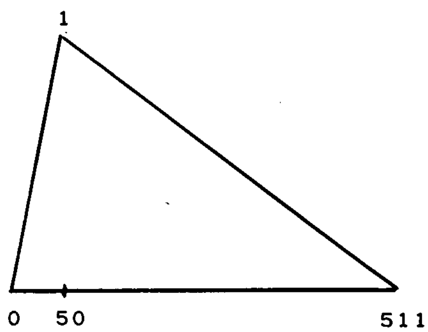


Fig. 18. Plucked sound.



$P_4 = 3$ seconds
$P_6 = 600$ Hz
$P_7 = 300$ Hz

$P(x) = 1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10}$

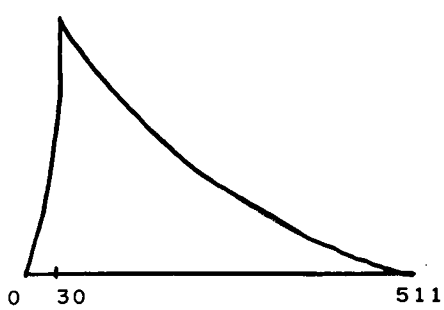
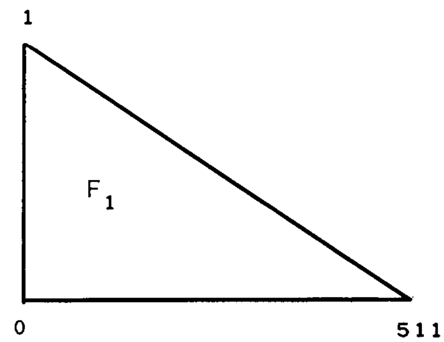


Fig. 19. Sound with odd harmonics.



$P_4 = 5$ seconds
$P_6 = 282.8$ Hz
$P_7 = 200$ Hz

$P(x) = 1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10}$

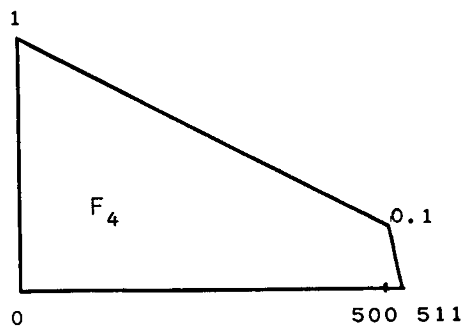


Fig. 20. Bell imitation.

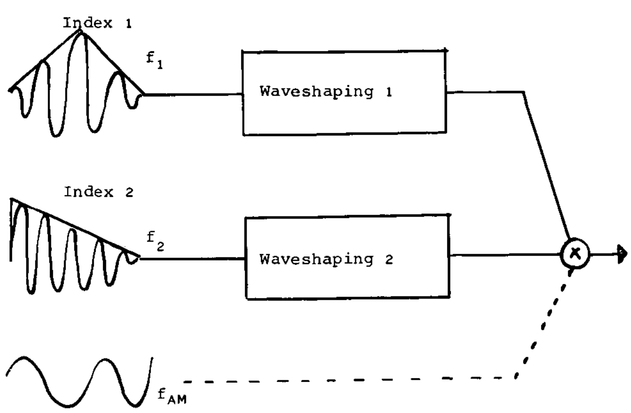


Fig. 21. Double modulation.

5.1 Side Aspects of the Double Modulation Technique

1) A subset of the double modulation instrument described in Fig. 21 is obtained by making index 2 constant, that is, the output of the distortion is a complex but fixed waveform. This is therefore equivalent to using the normal amplitude modulation instrument shown in Fig. 12, the sinusoidal modulation being changed to a more complex one. The resulting spectrum is the convolution of the distorted spectrum by the modulating one. It is obtained by replacing each component of the first spectrum by the entire second spectrum. In the case where f_1 is greater than f_{AM} the spectrum takes the appearance of Fig. 22.

2) If $f_1 = f_2$, the output of the double distortion standard instrument is equivalent to the distortion of a sine wave by a distortion polynomial that is the product of the two distortion polynomials used. This is a way to calculate a polynomial with a large order. More trivially it indicates that taking the square or the cube of the output of a standard instrument extends its spectrum by a factor of 2 or 3. However, more generally there is a difference between using two polynomials and one, as in the first case there are two different indexes that can do different things.

5.2 Other Ways

We may ask what happens to our previous schemata if one uses other waves than sinusoids as inputs in a distortion unit. We can predict that a waveform made up of m harmonics will be distorted to produce a sound with $(m \times n)$ harmonics, but the analytical solution is not simple when m and n are large. A particular and interesting case is when the input is in the form $I/2\ (1 - \cos \omega t) = I \cos^2(\omega/2t)$, for using the distortion polynomial $d_0 + d_1x + d_2x^2 + d_3x^3 \cdots$ is equivalent to using the distortion of $\sqrt{I} \cos \omega/2t$ by $d_0 + d_1x^2 + d_2x^4 + d_3x^6 \cdots$. In this particular case we see that it is equivalent to use this input or to change the distortion. With that kind of input (or seemingly, with the second distortion), the odd and even harmonics of the fundamental are no longer independent. This can be a useful feature if one gets into trouble with odd harmonics going in one way and even ones in another with a usual input. (In this case these two parts may be heard as being two separate tones.) It can even be the basis of a new system where all inputs are of that form, so the new polynomial generator only has to provide output for positive values of input, and the ADD generator has to be excluded from the instruments. This can be advantageous for hardware implementations.

6 A CONVERSATIONAL IMPLEMENTATION

We now give some details on a conversational implementation of the multiplication of distorted sine waves on a minicomputer. The Music V program has been implemented by F. Nayroles at the computer and musical acoustics department of the Laboratoire de Mécanique et d'Acoustique Appliquée in Marseille.

While this implementation on a small 16-bit minicomputer is useful, the turn-around time is quite slow for trial-and-error tests. In addition, there are four 14-bit digital-to-analog converters (using direct memory access) and a real-time external clock, all entirely designed and constructed by P. Karatchenzef. The sampling frequency, the start and stop of conversion, and other parameters are software controlled. An interesting feature is the ability to cycle on a buffer. It has been implemented as follows: the buffer (taken as 100 points) is continuously scanned and converted by hardware. It contains the values of a distorted sine wave, and it can be renewed as fast as a new waveform can be computed (Fig. 23). This takes 17 milliseconds on our minicomputer (floating-point multiplication 25 microseconds). A FORTRAN interactive program has been written (with an internal assembly language inner loop) for performing nonlinear distortion. It permits the user to define sounds generated by a standard or an amplitude-modulated instrument. f_{AM}/f_1 must be a rational fraction. This is possible because the rate of scanning of the buffer can be programmed and is 100 times (100 is the length of the buffer) the fundamental frequency.

Specifically, this is a conversational program to calculate a distortion, display spectra, accept amplitude compensation, index functions, values of amplitude-modulation and input frequencies (in terms of N_1f_0 and N_2f_0), and produce sounds in real time. (The sound is calculated during its execution, but the parameters are prepared in advance.) This gives a nonhifi quality sound (a 17-millisecond calculation time gives some roughness in transients), and one must adjust the sampling filters to 50 times the fundamental frequency. However, it is very efficient as a trial-and-error method, and the Music V values can be directly derived therefrom.

7 CONCLUSION

The general principles presented here can lead to many

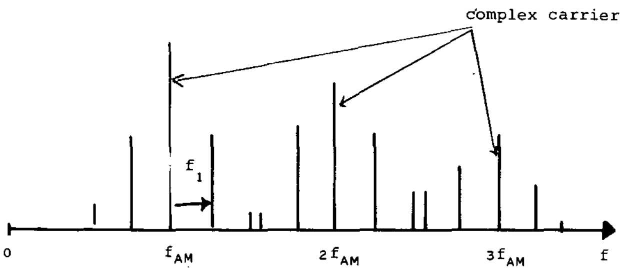


Fig. 22. Modulation with a complex wave.

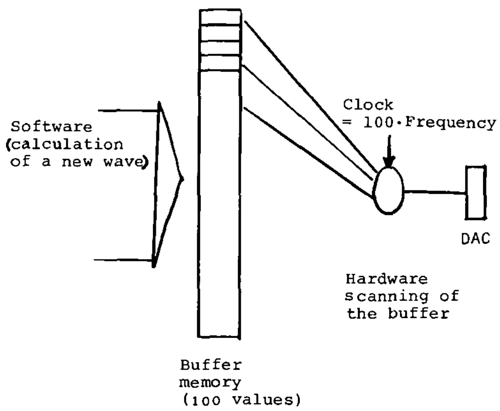


Fig. 23. Real-time implementation.

developments in digital synthesis of music. The description and use of the instruments is fairly simple, so one can concentrate on musical aspects of the sounds. For example, one can use a "timbre vibrato" instead of a frequency or amplitude vibrato by adding a sinusoidal function to the index. A little offset in amplitude-modulation or input frequencies provides a slight inharmonicity (if f_{AM} is high, the interval between harmonics is reduced or expanded) or even beats (if f_{AM} is low, folded components can beat with direct ones). Certainly adding planned or random modulation upon the frequency or portamento glides (as Morrill [10] did with a frequency-modulation trumpet) helps give life to a sound, and changing this from one note to another prevents monotony. For pianolike sounds or bells, doubling or tripling the notes with small and different frequency differences makes the sound more natural. Much work can be done in this direction.

In comparison with the classical frequency-modulation technique [2] there are some differences: the nonlinear distortion is exactly band-limited, whereas frequency modulation is not; the frequency modulation with sine-wave oscillations is unique, the nonlinear distortion can be varied by changing the distortion polynomial (to the extent that only changing this polynomial in a Music V score gives another "orchestration" feeling); also the missing carrier method has been widely exploited (under the name standard) for the nonlinear distortion, but not so much for frequency modulation. (However, it exists as $\sin(I \sin(\omega t + \psi))$ and is simply another type of nonlinear distortion.) Clearly, other trigonometric modulations have overcome the limitations of the classical frequency modulation [11] and are conceptually very similar to the nonlinear distortion [12]. The most important difficulty now is to find parameters corresponding to a desired musical effect [13]. The nonlinear distortion can be helpful in achieving this goal.

8 ACKNOWLEDGMENT

The author is indebted to the whole computer music community, and especially to J. C. Risset whose concern about timbre problems has been inspiring (and a constant dialogue with him has proved to be very helpful), and to Andy Moorer and Marc Le Brun (who independently has done a parallel work on wave shaping at Stanford University—the two works are in fact complementary) for their comments and reviewing.

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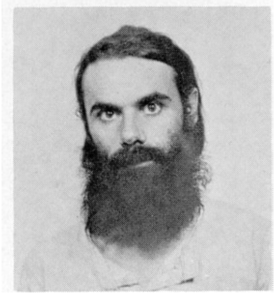

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