Player–Instrument Interaction Models for Digital Waveguide Synthesis of Guitar: Touch and Collisions

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***Abstract*—Physically inspired sound synthesis techniques have been devised for several instruments including guitar. Less well studied are methods to model the interactions of the player with strings and of the strings with other mechanical parts of the instrument. In this paper, we present methods based on simple scattering junctions to be inserted at various points along a digital waveguide simulating transversal wave propagation along a string. These junctions, which are derived using a balanced perturbation method for displacement wave variables, connect the waveguide with the external stimuli provided by the player or with objects that are part of the instrument. A model for plucking is revisited and improved. New nonlinear structures for the accurate model of collisions are developed. The proposed junctions achieve realistic synthesis of the interaction of the plucking finger or pick with the string, of the interaction of the fingers on the neck-side hand with strings, such as in the production of harmonics, and of the strings themselves with frets or fingerboard in a fretless instrument.**

***Index Terms*—Digital waveguides, guitar, harmonics, player's touch, pluck models, physical models, popping and slapping, scattering, sound synthesis, string.**

I. Introduction

IN this paper, we focus on aspects of the synthesis of guitar based on physically inspired models. While several excellent works have been published in this domain, e.g., [1]–[5], only a few deal with the play mode interaction of the player with the strings [6], [7] and of the strings with other parts of the instrument, such as the fingerboard and the frets [8], [9]. While ideal strings are among the simplest mechanical systems in musical acoustics, several other ingredients are necessary for the accurate synthesis of the sounds from real world string instruments. To name a few, suitable models of the string terminations and of the soundboard of the instrument are essential in order to produce realistic sounds. Moreover, real strings depart from the linear flexible string idealization, which results in frequency dependent propagation of the transversal modes, in the excitation of longitudinal modes of vibration and in the nonlinear coupling of polarization modes. Other essential ingredients are models of the other mechanical systems with which the strings come into contact during playing, including the player's fingers or hands.

Computationally efficient methods for modeling interaction and collisions are the main objectives of this work. Some of the results of this paper have been implemented in a noncommercial evolving plugin freely available at http://staffwww.itn.liu.se/~giaev/soundexamples.html.

Modeling and controlling the player's touch on the strings is a complex problem that has only found partial solution to date. The plucking action of the player on the string is often replaced by the injection of predetermined excitation and damping signals, e.g., see [3]. In applications where the interaction of the player with the string plays an essential role in the sound control interface, a model of the player's touch based on a few relevant parameters is desired in order to generate a wide palette of play modes, which can be modified at will. In [10], the authors proposed a simple but effective linear system to be inserted at the plucking point in a digital waveguide (DW) [11], as model of the coupling of the finger/pick with the string. The model was derived from a previous work by Cuzzucoli and Lombardo [6] and fixed a problem concerning the control of the interaction found in their approach. The linear system is described by a scattering junction [11] to be inserted at one or more points about the location of the pluck. The junction is derived by equally distributing, i.e., balancing the pluck perturbation on the two rails of the DW.

Mechanical collisions involving strings and other parts of the instrument such as the fingerboard and the frets occur in special play modes or styles, such as slapping (hitting the string very hard with the knuckle of the fingers) and popping (pulling the string very hard in plucking) styles in the fretted electric bass. The problem of modeling string collisions has been previously addressed by Rank and Kubin [9]. There, at a point of collision, a DW modeling transversal displacement wave propagation along the string is split into two separate DWs with perfectly reflecting boundaries at each side of the collision point. The two DWs are re-merged when the string is no longer in contact with the fret at the given point. In [8], it is shown that the Rank–Kubin model introduces undesired jumps of the output signal, which can be eliminated at extra computational cost by readjusting the offset constants along the DW or by means of time-domain finite difference schemes working at higher sampling rates.

Modeling collisions of the strings with the neck is even more crucial in fretless basses, guitars and banjos, or other similar instruments, where the action, i.e., the distance at rest of the string to the neck, is quite low, especially in the proximity of the finger pushing the string directly over the neck. There, the control of the string collisions is a fundamental part of the players' interaction with the instrument.

In this paper, we show that, whether in limit cases or extended forms, similar junctions can be successfully employed for modeling the stopping of the string due to imperfect clamping of the string by the finger on the fingerboard, the production of

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harmonics due to the proximity of the finger with the string at selected points, together with string collisions with the fingerboard, e.g., in popping and slapping styles and/or in fretless instruments. We propose a new model for string collisions based on nonlinear scattering junctions that are temporarily inserted at the collision points. These junctions keep connected the two portions of the DW lying on each side of a collision point, while proper initial conditions are set to handle the post-collision dynamics.

In sound synthesis based on DWs, different quantities, such as displacement, velocity and force, can be used as wave variables. Generally, the equations become simpler and the implementation easier if velocity waves are considered, given by the first time derivative of displacement. In this case, the coupling of the string with other systems such as the bridge or the nut can be directly modeled by the mechanical impedances of the string and of these systems. In this paper, we consider implementations based on displacement waves. The equation of motion or conservation laws are employed to model exceptional string segments such as the pluck point, finger contact in harmonics and collision points with the neck or frets. The main reason for this approach is to provide a coherent set of models valid for several types of interactions in a single framework. It turns out that the collision models are easily implemented if displacement waves are considered. In fact, testing the deformation of the string to check if it collided is crucial, which could only be achieved by point-to-point integration if velocity waves were employed. As this test must be performed on a sample-by-sample basis everywhere on the string or at least on large portions like the neck length, controlling collision models in waveguides based on velocity waves is costly. Therefore, we opted for displacement waves for the whole synthesis process, as in [9] and [8].

The paper is organized as follows. In Section II, we introduce a model for the plucking of strings, which uses force and finger/pick mass and stiffness as time-varying parameters. In Section III, we propose a new model for the collisions of the string with other objects. These could be other parts of the instrument, as the mentioned frets, or ingredients of the playing action, such as a slide or "bottleneck." In Section IV, we demonstrate the use of the proposed methods in various playing scenarios. For example, we adapt the plucking model to other frequent finger-string interactions, such as stopping and production of harmonics or the slap play mode. In Section V, we draw our conclusions.

II. Interaction Model at Pluck Point

The foundations of the interaction model at the point where the string is plucked by the player are quite simple. Following [6], the finger is modeled as a damped spring-mass system that comes in contact with the string. The model is controlled by the mass M of the finger or of the plectrum, by its stiffness K, by a damping factor R and by the force F_0 exerted by the player on the string. All these parameters can be considered as time-varying during the interaction to simulate the dynamics and the variable contact of the finger/pick with the string in the various phases of the plucking action. Initially, the string is at rest along the x-axis. During pluck, the finger comes in contact with a segment of the string of length $\Delta = x_2 - x_1$, positioned on the interval $[x_1, x_2]$. The string segment moves together with the finger, together forming a system of mass $M + \mu\Delta$, where μ is the linear mass density of the string. The equilibrium equation can be written as follows:

$$(M + \mu\Delta)\frac{\partial^2 u}{\partial t^2} = -R\frac{\partial u}{\partial t} - Ku + F(t) + F_0(t) \quad (1)$$

where $u(x,t)$ is the deformation of the string at point x and time t, $-Ku$ is the restoring force and $-R(\partial u)/(\partial t)$ the damping term of the finger/pick model. The force $F(t)$ is the resultant of the transversal component of the tensile force of the string acting at the extreme points of the plucking segment. In the linear approximation of the string, valid for small deformations, we have [12]:

$$F(t) = K_0\left(\left.\frac{\partial u}{\partial x}\right|_{x=x_2} - \left.\frac{\partial u}{\partial x}\right|_{x=x_1}\right) \quad (2)$$

where K_0 is the tension of the string, which is assumed to be constant.

A discrete counterpart of (1) can be obtained by replacing the partial derivatives with finite centered differences. A detailed derivation can be found in [6], [13] and corrections in [10]. Fixing the temporal sampling interval T, the spatial sampling interval is chosen as $X = cT$, where $c = \sqrt{K_0/\mu}$ is the propagation velocity along the string. The time–space sampling grid is given by $t_m = mT$ and $x_n = nX$, where the integers m and n, respectively, denote discrete time and space. Assuming, for simplicity, that $\Delta = X$ and that the plucking segment coincides with the interval $[(n_p - 1/2)X, (n_p + 1/2)X]$, where $n_p < N$ is a positive integer, we obtain

$$\begin{aligned} &c_1 u(n_p, m+1) - c_0 u(n_p, m) + c_{-1} u(n_p, m-1) \\ &\quad = u(n_p+1, m) + u(n_p-1, m) + \frac{X}{K_0}F_0(m) \end{aligned} \quad (3)$$

where

$$\begin{aligned} c_1 &= \left(1 + \frac{M}{\mu X} + \rho\right) \\ c_0 &= \left(\frac{2M}{\mu X} - \kappa\right) \\ c_{-1} &= \left(1 + \frac{M}{\mu X} - \rho\right). \end{aligned} \quad (4)$$

The dimensionless parameters

$$\begin{aligned} \rho &= \frac{R}{2\sqrt{\mu K_0}} \\ \kappa &= \frac{KX}{K_0} \end{aligned} \quad (5)$$

respectively, are proportional to the damping coefficient and the stiffness constant of the finger.

Away from the plucking segment, the classical wave equation holds for small deformations of the flexible string:

$$c^2\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} \quad (6)$$

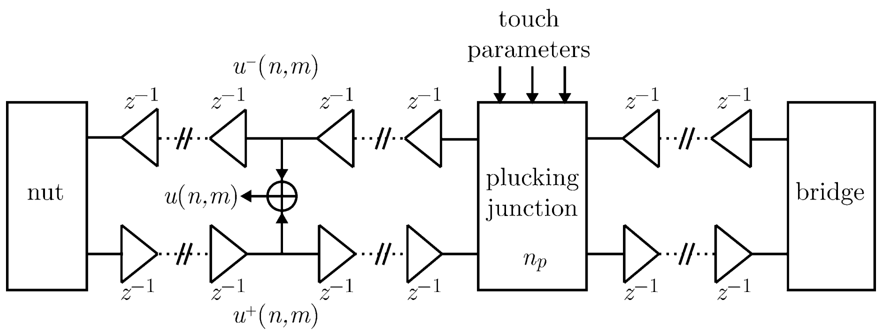

Fig. 1. Diagram of a DW with the insertion of a plucking scattering junction. The blocks "nut" and "bridge" denote suitable terminations to satisfy boundary conditions and coupling with the soundboard of the instrument at the string endpoints.

Thus, the solution $u(x,t)$ can be written in D'Alembert's form as the sum of a progressive (right going) $u^+(x,t) = f_r(t-x/c)$ and a regressive (left going) $u^-(x,t) = f_l(t+x/c)$ propagating waves, where the functions $f_r(t)$ and $f_l(t)$ are established by the initial conditions. In discrete terms, this means that

$$\begin{aligned} u(n,m) &= u^+(n,m) + u^-(n,m) \\ u^+(n+1,m) &= u^+(n,m-1) \\ u^-(n-1,m) &= u^-(n,m-1) \end{aligned} \tag{7}$$

for $n \neq n_p$, where we have normalized the sampling interval T to 1. It is well-known that the synthesis of the propagating wave solution (7) can be achieved by means of DWs consisting of two delay lines, each oriented in one direction of propagation.

As shown in Fig. 1, a block representing the player's touch interaction with the string, based on the linear model (3), can be inserted in a DW simulating the propagation of transversal displacement waves along the other portions of the string. In [6], implementation is achieved with the help of three delay lines. In [10], we showed that implementation in a more conventional and efficient DW based on two delay lines is possible. We also proposed corrections to the Cuzzucoli–Lombardo method in order to avoid inconsistencies that required the input to the DW to be nonzero even after the plucking action is terminated, with consequent negative implications on the control of repeated plucking of the same string in order to handle clicks introduced by the amplitude jumps.

A. Balanced Perturbation

In this section, we are going to derive the structure of a scattering junction useful for the implementation of the player's touch interaction with the string. We refer to the method we are going to use as the balanced perturbation. While similar tricks may be found in the literature, as applied to the synthesis of other instruments [14], [15], the method is strikingly simple and obtains proper equations describing the interaction of other mechanical systems with the waveguide. It rests upon D'Alembert's solution for the wave equation with arbitrary condition on initial displacement and zero initial velocity, where it is well-known that the progressive and regressive waves at initial time each equals half of the initial displacement.

In order to illustrate the method and obtain the pluck interaction junction we make the following considerations. At the plucking point n_p one can look at the interaction as a perturbation $h(m)$ of the deformation of the string that is added to the incoming waves

$$\begin{aligned} u_{\mathrm{in}}^-(m) &= u^-(n_p+1,m-1) \\ u_{\mathrm{in}}^+(m) &= u^+(n_p-1,m-1) \end{aligned} \tag{8}$$

in order to produce the outgoing waves

$$\begin{aligned} u_{\mathrm{out}}^-(m) &= u^-(n_p-1,m+1) \\ u_{\mathrm{out}}^+(m) &= u^+(n_p+1,m+1) \end{aligned} \tag{9}$$

in such a way that

$$u_{\mathrm{out}}(m) = u_{\mathrm{in}}(m) + h(m) \tag{10}$$

where we defined

$$\begin{aligned} u_{\mathrm{out}}(m) &= u_{\mathrm{out}}^-(m) + u_{\mathrm{out}}^+(m) \\ u_{\mathrm{in}}(m) &= u_{\mathrm{in}}^-(m) + u_{\mathrm{in}}^+(m) \end{aligned} \tag{11}$$

Since (10) is the only constraint, there are infinite ways of distributing $h(m)$ among the outgoing waves $u_{\mathrm{out}}^+(m)$ and $u_{\mathrm{out}}^-(m)$ at the pluck point. However, since there is no privileged direction, the equal distribution of the perturbation in the two directions is the most reasonable assumption

$$\begin{aligned} u_{\mathrm{out}}^-(m) &= u_{\mathrm{in}}^-(m) + \frac{h(m)}{2} \\ u_{\mathrm{out}}^+(m) &= u_{\mathrm{in}}^+(m) + \frac{h(m)}{2}. \end{aligned} \tag{12}$$

A diagram of the plucking perturbation injection node is shown in Fig. 2. We refer to the operation of equally distributing the perturbation on the DW rails as balanced injection. It must be pointed out that other unbalanced choices of unequal weights adding up to 1 would not affect the solution at the injection point. However, different weights lead to different propagation of the wave variables to the left and to the right of the plucking point, which results in a different global solution.

A more formal justification of the balanced perturbation method follows from the following ideas. From the dependency of D'Alembert's solution on initial position and velocity, it can

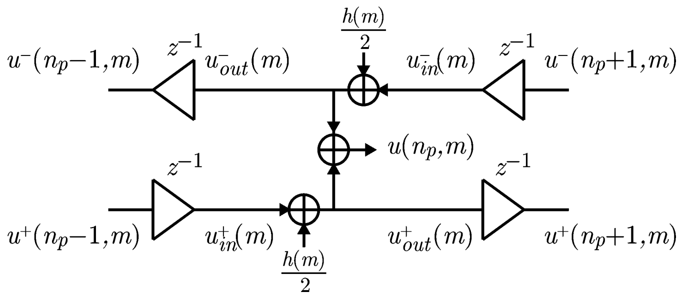

Fig. 2. Diagram of the balanced injection of the perturbation in a DW.

be shown (e.g., see [16]) that the displacement wave components u^- and u^+ in a string are related to the state of the string as follows:

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} u^-(x,t) \\ u^+(x,t) \end{bmatrix} = \begin{bmatrix} u(x,t) \\ w(x,t) \end{bmatrix} \tag{13}$$

where $u(x,t)$ is string displacement, which is both a state and an output variable, and

$$w(x,t) = \frac{1}{c} \int_0^x \frac{\partial u(\xi,t)}{\partial t} d\xi \tag{14}$$

is a state variable proportional to the spatial integral of velocity from one string end, up to position x. Applying (13) on both sides of the input–output relationship (12), we re-obtain (10) but we also discover that

$$w_{\text{out}}(m) = u^-_{\text{out}}(m) - u^+_{\text{out}}(m) = u^-_{\text{in}}(m) - u^+_{\text{in}}(m) \tag{15}$$

so that the velocity integral variable w is continuous across the interface with the perturbation $h(m)$. This is only true if the perturbation $h(m)$ is equally distributed among the two rails of the DW. Clearly, in a DW system the velocity integral is discretized. While it is possible to have a finite increment of the velocity by means of finite external stimuli—we will face this problem when we will discuss collisions—this same increment should be added to the velocity integral state variable throughout the portion of waveguide to the right of the perturbed point, so that (14) still holds. The introduction of the balanced perturbation avoids alteration of this variable and is particularly useful in dealing with displacement wave components that are not explicitly stimulated by a velocity blow. Initial or incremental velocity can be still injected in the waveguide with careful handling of the velocity integral, in turn using balanced injection for the velocity perturbation.

B. Plucking Scattering Junction

The equations in (12) are the foundation of the balanced perturbation method. It remains to specify the nature of the perturbation $h(m)$, which is the final step of the balanced perturbation method. We need to describe the dependency of the perturbation on the external force $F_0(m)$ and on the state of the string. This is readily achieved by substituting (7)–(11) in (3), obtaining

$$\begin{aligned} c_1 h(m+1) - c_0 h(m) + (c_{-1} - 1)h(m-1) \\ = (1-c_1)u_{\text{in}}(m+1) + c_0 u_{\text{in}}(m) \\ + (1-c_{-1})u_{\text{in}}(m-1) \\ + \frac{X}{K_0} F_0(m). \end{aligned} \tag{16}$$

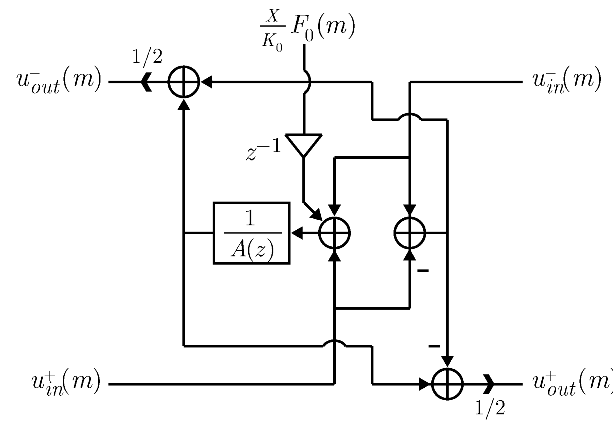

Fig. 3. Diagram of the plucking scattering junction.

In the z-transform of (16), we obtain

$$H(z) = \frac{B(z)}{A(z)} U_{\text{in}}(z) + \frac{Xz^{-1}}{K_0 A(z)} \tilde{F}_0(z) \tag{17}$$

where $U_{\text{in}}(z)$ and $\tilde{F}_0(z)$, respectively, are the z-transforms of $u_{\text{in}}(m)$ and $F_0(m)$ and the polynomials $A(z)$ and $B(z)$ are given as follows:

$$\begin{aligned} A(z) &= c_1 - c_0 z^{-1} - (1-c_{-1})z^{-2} \\ &= \frac{M}{\mu X}(1-z^{-1})^2 + \rho(1-z^{-2}) + \kappa z^{-1} + 1 \end{aligned} \tag{18}$$

and

$$B(z) = 1 - A(z). \tag{19}$$

Substituting (17) in a z-transformed version of (12) and taking (19) into account, we obtain the vector form

$$\begin{bmatrix} U^-_{\text{out}}(z) \\ U^+_{\text{out}}(z) \end{bmatrix} = \mathbf{S} \begin{bmatrix} U^-_{\text{in}}(z) \\ U^+_{\text{in}}(z) \end{bmatrix} + \frac{Xz^{-1}}{2K_0 A(z)} \tilde{F}_0(z) \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{20}$$

where $U^{\pm}_{\text{in/out}}(z)$ are the z-transforms of the corresponding lowercase signals $u^{\pm}_{\text{in/out}}(m)$ and

$$\mathbf{S} = \mathbf{S}(z) = \frac{1}{2} \begin{bmatrix} \frac{1}{A(z)} + 1 & \frac{1}{A(z)} - 1 \\ \frac{1}{A(z)} - 1 & \frac{1}{A(z)} + 1 \end{bmatrix} \tag{21}$$

is the scattering matrix of the plucking junction.

The diagram of the plucking scattering junction is shown in Fig. 3. This structure is more efficient than the one presented in [10] as only the second order all-pole filter $1/A(z)$ needs to be calculated.

It can be shown [10] that in the underdamped regime, essentially obtained when the mass ratio $M/\mu X$ dominates the dimensionless stiffness and damping parameters κ and ρ so that the discriminant of the polynomial $A(z)$ is negative, the two complex conjugated poles of the scattering matrix always lie within the unit circle. The filter $1/A(z)$ is stable bandpass, the frequencies $f_{\pm}$ of the poles being approximately proportional to the square root of the inverse of the mass ratio

$$f_{\pm} \approx \pm \frac{1}{2\pi T}\sqrt{\frac{\mu X}{M}}. \tag{22}$$

It must be pointed out that the scattering junction described by (21) is not necessarily passive. In fact, depending on the finger and string parameters, the power gain

$$G(e^{j\omega}) = \left|\det(\mathbf{S}(e^{j\omega}))\right| = \frac{1}{|A(e^{j\omega})|} \tag{23}$$

can be larger than 1, especially in the neighborhood of pole frequencies. However, the finger-string pluck interaction is only lasting a short interval of time of the order of 10 ms. The waves reflected at the nut and bridge originating from the initial perturbation, only manage to return at most a few times to the plucking point while plucking is active. Thus, the effect of the wave propagation loop is time limited so that the non-passivity of the scattering junction during plucking is not crucial for the overall waveguide stability.

Another point of concern is the stability of the time-varying filters in the scattering matrix entries. In fact, in normal operation, the finger parameters M, κ and ρ asynchronously transition from zero to their maximum values following arbitrary curves, to all return to zero at string release. The stability of the resulting time-varying scattering filters can be studied, as in [17], in terms of their state-space realizations of the type

$$\begin{cases} \boldsymbol{\xi}(n+1) = \mathbf{A}(n)\boldsymbol{\xi}(n) + \mathbf{b}(n)x(n) \\ y(n) = \mathbf{c}^T(n)\boldsymbol{\xi}(n) + d(n)x(n) \end{cases} \tag{24}$$

where ^{T} denotes transposition, $\boldsymbol{\xi}(n)$ is the state vector, $\mathbf{A}(n)$ the state transition matrix, $\mathbf{b}(n)$ the input-state coupling vector, $\mathbf{c}(n)$ the state-output coupling vector, $d(n)$ the input-output coupling scalar, $y(n)$ the output and $x(n)$ the input of any filter. The stability criterion formulated in [17] requires the quantity

$$|d(n)| + \sum_{p=-\infty}^{n-1} \left| \mathbf{c}^T(n) \left(\prod_{q=p+1}^{n-1} \mathbf{A}(q) \right) \mathbf{b}(p) \right| \tag{25}$$

to be upper bounded by a finite constant. Since the finger interacts with the string for a finite time interval only, the interval in which the scattering matrix (21) differs from the identity matrix is finite. Therefore, one can always find a state-space realization for its entry filters in which, outside the plucking interval, the input–output coupling coefficient $d(n) = 1$ for the filters on the diagonal and $d(n) = 0$ for the antidiagonal filters, while $\mathbf{A}(n), \mathbf{b}(n)$ and $\mathbf{c}(n)$ are all zero. In this case, (25) is trivially finite and BIBO stability is guaranteed for the time-varying filters.

In fixed point implementations, proper scaling of the filters is required in order to prevent overflow; the scaling factor can be retrieved from the maximum of the upper bound on (25) for each filter [17].

III. Collisions

Models for the synthesis of the sounds of strings colliding with frets or other object have been the object of a few studies [9], [8]. Strings are considered as a set of particles, each of mass μX, centered at the spatial samples nX and potentially colliding with objects, such as the frets or the neck, of much larger mass that are at rest in the center of mass frame. Collisions are detected by testing the value of the local deformation of the string against the coordinates of the other neighboring parts of the instrument. If u_{ref} denotes the perpendicular coordinate of the point in the obstacle closer to the string, a collision is declared to have happened in the time interval $](m-1)T, mT]$ if one of the following conditions are satisfied:

$$\begin{aligned} u_{\mathrm{in}}(m) &\leqslant u_{\mathrm{ref}} < 0 \\ u_{\mathrm{in}}(m) &\geqslant u_{\mathrm{ref}} > 0. \end{aligned} \tag{26}$$

The first case corresponds to an obstacle lying below the string rest line, while the second case corresponds to an obstacle lying above the string rest line.

In the next sections, we introduce new models for inelastic and semi-elastic collisions and compare them with the models described in [9] and [8].

A. Inelastic Collisions

In cases where the collisions are inelastic, the string sticks to the obstacle until the deformation propagating from other sections of the string pulls it away from the obstacle; momentum is conserved but not kinetic energy. Inelastic collisions were modeled in [9] by splitting the string DW into two DWs, each subject to perfect reflection boundary conditions on both sides of the collision point, with the addition of the value u_{ref} to ensure that the string is kept in touch with the obstacle without trespassing. The two DWs are then merged again once the collision conditions are not met anymore.

Problems with the Rank–Kubin approach have been pointed out in [8], where alternative methods are introduced. In one method, the instantaneous amplitude jumps due to the re-merging of DWs that have separately evolved is tackled by removing offset values in one of the portions of the string at release time. In a second method, a time-domain finite differences (TDFD) scheme is proposed based on a nonuniform time-space stencil. In [8], the TDFD section, solving the wave equation, connects the two DWs at a collision point. Proper initial conditions can be set to handle the post-collision dynamics. This second approach has greater flexibility but higher computational cost, also considering the fact that the string may hit obstacles at several different locations during oscillations. Moreover, in fretless instruments it is hard to select the most likely location of the points of collisions where to allocate the TDFD sections.

In this section, we propose a third alternate approach which, in spite of its simplicity, does not suffer from discontinuities problems and provides extremely realistic acoustic results. The collision point is modeled by means of a nonlinear scattering element where the nonlinearity is given by a decision switch testing whether there is collision and adding a balanced perturbation on both rails of the DW. Consequently, the sections of the DW on both sides of the collision point are always connected to each other. We will show that, when the string sticks to the obstacle, partial transmission and reflection occurs. Indeed, perfect reflection of deformation waves would actually imply an unnatural clamping of the string, which actually only comes to temporary rest on a rigid obstacle during inelastic collision. Moreover, at release time, the perturbation signal is gently lifted by the traveling deformation.

In the context of TDFD, the possibility to regard the difference between the freely oscillating string and the limited displacement at the collision point as a stimulus propagating along the string, which is closely related to our point of view, was briefly pointed out in an example contained in [18]. However, our technique is completely developed in the DW domain and easily extends the simulation to semi-elastic collisions.

B. New Collision Scattering Element

The procedure to derive the proper collision scattering element is extremely simple. The starting point is the foundation of the balanced perturbation method in (12). Here $h(m)$ is the perturbation fixing the string deformation at the perpendicular coordinate $u_{\rm ref}$ of the obstacle and is distributed to the rails of the DW by balanced injection. Obviously, in order to fix the string deformation at value $u_{\rm ref}$, we need to satisfy the equation

$$u_{\rm in}(m) + h(m) = u_{\rm ref} \tag{27}$$

where $u_{\rm in}(m)$ is the sum of the propagating waves entering the scattering junction. Substituting $h(m) = u_{\rm ref} - u_{\rm in}(m)$ in (12), obtains the following equation in matrix-vector form:

$$\begin{bmatrix} u_{\rm out}^{-}(m) \\ u_{\rm out}^{+}(m) \end{bmatrix} = \mathbf{S_c} \begin{bmatrix} u_{\rm in}^{-}(m) \\ u_{\rm in}^{+}(m) \end{bmatrix} + \frac{u_{\rm ref}}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{28}$$

where

$$\mathbf{S_c} = \frac{1}{2} \begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix} \tag{29}$$

is the scattering matrix of the collision. Although, in contrast with (20), we operated in time domain, the procedure is perfectly equivalent since the scattering matrix $\mathbf{S_c}$ does not depend on frequency. Notice that the scattering matrix is singular and such that waves are partly, indeed equally, transmitted and reflected by the junction. In fact, temporarily disregarding the term in $u_{\rm ref}$, we have that the progressive and regressive waves $u_{\rm out}^{+/-}$ coming out of the junction equal, with different signature, the semi-difference of the incoming progressive and regressive waves. Thus, half the progressive wave $u_{\rm in}^{+}$ is transmitted to the right and half of it is reflected to the left and the same is true for $u_{\rm in}^{-}$. If $u_{\rm ref}$ were zero then also $u_{\rm out}$ would be zero and the given scattering matrix is a way to enforce zero displacement at a point along the string without having perfect reflection. The scattering matrix (29) can be obtained from the scattering matrix of the plucking junction (21) in the limit as the ratio of the mass of the finger over the mass of the string segment $M/\mu X$ goes to infinity.

During propagation along the string, in particular on the neck portion of it, a decision has to be taken whether the string collides with an obstacle. This is readily done by testing if the sum of the incoming displacement waves $u_{\rm in}(m)$ is reaching or exceeding the perpendicular coordinate of the obstacle $u_{\rm ref}$. During collisions, the scattering matrix (29) operates on the incoming wave variables to produce the outgoing wave variables.

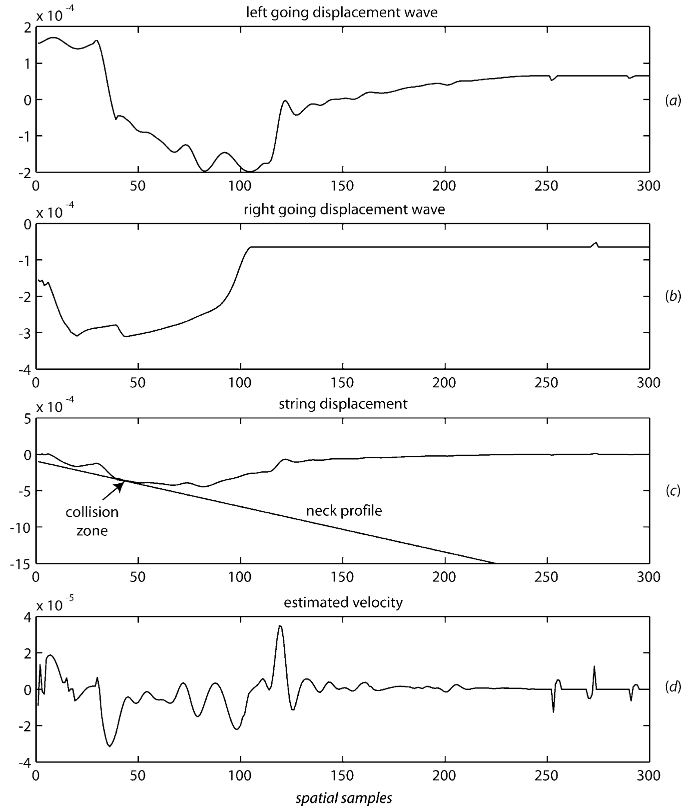

Fig. 8. String deformation (slapping case) at an instant of collision after a few previous collisions. (a) Left-going wave u^{-}. (b) Right-going wave u^{+}. (c) String displacement touching neck profile at a point. (d) Estimated velocity.

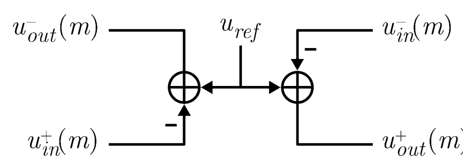

Fig. 4. Diagram of the split-string scattering junction in the Rank–Kubin model of inelastic collisions.

Otherwise, away from collision points, the scattering matrix becomes the identity matrix for unperturbed wave propagation.

A diagram of the nonlinear scattering junction modeling inelastic collisions is shown in Fig. 8. There, a switch is controlled by a hard limiter operating on the variable $h(m) = u_{\rm ref} - u_{\rm in}(m)$. To fix our ideas, we assume that the obstacle is below the string. Then, if $h(m) \geq 0$ a collision is flagged and the switch is closed. As a result, the scattering matrix switches from the identity matrix to the matrix $\mathbf{S_c}$. This is achieved simply by adding $h(m)/2$ to both incoming waves to obtain the proper outgoing waves. The term $u_{\rm ofs}$ allows to set additional initial conditions and will be discussed in Section III-C.

It is interesting to compare the input–output relationships of the proposed inelastic collision model with those deriving from the split-string model in [9]. When one of the collision conditions (26) is met, in our model (28) holds, while in the split-string model, diagrammed in Fig. 4 we have

$$\begin{bmatrix} u_{\rm out}^{-}(m) \\ u_{\rm out}^{+}(m) \end{bmatrix} = \widehat{\mathbf{S}}_{\mathbf{c}} \begin{bmatrix} u_{\rm in}^{-}(m) \\ u_{\rm in}^{+}(m) \end{bmatrix} + u_{\rm ref} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{30}$$

where

$$\widehat{\mathbf{S}}_{\mathbf{c}} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \tag{31}$$

is the scattering matrix of the collision deriving from two-sided perfect reflection. We then observe that, while in our model we have

$$\begin{bmatrix} u_{\mathrm{out}}^{-}(m) + u_{\mathrm{out}}^{+}(m) \\ u_{\mathrm{out}}^{-}(m) - u_{\mathrm{out}}^{+}(m) \end{bmatrix} = \begin{bmatrix} u_{\mathrm{ref}} \\ u_{\mathrm{in}}^{-}(m) - u_{\mathrm{in}}^{+}(m) \end{bmatrix} \tag{32}$$

for the split-string model we have

$$\begin{bmatrix} u_{\mathrm{out}}^{-}(m) + u_{\mathrm{out}}^{+}(m) \\ u_{\mathrm{out}}^{-}(m) - u_{\mathrm{out}}^{+}(m) \end{bmatrix} = \begin{bmatrix} 2u_{\mathrm{ref}} - u_{\mathrm{in}}^{-}(m) - u_{\mathrm{in}}^{+}(m) \\ u_{\mathrm{in}}^{-}(m) - u_{\mathrm{in}}^{+}(m) \end{bmatrix}. \tag{33}$$

Therefore, while in our model we have the equality $u_{\mathrm{out}} = u_{\mathrm{ref}}$ from the first vector component in (32), in the split-string collision model, in view of (26), we only have a constraining inequality, i.e., $u_{\mathrm{out}} \geq u_{\mathrm{ref}}$ if the obstacle is below the string and $u_{\mathrm{out}} \leq u_{\mathrm{ref}}$ if the obstacle is above the string. This helps explaining the differences in the two methods. It is well-known that D'Alembert's solution is invariant by transformation of the wave variables of the following kind:

$$\begin{aligned} u^{-} &\leftarrow u^{-} - C \\ u^{+} &\leftarrow u^{+} + C \end{aligned} \tag{34}$$

where one adds a constant C to one variable and subtracts the same constant from the other wave variable. In the Rank–Kubin update (33) a change on the wave variables (34) on one side of the collision point does not necessarily reflect to a similar change on the other side. For example,

$$\begin{aligned} u_{\mathrm{in}}^{-} &\leftarrow u_{\mathrm{in}}^{-} - C \\ u_{\mathrm{out}}^{+} &\leftarrow u_{\mathrm{out}}^{+} + C \end{aligned} \tag{35}$$

is compatible with the unchanged variables u_{out}^{-} and u_{in}^{+} on the other side of the collision point, as long as the collision conditions persist. In our model, the update (32) implies an identical change to the wave variables on the other side, i.e., one must have

$$\begin{aligned} u_{\mathrm{out}}^{-} &\leftarrow u_{\mathrm{out}}^{-} - C \\ u_{\mathrm{in}}^{+} &\leftarrow u_{\mathrm{in}}^{+} + C. \end{aligned} \tag{36}$$

The change in wave variables does not affect the solution as long as the collision conditions persist. However, an unequal change on the two sides will result in an amplitude jump of the solution when propagation switches back to ordinary at the end of the collision.

Consequently, in our model there is no need to control the invariance constants on the two sides of the collision points. On the contrary, this control is advocated in [8] for the improvement of the Rank–Kubin method.

C. Semi-Elastic Collisions

Since no collision is perfectly inelastic, in the model one must provide for a method to introduce partial elasticity. In semi-elastic collisions, particles much lighter than the obstacles bounce off with speed proportional to the impinging velocity and opposite direction. The proportionality constant is the coefficient of restitution C_R with value between 0 and 1. Especially in collisions with frets, due to their form factor and material, the string performs repeated small amplitude bounces before being fully released. Bouncing is responsible for the bright "metallic" sound heard in pop or slapping in fretted instruments. In a DW, the impinging velocity v_{in} can be simply estimated as the first order finite difference of u_{in}. However, difficulties can arise from geometrical factors. For example, the fact that the neck of the instrument is usually slanted with respect to the resting string implies that the string can hit the neck or the frets even when the velocity v_{in} is positive, i.e., already directed away from the neck. This is a consequence of the fact that displacement waves can travel undisturbed until they find a point of lower action toward the nut of the instrument. In order to handle these cases, a test on the sign of the velocity must be performed and only the negative values are partly reflected by the collision.

Coherently with D'Alembert's solution for displacement waves [12], in order to provide the colliding string segment with a post-collision velocity

$$v_{\mathrm{out}} = -C_R v_{\mathrm{in}} \tag{37}$$

at the hit point $n_h X$, one has to update the whole DW at and on the right of this point, where v_{in} is the updated velocity at time mT as if no collision occurred. This is achieved by adding to the wave variables on both rails of the DW two step functions of amplitude $(v_{\mathrm{out}} - v_{\mathrm{in}})(T)/(2)$ and opposite sign

$$\begin{aligned} u^{-}(n,m) &= u_{\mathrm{in}}^{-}(n,m) + (v_{\mathrm{out}} - v_{\mathrm{in}})\frac{T}{2}\mathfrak{H}(n - n_h - 1) \\ u^{+}(n,m) &= u_{\mathrm{in}}^{+}(n,m) - (v_{\mathrm{out}} - v_{\mathrm{in}})\frac{T}{2}\mathfrak{H}(n - n_h - 1) \end{aligned} \tag{38}$$

where $\mathfrak{H}(n) = 1$ if $n \geq 0$ and zero otherwise denotes the Heaviside step function and $u_{\mathrm{in}}^{-}(n,m)$ and $u_{\mathrm{in}}^{+}(n,m)$ denote the values that the wave variables would take at time mT if no collision had occurred. These traveling step waves ensure that, after any collision, the string velocity at the collision point switches from v_{in} to v_{out}, while the velocity integral state w is coherently altered elsewhere. The additional traveling steps evolve in time by generating a widening displacement pulse centered on the collision point, superposed to the waves generated by the pluck and propagated by the DW dynamics.

At the collision point, in order to take into account the displacement of the colliding string segment from collision time to current time, one also has to set the initial post collision displacement to $u_{\mathrm{ref}} + v_{\mathrm{out}}\Delta T$, which represents the post-collision position of the colliding segment of the string, assuming that the collision occurred $\Delta T < T$ seconds before update time mT. This is readily achieved by setting $u_{\mathrm{ofs}} = v_{\mathrm{out}}\Delta T$, in Fig. 5.

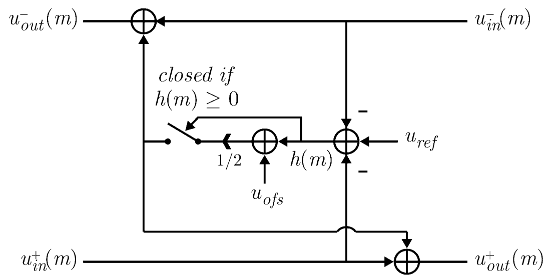

Fig. 5. Diagram of the scattering junction modeling inelastic collisions with underlying objects.

As we actually ignore the exact time of collision within a time interval of length T, reasonable assumptions are either to set $\Delta T < T$ to a random number uniformly distributed in $[0, 1]$ or to set $\Delta T = T/2$, which provides the average value of the collision time uncertainty. Other methods to estimate $\Delta T < T$ require interpolation. The update equations for the collision point are

$$\begin{bmatrix} u_{\text{out}}^{-}(m) \\ u_{\text{out}}^{+}(m) \end{bmatrix} = \mathbf{S_c} \begin{bmatrix} u_{\text{in}}^{-}(m) \\ u_{\text{in}}^{+}(m) \end{bmatrix} + \frac{u_{\text{ofs}}}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + (v_{\text{out}} - v_{\text{in}}) \frac{T}{2} \begin{bmatrix} +1 \\ -1 \end{bmatrix} \quad (39)$$

where $\mathbf{S_c}$ is the scattering matrix (29).

In the implementations, the velocity increment update procedure (38) can be too demanding. Switching to velocity wave variables would certainly make it very easy to enforce restitution velocity conditions but it would also make testing for collisions more demanding. In the displacement wave model, one can resort to an approximate procedure in which at the collision points, and only during collision, the colliding string segment is only displaced by the equivalent amount traveled in an interval of time ΔT with velocity v_{out}, i.e., it is only given a small incremental push for a fraction of time. The push is converted to displacement and added to the waveguide by balanced injection. It is very easy to modify the collision scattering junction in order to achieve this result. One just needs to add the quantity $u_{\text{ofs}} = -C_R v_{\text{in}} \Delta T$ to the signal deriving from $h(m)$ after the switch in the signal path in Fig. 5. The added value does not take part in the collision test, so it does not affect the switch control signal in the diagram.

In an even simpler implementation, one can decide to add just a small constant value for u_{ofs} to the displacement at collision point. This generates the effect of giving the string a small push without having to estimate and test the sign of the impinging velocity. The points of the string with negative velocity will push again toward the obstacle, generating a new collision. The sounds obtained with this simplified method are generally brighter and less accurate than those generated with velocity control. Their quality can be controlled by the displacement offset u_{ofs}, from inaudible to extremely audible and "metallic." The offset value can be made to depend on the neck profile so that strings bounce away from the neck to equal distance from this. Clearly, since the velocity profile is not modified, the string can collide again with the neck even in the immediate next step. This produces sounds rich in higher harmonics typical of string-fret collisions.

IV. Playing With Interaction Models

In this section, we illustrate the use in musical applications of the interaction models described in this paper, together with the results of our experimentation. Sound examples produced with the PluckSynth plugin or with offline simulations are available online at http://staffwww.itn.liu.se/~giaev/soundexamples.html.

A. Plucking

The finger/pick interaction model in Section II depends on the force applied by the player and on stiffness and damping parameters of the finger/pick. These physical parameters can be dynamically changed in order to simulate the action of the player and the variable contact of the finger/pick with the string and/or the variation of mechanical characteristics. For example, in plucking a string with the finger, the flesh comes first in contact with the string, introducing an increasing amount of damping (pre-damping phase), while in the last instants before release (excitation phase), the nail, drives the string until release (detach phase).

The main function of the pre-damping phase, lasting a few milliseconds, is to control the plucking style: a longer pre-damping phase features a "softer" playing style. In the excitation phase following pre-damping, a gradually increasing force $F_0(t)$ is applied to the string and, simultaneously, both the finger/pick inertia, parametrized by the mass $M(t)$, and elastic recoil forces, characterized by the stiffness coefficient $K(t)$, are applied to the string. These quantities vary in an arbitrary fashion and much depend on the pluck style (apoyando, tirando, etc.) and medium (finger flesh, nail, or plectrum). Constant stiffness and mass are not representative of real-life plucking. For example, the stiffness can be considered to increase as a soft plectrum is deformed during contact, or vary when the player switches from driving the string with the finger to grab it with the nail, a situation when also the mass parameter can be assumed to change. Finally, in the detach phase, all external forces abruptly decay to zero, leaving the string free to vibrate.

Mass, damping coefficient and stiffness can be modeled with the help of simple Attack-Decay-Sustain-Release (ADSR) curves that are easy to generate, require a small number of parameters and, at the same time, produce good acoustic results [13], [10].

The force exerted on a guitar string may be represented by a two-segment signal. During the initial attack phase, the force rises from zero to a final maximum and then rapidly goes back to zero as the string is released. Depending on how the player approaches the string, the force curve is here assumed to take on different shapes and time durations. Two types of parametric curves are combined for the construction of force signal segments: "S-shaped" and exponential [10]. The desired attack and release time intervals control the time scale of the curves while the maximum force magnitude A_{F_0} controls their amplitude. Two additional parameters s_a and s_a, respectively, control the attack and release slopes of the exponential ramps.

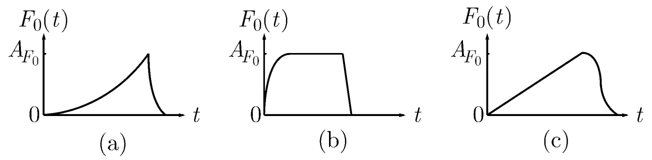

Fig. 6. (a) Exponential segments with s_a and $s_r < 1$. (b) Exponential attack and linear release with $s_a > 1$ and $s_r = 1$. (c) Linear attack segment with "S-shaped" release segment.

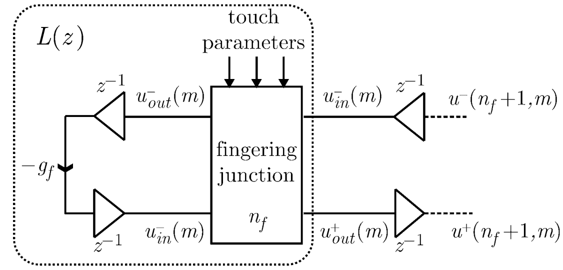

Fig. 7. Diagram of the fingering termination modeling the action of the player's fingers on the fretboard.

Modifying these parameters, a range of differently shaped force inputs can be generated. Three possible variations are depicted in Fig. 6.

In order to avoid unnatural damping during the detach phase, release time is in the range of one half a period of the strings' fundamental frequency [6]. The addition of a small amount of noise in detach phase allows for a close simulation of the rattle noise of the pick or of the fingernail and friction.

In our model, re-plucking the string, even at different positions, does not require to drive the string to a rest state. In fact, when force, finger/pick mass, damping, and stiffness control signals all go to zero, the polynomial $A(z)$ in the scattering matrix $\mathbf{S}$ in (21) becomes equal to 1 so that $\mathbf{S}$ reverts to the identity matrix and the wave propagation proceeds unperturbed henceforth. Therefore, re-plucking is obtained by driving the physical parameters of the interaction to zero at the original location and by inserting a new plucking junction at other or same location.

While tension varies by a little amount from string to string—indeed, for better playability, string sets are generally selected to have almost equal tension for the given tuning—the linear mass density almost doubles from string to string. As a result, the propagation velocity c decreases as we move to thicker low-pitched strings and, with it, the spatial sampling interval $X = cT$ also decreases for a fixed temporal sampling rate. Therefore, for a fixed form factor of the finger/pick, the pluck interaction may involve more than one adjacent point on the string. In order to accurately model the pluck interaction for thicker strings, one must introduce a few adjacent plucking junctions in the plucking zone. To model different contact and influence of the pluck in lateral zones of the plucking zone, a tapering window is employed, which weighs the physical control parameters. The shape of the window is dictated by the shape of the plucking object: bell-like for a finger and triangular-like for a pick.

In particular play modes, the player also imparts a velocity blow to the string before applying force. This is simulated by assigning a nonzero incremental initial velocity to the string. The procedure is similar to that described in (38) and it involves adding two spatial ramp signals of opposite sign to the top and bottom rails of the DW at and on the right of the plucking point.

Realistic pluck sounds also require modeling of nonlinear effects such as tension modulation, which reproduces the typical initial modulation to higher pitch when plucking strings, and longitudinal modes. Inspired by the methods in [19], the plucking pitch alteration is implemented with two first-order real allpass filters replacing the unit delay on each rail of the DW located near the bridge termination. The parameters of the allpass filters are modulated by an amount depending on the elongation of the string. Since in a DW the string length at rest is constrained to be an integer multiple of the spatial sampling X, these allpass filters, implementing fractional delay, also serve for fine tuning the model to correct pitch or to introduce vibrato. Accurate modeling of longitudinal modes is very complex. In our implementation, we resorted to an approximate generation of the "phantom" partial following [20].

In accurate models, two independent DW are employed to simulate each string's horizontal and vertical polarization modes [1]. The two DWs are coupled at special points such as the plucking zone and the bridge. Combined, the two DWs produce sounds that depend on the angle applied by the player in the excitation of the string. The resilience of the material employed for the bridge varies with the angle of attack. An accurate model of the bridge is needed, especially in acoustic instruments, in order to faithfully reproduce this effect. This is the object of current investigation and is outside the scope of this paper.

B. Pitching, Note Stopping, and Harmonics

In order to simulate the action of the player's neck-side hand in pressing strings against the fretboard for playing the different notes, a fingering scattering junction is placed at pressing point $n_f X$. This junction is further terminated on the next string segment toward the nut by a damped reflecting element characterized by a gain $g_f \leq 1$ and a sign inversion, as shown in Fig. 7.

The structure of the fingering section is similar to the plucking scattering junction illustrated in Section II, with the exclusion of the external force term since the static player force is perfectly balanced by the reaction of the fret once the finger is pressed against it. Thus, the touch parameters are mass, stiffness and damping coefficient. Here the mass dynamically increments from the mass of the finger to the mass of the finger-neck system as the finger pushes the string against the neck. In a fretless instrument the finger achieves imperfect clamping of the string on the neck, for which perfect reflection is not an accurate model. Even in fretted instruments, the advantage of this type of junction is that it allows to model muted playing or stopping the string at the fret [13] before final clamping.

The overall transfer function of the terminated fingering junction

$$L(z) = \frac{U^+_{\rm out}(z)}{U^-_{\rm in}(z)} = \frac{1 - A(z) - 2g_f z^{-2}}{2A(z) + g_f z^{-2}(1 - A(z))} \tag{40}$$

can be used to replace the system of fingering junction and damped reflector in Fig. 7. The coefficients of $A(z)$, which depend on the physical parameters as defined in (18) and (4), are subject to dynamical change. The dimensionless damping coefficient ρ contained in $A(z)$ controls pre-damping due to the initial contact of the finger with the string. The gain g_f, generally close to 1, controls the final damping due to imperfect clamping.

As the finger is pressed quite rapidly on the neck, pre-damping and finger stiffness have only a subtle effect on the produced tone. In a simpler implementation, the fingering scattering junction can be replaced by its large mass (finger-neck system) limit, which is identical in form to the matrix in (29). In that case, it is easy to show that the overall transfer function for fingering is

$$L_{M \gg \mu X}(z) = \frac{-1}{2 - g_f z^{-2}} \tag{41}$$

For use in conjunction with the string collision model discussed in Section III, the action profile $u_{\mathrm{ref}}(n)$, which represents the coordinates of the neck with respect to the resting string, needs to be updated to reflect the new configuration of the string touching the neck at position $n_f X$. This is simply achieved by linear scaling. With imperfect clamping, the collision model reproduces quite faithfully the rattle noise of the string bouncing on the fret or on the neck in proximity of the finger.

Deprived of the damping reflector termination, the fingering scattering junction of Fig. 7 can be advantageously placed at selected positions along the string, i.e., at integer submultiples of the string length, for the production of harmonics. The player's finger comes in loose contact with the string, which feels the effect of the finger mass, stiffness and damping but is still free to vibrate at that point. The best sounding prolonged harmonics are obtained with higher stiffness values and lower damping. Unlike the plucking model, only one fingering section is sufficient here, independently of the string linear mass, since the ability to produce harmonics is to minimize the contact of the finger with the string.

A model for the production of harmonics based on a Wave Digital resistor is presented in [21], which introduces controlled damping at a specific location. The advantage of our model is that, being based on the equations for the mechanical interaction, it is possible to control the acoustic quality of the produced harmonics directly from the physical parameters of finger and string and their contact.

When placed at a non-integer multiple of the string length, the proposed fingering junction has the effect of gradually stopping the string. This can be used for simulating stopping notes with either the neck-side or the bridge-side hand, as common practice in the playing experience.

C. *Popping and Slapping*

With the models developed in Section III, the DW based simulation of extreme push or pulls of the string becomes feasible and very realistic.

Popping is the action of giving a large pull to the string and suddenly releasing it. As a consequence, the string behavior deviates from the nicely behaved linear model. Nevertheless, with the addition of fractional delays modulating the tension, as mentioned in Section III, it is possible to partly model the string collisions with the fingerboard with good accuracy. Nonlinear collision junctions like the one in Fig. 5 are employed, together with one or more plucking junctions located in the plucking zone, depending on string linear mass density. The required force signal for popping is a positive pulse, which means, in our notation, that the force is directed away from the instrument. The initial velocity at plucking point is 0.

The collision junctions are activated wherever the first collision condition in (26) is satisfied. Simultaneous collisions can occur at several places along the fretboard. The condition for collision is independently checked at each junction, disregarding collisions occurring at other parts of the fretboard. This requires testing exclusively the values of the wave variables updated as if no collisions occurred within the past sampling interval. In approximate models, a restricted number of collision junctions can be distributed along the string at selected locations, which are activated when a nearby collision is detected. As collisions are transitory, a strategy is possible so that one can fix a maximum number of junctions to be allocated when and where needed until saturation of the resources.

According to the type of material and form factor, the collisions can range from inelastic to elastic, as controlled by the coefficient of restitution C_R. In fretless instruments, collisions appear to be close to inelastic, which calls for a small value of C_R. The opposite is true for strings hitting metal, round shaped, frets.

Slapping is the action of giving a rapid impulse to the string with the knuckle of the finger so that the string strongly hits the fingerboard or a fret if available and bounces back producing a bright percussive sound. Also for the simulation of slapping we employ the nonlinear scattering junction shown in Fig. 5, but with a nonzero initial velocity and a pulse shaped driving force both directed downwards, i.e., from string to fretboard. Sharp, "metallic" sounds are obtained modeling collisions as semi-elastic with C_R close or equal to 1, together with low values of the damping coefficient ρ. On a fretless fingerboard, slapping produces less bright sounds, while a large portion of the string pushes against the fingerboard. A smaller, close to 0, value for C_R is again selected in this case.

An example of string displacement as a result of multiple collisions is shown in Fig. 8, where the deformation of the string is plotted together with the left-going and right-going waves and the estimated velocity (time difference). It can be observed that the left-going wave in the figure is a filtered version of the right-going wave, due to the bridge termination model (low-pass), in the few milliseconds following the onset of the slap action. The result of previous collisions can be observed in the fluctuation of the velocity on the right side of the graph (three small amplitude peaks that traveled towards the nut, two of which were reflected). The previous collisions can also be traced in the left-going and right-going displacement waves in the same figure, which respectively show two and one very small amplitude peaks. In spite of the small level of these peaks, the result is very audible and fits the expectations of the acoustic rendering of a slapped string.

V. Conclusion

In this paper, we explored models for the most common types of interactions of the player with the guitar such as plucking, fingering with imperfect clamping, string stopping, and the production of harmonics. The models are all developed in the displacement wave variables framework for DW-based synthesis. The original model for the collision of the strings with other objects here introduced allows for very realistic simulations of special play modes such as slapping and popping. As the companion sound examples can demonstrate, the ensemble of interaction models form consistent building blocks for the refined synthesis of fretted or fretless plucked string instruments.

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