A Transient-Free Structure for Lagrange-Type Variable Fractional-Delay Digital Filter

Parinya Soontornwong*, Sorawat Chivapreecha*, Chusit Pradabpet#

*Department of Telecommunication Engineering, Faculty of Engineering,
King Mongkut's Institute of Technology Ladkrabang, Bangkok 10520, Thailand
S4610104@kmitl.ac.th, sorawat@telecom.kmitl.ac.th

#Faculty of Science and Technology,
Phranakhon Si Ayutthaya Rajabhat University, Thailand
pchoos@aru.ac.th

Abstract—**This paper presents a new structure for Lagrange-type variable fractional-delay (VFD) filter. A new structure is formulated from discrete Pascal transform (DPT) and its Pascal interpolation. The computational complexity which obtained from proposed structure is linear function of VFD filter order as same as the so-called Taylor's series structure and also be modular structure. Unfortunately, Taylor's series structure is suffered from transient error that caused from changing delay parameter but the proposed structure can online adjust delay parameter without transient error. Therefore, a transient-free structure will be the new choice for Lagrange-type VFD filters structure which is suitable for real-time application such as sampling rate conversion (SRC) and timing recovery in digital receiver.**

Keywords-Variable fractional-delay (VFD) filter; Lagrange-type VFD filter; transient-free structure; discrete Pascal transform (DPT); Pascal interpolation.

I. Introduction

In many applications of digital signal processing (DSP) require delay elements which can delay the signal in fractional fashion. Such delay elements can be realized by the so called fractional delay filter and when delay can be adjusted by users we call variable fractional-delay (VFD) filter. VFD filter will have the ability to adjust the group delay or phase delay and it can be used in many applications, such as discrete-time signal interpolation, timing offset recovery for digital receiver and sampling-rate conversion (SRC). Generally, most of maximally flat VFD filters are the Lagrange-type VFD filter. The design and realization of Lagrange-type VFD filters have been extensively discussed [1-5]. The effective and well-known structure for Lagrange-type VFD filters is the Farrow structure [6]. In addition, we can offer the Lagrange-type VFD filter in other structure as in [7-9]. If VFD filter is used in the SRC, which can change the sampling rate, it is necessary to adjust the delay parameter in VFD filter. Adjustment of delay parameter in the SRC has to be changed always in order to keep the desired sampling rate correctly. Therefore, some VFD filter structures such as the structure in [7] suitable for some applications, but it is not suitable for real-time application such as SRC and timing recovery in digital receiver because changing the delay parameter to be the cause of transient error, which is undesirable.

In this paper, the transient-free Lagrange-type VFD filter structure can be illustrated; that VFD filter can adjust delay parameter without transient error to the output, and it structure is modular structure as same as the so-called Taylor's series structure in [7]. The simulation results are compared with Taylor's series structure in order to investigate the transient-free of proposed structure. The background of Lagrange-type VFD filter and its structures are explained in section II. The proposed transient-free structure of VFD filter is described in section III. The transient-free investigation of proposed structure can be shown by simulation results in section IV, and conclusion in section V.

II. Lagrange-type VFD Filter

In this section, the structure of Lagrange-type VFD filter will be described. Lagrange-type VFD filter is formulated from Lagrange interpolation that utilize the k^{th}-degree Lagrange polynomial to fit $(k+1)$ data points. The famous filter structure for realization VFD filter is Farrow structure which is commonly used in Lagrange-type VFD filter [3]. The output $y(n)$ of Lagrange-type VFD filter can be computed by

$$y(n) = x(n-D) = \sum_{i=0}^{k} h_i(D)x(n-i). \qquad (1)$$

where D is delay parameter and the filter coefficient $h_i(D)$ are computed by the Lagrange polynomial as

$$h_i(D) = \prod_{\substack{l=0 \\ l\neq i}}^{k} \frac{D-l}{i-l} \qquad (2)$$

where $i = 0, 1, 2,\ldots, k$, and k is the order of Lagrange-type VFD filter. From (1), we can obtain the transfer function of Lagrange-type VFD filter as follows

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$$H(z,D)=\sum_{i=0}^{k}h_i(D)z^{-i}. \tag{3}$$

Furthermore, the filter coefficient of sub-filters which are the fixed-filters in Farrow structure has relationship to the Vandermonde matrix [6] and the transfer function of Lagrange-type VFD filter in term of sub-filters is

$$H(z,D)=\sum_{i=0}^{k}V_i(z)D^i=\mathbf{D}^{\mathbf{T}}\mathbf{V}(z)\ . \tag{4}$$

Therefore, the Lagrange-type VFD filter can be realized by sub-filter $V_i(z)$ using Farrow structure [3] as shown in Fig. 1. The filter coefficients of sub-filters are fixed values and the fractional delay can be varied by controlling the delay parameter D, i.e., the filter can online tuning the delay.

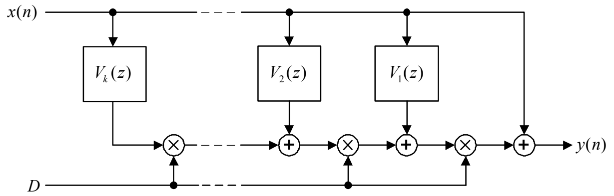

Figure 1. The Farrow structure.

However, the Farrow structure has more and more computational complexity when the order of VFD filter is higher both in term of number of additions and number of multiplications which are increased in exponential growth. Therefore, the symmetry properties of filter coefficient of sub-filters in Farrow structure which can reduce number of multiplications by 50% are proposed in [1-5] but computational complexity is still high and is exponential function of filter order. Consequently, Taylor's series structure in [7] is also proposed. The computational complexity of Taylor's series structure is linear function of order of VFD filter which has much less complexity than the Farrow structure and structure proposed in [1-5]. Taylor's series structure for Lagrange-type VFD filter of order k^{th} in [7] can be shown in Fig. 2.

Taylor's series structure has very low computational complexity and also be the modular structure. However, this structure is suffered from transient error that caused from changing delay parameter because computation path is in the same path of delay elements. Therefore, the state holding will be occurred during output calculation even the new delay parameter is changed. In practical, this structure is not suitable for the application that often change the delay especially in sampling rate conversion (SRC) application.

III. The Proposed VFD Filter Structure

Since the transient error will occur in Taylor's series structure during changing delay parameter although that structure can give the lowest computational complexity when compared with the existing structures. Then, transient-free structure is required. The proposed structure that also proposed in [8, 9] which can give the lowest computational complexity as same as Taylor's series structure is investigated the transient error when delay parameter is adjusted and the investigation result can show that this structure is a transient- free structure. Both Taylor's series structure and proposed structures have the same number of multiplications and number of additions. The transient-free structure for Lagrange-type VFD filter can be shown in Fig. 3.

The proposed transient-free structure is formulated from the so-called discrete Pascal transform (DPT) and its Pascal interpolation [8, 9]. Discrete-time signal $x(n)$ can be written in the form of weighted-sum of DPT basis functions $P_k(n)$ as shown in (5).

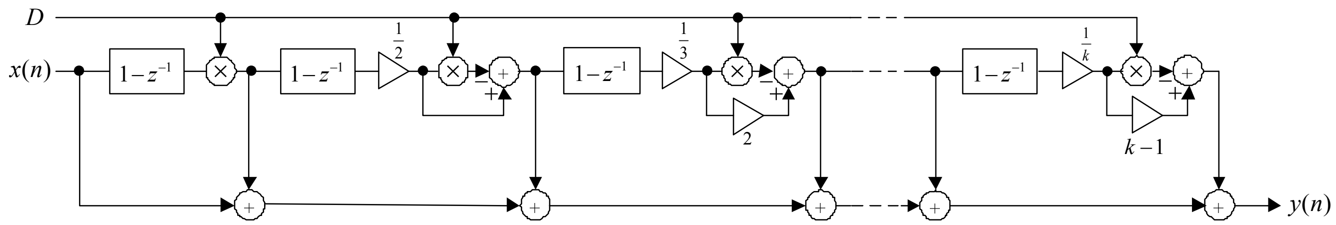

Figure 2. Taylor's series structure for Lagrange-type VFD filter of order k^{th}.

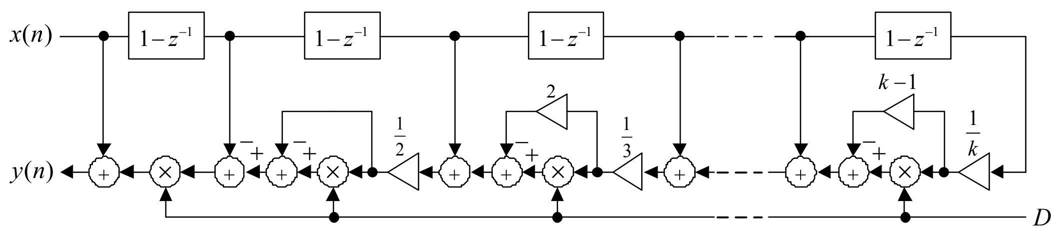

Figure 3. The proposed transient-free structure for Lagrange-type VFD filter of order k^{th}.

$$x(n)=\sum_{k=0}^{N-1} X_k P_k(n) \qquad ,0 \le n \le N-1 \quad (5)$$

where $P_k(n)$ is Pascal basis function [9] and the Pascal coefficient X_k can be calculated by the DPT as

$$X_k=\sum_{n=0}^{k}(-1)^n \binom{k}{n} x(n) \qquad ,0 \le k \le N-1. \quad (6)$$

From (5), in order to interpolate discrete-time input $x(n)$ by factor of L, we can obtain the interpolated discrete-time signal by changing basis function from $P_k(n)$ to $P_k\left(n/L\right)$. So $x_L(n)$ can be express as

$$x_L(n)=x\left(n/L\right)=\sum_{k=0}^{N-1} X_k P_k\left(n/L\right). \quad (7)$$

From (7), the proposed VFD filter structure consists of two sections: The first section (front-end) is developed from (6) by given $x(k) = x(n-k)$, and new form of front-end section can be expressed as

$$H_i(z)=\frac{Z\{X_i\}}{X(z)}=(1-z^{-1})^i \qquad ,i=0,1,2,...,k \quad (8)$$

where $Z\{\ \}$ is the z-transform. Therefore, we can consider (8) as the cascading of first-order digital differentiator. The front-end section is shown in Fig. 4.

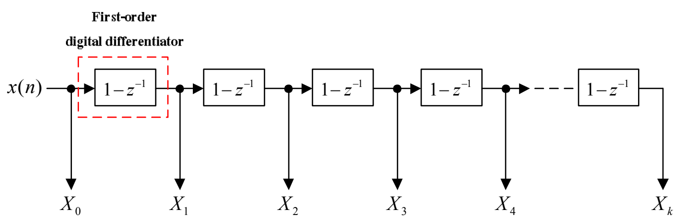

Figure 4. The front-end section.

The back-end section is developed from (7) by n/L is changed to delay parameter (D), the output $y(n)$ of proposed VFD filter can be expressed as

$$y(n)=\sum_{i=0}^{k} P_i(D) X_i \quad (9)$$

where the coefficients $P_i(D)$ are

$$P_i(D)=\frac{(-1)^i D^{(i)}}{i!} \quad (10)$$

and $D^{(i)} = D(D-1)(D-2)\ldots(D-i+2)(D-i+1)$.

The back-end realization can be obtained from (9) and (10). The Horner's rule can be used for sharing the common term of multiplications. The back-end section in [8, 9], if the filter order k^{th} is increased the value of the k^{th} constant multiplier and nearby constant multipliers will be very small values and may cannot be realized in practical. Therefore, the new back-end section of any k^{th} order can be improved as shown in Fig. 5.

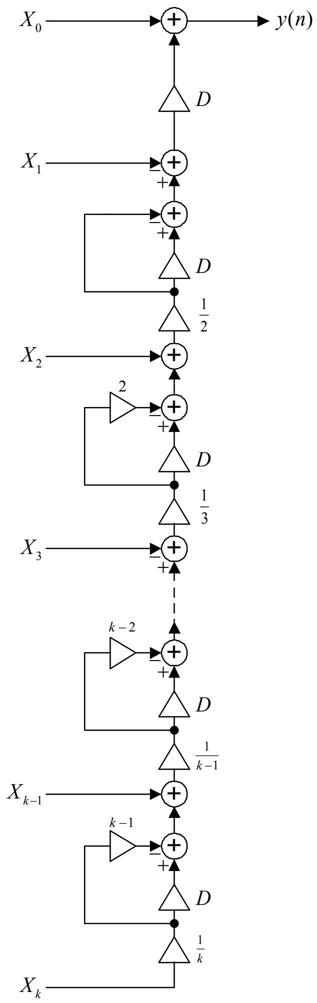

Figure 5. The new back-end section.

Finally, combining (8) and (9) yields the transfer function of the proposed VFD filter as

$$\tilde{H}(z,D)=\sum_{i=0}^{k} P_i(D)(1-z^{-1})^i. \quad (11)$$

Front-end section and back-end section will be combined to be the proposed VFD filter structure as shown in Fig. 3. The proposed structure also is the modular structure as the Taylor's series structure.

IV. TRANSIENT-FREE INVESTIGATION

Consider the interesting structure in [7] which is the 5^{th}-order VFD filter in Taylor's series structure as shown in Fig. 5. It can be online tuning, the fractional delay can be varied by controlling the delay parameter (D). However, when delay parameter is immediately adjusted the output $y(n)$ will be suffered from transient error because output of each state is still holding which is depend on old delay parameter. The correct output $y(n)$ can be obtained after waiting 5 samples passed, which is not suitable for some applications. Therefore, the performance of proposed VFD filter structure will be illustrated in this section, which the simulated results are compared with the Taylor's series structure in previous work [7].

In order to investigate the transient error that caused from changing delay parameter during on-line operation, both Taylor's series structure and proposed transient-free structure for Lagrange-type VFD filter of order 5^{th} and 11^{th} will be realized. Sinusoidal input signal $x(n)$ at frequency 100 Hz with sampling frequency 2 kHz is used to be the test signal. Test signal is feed to the 5^{th} order VFD filter using Taylor's series structure and proposed structure with the delay parameter is changed from D=2.0 to D=2.9 and to D=2.5, the simulation results can be shown as in Fig. 6 (upper trace is Taylor's series structure output, lower trace is proposed structure output). For the 11^{th} order VFD filter using Taylor's series structure and proposed structure, the delay parameter is changed from D=5.0 to D=5.9 and to D=5.5, the simulation results can be shown as in Fig. 7 (upper trace is Taylor's series structure output, lower trace is proposed structure output).

Form simulation results in Figs. 6-7, we can see that the results from Taylor's series structure has transient error during adjust delay parameter while the proposed structure can operate without transient error. Furthermore, we also notice that the amplitude of transient error in Taylor's series structure depend on the range of delay parameter which is adjusted and duration of transient error also depend on the order k^{th} of VFD filter.

Consider from Taylor's series structures in Fig. 2 and Fig. 5, transient error occurs from computation path which is embedded on the same path which contains delay elements then holding of state although the delay parameter is changed will be the cause of error. From Fig.3, the proposed structure can separate the computation path from the path that contains delay elements then when delay parameter is changed, the correct output can be calculated suddenly. Therefore, our proposed Lagrange-type VFD filter will be the transient-free structure, range of delay parameter which is adjusted and filter order do not affect to the occurrence of transient error in our proposed structure.

V. CONCLUSION

The transient-free structure for Lagrange-type VFD filter is proposed in this paper. The proposed structure is formulated from the so-called discrete Pascal transform (DPT) and its Pascal interpolation. Computational complexity of proposed structure is same as Taylor's series structure while proposed structure can operate without transient error during changing delay parameter (D). Consequently, the proposed structure is suitable for on-line tuning in the real-time applications.

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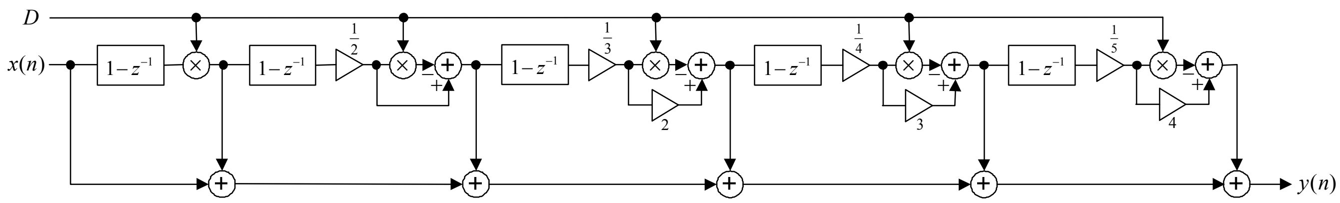

Fig. 5. Fifth-order VFD filter using Taylor's series structure.

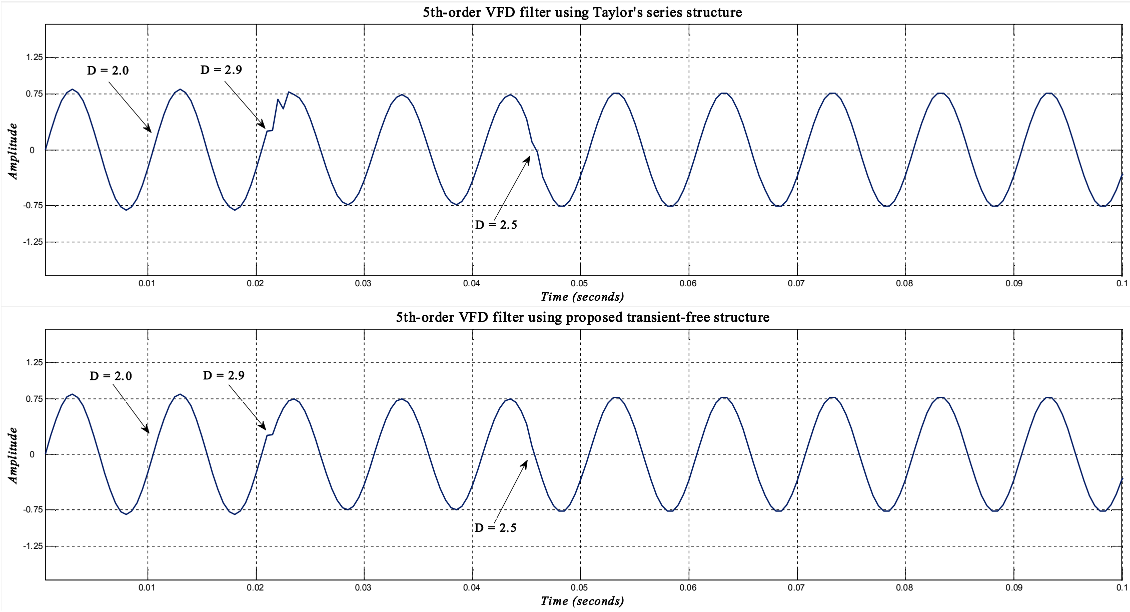

Figure 6. Simulation results of 5^{th} order VFD filter.

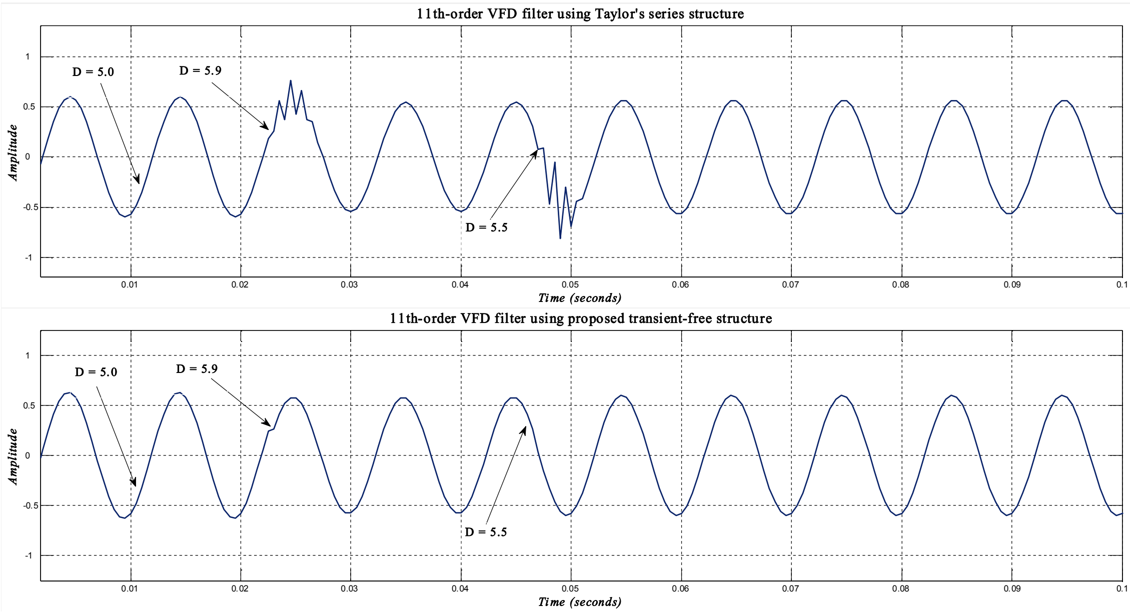

Figure 7. Simulation results of 11^{th} order VFD filter.