ANALYSIS, MODELING, AND REAL-TIME SOUND SYNTHESIS OF THE KANTELE, A TRADITIONAL FINNISH STRING INSTRUMENT

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ABSTRACT

The kantele is a unique old Finnish string instrument that has been found to have interesting acoustical principles of sound production, including nonlinear behavior and dual-mode vibrations of the strings. This paper describes results from a study where the acoustics of the instrument was first analyzed, its sound generation principles were then modeled computationally, and finally a real-time sound synthesis algorithm was developed for a floating-point signal processor.

1. KANTELE, THE INSTRUMENT

The kantele, a traditional Finnish string instrument, has a brilliant sound quality that is unique and characteristic to the instrument. The kantele has a very significant place in Finnish folklore as the instrument of the rune-singers. Thus the kantele has been given a place also in Finnish mythology, especially in the Kalevala, the collection of ancient Finnish runes [1].

There exist different versions of the kantele with 5 to 40 strings, with a unique way of coupling the strings to the wooden body. The body may have different shapes from a simple sound board to more complex bodies. The construction of the traditional five-string instrument, analyzed in this study, is depicted in Fig. 1. It resembles some zither instruments but shows originality of acoustical design. According to some estimates the kantele is about 2000 years old.

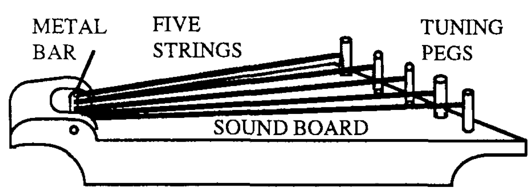

Fig. 1. *Structure of a five-string kantele.*

Our main interst was in the most traditional five-string kantele. It is a musical instrument tuned to the pentatonic scale. The modern concert kantele is a larger and complex instrument with many more strings and it requires more skill to play. Two special features in the timbre of the kantele were of main interest in our study. First, the second harmonic component is dominating in the radiated sound during the attack, even if virtually no second harmonic can be found in the vibration of the string if plucked in the middle [2]. Second, the sound from plucking a kantele string has a strong beat due to amplitude modulation.

Our hypothesis concerning the origin of the unique timbre and sound properties was that the unusual way the strings are fastened and supported may generate these effects. Figures 2a and 2b show a string fixed around the metal bar by a knot (left-hand side in Fig. 1) and a string wound around the tuning peg (right-hand side in Fig. 1).

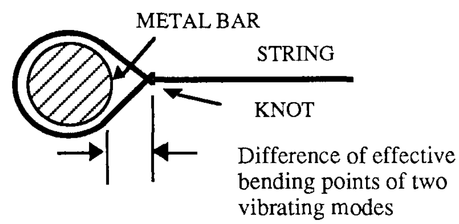

Fig. 2a. *String with a knot around the metal bar.*

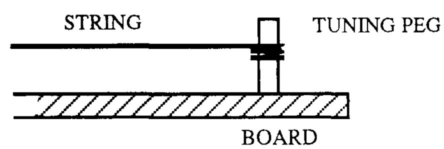

Fig. 2b. *String wound around the tuning peg.*

2. ACOUSTIC MEASUREMENTS OF THE KANTELE

To reveal the main principles of sound generation we used DSP analysis to find the temporal envelopes of the harmonic components of the sound from a single plucked string. Measurements were carried out in an anechoic chamber in different directions and according to various ways of plucking the string. Proper FIR bandpass filters tuned to each partial were used in the analysis. Fig. 3 shows the temporal envelopes of the first four harmonic components when plucking the lowest string in the middle point so that both horizontal (sound board plane) and vertical modes of vibration were excited.

The measurements indicate that the second harmonic is very dominant and other even harmonics coexist in the attack phase of the radiated sound contrary to the theory of string vibration when plucked in the middle. (Notice the different amplitude scales in the subpictures.) All measurements directly from the string by a laser vibrometer [3] and from the tuning peg in horizontal and vertical directions showed practically no even harmonics, as expected. The peg movements longitudinal to the string direction demonstrated, however, that the second harmon-

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ic is strongly generated. Model-based calculations also support this observation and suggest the existence of a square-law nonlinearity, as will be shown below.

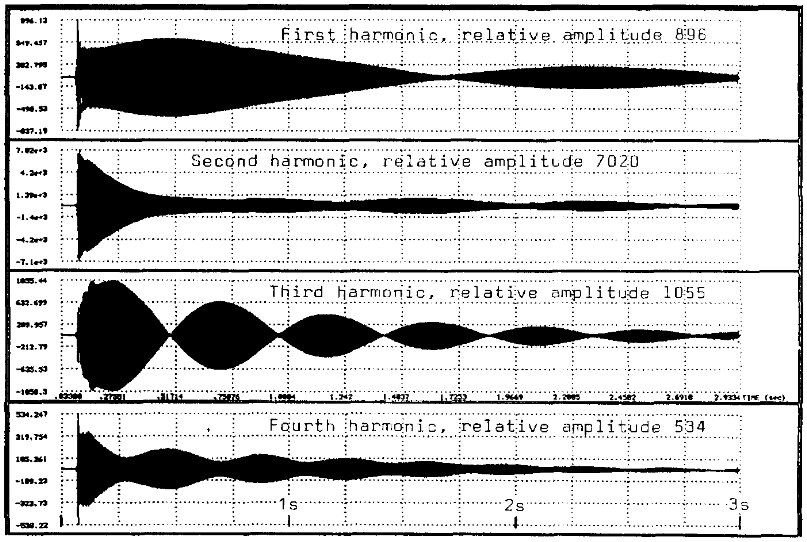

Fig. 3. *Envelopes of the four first harmonic components.*

The strong beat of the sound was found to originate from the knotted termination of the string at the other end. It is evident that the effective termination points of the string are different in the two main vibration planes (fig. 2a). This generates the beat when pairs of modes are summed up. The ratio of the estimated bending point distance and the length of the string (0.1 - 0.2 %) is close enough to the ratio of the beat frequency and the frequency of the corresponding partial. By plucking the string in a typical way the vibrational modes in the two planes are excited almost equally, leading often to somewhat circular polarization. This was proved by direct measurements from the string. The vibrational pattern was checked also visually by stroboscopic studies. In one more experiment we used a modified string termination with point support (no knot) and noticed the absense of the beats, as expected.

3. ANALYSIS OF THE STRING BEHAVIOR

Corresponding to the coordinate system of fig. 4 the transversal wave equation of an ideal string is [2]

$$\frac{\partial^2 d(x,t)}{\partial t^2} = c^2 \frac{\partial^2 d(x,t)}{\partial x^2}$$

where d is the displacement vector as a function of position x and time t, including y and z dimensions, and c is the wave velocity. The d'Alembert solution is the sum of two displacement waves propagating in opposite directions:

$$d(x,t) = d^+(x,t) + d^-(x,t)$$

where + is to the right and - is to the left. We may use four independent displacement terms d, their time derivatives (velocity terms) v, or second time derivatives (acceleration terms) a to describe the wave propagation:

$$d_y^+(x,t),\ d_y^-(x,t),\ d_z^+(x,t),\ d_z^-(x,t),$$
$$v_y^+(x,t),\ v_y^-(x,t),\ v_z^+(x,t),\ v_z^-(x,t), \text{ and}$$
$$a_y^+(x,t),\ a_y^-(x,t),\ a_z^+(x,t),\ a_z^-(x,t)$$

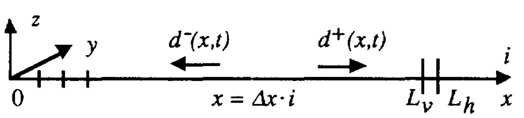

Fig. 4. *The string coordinate system.*

When plucked in the middle ($x = L/2$) the time behavior of an ideal string in the vertical plane is as depicted in fig. 5. Triangular displacement waves propagate to the left and right, reflecting with reversed polarity from the string termination points ($x =$ 0 and L). The resulting waveform will be a time-varying trapezoidal curve where the knee points travel with constant velocity c. The vibration of the middle point is a triangular one. The velocity waveforms are step-like functions and the acceleration is a pair of impulses. An ideal pluck corresponds to an acceleration impulse, so as to lead to a natural source-filter formulation [6].

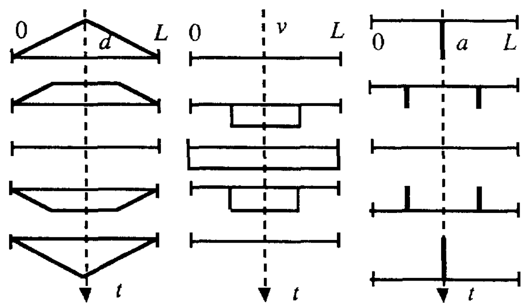

Fig. 5. *Displacement d, velocity v and acceleration a of a string (t = 0 to T/2) when plucked in the middle.*

The transversal forces to the peg and bar are proportional to the slopes ($\partial y/\partial x$ and $\partial z/\partial x$) at the termination points of the string, being square waves in the ideal case above (including only odd harmonics when plucked in the middle). When the waves travel in the string and reflect back at the ends the dispersion and losses of the system modify the waveforms. Higher modes attenuate faster so that finally the vibration approaches a sinusoidal halfwave standing wave of the fundamental mode.

Longitudinal waves are also generated in a string, although they are of little interest in most string instruments. In the kantele, however, they are very important. We may compute the lengthening dl of a short section dx of the string as

$$dl = \sqrt{dy^2 + dz^2 + dx^2} - dx$$

where dy and dz are the displacement differentials in horizontal and vertical dimensions. Because the velocity of longitudinal waves is tens of times higher than of the transversal waves we may assume that the excitation due to local lengthening spreads immediately over the string and the longitudinal forces at the termination points can be estimated from the total lengthening

$$\Delta l(t) = \int_0^L \left(\sqrt{\left(\frac{\partial y(x,t)}{\partial x}\right)^2 + \left(\frac{\partial z(x,t)}{\partial x}\right)^2 + 1} - 1 \right) dx$$

The derivatives are the string slope functions, written as $s_y(x,t)$ and $s_z(x,t)$. The tension $f(t)$ for small amplitudes is now

$$f(t) = SE \cdot \frac{\Delta l(t)}{L} = \frac{SE}{2L} \int_0^L \left\{ s_y(x,t)^2 + s_z(x,t)^2 \right\} dx$$

where S is the string diameter and E is Young's modulus. The square-law form in the equation indicates the nonlinearity that generates odd harmonics, as measured from the peg, the body, and the radiated sound.

Consider the tension $f(t)$ of a string in a motion as shown in figure 5. If the initial value of displacement in the middle point is D then $f(t)$ is easily integrated and found to be also a triangular wave but now having double frequency and amplitude proportional to D squared. Fig. 6 shows both the displacement and the tension as time functions. In a sinusoidal vibration (fundamental mode) the tension signal is sinusoidal and doubled in frequency (sine squared), unless the case is circular polarization which leads to a constant value of $f(t)$. Notice that due to the square-law behavior the decay rate of the longitudinal signal components is twice as fast as for the transversal vibrations.

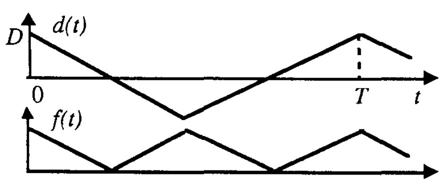

Fig. 6. *Middle point displacement and longitudinal force of ideal string vibration.*

4. OTHER PARTS OF THE INSTRUMENT

An assumption of rigid end supports was made in the previous analysis of the string behavior. In reality the tuning pegs and the bar at the other end do not have a zero driving-point admittance, so as to influence the effective length (when reactive) or to add damping to the string (when resistive).

A major source of complexity in string instruments is due to the mechanically and acoustically resonating body. In this study we didn't pay any special attention to it because our experiments have shown that the audible characteristics of the kantele are due more to the string behavior and sympathetic vibrations. Yet the body model, though relatively simple in the five-string kantele, is important as a computationally expensive part of the instrument. Modal analysis and FEM modeling [3],[4] as well as vibration measurements characterize and visualize the body behavior. For real-time sound synthesis purposes, however, the impulse response of the body, e.g. from the peg movement to the radiated sound field, is of major interest. Figure 7 illustrates the power spectrum of one such response measurement for the five-string kantele; the body acts as a resonant high-pass filter. In the time domain the effective decay constant of the impulse response is about 10 - 20 ms.

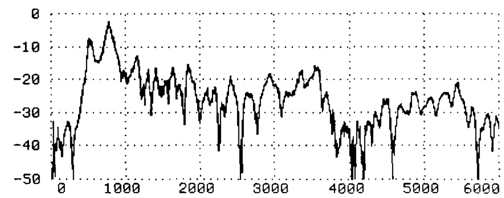

Fig. 7. *Power spectrum (dB, Hz) of a body response for the five-string kantele.*

Sympathetic vibrations of free strings are characteristic to most string instruments. In the kantele they are particularly important. The reverberant brilliance of timbre is due much to the availability of many undamped strings. This phenomenon is generally difficult to analyze and synthesize; several strings are coupled to each other through a bridge or some other part and this coupling is influenced by the numerous resonances of the body. In the kantele this is complicated further; sympathetic vibrations are effected also through the nonlinearities analyzed above.

4. SOUND SYNTHESIS MODELS

High-quality model-based real-time sound synthesis of string instruments is possible by using modern signal processors, as was shown e.g. in [5] for the acoustic guitar. In the present study we planned to follow the same guidelines but found that the two special properties of the kantele, as analyzed above, make it a much more complex task.

For DSP-based sound synthesis the wave behavior of a string must be discretized in time and place. Digital waveguide filters [7] are a proper way of implementing this. (For a good tutorial see Smith [6]). As shown by Smith the computational complexity may be reduced by orders of magnitude when the string is modeled as an ideal lossless waveguide and all losses, dispersion, and other distributed linear effects are lumped into a few special points such as the string terminations. This assumption is valid for example in the case of the guitar [5] but not for the kantele, where, e.g. to compute the longitudinal force, integration over the entire length of the string is needed as shown above.

Consider first at the synthesis of the dual-mode behavior of a kantele string. Instead of a simple dual-rail waveguide we have to add degrees of freedom by including four delay lines as a pair of mono-mode strings (fig. 8), being slightly different in length in horizontal and vertical dimensions. This is easy to implement but it doubles the computational burden. Simple simulation shows that the beat effect is generated as desired. All losses of the string model loop are lumped to reflection filters F_v and F_h. A natural wave variable to be used here is the slope function (spatial derivative of displacement) since both the transverse and longitudinal forces at string terminations are dependent on the slope functions.

Fig. 8 shows the full-scale synthesis model (one string and the body) based on the analysis of the instrument. Submodels for the vertical and horizontal modes are coupled through corresponding body filters to the output as well as the square-law longitudinal force through its body filter. According to measurements the three body filters (B_v, B_h and B_l in fig. 8) are quite different in details but as a first approximation they may be combined to a single resonant high-pass filter (see fig. 7). So far we have not used high-order body filters that are needed for the highest sound quality.

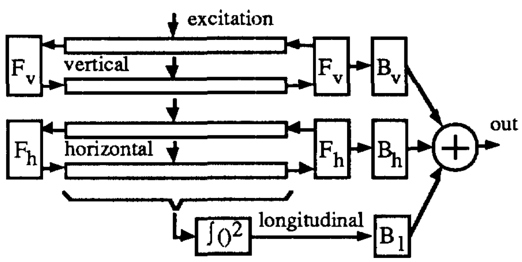

Fig. 8. *Block diagram for the synthesis model.*

Due to distributed nonlinearities in the string there is one more aspect in the kantele that has not drawn enough attention in linear waveguide models: the initial state of a plucked string (ref. to fig. 5). If e.g. the displacement is used as a wave variable the initial state is a pair of triangularly shaped waves traveling to the left and right. To create this we have three alternatives:

1) A triangular initial state is computed or loaded before starting the synthesis. This is straightforward but doesn't fit naturally to the idea of filter-based modeling and data stream computation.

2) A half-wave triangular signal that builds the desired initial state is fed to the delay lines in a single excitation point. This fits to the idea of filter-based modeling but the parameters of the excitation signal still depend on the global properties of the string.

3) A step function excitation together with high damping, followed by a sudden release, is connected to the excitation point. To create the desired triangular initial state a small damping should be included in every element of the delay lines. This is a natural and localized way to apply excitation but it leads to computationally expensive distributed processing of the string.

Simplified models

Full computation of the distributed string properties means that only one short string could be processed in real time on a modern signal processor. To have 5 to 40 strings and a body model the approach must be simplified but so that the perceptual quality of sound synthesis is not degraded too much.

For real-time synthesis of a five string kantele we simplified the model of fig. 8 so that the spatial integral of the squared slope was approximated by a signal that was derived by squaring the acceleration in the middle of the string and filtering this by a leaky integrator. In the ideal pluck of fig. 8 the acceleration is an impulse sequence of alternating polarity. The squaring makes all pulses positive so as to double the fundamental frequency, as desired. The leaky integrator creates a ramp-like signal that spectrally (not in time) resembles the desired longitudinal force at the tuning peg. By an experimentally designed extra filter the spectral properties of the nonlinear components may be improved further. Notice that the square-law works also when the acceleration waveform is a sine wave and generates no harmonics with a circularly polarized sine wave (since the squared accelerations in horizontal and vertical planes are summed). This formulation requires only minimal additions to the guitar synthesis model developed earlier.

Implementation aspects

The TMS320C30 floating-point signal processor and the QuickC30 programming environment [8] was used to achieve real-time performance. An important detail is the approximation of fractional delays that is needed to achieve any desired pitch value. First the Lagrange interpolation was applied as in [5]. The solution by all-pass filtering was found, however, more ideal for the constant string length of the instrument. The real-time kantele synthesis may be controlled from a MIDI controller (e.g. a keyboard), a sequencer program, or a special control language written for the purpose.

Results

The simplified model of the kantele, when properly adjusted, leads to synthetic sounds that well resemble the original instrument. Many challenges remain to improve the model both from a theoretical point of view and to add computational efficiency to be able to approach the physics of the real instrument. By this work we have analyzed and synthesized the unique dual-mode and nonlinear behaviors of the kantele strings. We have also demonstrated that by proper simplifications the physical modeling approach is applicable to real-time synthesis of a complex string behavior.

ACKNOWLEDGEMENTS

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REFERENCES

[1] *New Larousse Encyclopedia of Mythology*. Hamlyn, London 1983.

[2] N.H. Fletcher and T.D. Rossing, *The Physics of Musical Instruments*. Springer-Verlag, New York, 1991.

[3] J. Linjama, Personal communication on unpublished measurements.

[4] V. Siekkinen, Unpublished measurements and analysis. Tampere University of Technology, Finland.

[5] M. Karjalainen and U.K. Laine, "A Model for Real-Time Sound Synthesis of Guitar on a Floating-Point Signal Processor", Proc. of IEEE ICASSP-91, Toronto 1991.

[6] J.O. Smith, "Waveguide Synthesis Tutorial". (DRAFT 9/26/92) CCRMA, to appear in Computer Music Journal.

[7] J.O. Smith, "Music Applications of Digital Waveguides". CCRMA Report No. STAN-M-39, Stanford University, 1987.

[8] M. Karjalainen, "Object-Oriented Programming of DSP Processors: A Case Study of QuickC30". Proc. of IEEE ICASSP-92, San Francisco, 1992.