Algorithm: Molecular Graph Realizability Check

Input: A sequence of integer sets $S = \{V_1, V_2, \ldots, V_n\}$, where each $V_i = \{v_{i1}, v_{i2}, \ldots, v_{im}\}$ is a set of possible valences for chemical element i, sorted in ascending order.
Output: Boolean value indicating whether the sequence is realizable as a molecular graph.

Procedure:

Initialization:
Select the maximum valence for each element: $d_i \leftarrow \max(V_i)$ for all i
Compute total valence sum: $D = \sum_{i=1}^{n} d_i$

Step 1: Handshake Lemma Check
if D is odd **then**
Find the smallest Δ such that $\Delta = d_i - v_j$ for some $v_j \in V_i \setminus \{d_i\}$, sequentially checking the largest possible v_j until Δ is odd.
if such a Δ exists for any i **then**
Update valence: $d_i \leftarrow d_i - \Delta$, update $D \leftarrow D - \Delta$
else
Reject sequence
end if
end if

Step 2: Connectivity Check
if $D < 2(n-1)$ **then**
Reject sequence
end if

Step 3: Loop Prevention Check
if $D \geq 2\max\{d_1, d_2, \ldots, d_n\}$ **then**
Accept sequence as realizable
else
Identify the set of indices $I = \{j \mid d_j = \max(d_i)\}$
Select an index $i \in I$ where $d_i \neq d_{i+1}$, if such exists
if a lower valence exists in V_i **then**
Reduce d_i to the next largest available valence in V_i
Update D and recurse with the modified sequence
else
Reject sequence
end if
end if