Problem 1

Download data on five of your favorite stocks, risk-free rate, and a suitable "market portfolio". We are going to estimate the following model:

$$R_t^i = \alpha_i + \beta_i R_t^M.$$

1. Plot the time-series of prices. Comment on what you see (max three sentences).
2. Compute log-returns. Estimate the covariance matrix. Why do we sometimes prefer to work with returns instead of prices? (max three sentences).
3. Estimate the model and interpret the coefficient estimates. Do you think the model is appropriate? Why/Why not?
4. Given the model assumptions, the estimate of the intercept should be zero. Formulate the hypothesis to test this assumption and perform the test. Do we reject the model?
5. Can you list some "risks" that this model might fail to capture?

Problem 2

The purpose of this exercise is to show the effect of measurement error and to practice backtesting. We will construct some portfolios and evaluate their performance against an equal-weighted portfolio. Download data on 50 of your favorite stocks and the the risk-free rate, and do any transformations you think is necessary. You can choose the frequency and evaluation horizon.

1. Compute the covariance matrix. How many estimates are computed?
2. State the optimization problem for the **Global Minimum** Variance Portfolio. Solve this problem by hand for the two-asset problem where $Er_1 = 8\%, \quad \sigma_1 = 12\%, \quad Er_2 = 13\%, \quad \sigma_2 = 20\%, \quad \rho_{1,2} = 0.30$. What is the expected return for this portfolio? What is the variance? What is the Sharpe-ratio? Remember to show that this solution actually solves the **minimization** problem.
3. State the optimization problem for the **Optimal Risky** Portfolio. Solve this problem by hand for the two-asset problem where $Er_1 = 8\%, \quad \sigma_1 = 12\%, \quad Er_2 = 13\%, \quad \sigma_2 = 20\%, \quad \rho_{1,2} = 0.30$. What is the expected return for this portfolio? What is the variance? What is the Sharpe-ratio?
4. Construct the two portfolios above using five and 50 stocks using Python. Provide some summary statistics for each of the four portfolios you have constructed. Does the portfolios improve when we add more assets? Why is this?
5. Compare the out of sample performance of these portfolios against an equal-weighted portfolio. Comment on the results.
6. Are you able to improve upon the estimation methodology above to improve your portfolio performance? You can also try your own recipe to try to beat the equal-weighted portfolio.