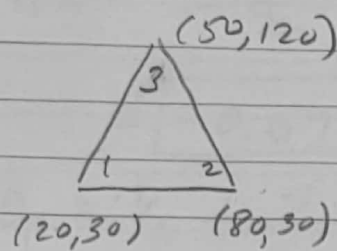


Exam 2 - Practice Problems 6.7a, 8a, 12b; 8.5; 10.3a, 11c, 15

6.7a



$$\{d\} = \begin{Bmatrix} 2 \\ 1 \\ -5 \\ 0 \\ 3 \\ 1 \end{Bmatrix} \text{ mm}$$

$$\beta_1 = y_2 - y_3 = -90$$

$$\beta_2 = y_3 - y_1 = 90$$

$$\beta_3 = y_1 - y_2 = 0$$

$$\gamma_1 = x_2 - x_3 = -30$$

$$\gamma_2 = x_1 - x_3 = -30$$

$$\gamma_3 = x_2 - x_1 = 60$$

$$\frac{1}{2A} = \begin{vmatrix} 1 & 20 & 30 \\ 1 & 80 & 30 \\ 1 & 50 & 120 \end{vmatrix} = 5400$$

$$[B] = \frac{1}{5400} \begin{bmatrix} -90 & 0 & 90 & 0 & 0 & 0 \\ 0 & -30 & 0 & -30 & 0 & 60 \\ -30 & -90 & -30 & 90 & 60 & 0 \end{bmatrix}$$

$$[D] = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} = \frac{10529 \text{ Pa}}{1-(0.25)^2} \begin{bmatrix} 1 & .25 & 0 \\ .25 & 1 & 0 \\ 0 & 0 & .375 \end{bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_{3 \times 1} = \begin{bmatrix} D \end{bmatrix}_{3 \times 3} \begin{bmatrix} B \end{bmatrix}_{3 \times 6} \{d\}_{6 \times 1} = \dots = \begin{Bmatrix} -2.645 \\ -.078 \\ 0.1165 \end{Bmatrix} \text{ GPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \boxed{-0.07249 \text{ MPa}, -2.650 \text{ MPa}}$$

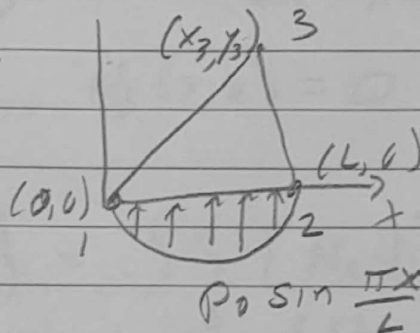
$$\theta_p = \frac{1}{2} \tan^{-1} \left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y} \right) = \boxed{-2.597^\circ}$$

6.8

$$\sigma_{VM} = \sqrt{\sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3\tau_{xy}^2}$$

$$= \boxed{2.614 \text{ MPa}}$$

6.12b



$$A = \frac{1}{2} L y_3 \Rightarrow 2A = L y_3$$

$$\alpha_1 = x_2 y_3 - y_2 x_3 = L y_3$$

$$\alpha_2 = x_3 y_1 - x_1 y_3 = 0$$

$$\alpha_3 = x_1 y_2 - y_2 x_1 = 0$$

$$\beta_1 = y_2 - y_3 = -y_3$$

$$\beta_2 = y_3 - y_1 = y_3$$

$$\beta_3 = y_1 - y_2 = 0$$

$$\gamma_1 = x_3 - x_2 = x_3$$

$$\gamma_2 = x_1 - x_3 = -x_3$$

$$\gamma_3 = x_2 - x_1 = L$$

$$\begin{aligned}
 N_1(x, 0) &= \frac{1}{2A} (\alpha_1 + \beta_1 x + \gamma_1) \quad \text{---} \\
 &= \frac{1}{L\gamma_3} [L\gamma_3 + (-\gamma_3)x] \\
 &= 1 - x/L
 \end{aligned}$$

$$\begin{aligned}
 N_2(x, 0) &= \frac{1}{2A} (\alpha_2 + \beta_2 x + \gamma_2) \quad \text{---} \\
 &= \frac{1}{L\gamma_3} [0 + \gamma_3 x] \\
 &= x/L
 \end{aligned}$$

$$N_3(x, 0) = \frac{1}{2A} (\alpha_3 + \beta_3 x + \gamma_3) \quad \text{---}$$

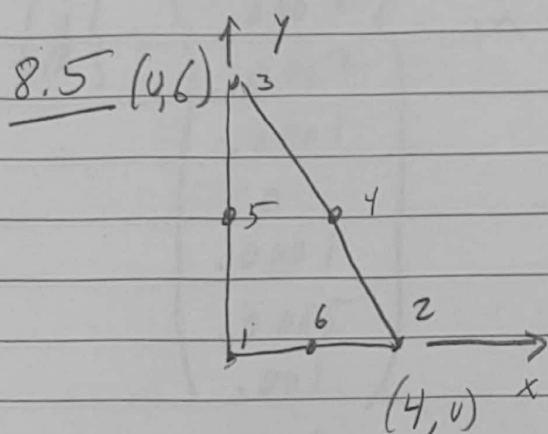
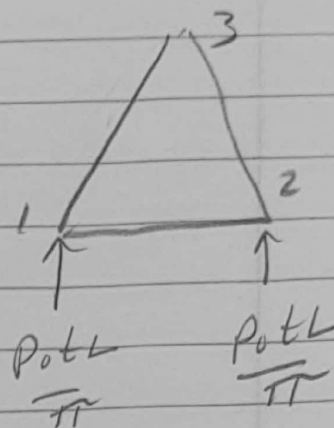
$$N_3(x, 0) = 0$$

$$\{f_s\}_{6 \times 1} = \int_S \int [N_s]_{6 \times 2}^T \{T_s\}_{2 \times 1} ds$$

$$= t \int_0^L \begin{bmatrix} N_1 & 0 \\ 0 & N_1 \\ N_2 & 0 \\ 0 & N_2 \\ N_3 & 0 \\ 0 & N_3 \end{bmatrix} \begin{Bmatrix} 0 \\ p_0 \sin \frac{\pi x}{L} \end{Bmatrix} dx$$

$$= p_0 t \int_0^L \begin{Bmatrix} 0 \\ (1-x/L) \sin \frac{\pi x}{L} \\ 0 \\ (x/L) \sin \frac{\pi x}{L} \\ 0 \\ 0 \end{Bmatrix} dx$$

$$\{f_s\} = \begin{pmatrix} 0 \\ P_0 t L / \pi \\ 0 \\ P_0 t L / \pi \\ 0 \\ 0 \end{pmatrix}$$



$$b = 4 \text{ in}$$

$$h = 6 \text{ in}$$

$$2A = 24 \text{ in}^2$$

$$\text{centroid} = (4/3, 2)$$

Find stress and strain at centroid

Eq 8.2.8 - evaluate at $(x, y) = (4/3, 2)$

to show

$$\beta_1 = -2$$

$$\delta_1 = -4/3$$

$$\beta_2 = 2$$

$$\delta_2 = 0$$

$$\beta_3 = 0$$

$$\delta_3 = 4/3$$

$$\beta_4 = 8$$

$$\delta_4 = 16/3$$

$$\beta_5 = -8$$

$$\delta_5 = 0$$

$$\beta_6 = 0$$

$$\delta_6 = -16/3$$

$$\begin{Bmatrix} C_x \\ C_y \\ \epsilon_{xy} \end{Bmatrix} = \frac{1}{24} \begin{bmatrix} -2 & 0 & 2 & 0 & 0 & 0 & 8 & 0 & -8 & 0 & 0 & 0 \\ 0 & -4/3 & 0 & 0 & 0 & 7/3 & 0 & 16/3 & 0 & 0 & 0 & -16/3 \\ -4/3 & -2 & 0 & 2 & 4/3 & 0 & 16/3 & 8 & 0 & -8 & -16/3 & 0 \end{bmatrix} \begin{Bmatrix} d \end{Bmatrix}$$

3x1 3x12 12x1

where

$$\{d\} = \begin{Bmatrix} 0 \\ 0 \\ .001 \\ .002 \\ .0005 \\ .0002 \\ .0002 \\ .0001 \\ 0 \\ .0001 \\ .0005 \\ .001 \end{Bmatrix} \text{ in}$$

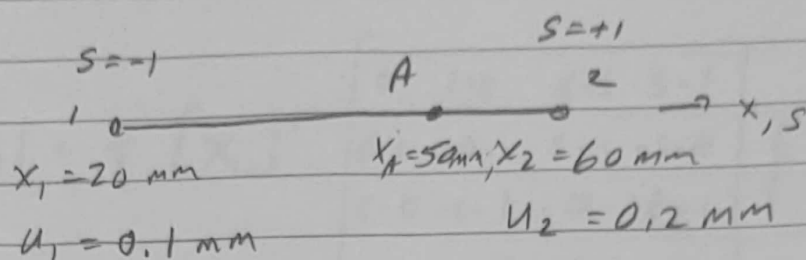
12x1

$$\text{Matlab} \Rightarrow \{\epsilon\} = \begin{Bmatrix} 1.5 \text{e-}4 \\ -1.889 \text{e-}4 \\ 1.278 \text{e-}4 \end{Bmatrix}$$

Stresses at centroid =

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} 3289 \\ -4844 \\ 1533 \end{Bmatrix} \text{ psi}$$

10.39



$$a) \quad x(s) = N_1(s) x_1 + N_2(s) x_2$$

$$\text{where } N_1 = \frac{1-s}{2}, \quad N_2 = \frac{1+s}{2}$$

Point A

$$50 \text{ mm} = \left(\frac{1-s}{2} \right) 20 \text{ mm} + \left(\frac{1+s}{2} \right) 60 \text{ mm}$$

$$100 = 20 - 20s_A + 60 + 60s_A$$

$$20 = 40s_A \Rightarrow s_A = \boxed{0.5}$$

$$b) \quad N_1(s_A) = \frac{1-0.5}{2} = \boxed{\frac{1}{4}}$$

$$N_2(s_A) = \frac{1+0.5}{2} = \boxed{\frac{3}{4}}$$

$$c) \quad u_A = N_1(s_A) u_1 + N_2(s_A) u_2$$

$$u_A = \frac{1}{4} (0.1 \text{ mm}) + \frac{3}{4} (0.2 \text{ mm})$$

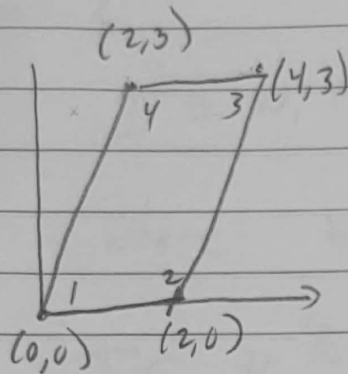
$$\boxed{u_A = 0.175 \text{ mm}}$$

$$d) \quad \epsilon = [B] \{d\} = \left[-\frac{1}{L} \quad \frac{1}{L} \right] \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \frac{u_2 - u_1}{L}$$

$$\boxed{\epsilon = 0.0025}$$

10, 11 c

$$|J| = \frac{1}{8} [X_c]^T \begin{bmatrix} 0 & 1-t & t-s & s-1 \\ t-1 & 0 & s+1 & -s-t \\ s-t & -s-1 & 0 & t+1 \\ 1-s & s+t & -t-1 & 0 \end{bmatrix} \begin{Bmatrix} y_c \end{Bmatrix}$$



$$[X_c]^T = [0 \ 2 \ 4 \ 2]$$

$$[y_c] = \begin{Bmatrix} 0 \\ 0 \\ 3 \\ 3 \end{Bmatrix}$$

$$|J| = \frac{1}{8} [0 \ 2 \ 4 \ 2] \begin{Bmatrix} 3(t-s) + 3(s-1) \\ 3(s+1) + 3(-s-t) \\ 3t+1 \\ 3(-t-1) \end{Bmatrix}$$

$$= \frac{3}{8} [0 \ 2 \ 4 \ 2] \begin{Bmatrix} t-1 \\ 1-t \\ t+1 \\ -t-1 \end{Bmatrix}$$

$$= \frac{3}{8} [2(1-t) + 4(t+1) + 2(-t-1)]$$

$$|J| = \boxed{3/2}$$

$$A = (2)(3) = 6 \Rightarrow |J| = \frac{A}{4} = \frac{3}{2} \checkmark$$

10.15 g

$$\int_{-1}^1 4^s - 2s \, ds$$

 $n=2$

$$I \approx w_1 f(s_1) + w_2 f(s_2)$$

$$\text{where } w_1 = w_2 = 1, \quad s_1, s_2 = \pm 1/\sqrt{3}$$

$$\begin{aligned} I &\approx 4^{-1/\sqrt{3}} - 2(-1/\sqrt{3}) + 4^{1/\sqrt{3}} - 2(1/\sqrt{3}) \\ &= \boxed{2.6755} \end{aligned}$$

 $n=2$

$$I \approx w_1 f(s_1) + w_2 f(s_2) + w_3 f(s_3)$$

where

$$w_1 = w_3 = 5/9, \quad w_2 = 8/2$$

$$s_1, s_3 = \pm \sqrt{3}/5, \quad s_2 = 0$$

$$I \approx \left(\frac{5}{9}\right) \left[4^{\sqrt{3}/5} - 2(\sqrt{3}/5)\right] + \left(\frac{8}{2}\right) [4^0 - 2(0)] + \left(\frac{5}{9}\right) \left[4^{-\sqrt{3}/5} - 2(-\sqrt{3}/5)\right]$$

$$I \approx \boxed{2.70457}$$

Note

$$I_{\text{exact}} = 2.7051$$


```

% exam 2 practice problems
clc; clear; format compact; format short
%
% 6.7a
%
disp('----- 6.7a -----')
d=1e-3*[2 1 .5 0 3 1]' % m
A=det([1 20 30; 1 80 30; 1 50 120])/2
B=(1/(2*A))*[-90 0 90 0 0 0;
    0 -30 0 -30 0 60;
    -30 -90 -30 90 60 0] % 1/m
E=105e9; %N/m^2
nu=0.25
D=(E/(1-nu^2))*[1 nu 0; nu 1 0; 0 0 (1-nu)/2] % N/m^2
sigma=D*B*d % N/m^2
sx=sigma(1);
sy=sigma(2);
txy=sigma(3);

sig_1=(sx+sy)/2+sqrt(((sx-sy)/2)^2+txy^2)
sig_2=(sx+sy)/2-sqrt(((sx-sy)/2)^2+txy^2)
theta_p=(1/2)*atand(2*txy/(sx-sy))
%
% 6.8
%
disp('----- 6.8 -----')
s_vm=sqrt(sx^2-sx*sy+sy^2+3*txy^2)

%
% 8.5
%
disp('----- 8.5 -----')
clear
A=12;
B=[ -2    0 2 0    0    0    8    0 -8 0    0    0;
    0 -4/3 0 0    0 4/3    0 16/3 0 0    0 -16/3;
   -4/3   -2 0 2 4/3    0 16/3    8 0 -8 -16/3    0];
d=[0 0 .001 .002 .0005 .0002 .0002 .0001 0 .0001 .0005 .001]';
eps=(1/(2*A))*B*d
E=30e6;
nu=0.25
D=(E/(1-nu^2))*[1 nu 0; nu 1 0; 0 0 (1-nu)/2]
sigma=D*eps
%
% 10.15g
%
disp('----- 10.15g -----')
clear; format long
disp('n=2')
s1=-1/sqrt(3);
s2=1/sqrt(3);
I=4^s1-2*s1+4^s2-2*s2
disp('n=3')
s1=-sqrt(3/5);

```

$s_2=0;$

$s_3=\sqrt{3/5};$

$I=(5/9) * (4^{s_1}-2*s_1) + (8/9) * (4^{s_2}-2*s_2) + (5/9) * (4^{s_3}-2*s_3)$

----- 6.7a -----

d =

0.0020
0.0010
0.0005
0
0.0030
0.0010

A =

2700

B =

-0.0167	0	0.0167	0	0	0
0	-0.0056	0	-0.0056	0	0.0111
-0.0056	-0.0167	-0.0056	0.0167	0.0111	0

nu =

0.2500

D =

1.0e+11 *		
1.1200	0.2800	0
0.2800	1.1200	0
0	0	0.4200

sigma =

1.0e+06 *
-2.6444
-0.0778
0.1167

sig_1 =

-7.2486e+04

sig_2 =

-2.6497e+06

theta_p =

-2.5972

----- 6.8 -----

s_vm =

2.6142e+06

----- 8.5 -----

eps =

1.0e-03 *
0.1500
-0.1889
0.1278

nu =

0.2500

D =

32000000	8000000	0
8000000	32000000	0
0	0	12000000

sigma =

1.0e+03 *
3.2889
-4.8444
1.5333

----- 10.15g -----

n=2

```
I =  
    2.675540465622292  
n=3  
I =  
    2.704575374345964  
>>
```