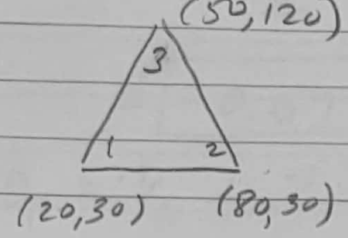
1

Exam 2 - Practice Problems 6.7a, 8a, 12b; 8.5; 10.3a, 11c, 15

<u>6.7a</u>



$$\{d\} = \begin{Bmatrix} 2 \\ 1 \\ .5 \\ 0 \\ 3 \\ 1 \end{Bmatrix} \text{ mm}$$

$$\beta_1 = y_2 - y_3 = -90$$
$$\beta_2 = y_3 - y_1 = 90$$
$$\beta_3 = y_1 - y_2 = 0$$
$$\gamma_1 = x_3 - x_2 = -30$$
$$\gamma_2 = x_1 - x_3 = -30$$
$$\gamma_3 = x_2 - x_1 = 60$$

$$\frac{1}{2A} = \begin{vmatrix} 1 & 20 & 30 \\ 1 & 80 & 30 \\ 1 & 50 & 120 \end{vmatrix} = 5400$$

$$[B] = \frac{1}{5400} \begin{bmatrix} -90 & 0 & 90 & 0 & 0 & 0 \\ 0 & -30 & 0 & -30 & 0 & 60 \\ -30 & -90 & -30 & 90 & 60 & 0 \end{bmatrix}$$

$$[D] = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} = \frac{105e9 \text{ Pa}}{1-(0.25)^2} \begin{bmatrix} 1 & .25 & 0 \\ .25 & 1 & 0 \\ 0 & 0 & .375 \end{bmatrix}$$

$$\underset{3\times1}{\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}} = \underset{3\times3}{[D]} \underset{3\times6}{[B]} \underset{6\times1}{\{d\}} = \ldots = \begin{Bmatrix} -2.646 \\ -.078 \\ 0.1165 \end{Bmatrix} \text{ GPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \boxed{-0.07249 \text{ MPa}, -2.650 \text{ MPa}}$$

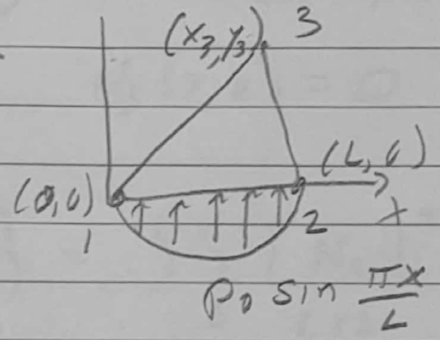
$$\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) = \boxed{-2.597°}$$

6.8

$$\sigma_{VM} = \sqrt{\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2}$$

$$= \boxed{2.614 \text{ MPa}}$$

6.12b



$A = \frac{1}{2} L y_3 \Rightarrow 2A = L y_3$

$$\alpha_1 = x_2 y_3 - y_2 x_3 = L y_3$$
$$\alpha_2 = x_3 y_1 - x_1 y_3 = 0$$
$$\alpha_3 = x_1 y_2 - y_2 x_1 = 0$$

$$\beta_1 = y_2 - y_3 = -y_3$$
$$\beta_2 = y_3 - y_1 = y_3$$
$$\beta_3 = y_1 - y_2 = 0$$

$$\gamma_1 = x_3 - x_2 = x_3$$
$$\gamma_2 = x_1 - x_3 = -x_3$$
$$\gamma_3 = x_2 - x_1 = L$$

$$N_1(x,0) = \frac{1}{2A}\left(\alpha_1 + \beta_1 x + \gamma_1 \cancel{y}^{\,0}\right)$$

$$= \frac{1}{L y_3}\left[L y_3 + (-y_3)x\right]$$

$$= 1 - x/L$$

$$N_2(x,0) = \frac{1}{2A}\left(\alpha_2 + \beta_2 x + \gamma_2 \cancel{y}^{\,0}\right)$$

$$= \frac{1}{L y_3}\left[0 + y_3 x\right]$$

$$= x/L$$

$$N_3(x,0) = \frac{1}{2A}\left(\cancel{\alpha_3}^{\,0} + \cancel{\beta_3}^{\,0} x + \gamma_3 \cancel{y}^{\,0}\right)$$

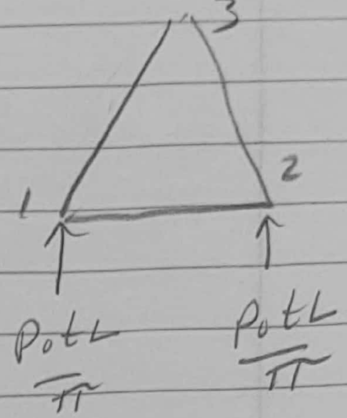
$$N_3(x,0) = 0$$

$$\underset{6\times 1}{\{f_s\}} = \iint_S \underset{6\times 2}{[N_s]^T} \underset{2\times 1}{\{T_s\}}\, ds$$

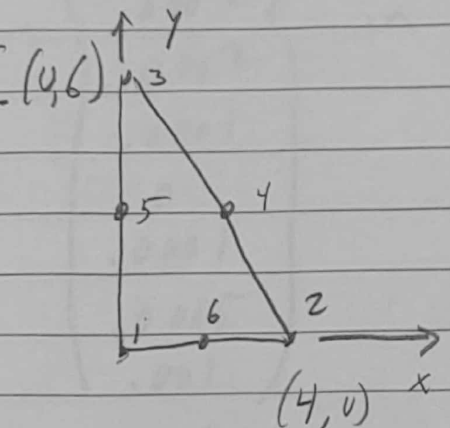
$$= t\int_0^L \begin{bmatrix} N_1 & 0 \\ 0 & N_1 \\ N_2 & 0 \\ 0 & N_2 \\ N_3 & 0 \\ 0 & N_3 \end{bmatrix} \begin{Bmatrix} 0 \\ p_0 \sin \pi x/L \end{Bmatrix} dx$$

$$= p_0 t \int_0^L \begin{Bmatrix} 0 \\ (1 - x/L)\sin \pi x/L \\ 0 \\ (x/L)\sin \pi x/L \\ 0 \\ 0 \end{Bmatrix} dx$$

$$\{f_s\} = \begin{Bmatrix} 0 \\ P_0 t L/\pi \\ 0 \\ P_0 t L/\pi \\ 0 \\ 0 \end{Bmatrix}$$



8.5



$b = 4$ in

$h = 6$ in

$2A = 24$ in^2

centroid - $(4/3, 2)$

Find stress and strain at centroid

Eq 8.2.8 - evaluate at $(x,y) = (4/3, 2)$ to show

$\beta_1 = -2$ $\quad\quad$ $\gamma_1 = -4/3$

$\beta_2 = 2$ $\quad\quad$ $\gamma_2 = 0$

$\beta_3 = 0$ $\quad\quad$ $\gamma_3 = 4/3$

$\beta_4 = 8$ $\quad\quad$ $\gamma_4 = 16/3$

$\beta_5 = -8$ $\quad\quad$ $\gamma_5 = 0$

$\beta_6 = 0$ $\quad\quad$ $\gamma_6 = -16/3$

$$\underset{3\times1}{\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}} = \frac{1}{24} \underset{3\times12}{\begin{bmatrix} -2 & 0 & 2 & 0 & 0 & 0 & 8 & 0 & -8 & 0 & 0 & 0 \\ 0 & -4/3 & 0 & 0 & 0 & 4/3 & 0 & 16/3 & 0 & 0 & 0 & -16/3 \\ -4/3 & -2 & 0 & 2 & 4/3 & 0 & 16/3 & 8 & 0 & -8 & -16/3 & 0 \end{bmatrix}} \underset{12\times1}{\{d\}}$$

where

$$\{d\} = \begin{Bmatrix} 0 \\ 0 \\ .001 \\ .002 \\ .0005 \\ .0002 \\ .0002 \\ .0001 \\ 0 \\ .0001 \\ .0005 \\ .001 \end{Bmatrix}_{12\times1} \text{ in}$$

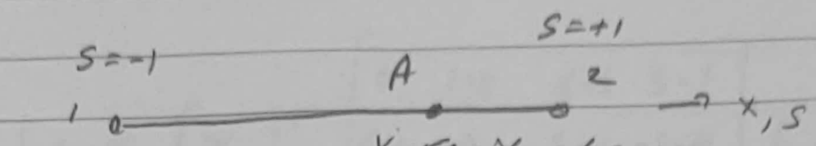
Matlab $\Rightarrow$ $\{\epsilon\} = \begin{Bmatrix} 1.5\text{e-}4 \\ -1.889\text{e-}4 \\ 1.278\text{e-}4 \end{Bmatrix}$

stresses at centroid

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} 3289 \\ -4844 \\ 1533 \end{Bmatrix} \text{psi}$$

6

10.3a



$x_1 = 20$ mm $\quad x_A = 50$ mm, $x_2 = 60$ mm

$u_1 = 0.1$ mm $\quad u_2 = 0.2$ mm

a) $x(s) = N_1(s)\,x_1 + N_2(s)\,x_2$

where $N_1 = \frac{1-s}{2}$, $N_2 = \frac{1+s}{2}$

Point A

$50\text{ mm} = \left(\frac{1-s}{2}\right) 20\text{ mm} + \left(\frac{1+s}{2}\right) 60\text{ mm}$

$100 = 20 - 20s_A + 60 + 60s_A$

$20 = 40s_A \Rightarrow s_A = \boxed{0.5}$

b) $N_1(s_A) = \frac{1-.5}{2} = \boxed{1/4}$

$N_2(s_A) = \frac{1+.5}{2} = \boxed{3/4}$

c) $u_A = N_1(s_A)\,u_1 + N_2(s_A)\,u_2$

$u_A = 1/4\,(0.1\text{ mm}) + 3/4\,(0.2\text{ mm})$

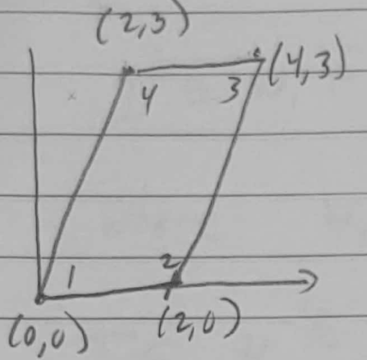
$\boxed{u_A = 0.175\text{ mm}}$

d) $\epsilon = [B]\{d\} = [-1/L \;\; 1/L]\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \frac{u_2 - u_1}{L}$

$\boxed{\epsilon = 0.0025}$

10.11 c

$$|[J]| = \frac{1}{8}[X_c]^T \begin{bmatrix} 0 & 1-t & t-s & s-1 \\ t-1 & 0 & s+1 & -s-t \\ s-t & -s-1 & 0 & t+1 \\ 1-s & s+t & -t-1 & 0 \end{bmatrix} \{Y_c\}$$



$$[X_c]^T = [0 \quad 2 \quad 4 \quad 2]$$

$$[Y_c] = \begin{Bmatrix} 0 \\ 0 \\ 3 \\ 3 \end{Bmatrix}$$

$$|[J]| = \frac{1}{8}[0 \; 2 \; 4 \; 2] \begin{Bmatrix} 3(t-s)+3(s-1) \\ 3(s+1)+3(-s-t) \\ 3\, t+1 \\ 3(-t-1) \end{Bmatrix}$$

$$= \frac{3}{8}[0 \; 2 \; 4 \; 2] \begin{Bmatrix} t-1 \\ 1-t \\ t+1 \\ -t-1 \end{Bmatrix}$$

$$= \frac{3}{8}\left[2(1-\cancel{t}) + 4(\cancel{t}+1) + 2(-\cancel{t}-1)\right]$$

$$|[J]| = \boxed{3/2}$$

$$A = (2)(3) = 6 \implies |[J]| = A/4 = 3/2 \checkmark$$

10.15 g

$$\int_{-1}^{1} 4^s - 2s \, ds$$

$n=2$

$$I \cong w_1 f(s_1) + w_2 f(s_2)$$

where $W_1 = W_2 = 1$, $S_1, S_2 = \pm 1/\sqrt{3}$

$$I \cong 4^{-1/\sqrt{3}} - 2(-1/\sqrt{3}) + 4^{1/\sqrt{3}} - 2(1/\sqrt{3})$$

$$= \boxed{2.6755}$$

$n=2$

$$I \cong w_1 f(s_1) + W_2 f(s_2) + W_3 f(s_3)$$

where

$W_1 = W_3 = 5/9$, $W_2 = 8/2$

$S_1, S_3 = \pm\sqrt{3/5}$, $S_2 = 0$

$$I \cong \left(\tfrac{5}{9}\right)\left[4^{-\sqrt{3/5}} - 2\left(-\sqrt{\tfrac{3}{5}}\right)\right] + \left(\tfrac{8}{9}\right)\left[4^0 - 2(0)\right] + \left(\tfrac{5}{9}\right)\left[4^{\sqrt{3/5}} - 2\left(\sqrt{\tfrac{3}{5}}\right)\right]$$

$$I \simeq = \boxed{2.70457}$$

Note

$$I_{exact} = 2.7051$$

```
------------ 6.7a ---------------
d =
    0.0020
    0.0010
    0.0005
         0
    0.0030
    0.0010
A =
        2700
B =
   -0.0167         0    0.0167         0         0         0
         0   -0.0056         0   -0.0056         0    0.0111
   -0.0056   -0.0167   -0.0056    0.0167    0.0111         0
nu =
    0.2500
D =
   1.0e+11 *
    1.1200    0.2800         0
    0.2800    1.1200         0
         0         0    0.4200
sigma =
   1.0e+06 *
   -2.6444
   -0.0778
    0.1167
sig_1 =
  -7.2486e+04
sig_2 =
  -2.6497e+06
theta_p =
   -2.5972
------------ 6.8 ---------------
s_vm =
   2.6142e+06
------------ 8.5 ---------------
eps =
   1.0e-03 *
    0.1500
   -0.1889
    0.1278
nu =
    0.2500
D =
    32000000     8000000           0
     8000000    32000000           0
           0           0    12000000
sigma =
   1.0e+03 *
    3.2889
   -4.8444
    1.5333
------------ 10.15g ---------------
n=2
```

```
I =
   2.675540465622292
n=3
I =
   2.704575374345964
>>
```

```
% exam 2 practice problems
clc; clear; format compact; format short
%
% 6.7a
%
disp('------------ 6.7a ---------------')
d=1e-3*[2 1 .5 0 3 1]' % m
A=det([1 20 30; 1 80 30; 1 50 120])/2
B=(1/(2*A))*[-90 0 90 0 0 0;
    0 -30 0 -30 0 60;
    -30 -90 -30 90 60 0] % 1/m
E=105e9; %N/m^2
nu=0.25
D=(E/(1-nu^2))*[1 nu 0; nu 1 0; 0 0 (1-nu)/2]  % N/m^2
sigma=D*B*d % N/m^2
sx=sigma(1);
sy=sigma(2);
txy=sigma(3);

sig_1=(sx+sy)/2+sqrt(((sx-sy)/2)^2+txy^2)
sig_2=(sx+sy)/2-sqrt(((sx-sy)/2)^2+txy^2)
theta_p=(1/2)*atand(2*txy/(sx-sy))
%
% 6.8
%
disp('------------ 6.8 ---------------')
s_vm=sqrt(sx^2-sx*sy+sy^2+3*txy^2)

%
% 8.5
%
disp('------------ 8.5 ---------------')
clear
A=12;
B=[   -2    0 2 0    0   0    8    0 -8  0     0     0;
       0 -4/3 0 0   0 4/3    0 16/3  0  0     0 -16/3;
    -4/3   -2 0 2 4/3   0 16/3    8  0 -8 -16/3     0];
d=[0 0 .001 .002 .0005 .0002 .0002 .0001 0 .0001 .0005 .001]';
eps=(1/(2*A))*B*d
E=30e6;
nu=0.25
D=(E/(1-nu^2))*[1 nu 0; nu 1 0; 0 0 (1-nu)/2]
sigma=D*eps
%
% 10.15g
%
disp('------------ 10.15g ---------------')
clear; format long
disp('n=2')
s1=-1/sqrt(3);
s2=1/sqrt(3);
I=4^s1-2*s1+4^s2-2*s2
disp('n=3')
s1=-sqrt(3/5);
```

```
s2=0;
s3=sqrt(3/5);
I=(5/9)*(4^s1-2*s1)+(8/9)*(4^s2-2*s2)+(5/9)*(4^s3-2*s3)
```