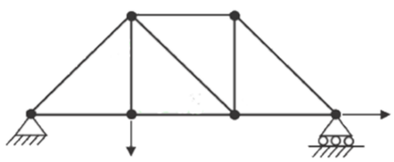
Exam 1 Solution

1a)

$$\begin{Bmatrix} f_{1x} \\ f_{2x} \end{Bmatrix} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \qquad U = \frac{1}{2}\{d\}^T[k]\{d\} = \frac{1}{2}\begin{bmatrix} u_1 & u_2 \end{bmatrix} \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

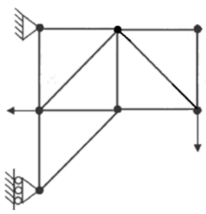
Version	f_{1x} (lb)	f_{1x} (lb)	T / C ?	U (lb-in)
1	500	-500	compression	125
2	-160	160	tension	16
3	-250	250	Tension	62.5

b) Version 1



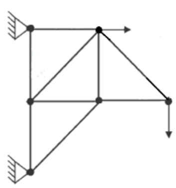
$[k]$	$\{F\}$	$[K]$	$\{D_{reduced}\}$	$[K_{reduced}]$
4×4	12×1	12×12	9×1	9×9

Version 2



$[k]$	$\{F\}$	$[K]$	$\{D_{reduced}\}$	$[K_{reduced}]$
4×4	14×1	14×14	11×1	11×9

Version 3



$[k]$	$\{F\}$	$[K]$	$\{D_{reduced}\}$	$[K_{reduced}]$
4×4	12×1	12×12	8×1	8×8

2. All versions

Theta (deg)	c	s	c^2	s^2	cs
135	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
180	-1	0	1	0	0

$$\left[k^{(1)}\right] = \frac{E_1 A}{L}\begin{bmatrix} 1/2 & -1/2 & -1/2 & 1/2 \\ -1/2 & 1/2 & 1/2 & -1/2 \\ -1/2 & 1/2 & 1/2 & -1/2 \\ 1/2 & -1/2 & -1/2 & 1/2 \end{bmatrix} \qquad \left[k^{(2)}\right] = \frac{E_2 A}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Global Equations (after applying loads and boundary conditions):

$$\begin{Bmatrix} 0 \\ -P \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{12} & K_{22} \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \end{Bmatrix}$$

where $K_{11} = \frac{E_1 A}{2L} + \frac{E_2 A}{L}$, $K_{12} = -\frac{E_1 A}{2L}$ and $K_{22} = \frac{E_1 A}{2L}$

a) Solve global equations for u_1 and v_1.

b) Stress in element 1

$$\sigma^{(1)} = \frac{E_1}{L}\begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ 0 \\ 0 \end{Bmatrix}$$

c) Reaction forces

$$\begin{Bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ F_{3x} \\ F_{3y} \end{Bmatrix} = \begin{Bmatrix} K_{11} & K_{12} \\ K_{12} & K_{22} \\ K_{13} & K_{23} \\ K_{14} & K_{24} \\ K_{15} & K_{25} \\ K_{16} & K_{26} \end{Bmatrix} \begin{Bmatrix} u_1 \\ v_1 \end{Bmatrix}$$

where $K_{11} = \frac{E_1 A}{2L} + \frac{E_2 A}{L}$, $K_{12} = -\frac{E_1 A}{2L}$, $K_{22} = \frac{E_1 A}{2L}$, $K_{13} = -\frac{E_2 A}{L}$, $K_{23} = 0$, $K_{14} = 0$, $K_{24} = 0$, $K_{15} = -\frac{E_1 A}{2L}$, $K_{25} = \frac{E_1 A}{2L}$, $K_{16} = \frac{E_1 A}{2L}$ and $K_{26} = -\frac{E_1 A}{2L}$

Version	u_1 (in)	v_1 (in)	$\sigma^{(1)}$ (psi)	Reaction at Node #	F_x (lb)	F_y (lb)
1	-.004	-.0028	1,414 (T)	3	-2,000	2,000
2	-.015	-.025	1,768 (T)	2	2,500	0
3	-.009	-.0045	2,121 (T)	3	-3,000	3,000

3. a) Since $v_1 = \emptyset_1 = v_2 = 0$, equation 4 gives $M_2 = \frac{EI}{L^3}(4L^2)\,\emptyset_2$ and, from table D-1, $M_2 = \frac{wL^2}{30}$, we can solve for $\emptyset_2 = \frac{wL^3}{120\,EI}$

$$[k] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

b) Reaction forces given by $\{F\} = [K]\{D\} - \{F_0\}$, or

$$\begin{Bmatrix} F_{1y} \\ M_1 \\ F_{2y} \\ M_2 \end{Bmatrix} = [k]\begin{Bmatrix} 0 \\ 0 \\ 0 \\ \emptyset_2 \end{Bmatrix} - \begin{Bmatrix} -\frac{7wL}{20} \\ -\frac{wL^2}{20} \\ -\frac{3wL}{20} \\ \frac{wL^2}{30} \end{Bmatrix}$$

which gives $F_{1y} = \left(\frac{EI}{L^3}\right)(6L)\emptyset_2 - \left(-\frac{7wL}{20}\right)$ and $M_1 = \left(\frac{EI}{L^3}\right)(2L^2)\emptyset_2 - \left(-\frac{wL^2}{20}\right)$.

Results:

Version	$\emptyset_2$ (radians)	F_{1y} (N)	M_1 (N-m)
1	4.167e-4	12,000	20,000
2	4.167e-4	8,000	13,333
3	1.190e-3	4,000	6,667