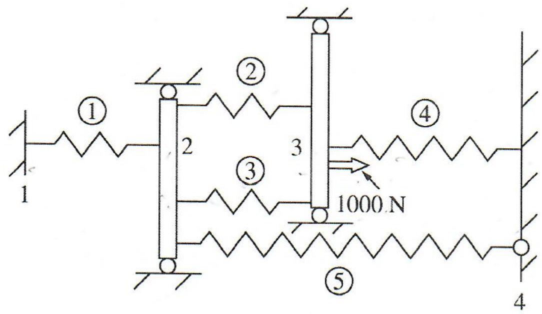
2.17 For the five-spring assemblage shown in Figure P2–17, determine the displacements at nodes 2 and 3 and the reactions at nodes 1 and 4. Assume the rigid vertical bars at nodes 2 and 3 connecting the springs remain horizontal at all times but are free to slide or displace left or right. There is an applied force at node 3 of 1000 N to the right.



■ Figure P2–17

Let $k^{(1)} = 500$ N/mm, $k^{(2)} = k^{(3)} = 300$ N/mm, and $k^{(4)} = k^{(5)} = 400$ N/mm.

1

$$\begin{Bmatrix} f_{1x} \\ f_{2x} \end{Bmatrix} = \begin{array}{c} \begin{matrix} u_1 & u_2 \end{matrix} \\ \underbrace{\begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix}}_{2\times2} \end{array} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$[k^{(1)}]$

$$[k^{(2)}] = [k^{(3)}] = \begin{array}{c} \begin{matrix} u_2 & u_3 \end{matrix} \\ \begin{bmatrix} 300 & -300 \\ -300 & 300 \end{bmatrix} \end{array}$$

$$[k^{(4)}] = \begin{array}{c} \begin{matrix} u_3 & u_4 \end{matrix} \\ \begin{bmatrix} 400 & -400 \\ -400 & 400 \end{bmatrix} \end{array}$$

$$[k^{(5)}] = \begin{array}{c} \begin{matrix} u_2 & u_4 \end{matrix} \\ \begin{bmatrix} 400 & -400 \\ -400 & 400 \end{bmatrix} \end{array}$$

$$
\underset{4\times4}{[K]} = \begin{array}{c}
\begin{array}{cccc} u_1 & u_2 & u_3 & u_4 \end{array} \\
\begin{bmatrix}
500 & -500 & 0 & 0 \\
-500 & \begin{pmatrix} 500+300 \\ +300 \\ +400 \end{pmatrix} & \begin{pmatrix} -300 \\ -300 \end{pmatrix} & -400 \\
0 & \begin{pmatrix} -300 \\ -300 \end{pmatrix} & \begin{pmatrix} 300 \\ +300 \\ +400 \end{pmatrix} & -400 \\
0 & -400 & -400 & \begin{pmatrix} 400+ \\ 400 \end{pmatrix}
\end{bmatrix}
\end{array}
$$

$$
\begin{Bmatrix} F_{1x} \\ \cancel{F_{2x}} \xrightarrow{} 0 \\ \cancel{F_{3x}} \xrightarrow{} 1000\,N \\ F_{4x} \end{Bmatrix} = \begin{bmatrix} 500 & -500 & 0 & 0 \\ -500 & 1500 & -600 & -400 \\ 0 & -600 & 1000 & -400 \\ 0 & -400 & -400 & 800 \end{bmatrix} \begin{Bmatrix} \cancel{u_1} \xrightarrow{} 0 \\ u_2 \\ u_3 \\ \cancel{u_4} \xrightarrow{} 0 \end{Bmatrix}
$$

eqs. 2 & 3

$$
\begin{Bmatrix} 0 \\ 1000 \end{Bmatrix} = \begin{bmatrix} 1500 & -600 \\ -600 & 1000 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}
$$

Solve w/ Matlab

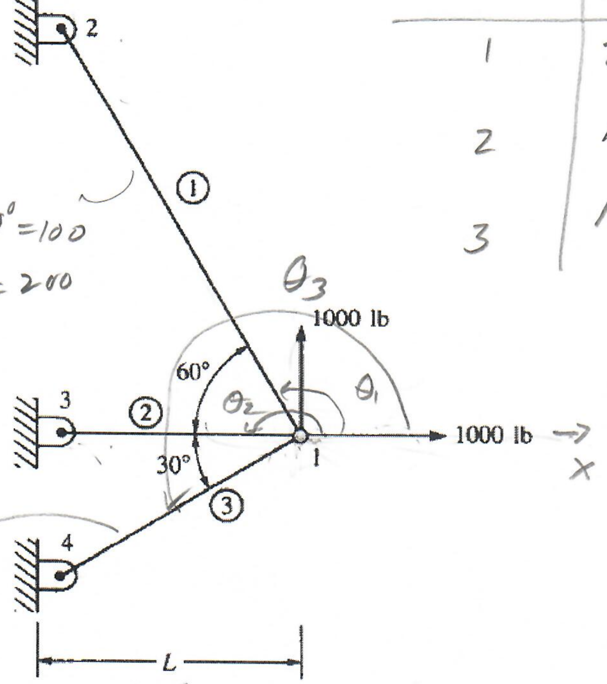
$$
\boxed{\begin{aligned} u_2 &= 0.5263 \text{ mm} \\ u_3 &= 1.316 \text{ mm} \end{aligned}}
$$

Reactions

$$F_{1x} = -500\,u_2 = \boxed{-263.2\ N}$$

$$F_{4x} = -400\,u_2 - 400\,u_3 = \boxed{-736.8\ N}$$

3.22 For the truss shown in Figure P3–22 solve for the horizontal and vertical components of displacement at node 1 and determine the stress in each element. Also verify force equilibrium at node 1. All elements have $A = 1$ in.^{2} and $E = 10 \times 10^6$ psi. Let $L = 100$ in.



$L_1 \cos 60° = 100$

$L_1 = 200$

$L_3 \cos 30 = 100$

$L_3 = 115.5$

Element	L	θ	c	s	c^2	s^2	cs
1	200	120	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$-\frac{\sqrt{3}}{4}$
2	100	180	-1	0	1	0	0
3	115.5	210	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{4}$	$-\frac{\sqrt{3}}{4}$

each element

$$[k] = \frac{AE}{L}\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$$

Global terms

$$\begin{Bmatrix} F_{1x} = 1000 \text{ lb} \\ F_{1y} = 1000 \text{ lb} \\ F_{2x} \\ F_{2y} \\ F_{3x} \\ F_{3y} \\ F_{4x} \\ F_{4y} \end{Bmatrix} = \begin{array}{c} u_1 \quad v_1 \quad u_2 \quad v_2 \quad u_3 \quad v_3 \quad u_4 \quad v_4 \\ \left[\begin{array}{cccccccc} \begin{bmatrix} K_{11} & K_{12} \\ K_{12} & K_{22} \end{bmatrix} & & [\;] & & [\;] & \\ & & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 \\ [\;] & 0 & 0 & [\;] & 0 & 0 \\ & 0 & 0 & & 0 & 0 \\ [\;] & 0 & 0 & 0 & 0 & [\;] \\ & 0 & 0 & 0 & 0 & \end{array}\right] \end{array} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \to 0 \\ v_2 \to 0 \\ u_3 \to 0 \\ v_3 \to 0 \\ u_4 \to 0 \\ v_4 \to 0 \end{Bmatrix}$$

8×1 $\qquad$ 8×8

$$K_{11} = \overset{(1)(10\times10^6)}{AE}\left[\frac{c_1^2}{L_1} + \frac{c_2^2}{L_2} + \frac{c_3^2}{L_3}\right] = 1.775e5$$

$$K_{12} = AE\left[\frac{c_1 s_1}{L_1} + \frac{c_2 s_2}{L_2} + \frac{c_3 s_3}{L_3}\right] = 1.585e4$$

$$K_{22} = AE\left[\frac{s_1^2}{L_1} + \frac{s_2^2}{L_2} + \frac{s_3^2}{L_3}\right] = 5.915e4$$

$$\begin{Bmatrix} 1000 \\ 1000 \end{Bmatrix} = 10^6 \begin{bmatrix} 1.775e5 & 1.585e4 \\ 1.585e4 & 5.915e4 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \end{Bmatrix}$$

Solving with Matlab

$$\boxed{\begin{aligned} u_1 &= 0.004226 \text{ in} \\ v_1 &= 0.01577 \text{ in} \end{aligned}}$$

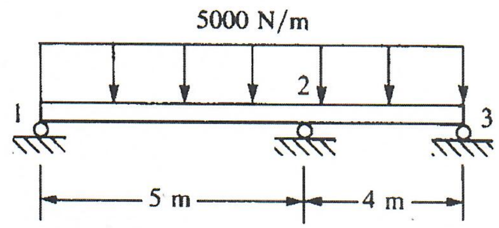
Stresses

$$\sigma^{(1)} = \frac{E}{L_1}\left[-c_1 \;\; -s_1 \;\; c_1 \;\; s_1\right] \begin{Bmatrix} u_1 \\ v_1 \\ \cancel{u_2} \to 0 \\ \cancel{v_2} \to 0 \end{Bmatrix} = -577.4 \text{ psi} \quad (C)$$

$$\sigma^{(2)} = \frac{E}{L_2}\left[-c_2 \;\; -s_2 \;\; c_2 \;\; s_2\right] \begin{Bmatrix} u_1 \\ v_1 \\ \cancel{u_3} \to 0 \\ \cancel{v_3} \to 0 \end{Bmatrix} = 422.7 \text{ psi} \quad (T)$$

$$\sigma^{(3)} = \frac{E}{L_3}\left[-c_3 \;\; -s_3 \;\; c_3 \;\; s_3\right] \begin{Bmatrix} u_1 \\ v_1 \\ \cancel{u_4} \to 0 \\ \cancel{v_4} \to 0 \end{Bmatrix} = 1000 \text{ psi} \quad (T)$$

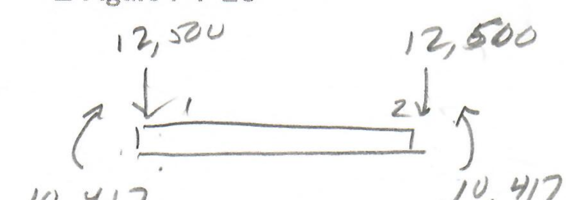
4.21–4.26 For the beams shown in Figures P4–21 through P4–26, determine the nodal displacements and slopes, the forces in each element, and the reactions.



$E = 210$ GPa $= 210 \times 10^9$ N/M^2
$I = 2 \times 10^{-4}$ m^4

■ Figure P4–26

(1)



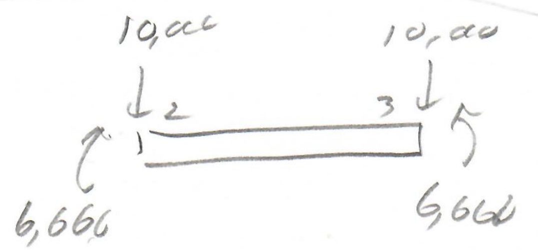
$wL/2 = \dfrac{(5000\ \text{N/m})(5\text{m})}{2}$

$= 12{,}500$ N

$wL^2/12 = \dfrac{(5000\ \text{N/m})(5\text{m})^2}{12}$

$= 10{,}417$ N-M

(2)



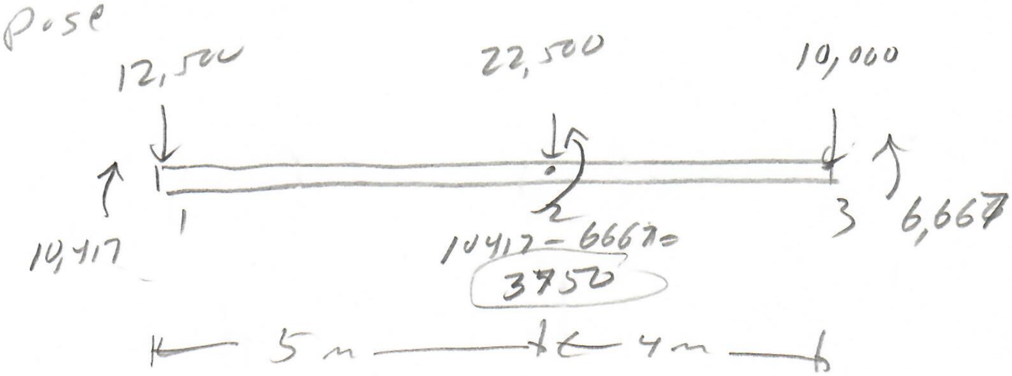
$wL/2 = \dfrac{(5000\ \text{N/m})(4\text{m})}{2}$

$= 10{,}000$ N

$wL^2/12 = \dfrac{(5000\ \text{N/m})(4\text{m})^2}{12}$

$= 6{,}666$ N-m

Superpose



$$[k] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ & 4L^2 & -6L & 2L^2 \\ \text{sym} & & 12 & -6L \\ & & & 4L^2 \end{bmatrix}$$

Global eq's

$$\begin{Bmatrix} F_{1y} \\ \cancel{M_1} \to 10{,}417 \\ F_{2y} \\ \cancel{M_2} \to 3750 \\ F_{3y} \\ \cancel{M_3} \to 6{,}667 \end{Bmatrix} = \begin{array}{c} \begin{matrix} v_1 & \phi_1 & v_2 & \phi_2 & v_3 & \phi_3 \end{matrix} \\ \begin{bmatrix} & & & & & \\ & K_{22} & & K_{24} & & K_{26} \\ & & & & & \\ & & & K_{44} & & K_{46} \\ & & & & & \\ & & & & & K_{66} \end{bmatrix} \end{array} \begin{Bmatrix} \cancel{v_1} \to 0 \\ \phi_1 \\ \cancel{v_2} \to 0 \\ \phi_2 \\ \cancel{v_3} \to 0 \\ \phi_3 \end{Bmatrix}$$

Solve eqs 2, 4, 6

$$K_{22} = \frac{EI}{L_1^3}\left(4L_1^2\right) = \frac{4EI}{L_1} = 33.6\,e6$$

$$K_{24} = \frac{EI}{L_1^3}\left(2L_1^2\right) = \frac{2EI}{L_1} = 16.8\,e6$$

$$K_{26} = 0$$

$$K_{44} = \frac{EI}{L_1^3}\left(4L_1^2\right) + \frac{EI}{L_2^3}\left(4L_2^2\right) = 4EI\left(\frac{1}{L_1} + \frac{1}{L_2}\right)$$
$$= 75.6\,e6$$

$$K_{46} = \frac{EI}{L_2^3}\left(2L_2^2\right) = \frac{2EI}{L_2} = 21.0\,e6$$

$$K_{66} = \frac{EI}{L_2^3}\left(4L_2^2\right) = \frac{4EI}{L_2} = 42.0e6$$

eqs 2, 4, 6

$$\begin{Bmatrix} -10{,}417 \\ 3750 \\ 6667 \end{Bmatrix} = 1e6 \begin{bmatrix} 33.6 & 16.8 & 0 \\ 16.8 & 75.6 & 21.0 \\ 0 & 21.0 & 420 \end{bmatrix} \begin{Bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{Bmatrix}$$

Solve w/ Matlab

$$\boxed{\begin{aligned} \phi_1 &= -3.596e-4 \text{ rad} \\ \phi_2 &= 0.992e-4 \text{ rad} \\ \phi_3 &= 1.091e-4 \text{ rad} \end{aligned}}$$

Reactions

$$\begin{Bmatrix} F_{1y} \\ M_1 \\ F_{2y} \\ M_2 \\ F_{3y} \\ M_3 \end{Bmatrix} = \begin{array}{c} \begin{array}{cccccc} v_1 & \phi_1 & v_2 & \phi_2 & v_3 & \phi_3 \end{array} \\ \begin{bmatrix} & K_{12} & & K_{14} & & K_{16} \\ & K_{22} & & K_{24} & & K_{26} \\ & K_{23} & & K_{34} & & K_{36} \\ & K_{24} & & K_{44} & & K_{46} \\ & K_{25} & & K_{45} & & K_{56} \\ & K_{26} & & K_{46} & & K_{66} \end{bmatrix} \end{array} \begin{Bmatrix} \cancel{v_1}? \\ \phi_1 \\ \cancel{v_2}? \\ \phi_2 \\ \cancel{v_3}? \\ \phi_3 \end{Bmatrix} - \begin{Bmatrix} -12{,}500 \\ -10{,}417 \\ -22{,}500 \\ 3750 \\ -10{,}000 \\ 6667 \end{Bmatrix}$$

6x6

$[K][D] - \{F_0\}$

Solving with Matlab

$$\begin{Bmatrix} F_{1y} \\ M_1 \\ F_{2y} \\ M_2 \\ F_{3y} \\ M_3 \end{Bmatrix} = \begin{Bmatrix} 9875 \text{ N} \\ 0 \\ 28,406 \text{ N} \\ 0 \\ 6719 \text{ N} \\ 0 \end{Bmatrix}$$

$$F_{Total} = F_{1y} + F_{2y} + F_{3y} = 45,000 \text{ N}$$

```
%
% 4-26
%
clc; ; clear; format compact; format long
%
E=210e9;
I=2e-4;
w=5000;
L1=5;
L2=4;
F0=[-w*L1/2; -w*L1^2/12; -(w*L1/2+w*L2/2); w*(L1^2-L2^2)/12; -w*L2/2; w*L2^2/12]
%
% Element Stiffness Matrices
%
for L=[5 4]
    k=(E*I/L^3)*[12 6*L -12 6*L
        6*L 4*L^2 -6*L 2*L^2
        -12 -6*L 12 -6*L
        6*L 2*L^2 -6*L 4*L^2];
    if L==5
        k1=k
    else
        k2=k
    end
end
%
% Global Stiffness matrix
%
K1=zeros(6,6);
K1(1:4,1:4)=k1;
K2=zeros(6,6);
K2(3:6,3:6)=k2;
K=K1+K2
%
% impose BC (v1=v2=v3)
%
Kreduced=K([2 4 6],[2 4 6])
Freduced=[-10417; 3750; 6667]
% Freduced=[F0(2);F0(4);F0(6)]
%
% solve
%
D=Kreduced\Freduced
phi_1=D(1)
phi_2=D(2)
phi_3=D(3)
%
% Reactions
%
Dfull=[0;phi_1;0;phi_2;0;phi_3]
F=K*Dfull-F0
Net_force=F(1)+F(3)+F(5)
```