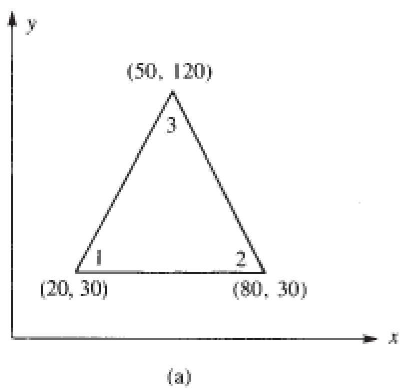
MCE 466
Exam 2 Practice Problems – 6.7a, 8a, 12b; 8.5; 10.3a, 11c, 15g

6.7a

6.7 For the elements given in Problem 6.6, the nodal displacements are given as

$$u_1 = 2.0 \text{ mm} \quad v_1 = 1.0 \text{ mm} \quad u_2 = 0.5 \text{ mm}$$
$$v_2 = 0.0 \text{ mm} \quad u_3 = 3.0 \text{ mm} \quad v_3 = 1.0 \text{ mm}$$



■ Figure P6–6

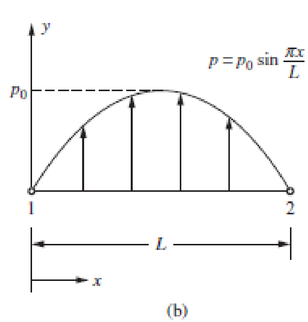
Determine the element stresses σ_x, σ_y, τ_{xy}, σ_1, and σ_2 and the principal angle θ_p. Use the values of E, ν, and t given in Problem 6.6.

6.8a

6.8 Determine the von Mises stress for Problem 6.7.

6.12b

6.12 Determine the nodal forces for (1) the quadratic varying pressure loading shown in Figure P6–12(a) and (2) the sinusoidal varying pressure loading shown in Figure P6–12(b) by the work equivalence method [use the surface integral expression given by Eq. (6.3.7)]. Assume the element thickness to be t.



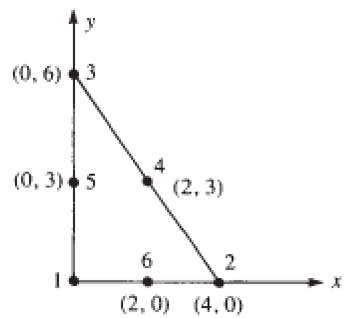
■ Figure P6–12

8.5

8.5 For the linear-strain elements shown in Figure P8–5, determine the strains ε_x, ε_y, and γ_{xy}. Evaluate the stresses σ_x, σ_y, and τ_{xy} at the centroids. The coordinates of the nodes are shown in units of inches. Let $E = 30 \times 10^6$ psi, $\nu = 0.25$, and $t = 0.25$ in. for both elements. Assume plane stress conditions apply. The nodal displacements are given as

$$
\begin{array}{ll}
u_1 = 0.0 \text{ in.} & v_1 = 0.0 \text{ in.} \\
u_2 = 0.001 \text{ in.} & v_2 = 0.002 \text{ in.} \\
u_3 = 0.0005 \text{ in.} & v_3 = 0.0002 \text{ in.} \\
u_4 = 0.0002 \text{ in.} & v_4 = 0.0001 \text{ in.} \\
u_5 = 0.0 \text{ in.} & v_5 = 0.0001 \text{ in.} \\
u_6 = 0.0005 \text{ in.} & v_6 = 0.001 \text{ in.}
\end{array}
$$

(*Hint*: Use the results of Section 8.2.)



(a)

■ Figure P8–5

10.3 Note: For Figure 10.3a, do all four parts of problem 10.2 for the problem shown in 10.3 (a):

a) Find the instrinsic coordinate, *s*, at point *A*.
b) Find the shape functions N_1 and N_2 at point *A*.
c) If the displacements at nodes 1 and 2 are u_1=.005 and $u2$=-.005, determine the displacement of point *A*.
d) If the displacements at nodes 1 and 2 are u_1=.005 and $u2$=-.005, determine the strain in the element .

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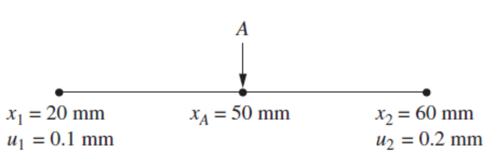
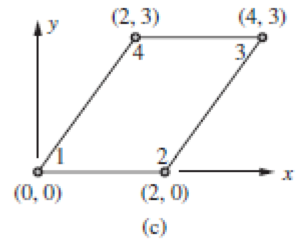
10.3 Answer the same questions as posed in problem 10.2 with the data listed under the Figure P10–3.



(a)

10.11 c

10.11 Determine the Jacobian matrix $[J]$ and its determinant for the elements shown in Figure P10–11. Show that the determinant of $[J]$ for rectangular and parallelogram shaped elements is equal to $A/4$, where A is the physical area of the element and 4 actually represents the area of the rectangle of sides 2×2 when $b = 1$ and $h = 1$ in Figure 6–20.



■ Figure P10–11

10.15g (Gauss quadrature only)

10.15 Use Gaussian quadature with two and three Gauss points and Table 10–2 to evaluate the following integrals:

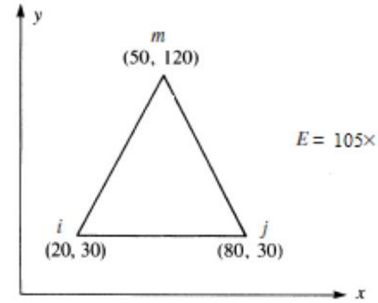
(g) $\int_{-1}^{1} (4^s - 2s)\,ds$

■ Figure P10–11

Solutions

6.7 (a) By Equation (6.2.36)

$$\{\sigma\} = [D]\,[B]\,\{d\}$$



(a)

$E = 105\times 10^{9}$ Pa, $\nu = 0.25$, $t = 0.010$ m

$$\beta_i = y_j - y_m = 30 - 120 = -90 \qquad \gamma_i = x_m - x_j = 50 - 80 = -30$$

$$\beta_j = y_m - y_i = 120 - 30 = 90 \qquad \gamma_i = x_i - x_m = 20 - 50 = -30$$

$$\beta_m = y_i - y_j = 30 - 30 = 0 \qquad \gamma_m = x_j - x_i = 80 - 20 = 60$$

$$2A = x_i(y_j - y_m) + x_j(y_m - y_i) + x_m(y_i - y_j)$$

$$= 20(-90) + 80(90) + 50(0) = 5400 \text{ mm}^2$$

$$[B] = \frac{1}{5400}\begin{bmatrix} -90 & 0 & 90 & 0 & 0 & 0 \\ 0 & -30 & 0 & -30 & 0 & 60 \\ -30 & -90 & -30 & 90 & 60 & 0 \end{bmatrix}$$

$$[D] = \frac{105\times 10^{9}}{1-(0.25)^2}\begin{bmatrix} 1 & 0.25 & 0 \\ 0.25 & 1 & 0 \\ 0 & 0 & 0.375 \end{bmatrix} = 1.12\times 10^{11}\begin{bmatrix} 1 & 0.25 & 0 \\ 0.25 & 1 & 0 \\ 0 & 0 & 0.375 \end{bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{1.12\times 10^{11}}{5400\times 10^{-3}}\begin{bmatrix} 1 & 0.25 & 0 \\ 0.25 & 1 & 0 \\ 0 & 0 & 0.375 \end{bmatrix}\begin{bmatrix} -90 & 0 & 90 & 0 & 0 & 0 \\ 0 & -30 & 0 & -30 & 0 & 60 \\ -30 & -90 & -30 & 90 & 60 & 0 \end{bmatrix}$$

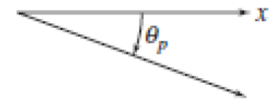
$$\times 10^{-3}\ \frac{\text{m}}{\text{mm}}\begin{Bmatrix} 0.002 \\ 0.001 \\ 0.0005 \\ 0 \\ 0.003 \\ 0.001 \end{Bmatrix}$$

$$= \begin{Bmatrix} -2.645 \text{ GPa} \\ -0.078 \text{ GPa} \\ 0.1165 \text{ GPa} \end{Bmatrix}$$

$$\sigma_{1,2} = \frac{-2.645 + (-0.078)}{2} \pm \sqrt{\left(\frac{-2.645 + -0.078}{2}\right)^2 + 0.1165^2}$$

$$= -1.86 \pm 1.29$$

$$\sigma_1 = -0.07 \text{ GPa} \qquad \sigma_2 = -2.65 \text{ GPa}$$



$$\tan 2\theta_{p_1} = \frac{2(0.1165)}{-2.645 + 0.078} = -0.091$$

$$\theta_p = -2.59°$$

6.8 Von Mises stress

(a) $\sigma_1 = -0.07$ $\qquad$ $\sigma_2 = -2.65$

$$\sigma_e = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2} \qquad \sigma_e = 2.615 \text{ GPa}$$

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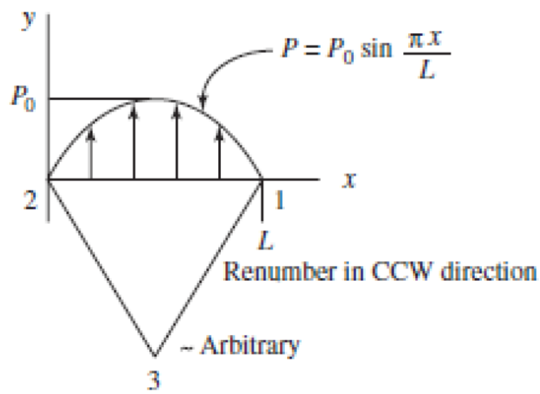
6.12

(b)



$$\{f_s\} = \int_s\int N_s^{\ T}\ [T_s]\ ds$$

$[T_s]$ = Surface tractions

$$= \begin{Bmatrix} T_{sx} \\ T_{sy} \end{Bmatrix} = \begin{Bmatrix} 0 \\ P_0 \sin \pi \frac{N}{L} \end{Bmatrix}$$

$[N_s]$ = Shape function matrix evaluated along edge 1-2

$$= \begin{bmatrix} N_i & 0 & N_j & 0 & N_m & 0 \\ 0 & N_i & 0 & N_j & 0 & N_m \end{bmatrix}$$

Let $i = 1$

$j = 2$

$m = 3$

$$N_i = N_1 = \frac{1}{2A}\ (\alpha_i + \beta_i x + \gamma_i y)$$

$$N_1(y=0) = \frac{1}{2A}\,(\alpha_i + \beta_i x)$$

$$\alpha_i = x_j y_m - y_j x_m$$

$$= 0(y_m) - 0(x_m) = 0$$

$$\beta_i = y_j - y_m$$

$$= 0 - y_m = -\,y_m$$

$$N_1(y=0) = \frac{1}{2A}\,(0 - y_m x) = \frac{-y_m x}{2A}$$

$$N_j = N_2 = \frac{1}{2A}\,(\alpha_j + \beta_j x + \gamma_j y)$$

$$N_2(y=0) = \frac{1}{2A}\,(\alpha_i + \beta_i x)$$

$$\alpha_j = y_j x_m - x_j y_m$$

$$= 0(x_m) - L y_m = -\,L y_m$$

$$\beta_j = y_m - y_i$$

$$= y_m - 0 = y_m$$

$$N_2(y=0) = \frac{1}{2A}\,[-L y_m + y_m x] = \frac{y_m}{2A}\,[x - L]$$

$$N_m = N_3 = \frac{1}{2A}\,[\alpha_m + \beta_m x + \gamma_m y]$$

$$N_3(y=0) = \frac{1}{2A}\,[\alpha_m + \beta_m x]$$

$$\alpha_m = x_i y_j - y_i x_j = L(0) - 0(0) = 0$$

$$\beta_m = y_i - y_j = 0$$

$\therefore\ N_m(y=0) = 0$ As expected

$$\{f_s\} = \int_0^{x=L}\int_0^{z=t} \begin{bmatrix} N_1 & 0 \\ 0 & N_1 \\ N_2 & 0 \\ 0 & N_2 \\ N_3 & 0 \\ 0 & N_3 \end{bmatrix} \begin{Bmatrix} 0 \\ P_0 \sin\frac{\pi x}{L} \end{Bmatrix} dz\,dx$$

$$= t\int_0^{x=L}\begin{bmatrix}0\\ N_1P_0\sin\frac{\pi x}{L}\\ 0\\ N_2P_0\sin\frac{\pi x}{L}\\ 0\\ N_3P_0\sin\frac{\pi x}{L}\end{bmatrix}dx = t\int_0^{x=L}\begin{bmatrix}0\\ \frac{-y_m x}{2A}P_0\sin\frac{\pi x}{L}\\ 0\\ \frac{y_m}{2A}\ x-L\ \ P_0\sin\frac{\pi x}{L}\\ 0\\ 0\end{bmatrix}dx \qquad \text{(A)}$$

2nd term in (A) ($y_m = y_3$)

$$f_{s1y} = \frac{-t\,y_3P_0}{2A}\int_0^{x=L} x\sin\left(\frac{\pi x}{L}\right)\,dx$$

$$\int u\,dv = uv - \int v\,du$$

$$u = x \quad du = dx$$

$$dv = \sin\frac{\pi x}{L}\,dx \quad v = -\cos\frac{\frac{\pi x}{L}}{\frac{\pi}{L}}$$

$$= \frac{-t\,y_3P_0}{2A}\left[-\frac{xL}{\pi}\cos\pi\frac{x}{L} + \int_0^{x=L}\frac{L}{\pi}\cos\left(\pi\frac{x}{L}\right)dx\right]$$

$$= \frac{-t\,y_3P_0}{2A}\left[-\frac{xL}{\pi}\cos\left(\pi\frac{x}{L}\right) + \frac{L^2}{\pi^2}\sin\left(\pi\frac{x}{L}\right)\right]\Bigg|_0^L$$

$$= \frac{-t\,y_3P_0}{2A}\left[\frac{-L^2}{\pi}(-1) + 0 - 0 - 0\right]$$

$$f_{s1y} = \frac{-t\,y_3P_0}{2A}\left(\frac{L^2}{\pi}\right) = \frac{t\,P_0L}{\pi}$$

$$A = \frac{-1}{2}\,Ly_3$$

4th term in (A)

$$f_{s2y} = t\int_0^L\left[\frac{y_3}{2A}P_0\,x\sin\frac{\pi x}{L} - \frac{y_3}{2A}P_0L\sin\left(\frac{x\pi}{L}\right)\right]dx$$

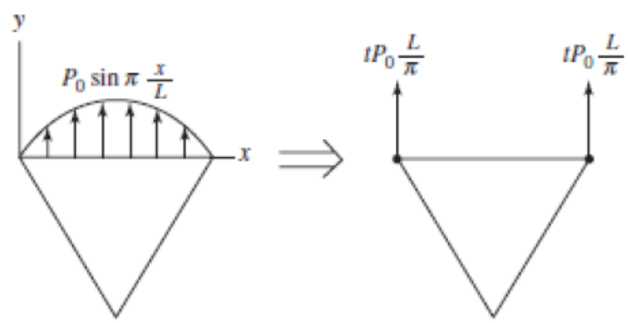
$$= \frac{t\,y_3}{2A}P_0\ \underbrace{\int_0^L x\sin\frac{\pi x}{L}\,dx}_{\text{DONE IN 2nd TERM}} - \frac{t\,y_3}{2A}P_0L\ \int_0^L\sin\frac{\pi x}{L}dx$$

$$= \frac{t\,y_3P_0}{2A}\left[\frac{L^2}{\pi}\right] + \frac{t\,y_3P_0L}{2A}\left[\frac{L}{\pi}\cos\pi\frac{x}{L}\right]\Bigg|_0^L$$

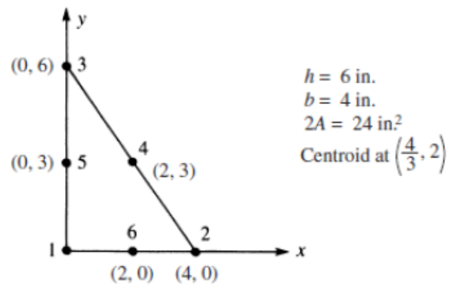
$$= \frac{t\,y_3P_0}{2A}\,\frac{L^2}{\pi} + \frac{t\,y_3P_0L}{2A}\,\frac{L}{\pi}\ [-1+1]$$

$$f_{s2y} = \frac{t\,y_3 P_0 L^2}{2A\,\pi} = \frac{t\,\cancel{y_3}\,L^{\cancel{2}} P_0}{\cancel{2}\left[\frac{1}{\cancel{2}}\cancel{L}\,\cancel{y_3}\right]\pi} = \frac{t\,L\,P_0}{\pi}$$

$$\therefore \quad \vec{f}_s = \begin{Bmatrix} f_{s1x} \\ f_{s1y} \\ f_{s2x} \\ f_{s2y} \\ f_{s3x} \\ f_{s3y} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \frac{t\,P_0\,L}{\pi} \\ 0 \\ \frac{t\,P_0\,L}{\pi} \\ 0 \\ 0 \end{Bmatrix}$$



8.5 (a)



(a)

$$\{\varepsilon\} = [B]\{d\}$$

$$[B] = \frac{1}{2A}\begin{bmatrix} \beta_1 & 0 & \beta_2 & 0 & \beta_3 & 0 & \beta_4 & 0 & \beta_5 & 0 & \beta_6 & 0 \\ 0 & \gamma_1 & 0 & \gamma_2 & 0 & \gamma_3 & 0 & \gamma_4 & 0 & \gamma_5 & 0 & \gamma_6 \\ \gamma_1 & \beta_1 & \gamma_2 & \beta_2 & \gamma_3 & \beta_3 & \gamma_4 & \beta_4 & \gamma_5 & \beta_5 & \gamma_6 & \beta_6 \end{bmatrix}$$

Element is oriented as in Section 8.2

$\therefore$ β's and γ's as in Section 8.2, Eq. (8.2.8)

$$\beta_1 = -3h + \frac{4hx}{6} + 4y = 6x + 4y - 18$$

$$\beta_2 = -h + \frac{4hx}{6} = 6x - 6, \ \beta_3 = 0$$

$$\beta_4 = 4y, \ \beta_5 = -4y$$

$$\beta_6 = 4h - \frac{8hx}{b} - 4y = -12x - 4y + 24$$

$$\gamma_1 = -3b + 4x + \frac{4by}{h} = 4x + \frac{8}{3}y - 12$$

$$\gamma_2 = 0$$

$$\gamma_3 = -b + \frac{4by}{h} = \frac{8}{3}y - 4$$

$$\gamma_4 = 4x$$

$$\gamma_5 = 4b - 4x - \frac{8by}{h} = -4x - \frac{16}{3}y + 16$$

$$\gamma_6 = -4x$$

$\therefore$ $2A\,\varepsilon_x = \beta_2 u_2 + \beta_4 u_4 + \beta_6 u_6$

$$= 0.001\,(6x - 6) + 0.0002\,(4y) + 0.0005\,(-12x - 4y + 24)$$

$$2A\ \varepsilon_x = -0.0012y + 0.006$$

$$\therefore \qquad \varepsilon_x = -5 \times 10^{-5}\,y + 2.5 \times 10^{-4}$$

$$2A\ \varepsilon_y = \gamma_3 v_3 + \gamma_4 v_4 + \gamma_5 v_5 + \gamma_6 v_6$$

$$= 0.0002\left(\frac{8}{3}y - 4\right) + 0.0001\,(4x) + 0.0001\left(-4x - \frac{16}{3}y + 16\right) + 0.001\,(-4x)$$

$$2A\ \varepsilon_y = -0.004x + 0.0008$$

$$\therefore \qquad \varepsilon_y = -1.67 \times 10^{-4}x + 3.33 \times 10^{-5}$$

$$2A\ \gamma_{xy} = 0.002\,(6x - 6) + 0.0005\left(\frac{8}{3}y - 4\right) + 0.0002\,(4x) + 0.0001(4y)$$

$$+ 0.0001\,(-4y) + 0.0005\,(-4x) + 0.001\,(-12x - 4y + 24)$$

$$2A\ \gamma_{xy} = -0.0012x - 0.00267y + 0.01$$

$$\therefore \qquad \gamma_{xy} = -5 \times 10^{-5}x - 1.11 \times 10^{-4}\,y + 4.167 \times 10^{-4}$$

Evaluate stresses at centroid

$$\{\sigma\} = [D]\,\{\varepsilon\}$$

$$\{\varepsilon\}\Big|_{\left(\frac{4}{3},2\right)} = \begin{Bmatrix} 0.00015 \\ -1.89 \times 10^{-4} \\ 0.000128 \end{Bmatrix}$$

$$\{\sigma\} = \frac{E}{1 - v^2}\begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & \frac{1-v}{2} \end{bmatrix}\begin{Bmatrix} 1.5 \times 10^{-4} \\ -1.89 \times 10^{-4} \\ 1.28 \times 10^{-4} \end{Bmatrix}$$

$$\{\sigma\} = \begin{Bmatrix} 3288 \\ -4848 \\ 1536 \end{Bmatrix}\ \text{psi}$$

10.3

10.3 Using Equation (10.1.1 b)

Figure (a)

(a) $x = x_A = 50$ mm

$$s = \left[50 - \left(\frac{20+60}{2}\right)\right]\left(\frac{2}{60-20}\right)$$

$$s = [50 - 40]\left(\frac{2}{40}\right)$$

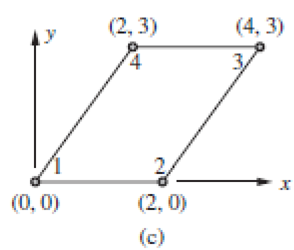
$$s = 0.5$$

(b) $N_1 = \dfrac{1-0.5}{2} = \dfrac{1}{4}$, $N_2 = \dfrac{1+0.5}{2} = \dfrac{3}{4}$

(c) $u_A = \begin{bmatrix} 1/4 & 3/4 \end{bmatrix} \begin{Bmatrix} 0.10 \\ 0.20 \end{Bmatrix} = 0.175 \text{ mm}$

(d) $\varepsilon_x = \begin{bmatrix} -\dfrac{1}{40} & \dfrac{1}{40} \end{bmatrix} \begin{Bmatrix} 0.10 \\ 0.20 \end{Bmatrix} = 0.0025$

10.11c



■ Figure P10–11

By Equation (10.2.22)

$$|[J]| = \frac{1}{8}\,[0\ 2\ 4\ 2]\begin{bmatrix} 0 & 1-t & t-s & s-1 \\ t-1 & 0 & s+1 & -s-t \\ s-t & -s-1 & 0 & t+1 \\ s-1 & s+t & -t-1 & 0 \end{bmatrix}\begin{Bmatrix} 0 \\ 0 \\ 3 \\ 3 \end{Bmatrix} = \frac{3}{2}$$

$$|[J]| = \frac{A}{4} = \frac{1}{4}\,(2 \times 3) = \frac{3}{2}$$

10.15g

Two Gauss points

$$\int_{-1}^{1}(4^{s}-2s)ds$$

$$I=4^{(-0.57735)}-2(-0.57735)$$

$$+4^{(0.57735)}-2(0.57735)$$

$$=2.6755$$

Three Gauss Points

$$x_1=x_3=\pm 0.77460, x_2=0, W_1=W_3=0.5555, W_2=0.8888$$

$$(4^{-0.7746}-2(-0.7746))0.5555$$

$$+(4^{0}-2(0))0.8888+(4^{0.7746}-2(0.7746))0.5555$$

$$=2.7045$$

Exact is 2.7051