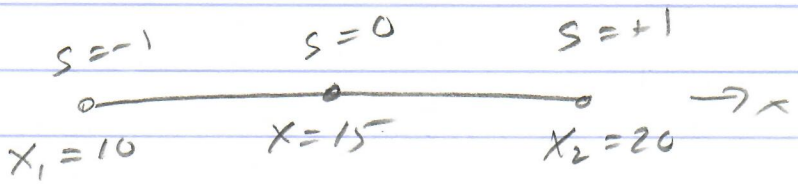
Hw 7

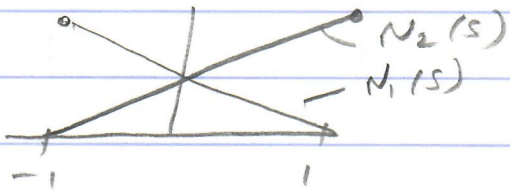
10.2a



$$x(s) = N_1(s)\,x_1 + N_2(s)\,x_2$$

$$N_1 = \frac{1-s}{2}$$

$$N_2 = \frac{1+s}{2}$$

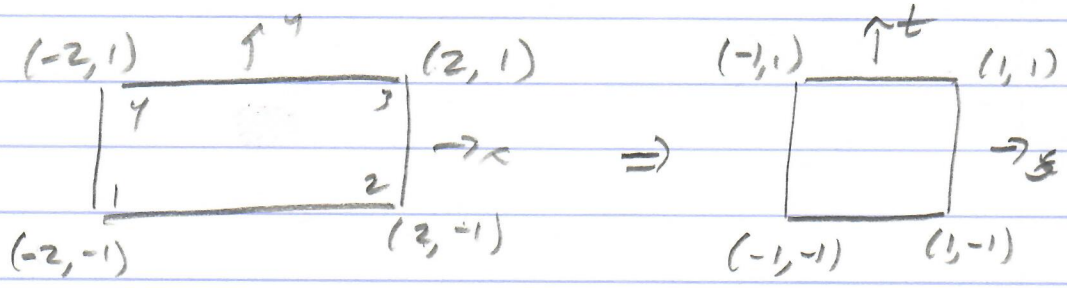


$$15 = \left(\frac{1-s}{2}\right)10 + \left(\frac{1+s}{2}\right)20$$

$$0 = -5s + 10s$$

$$0 = 5s \Rightarrow \boxed{s = 0}$$

10-11



$$J = \begin{bmatrix} \partial x/\partial s & \partial y/\partial s \\ \partial x/\partial t & \partial y/\partial t \end{bmatrix}$$

$$x(s,t) = \frac{1}{4}\left[(1-s)(1-t)x_1 + (1+s)(1-t)x_2 + (1+s)(1+t)x_3 + (1-s)(1+t)x_4\right]$$

$$y(s,t) = \frac{1}{4}\left[(1-s)(1-t)\,y_1 + (1+s)(1-t)\,y_2 + (1+s)(1+t)\,y_3 + (1-s)(1+t)\,y_4\right]$$

$$\frac{\partial x}{\partial s} = \frac{1}{4}\left[+(1-t)(+2) + (1-t)(2) + (1+t)(2) + (1+t)(2)\right]$$

$$= \frac{1}{4}(8+0) = 2$$

$$\frac{\partial x}{\partial t} = \frac{1}{4}\left[+(1-s)(+2) - (1+s)(2) + (1+s)(2) + (1-s)(-2)\right]$$

$$= \frac{1}{4}[0+0) = 0$$

$$\frac{\partial y}{\partial s} = \frac{1}{4}\left[+(1-t)(+1) + (1-t)(-1) + (1+t)(1) - (1+t)(1)\right]$$

$$= \frac{1}{4}[0+0] = 0$$

$$\partial y/\partial t = \frac{1}{4}\left[-(1-s)(-1) - (1+s)(-1) + (1+s)(1) + (1-s)(1)\right]$$

$$= \frac{1}{4}[4+0] = 1$$

$$[J] = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow |J| = 2$$

10.15 $I = \int_{-1}^{1} \cos\frac{s}{2}\, ds$

Analytically (exact)

$$\int_{-1}^{1} \cos\frac{s}{2}\, ds = 2\sin s/2 \Big|_{-1}^{1}$$

$$= 2\sin 1/2 - 2\sin(-1/2)$$

$$= 1.917702$$

n=2

$w_1 = w_2 = 1$

$x_1, x_2 = \pm 0.5773 = \pm 1/\sqrt{3}$

$$I \approx \sum_{i=1}^{2} w_i f(x_i) = (1)\cos\left(\frac{-1}{2\sqrt{3}}\right) + (1)\cos\left(\frac{+1}{2\sqrt{3}}\right)$$

$$= 1.91724$$

n=3

$x_1, x_3 = \pm .7745 = \pm\sqrt{3/5}$

$x_2 = 0$

$w_1 = w_3 = 5/9, \quad w_2 = 8/9$

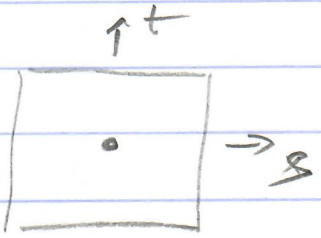
$$I \approx \sum_{i=1}^{3} w_i f(x_i)$$

$$= 5/9\cos\left(-\frac{1}{2}\sqrt{3/5}\right) + 8/9\cos(0) + \frac{5}{9}\cos\left(\frac{+1}{2}\sqrt{3/5}\right)$$

$$= 1.917703$$

4) $\int_{-1}^{1}\int_{-1}^{1} \cos s \cos t \, ds\, dt$

$n=1$ (1×1)



$w_1 = 2$

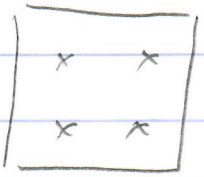
$x_1 = 0$

$$I \approx \sum_{i=1}^{1}\sum_{j=1}^{1} w_i w_j f(s_i, t_j)$$

$$= (2)(2) f(0,0)$$

$$= \boxed{4}$$

$n=2$ (2×2)



$x_1, x_2 = \pm \frac{1}{\sqrt{3}}$

$w_1 = w_2 = 1$

$$I \approx \sum_{i=1}^{2}\sum_{j=1}^{2} w_i w_j f(s_i, t_j)$$

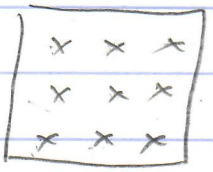
$$= \cos\left(-\tfrac{1}{\sqrt{3}}\right)\cos\left(-\tfrac{1}{\sqrt{3}}\right) + \cos\left(-\tfrac{1}{\sqrt{3}}\right)\cos\left(\tfrac{1}{\sqrt{3}}\right)$$
$$+ \cos\left(\tfrac{1}{\sqrt{3}}\right)\cos\left(\tfrac{1}{\sqrt{3}}\right) + \cos\left(\tfrac{1}{\sqrt{3}}\right)\cos\left(-\tfrac{1}{\sqrt{3}}\right)$$

$$= 4\cos^2\left(\tfrac{1}{\sqrt{3}}\right) = \boxed{2.809}$$

$n=3$ $\qquad w_1 = w_3 = 5/9, \quad w_2 = 8/9$

$x_1, x_3 = \pm\sqrt{3/5}, \quad x_2 = 0$

$$I \simeq \sum_{i=1}^{3}\sum_{j=1}^{3} w_i w_j f(s_i, t_j)$$



$$I \simeq 4\left(\tfrac{5}{9}\right)\left(\tfrac{5}{9}\right)\cos^2\left(\sqrt{\tfrac{3}{5}}\right) + 4\left(\tfrac{5}{9}\right)\left(\tfrac{8}{9}\right)\cos(0)\cos\left(\sqrt{\tfrac{3}{5}}\right) + \left(\tfrac{8}{9}\right)\left(\tfrac{8}{9}\right)\cos(0)\cos(0)$$

$$I = \boxed{2.833}$$

Exact = 2.8323