MCE 466 – Heat Transfer Analysis – Logan's Example 16.7

Consider the transient heat transfer problem described in Logan's Example Problem 16.7 (see Figure 1).

Example 16.7

A circular fin (Figure 16–23) is made of pure copper with a thermal conductivity of $K_{xx} = 400\ \mathrm{W/(m \cdot {}^\circ C)}$, $h = 150\ \mathrm{W/(m^2 \cdot {}^\circ C)}$, mass density $\rho = 8900\ \mathrm{kg/m^3}$, and specific heat $c = 375\ \mathrm{J/(kg \cdot {}^\circ C)}$ ($1\ \mathrm{J} = 1\ \mathrm{W \cdot s}$). The initial temperature of the fin is 25 °C. The fin length is 2 cm, and the diameter is 0.4 cm. The right tip of the fin is insulated. The base of the fin is then suddenly increased to a temperature of 85 °C and maintained at this temperature. ~~Use the consistent form of the capacitance matrix, a time step of 0.1 s, and $\beta = \frac{2}{3}$. Use two elements of equal length~~. Determine the temperature distribution up to 3 s.

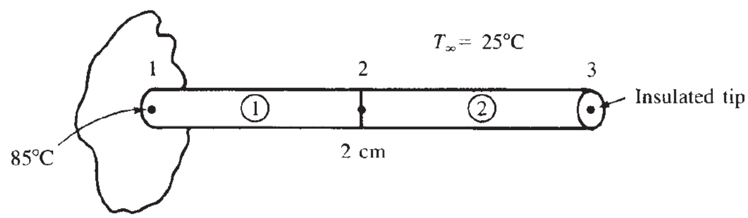

Figure 16–23 Rod subjected to time-dependent temperature

Figure 1. Example 16.7 (ref. "A First Course in the Finite Element Method," 6th Edition, Daryl L. Logan, 2017)

Abaqus Consistent Units

Quantity	SI	SI (mm)	US Unit (ft)	US Unit (inch)
Length	m	mm	ft	in
Force	N	N	lbf	lbf
Mass	kg	tonne (10^3 kg)	slug	lbf s^2/in
Time	s	s	s	s
Stress	Pa (N/m^2)	MPa (N/mm^2)	lbf/ft^2	psi (lbf/in^2)
Energy	J	mJ (10^{-3} J)	ft lbf	in lbf
Density	kg/m^3	tonne/mm^3	slug/ft^3	lbf s^2/in^4

Summary of the modeling steps:

1. Sketch Module – Create the rectangular shape shown in Figure 2. Be sure to convert dimensions to meters
2. Part Module – Create an axisymmetric solid part
3. Property Module – Set the following properties, create Section, and assign Section to full region
    - density=8900 kg/m^2 (General tab)
    - conductivity=400W/(m °C) (Thermal tab)
    - specific heat = 375 J/(kg °C) (Thermal tab)
4. Assembly Module – Create instance
5. Step Module – Create Step: Heat Transfer (Transient), Time period =3, Incrementation=Fixed, Maximum number of increments=30, Increment size=0.1; Output=>History Output Requests=>Create=>Every n increments, n=1
6. Interaction Module – Create Interaction on right edge: Type-Surface film condition, Definition-Embedded Coefficient, Film coefficient=150, Sink definition=Uniform, Sink temperature=25, Sink Amplitude=Instantaneous
7. Load Module – Predefined Field (pull down menu) – Step=Initial, Type=Temperature, select full region, Magnitude=25; Create Boundary Condition, select bottom edge, set Temperature=85.
8. Mesh Module – Global Seed=0.0005, Mesh Control=Quad, Element Type=8-node quadratic axisymmetric heat transfer, Mesh Part
9. Job Module – Create Job, Submit Job
10. Visualization Module – Examine contours of Nodal Temperature (NT11) at different times (using "VCR-type" Next and Previous buttons on upper right hand side). Record nodal temperature at z=2 cm at times 0, 1, 2 and 3 seconds on the Solution Summary Sheet (attached) and compare to the text solution shown in Appendix 1 (which may be slightly different due to different element type and time stepping algorithm)

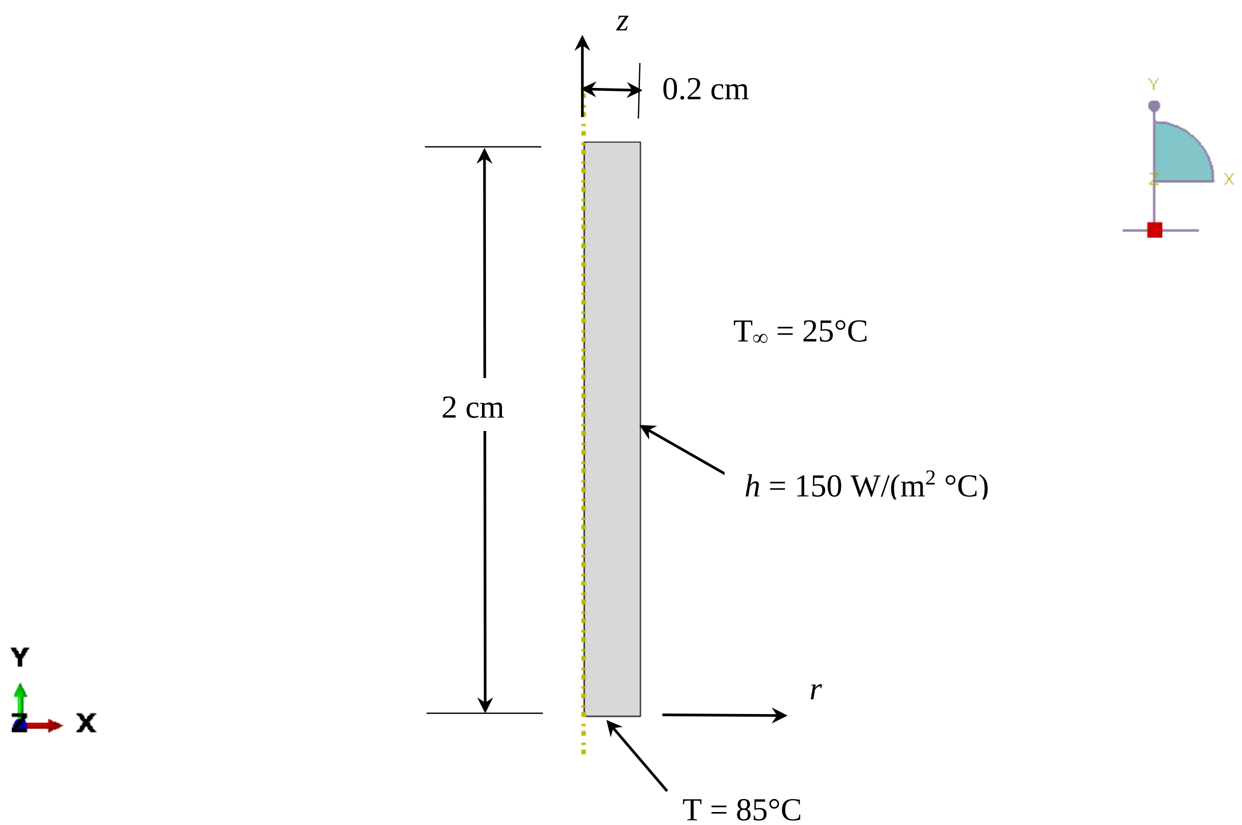

Figure 2. Axisymmetric Model

Appendix 1 – Logan's Example 16-7 solution

Table 16–4 Nodal temperatures at various times for Example 16.7

	Temperature of Node Numbers (°C)		
Time (s)	1	2	3
0.1	85	18.534	26.371
0.2	85	29.732	21.752
0.3	85	36.404	22.662
0.4	85	41.032	25.655
0.5	85	44.665	29.312
0.6	85	47.749	33.059
0.7	85	50.482	36.669
0.8	85	52.956	40.062
0.9	85	55.218	43.218
1.0	85	57.296	46.139
1.1	85	59.208	48.837
1.2	85	60.969	51.327
1.3	85	62.593	53.623
1.4	85	64.089	55.741
1.5	85	65.469	57.693
1.6	85	66.742	59.493
1.7	85	67.915	61.152
1.8	85	68.996	62.683
1.9	85	69.993	64.094
2.0	85	70.912	65.395
2.1	85	71.760	66.594
2.2	85	72.542	67.700
2.3	85	73.262	68.720
2.4	85	73.926	69.660
2.5	85	74.539	70.527
2.6	85	75.104	71.326
2.7	85	75.624	72.063
2.8	85	76.104	72.742
2.9	85	76.547	73.368
3.0	85	76.955	73.946