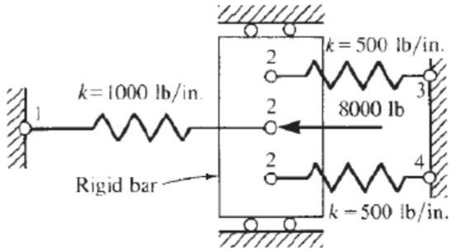
Exam 1 practice problems

2.10



$$[k^{(1)}] = \begin{bmatrix} 1000 & -1000 \\ -1000 & 1000 \end{bmatrix}$$

$$[k^{(2)}] = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix}$$

$$[k^{(3)}] = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix}$$

$$\{F\} = [K]\{d\}$$

$$\begin{Bmatrix} F_{1x} = ? \\ F_{2x} = -8000 \\ F_{3x} = ? \\ F_{4x} = ? \end{Bmatrix} = \begin{bmatrix} 1000 & -1000 & 0 & 0 \\ -1000 & 2000 & -500 & -500 \\ 0 & -500 & 500 & 0 \\ 0 & -500 & 0 & 500 \end{bmatrix} \begin{Bmatrix} u_1 = 0 \\ u_2 = ? \\ u_3 = 0 \\ u_4 = 0 \end{Bmatrix}$$

$$\Rightarrow \qquad u_2 = \frac{-8000}{2000} = -4 \text{ in.}$$

Reactions

$$\begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \\ F_{4x} \end{Bmatrix} = \begin{bmatrix} 1000 & -1000 & 0 & 0 \\ -1000 & 2000 & -500 & -500 \\ 0 & -500 & 500 & 0 \\ 0 & -500 & 0 & 500 \end{bmatrix} \begin{Bmatrix} 0 \\ -4 \\ 0 \\ 0 \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \\ F_{4x} \end{Bmatrix} = \begin{Bmatrix} 4000 \\ -8000 \\ 2000 \\ 2000 \end{Bmatrix} \text{lb}$$

Element (1)

$$\begin{Bmatrix} f_{1x}^{(1)} \\ f_{2x}^{(1)} \end{Bmatrix} = \begin{bmatrix} 1000 & -1000 \\ -1000 & 1000 \end{bmatrix} \begin{Bmatrix} 0 \\ -4 \end{Bmatrix} \Rightarrow \begin{Bmatrix} f_{1x}^{(1)} \\ f_{2x}^{(1)} \end{Bmatrix} = \begin{Bmatrix} 4000 \\ -4000 \end{Bmatrix} \text{lb}$$

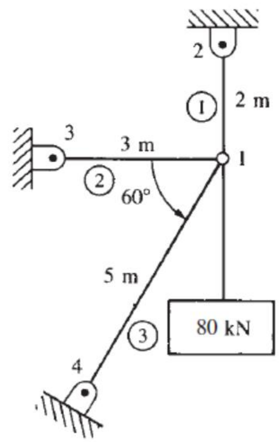
Element (2)

$$\begin{Bmatrix} f_{2x}^{(2)} \\ f_{3x}^{(2)} \end{Bmatrix} = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix} \begin{Bmatrix} -4 \\ 0 \end{Bmatrix} \Rightarrow \begin{Bmatrix} f_{2x}^{(2)} \\ f_{3x}^{(2)} \end{Bmatrix} = \begin{Bmatrix} -2000 \\ 2000 \end{Bmatrix} \text{lb}$$

Element (3)

$$\begin{Bmatrix} f_{2x}^{(3)} \\ f_{4x}^{(3)} \end{Bmatrix} = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix} \begin{Bmatrix} -4 \\ 0 \end{Bmatrix} \Rightarrow \begin{Bmatrix} f_{2x}^{(3)} \\ f_{4x}^{(3)} \end{Bmatrix} \begin{Bmatrix} -2000 \\ 2000 \end{Bmatrix} \text{lb}$$

3.30 For the plane trusses shown in Figures P3–29 and P3–30, determine the horizontal and vertical displacements of node 1 and the stresses in each element. All elements have $E = 210$ GPa and $A = 4.0 \times 10^{-4}$ m^2.



■ Figure P3–30

Element 1–2

$C = 0, S = 1$

$$[k_{1\text{-}2}] = (4 \times 10^{-4})(210 \times 10^{9}) \begin{array}{c} \begin{array}{cc} (1) & (2) \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} \end{array}$$

Element 1–3

$C = -1, S = 0$

$$[k_{1\text{-}3}] = 84 \times 10^{6} \begin{array}{c} \begin{array}{cc} (1) & (3) \end{array} \\ \begin{bmatrix} \frac{1}{3} & 0 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{1}{3} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Element 1–4

$C = -0.5, S = -\dfrac{\sqrt{3}}{2}$

$$[k_{1\text{-}4}] = 84 \times 10^{6} \begin{array}{c} \begin{array}{cc} (1) & (4) \end{array} \\ \begin{bmatrix} 0.05 & 0.0866 & -0.05 & -0.0866 \\ 0.0866 & 0.15 & -0.0866 & -0.15 \\ -0.05 & -0.0866 & 0.05 & 0.0866 \\ -0.0866 & -0.15 & 0.0866 & 0.15 \end{bmatrix} \end{array}$$

3.30 (cont.)

$$u_2 = v_2 = u_3 = v_3 = u_4 = v_4 = 0$$

$$\begin{Bmatrix} F_{1x} = 0 \\ F_{1y} = -80 \end{Bmatrix} = 84 \times 10^6 \begin{bmatrix} 0.3883 & 0.0866 \\ 0.0866 & 0.65 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \end{Bmatrix}$$

$$\Rightarrow \quad u_1 = 3.37 \times 10^{-4}\,\text{m}$$

$$v_1 = -15.1 \times 10^{-4}\,\text{m}$$

Element stresses

$$\sigma^{(1)} = \frac{210 \times 10^9}{2}\,[0 \quad -1 \quad 0 \quad 1] \begin{Bmatrix} 3.37 \times 10^{-4} \\ -15.1 \times 10^{-4} \\ 0 \\ 0 \end{Bmatrix}$$

$$\Rightarrow \sigma^{(1)} = 158.5\text{ MPa (T)}$$

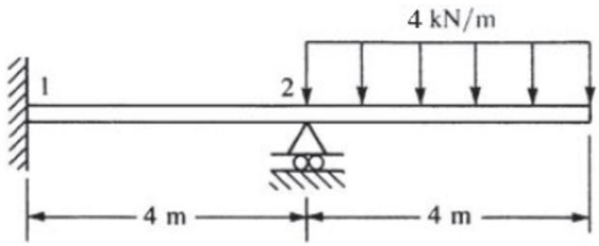
$$\sigma^{(2)} = \frac{210 \times 10^9}{3}\,[1 \quad 0 \quad -1 \quad 0] \cdot \begin{Bmatrix} 3.37 \times 10^{-4} \\ -15.1 \times 10^{-4} \\ 0 \\ 0 \end{Bmatrix}$$

$$\Rightarrow \sigma^{(2)} = 23.59\text{ MPa (T)}$$

$$\sigma^{(3)} = \frac{210 \times 10^9}{5} \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{Bmatrix} 3.37 \times 10^{-4} \\ -15.1 \times 10^{-4} \\ 0 \\ 0 \end{Bmatrix}$$

$$\Rightarrow \sigma^{(3)} = -47.85\text{ MPa (C)}$$

4-21 For the beams shown in Figures P4–21 through P4–26, determine the nodal displacements and slopes, the forces in each element, and the reactions.



$E = 70$ GPa
$I = 3 \times 10^{-4}$ m^4

■ Figure P4–21

$$[K] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 24 & 0 & -12 & 6L \\ 6L & 2L^2 & 0 & 8L^2 & -6L & 2L^2 \\ 0 & 0 & -12 & -6L & 12 & -6L \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

After imposing the boundary conditions and using work equivalence

$v_1 = \phi_1 = v_2 = 0$, we have in $\{F\} = [K]\{d\}$

$$\begin{Bmatrix} \frac{-wL^2}{12} \\ \frac{-wL}{2} \\ \frac{wL^2}{12} \end{Bmatrix} = \frac{EI}{L^3}\begin{bmatrix} 8L^2 & -6L & 2L^2 \\ -6L & 12 & -6L \\ 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{Bmatrix} \phi_2 \\ v_3 \\ \phi_3 \end{Bmatrix} \quad \begin{matrix} (1) \\ (2) \\ (3) \end{matrix}$$

From (1) and (3)

$$\frac{wL^2}{12} = \frac{EI}{L^3}\,[-8L^2\phi_2 + 6Lv_3 - 2L^2\phi_3]$$

$$\frac{wL^2}{12} = \frac{EI}{L^3}\,[2L^2\phi_2 - 6Lv_3 + 4L^2\phi_3]$$

$$\frac{wL^3}{6} = \frac{EI}{L^3}\,[-6L^2\phi_2 + 2L^2\phi_3] \quad (4)$$

From (2) and (3)

$$\frac{-wL^2}{4} = \frac{EI}{L^3}\,[-3L^2\phi_2 + 6Lv_3 - 3L^2\phi_3]$$

$$\frac{wL^2}{12} = \frac{EI}{L^3}\,[2L^2\phi_2 - 6Lv_3 + 4L^2\phi_3]$$

$$\frac{-wL^2}{6} = \frac{EI}{L^3}\,[-L^2\phi_2 + L^2\phi_3] \quad (5)$$

Adding (4) and (5) we have

$$\frac{wL^2}{6} = \frac{EI}{L^3}\ [-6L^2\phi_2 + 2L^2\phi_3]$$

$$\frac{2wL^2}{6} = \frac{EI}{L^3}\ [2L^2\phi_2 - 2L^2\phi_3]$$

$$\frac{wL^2}{2} = \frac{EI}{L^3}\ [-4L^2\phi_2] \ \Rightarrow\ \boxed{\phi_2 = \frac{-wL^3}{8EI} = -1.524\times10^{-3}\,\text{rad}}$$

Substituting in (5) $\Rightarrow \dfrac{-wL^2}{6} = \dfrac{EI}{L^3}\left[-L^2\left(\dfrac{-wL^3}{8EI}\right) + L^2\phi_3\right]$

$$\Rightarrow \boxed{\phi_3 = \frac{-7wL^3}{24EI} = -3.556\times10^{-3}\ \text{rad}}$$

Finally substituting in (1)

$$\Rightarrow \boxed{v_3 = \frac{-wL^4}{4EI} = -0.0122\ \text{m}}$$

Reactions can be found from the global equation

$$\{F\} = [K]\,\{d\} - \{F_0\}$$

$$F_{1y} = \frac{EI}{L^3}\,[6L\phi_2] - 0 = \frac{EI}{L^3}6L\left(\frac{-wL^3}{8EI}\right) = \frac{-3wL}{4} = -12\ \text{kN}$$

$$M_1 = \frac{EI}{L^3}\ [2L^2\phi_2] - 0 = \frac{EI}{L^3}2L^2\left(\frac{-wL^3}{8EI}\right) = \frac{-wL^2}{4} = -16\ \text{kN}\cdot\text{m}$$

$$F_{2y} = \frac{EI}{L^3}\ [-12v_3 + 6L\phi_3] - \left[-\frac{wL}{2}\right] = \frac{7wL}{4} = 28\ \text{kN}$$

and $M_2 = 0$

Numerical solution (Matlab)

```
function problem_4_21
%
clc; clear all; format long
%
% Units:
%   Force - Newton
%   Length - meters
%
E=70e9;
I=3e-4;
L=4;
w=4000;
%
% solve eqs. 1-3
%
F=[-w*L^2/12;-w*L/2;w*L^2/12];
K=(E*I/L^3)*[8*L^2 -6*L 2*L^2;
    -6*L 12 -6*L
    2*L^2 -6*L 4*L^2];
D=K\F;
phi2=D(1)
v3=D(2)
phi3=D(3)
%
% find reaction
%
clear K F
K=k_global(E,I,L);
D=[0; 0; 0; phi2; v3; phi3];
F0=[0; 0; -w*L/2; -w*L^2/12; -w*L/2;
w*L^2/12];
F=K*D-F0
%
%-------------------------------------
%
function K=k_global(E,I,L)
%
k1=zeros(6,6);
k2=zeros(6,6);
k1(1:4,1:4)=k_element(E,I,L);
k2(3:6,3:6)=k_element(E,I,L);
K=k1+k2;
%
%--------------------------------------
%
function k=k_element(E,I,L)
%
k=(E*I/L^3)*[12,6*L,-12,6*L;
    6*L,4*L^2,-6*L,2*L^2;
    -12,-6*L,12,-6*L;
    6*L,2*L^2,-6*L,4*L^2];
```

Command Window Output:

```
phi2 =
  -0.001523809523810
v3 =
  -0.012190476190476
phi3 =
  -0.003555555555556
F =
   1.0e+04 *
  -1.199999999999999
  -1.599999999999999
   2.799999999999999
   0.000000000000001
                   0
   0.000000000000002
```