HW#1 A. 1, 7, 9, 10 ; B. 3

$$[A] = \begin{bmatrix} 4 & 0 \\ 1 & 8 \end{bmatrix}, \quad [B] = \begin{bmatrix} 2 & 0 \\ 2 & 4 \end{bmatrix}, \quad [C] = \begin{bmatrix} 3 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$$

$$[D] = \begin{bmatrix} 5 & 2 & 1 \\ 2 & 10 & 0 \\ 1 & 0 & 5 \end{bmatrix}, \quad [E] = \begin{Bmatrix} 3 \\ 2 \\ 1 \end{Bmatrix}$$

A. 1 a) $[A] + [B] = \begin{bmatrix} 4 & 0 \\ 1 & 8 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 3 & 12 \end{bmatrix}$

b) $\underset{2\times2}{[A]} + \underset{3\times2}{[C]} \Rightarrow$ nonsense

c) $\underset{2\times2}{[A]}\underset{3\times2}{[C]^T} \Rightarrow$ nonsense

d) $\underset{3\times3}{[D]}\underset{3\times1}{[E]} = \begin{bmatrix} 5 & 2 & 1 \\ 2 & 10 & 0 \\ 1 & 0 & 5 \end{bmatrix} \begin{Bmatrix} 3 \\ 2 \\ 1 \end{Bmatrix} = \begin{Bmatrix} 20 \\ 26 \\ 8 \end{Bmatrix}$

e) $\underset{3\times3}{[D]}\underset{2\times3}{[C]} \Rightarrow$ nonsense

f) $\underset{2\times3}{[C]}\underset{3\times3}{[D]} = \begin{bmatrix} 3 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 5 & 2 & 1 \\ 2 & 10 & 0 \\ 1 & 0 & 5 \end{bmatrix}$

$$= \underset{2\times3}{\begin{bmatrix} 19 & 26 & 3 \\ -3 & -2 & 9 \end{bmatrix}}$$

A.7 $\left([A][B]\right)^T = [B]^T[A]^T$

$_{2\times2}\ _{2\times3}\quad _{3\times2}\ _{2\times2}$

$$[A] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad [B] = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}$$

$2\times2 \qquad 2\times3$

$$[A][B] = \begin{bmatrix} (a_{11}b_{11}+a_{12}b_{21}) & (a_{11}b_{12}+a_{12}b_{22}) & (a_{11}b_{13}+a_{12}b_{23}) \\ (a_{21}b_{11}+a_{22}b_{21}) & (a_{21}b_{12}+a_{22}b_{22}) & (a_{21}b_{13}+a_{22}b_{23}) \end{bmatrix}$$

$$\left([A][B]\right)^T = \begin{bmatrix} (a_{11}b_{11}+a_{12}b_{21}) & (a_{21}b_{11}+a_{22}b_{21}) \\ (a_{11}b_{12}+a_{12}b_{22}) & (a_{21}b_{12}+a_{22}b_{22}) \\ (a_{11}b_{13}+a_{12}b_{23}) & (a_{21}b_{13}+a_{22}b_{23}) \end{bmatrix}$$

3×2

$$[B]^T[A]^T = \begin{bmatrix} b_{11} & b_{21} \\ b_{12} & b_{22} \\ b_{13} & b_{23} \end{bmatrix} \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix}$$

$3\times2 \qquad 2\times2$

$$= \begin{bmatrix} (a_{11}b_{11}+a_{12}b_{21}) & (a_{21}b_{11}+a_{22}b_{21}) \\ (a_{11}b_{12}+a_{12}b_{22}) & (a_{21}b_{12}+a_{22}b_{22}) \\ (a_{11}b_{13}+a_{12}b_{23}) & (a_{21}b_{13}+a_{22}b_{23}) \end{bmatrix}$$

3×2

$$\left([A][B]\right)^T = [B]^T[A]^T \quad \checkmark$$

A.9 $[X] = \begin{bmatrix} x & y \\ 1 & x \end{bmatrix}$, $[A] = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$

$$[X]^T[A][X] = \underset{2\times2}{\begin{bmatrix} x & 1 \\ y & x \end{bmatrix}} \underset{2\times2}{\begin{bmatrix} a & b \\ b & c \end{bmatrix}} \underset{2\times2}{\begin{bmatrix} x & y \\ 1 & x \end{bmatrix}}$$

$$= \begin{bmatrix} ax+b & bx+c \\ ay+bx & by+cx \end{bmatrix} \begin{bmatrix} x & y \\ 1 & x \end{bmatrix}$$

$$= \begin{bmatrix} ax^2+bx+bx+c & axy+by+bx^2+cx \\ axy+bx^2+by+cx & ay^2+bxy+bxy+cx^2 \end{bmatrix}$$

$$= \begin{bmatrix} (ax^2+2bx+c) & (axy+by+bx^2+cx) \\ (axy+by+bx^2+cx) & (ay^2+2bxy+cx^2) \end{bmatrix}$$

↑ symmetric ✓

A.10 $$[K] = \int_0^L [B]^T E [B]\, dx$$

$$\underset{1\times2}{[B]} = \begin{bmatrix} -1/L & 1/L \end{bmatrix}$$

$$[K] = E \int_0^L \underset{2\times1}{\begin{bmatrix} -1/L \\ 1/L \end{bmatrix}} \underset{1\times2}{\begin{bmatrix} -1/L & 1/L \end{bmatrix}} dx$$

$$= E \int_0^L \begin{bmatrix} 1/L^2 & -1/L^2 \\ -1/L^2 & 1/L^2 \end{bmatrix} dx$$

$$[K] = E \begin{bmatrix} 1/L^2 & -1/L^2 \\ -1/L^2 & 1/L^2 \end{bmatrix} \int_0^L dx \qquad x: 0 \to L$$

$$[K] = E \begin{bmatrix} 1/L & -1/L \\ -1/L & 1/L \end{bmatrix}$$

$$[K] = \frac{E}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

B.3 Matrix form

$$\begin{bmatrix} 2 & -4 & -5 \\ 0 & 2 & 4 \\ 1 & -1 & 2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 6 \\ -1 \\ 2 \end{Bmatrix}$$

$$\left[\begin{array}{ccc|c} 2 & -4 & -5 & 6 \\ 0 & 2 & 4 & -1 \\ 1 & -1 & 2 & 2 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 2 & -4 & -5 & 6 \\ 0 & 2 & 4 & -1 \\ 0 & 1 & 9/2 & -1 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 2 & -4 & -5 & 6 \\ 0 & 2 & 4 & -1 \\ 0 & 0 & 5/2 & -1/2 \end{array}\right]$$

↖ elimination

Back substitution

$$5/2\, x_3 = -1/2 \quad \Rightarrow \quad x_3 = -1/5$$

$$2x_2 + 4\left(-\tfrac{1}{5}\right) = -1 \Rightarrow x_2 = -\tfrac{1}{10}$$

$$2x_1 - 4x_2 - 5x_3 = 6$$

$$2x_1 + \tfrac{2}{5} + 1 = 6$$

$$x_1 = \tfrac{23}{10}$$

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 23/10 \\ -1/10 \\ -1/5 \end{Bmatrix}$$