MCE 466 – Homework Assignment #6

1) Consider Example 6.1

> Evaluate the stiffness matrix for the element shown in Figure 6–11. The coordinates are shown in units of inches. Assume plane stress conditions. Let $E = 30 \times 10^6$ psi, $\nu = 0.25$, and thickness $t = 1$ in. Assume the element nodal displacements have been determined to be $u_1 = 0.0$, $v_1 = 0.0025$ in., $u_2 = 0.0012$ in., $v_2 = 0.0$, $u_3 = 0.0$, and $v_3 = 0.0025$ in. Determine the element stresses.

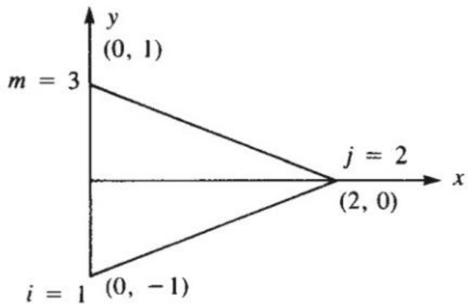

Figure 6–11 Plane stress element for stiffness matrix evaluation

Which can be solved using the following Matlab script:

```
clc; clear all; close all
%
E=30e6;
nu=0.25;
t=1;
plane_stress=true; % Note: use plane_stress=false for plane strain problems
x=[0 2 0];
y=[-1 0 1];
% stiffness matrix
k=k_cst(E,nu,t,x,y,plane_stress)
% stresses
u=[0 .0012 0];
v=[.0025 0 .0025];
sigma=sigma_cst(E,nu,t,x,y,plane_stress,u,v)
```

Write the two Matlab functions, `k_cst` and `sigma_cst`, that are needed for this script. Verify that your functions give the correct solution:

$$[k] = 4.0 \times 10^6 \begin{bmatrix} 2.5 & 1.25 & -2 & -1.5 & -0.5 & 0.25 \\ 1.25 & 4.375 & -1 & -0.75 & -0.25 & -3.625 \\ -2 & -1 & 4 & 0 & -2 & 1 \\ -1.5 & -0.75 & 0 & 1.5 & 1.5 & -0.75 \\ -0.5 & -0.25 & -2 & 1.5 & 2.5 & -1.25 \\ 0.25 & -3.625 & 1 & -0.75 & -1.25 & 4.375 \end{bmatrix} \frac{\text{lb}}{\text{in.}}$$

and

$$\sigma_x = 19{,}200 \text{ psi} \qquad \sigma_y = 4800 \text{ psi} \qquad \tau_{xy} = -15{,}000 \text{ psi}$$

2) Consider Example 6.2:

For a thin plate subjected to the surface traction shown in Figure 6–16, determine the nodal displacements and the element stresses. The plate thickness $t = 1$ in., $E = 30 \times 10^6$ psi, and $\nu = 0.30$.

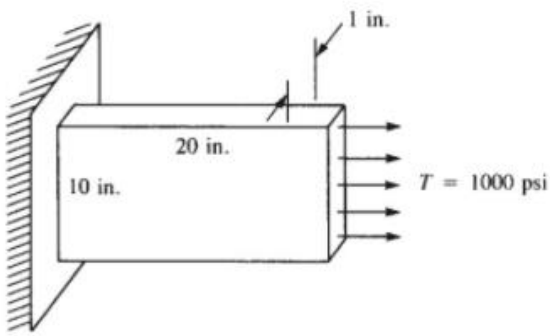

Figure 6–16 Thin plate subjected to tensile stress

Which can be solved using the following Matlab script:

```
function example_6_2
%
clc; clear all; close all
%
E=30e6;
nu=0.3;
t=1;
plane_stress=true; % Note: use plane_stress=false for plane strain problems
for element=1:2
    if element==1
        x=[0 20  0];
        y=[0 10 10];
        k1=k_cst(E,nu,t,x,y,plane_stress);
        % rearrange rows and columns
        k1p=[k1(:,1:2),k1(:,5:6),k1(:,3:4)];
        k1p=[k1p(1:2,:);k1p(5:6,:);k1p(3:4,:)]
    else
        x=[0 20 20];
        y=[0  0 10];
```

```
        k2=k_cst(E,nu,t,x,y,plane_stress);
        % rearrange rows and columns
        k2p=[k2(:,1:2),k2(:,5:6),k2(:,3:4)];
        k2p=[k2p(1:2,:);k2p(5:6,:);k2p(3:4,:)]
    end
end
K=zeros(8,8);
K(1:6,1:6)=K(1:6,1:6)+k1p;
K([1,2,5:8],[1,2,5:8])=K([1,2,5:8],[1,2,5:8])+k2p;
P_known=[5000; 0; 5000; 0];
d_unknown=K(5:8,5:8)\P_known
d=[0;0;0;0;d_unknown]
for element=1:2
    if element==1
        x=[0 20  0];
        y=[0 10 10];
        u=[d(1) d(5) d(3)];
        v=[d(2) d(6) d(4)];
        sigma=sigma_cst(E,nu,t,x,y,plane_stress,u,v)
    else
        x=[0 20 20];
        y=[0  0 10];
        u=[d(1) d(7) d(5)];
        v=[d(2) d(8) d(6)];
        sigma=sigma_cst(E,nu,t,x,y,plane_stress,u,v)
    end
end
```

Using the functions you wrote in problem 1, verify that your code gives the correct solution:

Nodal displacements

$$\begin{Bmatrix} u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix} = \begin{Bmatrix} 609.6 \\ 4.2 \\ 663.7 \\ 104.1 \end{Bmatrix} \times 10^{-6} \text{ in.}$$

Element 1 stresses

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{Bmatrix} 1005 \\ 301 \\ 2.4 \end{Bmatrix} \text{psi}$$

Element 2 stresses

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{Bmatrix} 995 \\ -1.2 \\ -2.4 \end{Bmatrix} \text{psi}$$

3) Solve text problem 8.3

8.3 For the element of Figure 8–3 (shown again as Figure P8–3) subjected to the uniform pressure shown acting over the vertical side, determine the nodal force replacement system using Eq. (6.3.7). Assume an element thickness of t.

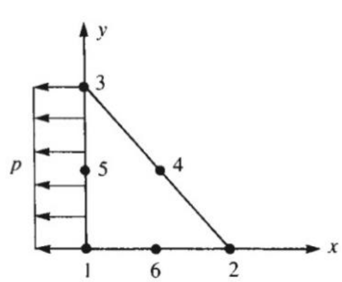

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Figure P8–3