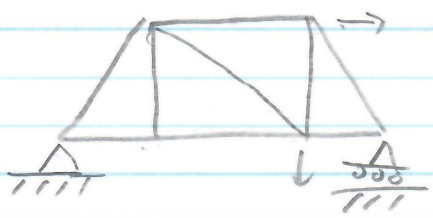
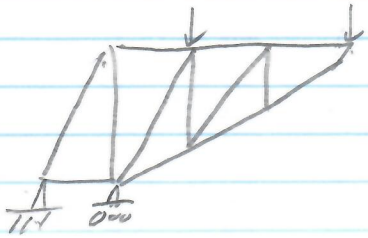
Exam 1 - Solution

Exam Versions

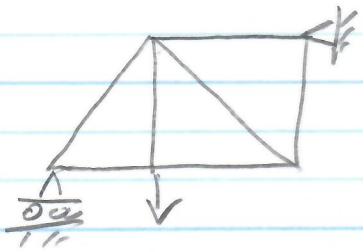
(1) 15



(2)



(3)



1a) $F = f_{2x} = k(u_2 - u_1) = 1000\ \frac{lb}{in}(u_2 - u_1)$

$U = \frac{1}{2} k (u_2 - u_1)^2$

①	②	③
$u_1 = .5$ in	$u_1 = .8$ in	$u_1 = .3$ in
$u_2 = .8$ in	$u_2 = .5$ in	$u_2 = .5$ in
$F = 300$ lb (T)	$F = -300$ lb (C)	$F = 200$ lb (T)
$U = 45$ lb-in	$U = 45$ lb-in	$U = 20$ lb-in

b)

unknowns = 2 × (# nodes) − (# constraints)

①	②	③
9	13	7

c) $u = [N]\{d\} = [N_1\ N_2\ N_3\ N_4] \begin{Bmatrix} u_2 \\ \phi_2 \\ u_3 \\ \phi_3 \end{Bmatrix}^0 = N_2 \phi_2$

where $\phi_2 = .0012$ rad (given)

$N_2 = \frac{1}{L^3}(x^3 L - 2x^2 L^2 + x L^3)$

①	②	③
$x = 60$	$x = 120$	$x = 180$
$L = 240$	$L = 240$	$L = 240$
$v = .0405$ in	$v = .0360$ in	$v = .0135$ in

2a) Element 1 $\theta_1 = 0,\ c_1 = 1,\ s_1 = 0$

$$[k^{(1)}] = \frac{AE_1}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Element 2 $\theta_2 = 270^\circ,\ c_2 = 0,\ s_2 = -1$

$$[k^{(2)}] = \frac{AE_2}{L}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

Assemble & apply B.C. $u_1 = v_1 = u_3 = v_3 = 0$

$$\begin{Bmatrix} F\cos 45^\circ \\ F\sin 45^\circ \end{Bmatrix} = \frac{A}{L}\begin{bmatrix} E_1 & 0 \\ 0 & E_2 \end{bmatrix}\begin{Bmatrix} u_2 \\ v_2 \end{Bmatrix}$$

Solving $u_2 = \frac{\sqrt{2}}{2}\frac{FL}{AE_1}$, $u_3 = \frac{\sqrt{2}}{2}\frac{FL}{AE_2}$

b)

$$\sigma^{(2)} = \frac{E_2}{L}\begin{bmatrix} -c_2 & -s_2 & c_2 & s_2 \end{bmatrix}\begin{Bmatrix} u_2 \\ v_2 \\ 0 \\ 0 \end{Bmatrix}$$

$$= \frac{E_2}{L}(-s_2)\,v_2 = \frac{E_2 v_2}{L}$$

①	②	③
$u_2 = .00212$ in	$u_2 = .00141$ in	$u_2 = .00849$ in
$v_2 = .00707$ in	$v_2 = .00424$ in	$v_2 = .00424$ in
$\sigma^{(2)} = 354$ psi	$\sigma^{(2)} = 707$ psi	$\sigma^{(2)} = 1414$ psi

3 a) one element model

$$\begin{Bmatrix} F_{1y} \\ M_1 \\ F_{2y} \\ M_2 \end{Bmatrix} = \frac{EI}{L^3} \begin{bmatrix} v_1 & \phi_1 & v_2 & \phi_2 \\ 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{Bmatrix} \cancel{\phi_1}\,0 \\ \phi_1 \\ \cancel{v_2}\,0 \\ \cancel{\phi_2}\,0 \end{Bmatrix}$$

4×4

Eq. 2

$$M_1 = \frac{EI}{L^3}(4L^2)\phi_1$$

where $M_1 = -wL^2/12$

$$\phi_1 = -\frac{wL^3}{48EI}$$

b)

$$\{F\} = [K]\{D\} - \{F_0\}$$

Eq. 1

$$F_{1y} = K_{12}\phi_1 - \left(-\frac{wL}{2}\right)$$

$$= \left(\frac{EI}{L^3}\right)(6L)\left(\frac{-wL^3}{48EI}\right) + \frac{wL}{2}$$

$$= \ldots = 3/8\, wL$$

①	②	③
$\phi_1 = -6.25e\text{-}6$ rad	$\phi_1 = -2.08e\text{-}6$ rad	$\phi_1 = -6.94e\text{-}7$ rad
$F_{1y} = 1125$ lb	$F_{1y} = 750$ lb	$F_{1y} = 375$ lb