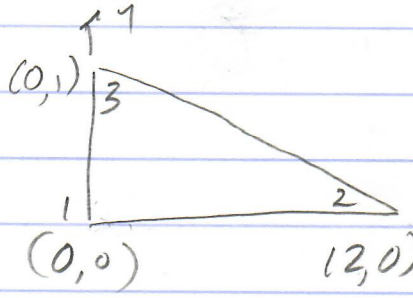
HW 5 6-3c, 6-11b, 6-13

6.3c



$i, j, m \Rightarrow 1, 2, 3$

$t = 1$ in

$A = 1$ in^2

$$[k] = tA\,[B]^T[D][B]$$

$6\times6 \quad 6\times3 \quad 3\times3 \quad 3\times6$

$$[D] = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix}$$

$$[B] = \frac{1}{2A}\begin{bmatrix} \beta_1 & 0 & \beta_2 & 0 & \beta_3 & 0 \\ 0 & \gamma_1 & 0 & \gamma_2 & 0 & \gamma_3 \\ \gamma_1 & \beta_1 & \gamma_2 & \beta_2 & \gamma_3 & \beta_3 \end{bmatrix}$$

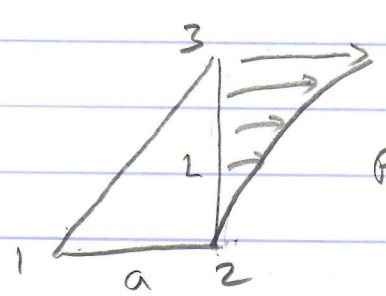
$$\beta_1 = y_2 - y_3 = -1, \quad \gamma_1 = x_3 - x_2 = -2$$
$$\beta_2 = y_3 - y_2 = +1, \quad \gamma_2 = x_1 - x_3 = 0$$
$$\beta_3 = y_1 - y_2 = 0, \quad \gamma_3 = x_2 - x_1 = 2$$

$$2A = \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 0 & 1 \end{vmatrix} = 2$$

$A = 1$ in^2

See Mathab script & output for results

6.11c



$$p(y) = p_0 \frac{y^2}{L^2}$$

$$\{f_s\} = t \int_0^L [N_s]^T \{T_s\} \, dy$$

6×1

$$= t \int_0^L \begin{bmatrix} N_1 & 0 \\ 0 & N_1 \\ N_2 & 0 \\ 0 & N_2 \\ N_3 & 0 \\ 0 & N_3 \end{bmatrix} \begin{Bmatrix} p_0 y^2/L^2 \\ 0 \end{Bmatrix} dy$$

6×2

$$N_1 = \frac{1}{2A}(\alpha_1 + \beta_1 x + \delta_1 y) = 0 \text{ along } 2\text{-}3$$

$$N_2 = \frac{1}{2A}(\alpha_2 + \beta_2 x + \delta_2 y) = \frac{a(L-y)}{2A}$$

$$N_3 = \frac{1}{2A}(\alpha_3 + \beta_3 x + \delta_3 y) = ay/2A$$

$$N_2 = \frac{a(L-y)}{2A} = \frac{(L-y)}{2L} = 1 - y/L$$

$$N_3 = \frac{ay}{aL} = y/L$$

$$\{f_s\} = t \int_0^L \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1-y/L & 0 \\ 0 & 1-y/L \\ y/L & 0 \\ 0 & y/L \end{bmatrix} \begin{Bmatrix} p_0 y^2/L^2 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ f_{s2x} \\ 0 \\ f_{s3x} \\ 0 \end{Bmatrix}$$

6×1

Node 2

$$f_{s2x} = t \int_0^L (1 - y/L)\, p_0 y^2/L^2 \, dy$$

$$= t \int_0^L \frac{p_0 y^2}{L^2} - \frac{p_0 y^3}{L^3} \, dy$$

$$= \frac{p_0 t}{L^3} \int_0^L L y^2 - y^3 \, dy$$

$$= \frac{p_0 t}{L^3} \left[L\left(\frac{L^3}{3}\right) - \frac{L^4}{4} \right]$$

$$= p_0 t L \left(\underset{4/12}{1/3} - \underset{3/12}{1/4} \right)$$

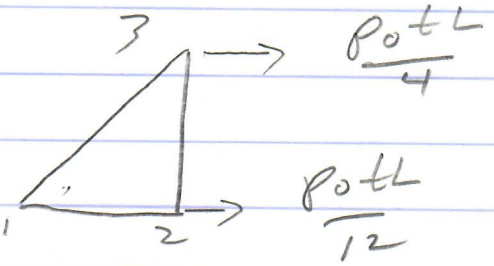
$$\boxed{f_{s2x} = \frac{p_0 t L}{12}}$$

Node 3

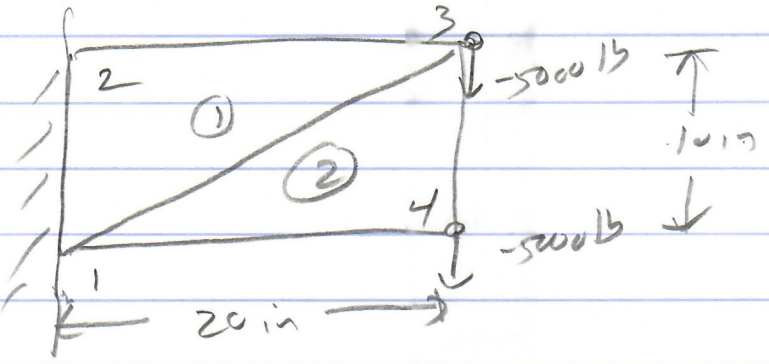
$$f_{s3x} = t \int_0^L \left(\frac{y}{L}\right)\left(p_0 y^2/L^2\right) dy$$

$$= \frac{p_0 t}{L^3} \int_0^L y^3 \, dy \qquad \frac{L^4}{4}$$

$$\boxed{f_{s3x} = \frac{p_0 t L}{4}}$$



6.13



Global eq (refer to ex. 6-2)

$$\begin{Bmatrix} 0 \\ -5000\,\text{lb} \\ 0 \\ -5000\,\text{lb} \end{Bmatrix} = \frac{375{,}000}{0.91} \begin{bmatrix} 48 & 0 & -28 & 14 \\ & 87 & 12 & -80 \\ & \text{sym} & 48 & -26 \\ & & & 87 \end{bmatrix} \begin{Bmatrix} u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix}$$

Element 1

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \underset{3\times3}{[D]}\underset{3\times6}{[B]} \begin{Bmatrix} 0 \\ 0 \\ u_3 \\ v_3 \\ 0 \\ 0 \end{Bmatrix}$$

6×1

Note
$u_3 = d(1)$
$v_3 = d(2)$

Element 2

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [D][B] \begin{Bmatrix} 0 \\ 0 \\ u_4 \\ v_4 \\ u_3 \\ v_3 \end{Bmatrix}$$

Note
$u_4 = d(3)$
$v_4 = d(4)$

See Matlab script & output for results