Homework Assignment #6 – Solution

1)

```
function k=k_cst(E,nu,t,x,y,plane_stress)
%
if plane_stress==true
    D=E/(1-nu^2)*[1 nu 0; nu 1 0; 0 0 (1-nu)/2];
else
    D=E/((1+nu)*(1-2*nu))*[1-nu nu 0; nu 1-nu 0; 0 0 (1-2*nu)/2];
end
%
A=.5*det([1 x(1) y(1); 1 x(2) y(2); 1 x(3) y(3)]);
beta=[y(2)-y(3) y(3)-y(1) y(1)-y(2)];
gamma=[x(3)-x(2) x(1)-x(3) x(2)-x(1)];
B=[ beta(1)     0       beta(2)     0        beta(3)     0;
      0       gamma(1)    0       gamma(2)     0       gamma(3);
    gamma(1) beta(1)  gamma(2)  beta(2)   gamma(3)  beta(3)]/(2*A);
k=t*A*B'*D*B;
```

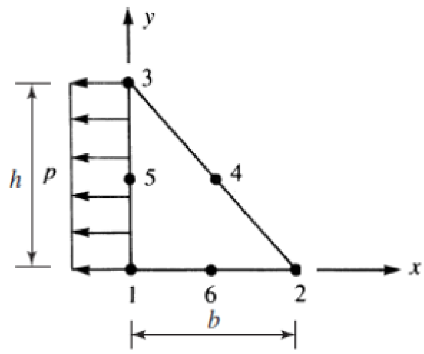
2)

```
function sigma=sigma_cst(E,nu,t,x,y,plane_stress,u,v)
%
if plane_stress==true
    D=E/(1-nu^2)*[1 nu 0; nu 1 0; 0 0 (1-nu)/2];
else
    D=E/((1+nu)*(1-2*nu))*[1-nu nu 0; nu 1-nu 0; 0 0 (1-2*nu)/2];
end
%
A=.5*det([1 x(1) y(1); 1 x(2) y(2); 1 x(3) y(3)]);
beta=[y(2)-y(3) y(3)-y(1) y(1)-y(2)];
gamma=[x(3)-x(2) x(1)-x(3) x(2)-x(1)];
B=[ beta(1)     0       beta(2)     0        beta(3)     0;
    0       gamma(1)    0       gamma(2)     0       gamma(3);
    gamma(1) beta(1)  gamma(2)  beta(2)   gamma(3)  beta(3)]/(2*A);

d=[u(1); v(1); u(2); v(2); u(3); v(3)];
sigma=D*B*d;
```

3)

8.3



The equation is $\{f_s\} = \int_s [N_s]^T \{T\}\, ds$ (1)

$$\{T\} = \begin{Bmatrix} p_x \\ p_y \end{Bmatrix} = \begin{Bmatrix} p \\ 0 \end{Bmatrix} \text{ …. is the surface traction} \quad (2)$$

$$[N] = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0 & N_5 & 0 & N_6 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0 & N_5 & 0 & N_6 \end{bmatrix} \quad (3)$$

Substituting (2) and (3) in Equation (1), we have

$$\{f_s\} = \int_0^t \int_0^h \begin{bmatrix} N_1 & 0 \\ 0 & N_1 \\ N_2 & 0 \\ 0 & N_2 \\ N_3 & 0 \\ 0 & N_3 \\ \vdots & \vdots \\ N_6 & 0 \\ 0 & N_6 \end{bmatrix} \begin{Bmatrix} p \\ 0 \end{Bmatrix} dy\, dz \quad \begin{matrix} \text{at } x = 0 \\ y = y \end{matrix}$$

$$\{f_s\} = t\int_0^h \left\{\begin{matrix} N_1 p \\ 0 \\ N_2 p \\ 0 \\ N_3 p \\ 0 \\ \vdots \\ N_6 p \\ 0 \end{matrix}\right\} dy \quad \begin{matrix} \\ \\ \\ \\ \\ \\ \\ \text{at } x = 0 \\ y = y \end{matrix} \tag{4}$$

From Section 8.2 for this particular element we have

$$N_1 = 1 - \frac{3x}{b} - \frac{3y}{h} + 2x^2 + 4xy + \frac{2y^2}{h^2}$$

$$N_2 = \frac{-x}{b} + \frac{2x^2}{b^2},\ N_3 = \frac{-y}{h} + \frac{2y^2}{h^2}$$

$$N_4 = \frac{4xy}{bh},\ N_5 = \frac{4y}{h} - \frac{4xy}{bh} - \frac{4y^2}{h^2} \tag{5}$$

$$N_6 = \frac{4x}{b} - \frac{4x^2}{b^2} - \frac{4xy}{bh}$$

Substitute (5) into (4) and evaluating N's at $x = 0$, $y = y$, we have

$$\{f_s\} = t\int_0^h \begin{Bmatrix} \left(1-\frac{3y}{h}+\frac{2y^2}{h^2}\right)p \\ 0 \\ 0 \\ 0 \\ \left(\frac{-y}{h}+\frac{2y^2}{h^2}\right)p \\ 0 \\ 0 \\ 0 \\ \left(\frac{4y}{h}-\frac{4y^2}{h^2}\right)p \\ 0 \\ 0 \\ 0 \end{Bmatrix} dy$$

$$f_{s1x} = pt\left(y-\frac{3y^2}{2h}+\frac{2y^3}{3h^2}\right)\Bigg|_0^h = \frac{pth}{6}$$

$$f_{s3x} = \left(\frac{-y^2}{2h}+\frac{2y^3}{3h^2}\right)pt = \frac{pth}{6}$$

$$f_{s5x} = pt\left(\frac{4y^2}{2h}-\frac{4y^3}{3h^2}\right)\Bigg|_0^h = \frac{2\,pth}{3}$$