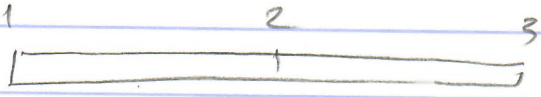
Hw #4

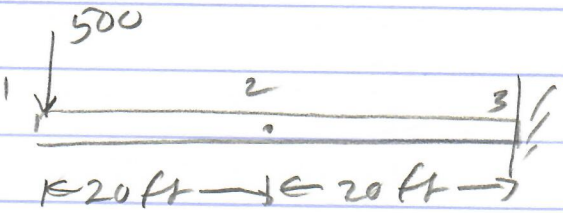
Problems 1-3 – Stiffness Matrices



$$[k] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ & 4L^2 & -6L & 2L^2 \\ & \text{Sym} & 12 & -6L \\ & & & 4L^2 \end{bmatrix}$$

$$[K] = \frac{EI}{L^3}\begin{bmatrix} \begin{bmatrix} 12 & 6L & -12 & 6L \\ & 4L^2 & -6L & 2L^2 \end{bmatrix} & \\ & \begin{bmatrix} 24 & 0 & -12 & 6L \\ & 8L^2 & -6L & 2L^2 \end{bmatrix} \\ & \begin{bmatrix} \text{Sym} & & 12 & -6L \\ & & & 4L^2 \end{bmatrix} \end{bmatrix}$$

4.7



$E = 30e6$ psi

$I = 200$ in^4

Apply loads & B.C. $V_2 = V_3 = \phi_3 = 0$

$$\begin{Bmatrix} F_{1y} \\ M_1 \\ F_{2y} \\ M_2 \\ F_{3y} \\ M_3 \end{Bmatrix} = \frac{EI}{L^3}\begin{bmatrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix}\begin{Bmatrix} V_1 \\ \phi_1 \\ V_2 \\ \phi_2 \\ V_3 \\ \phi_3 \end{Bmatrix}$$

$F_{1y} = -500$, $M_1 = 0$, $F_{2y} = 0$, $M_2 = 0$, $V_2 = 0$, $V_3 = 0$, $\phi_3 = 0$

6 x 6

Solve for $v_1 = -6.72\,e{-1}$ in

$\phi_1 = 3.6\,e{-3}$ rad

$\phi_2 = 1.2\,e{-3}$ rad

Reactions:

$F_{1y} = -500$ lb

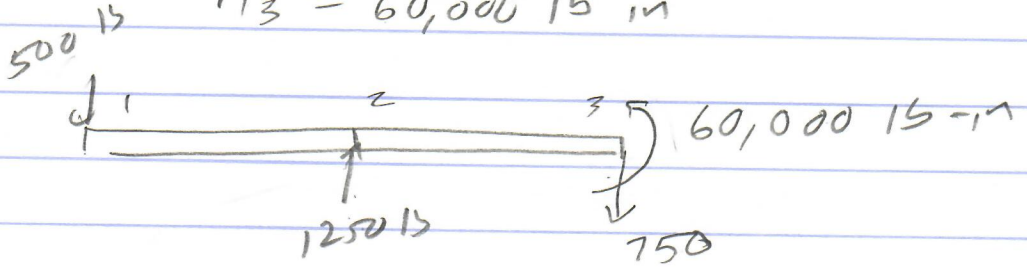
$M_1 = 0$ lb-in

$F_{2y} = 1,250$ lb

$M_2 = 0$

$F_{3y} = -750$ lb

$M_3 = 60,000$ lb in



element 1

$$\begin{Bmatrix} F_{1y}^{(1)} \\ M_1^{(1)} \\ F_{2y}^{(1)} \\ M_2^{(1)} \end{Bmatrix} = \underbrace{\begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}}_{4\times 4} \begin{Bmatrix} v_1 \\ \phi_1 \\ v_2 \\ \phi_2 \end{Bmatrix}$$

$f_{1y}^{(1)} = -500$ lb

$m_1^{(1)} = 0$ lb-in

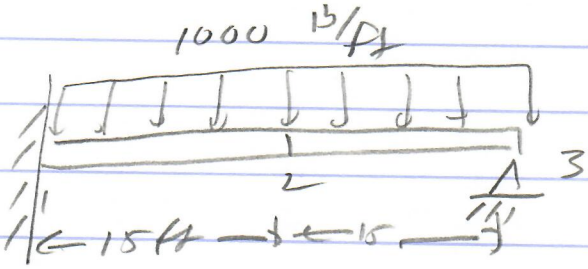
$f_{2y}^{(1)} = 500$ lb

$m_2^{(1)} = -120,000$ lb-in

element 2

$$\begin{Bmatrix} f_{z1}^{(2)} \\ m_2^{(2)} \\ f_{z2}^{(2)} \\ m_3^{(2)} \end{Bmatrix} = [K] \begin{Bmatrix} v_2 \\ \phi_2 \\ v_3 \\ \phi_3 \end{Bmatrix} = \begin{Bmatrix} 750 \text{ lb} \\ 120{,}000 \text{ lb-in} \\ -750 \text{ lb} \\ 60{,}000 \text{ lb} \end{Bmatrix}$$

4-23

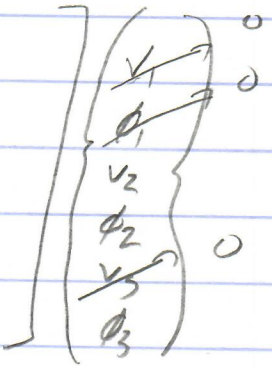


$E = 29\text{e}6$ psi

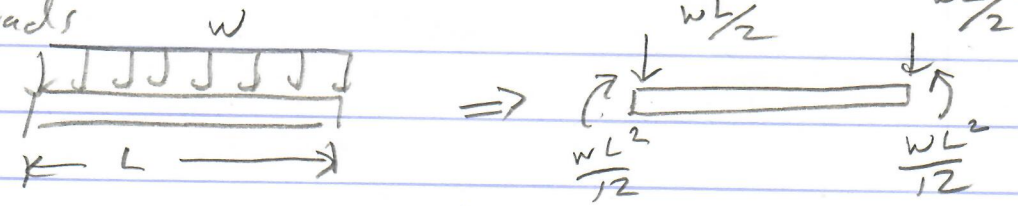
$I = 200 \text{ in}^4$

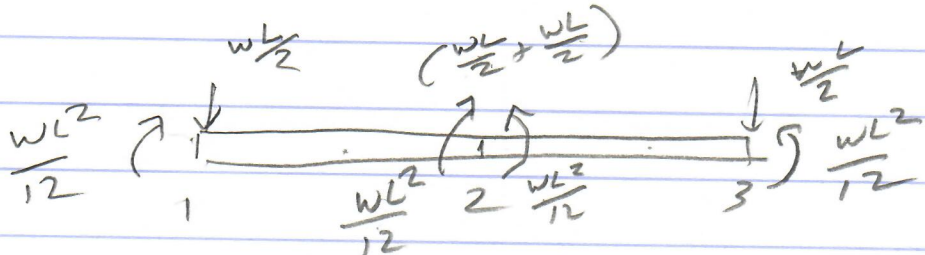
$L = 15$ ft

$$[K]_{6\times6} = \frac{EI}{L^3}$$



Dist. loads





$$\begin{Bmatrix} F_{z1} \\ M_2 \\ M_3 \end{Bmatrix} = \begin{Bmatrix} -wL \\ 0 \\ wL^2/12 \end{Bmatrix}$$

$$\begin{Bmatrix} -wL \\ 0 \\ wL^2/12 \end{Bmatrix} = \underset{3\times3}{[K_p]} \begin{Bmatrix} v_2 \\ \phi_2 \\ \phi_3 \end{Bmatrix}$$

Solving for $v_2 = -1.257$ in

$\phi_2 = -.003491$ rad

$\phi_3 = .01397$ rad

Reactions

$$\begin{Bmatrix} F_{1y} \\ M_1 \\ F_{2y} \\ M_2 \\ F_{3y} \\ M_3 \end{Bmatrix} = \underset{6\times6}{[K]} \begin{Bmatrix} 0 \\ 0 \\ -1.257 \text{ in} \\ -.003491 \\ 0 \\ 0.01397 \end{Bmatrix} - \begin{Bmatrix} -wL/2 \\ -wL^2/12 \\ -wL \\ 0 \\ -wL/2 \\ wL^2/12 \end{Bmatrix}$$

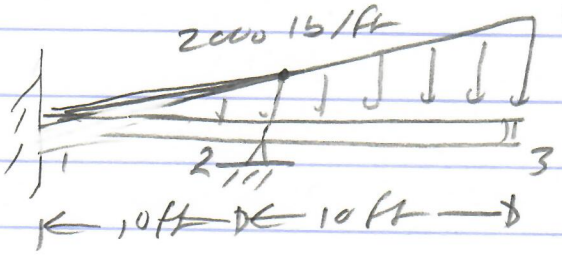
$$= \begin{Bmatrix} 18{,}750 \text{ lb} \\ 1.35e6 \text{ lb-in} \\ 0 \\ 0 \\ 11{,}250 \text{ lb} \\ 0 \end{Bmatrix}$$

element load

$$\begin{Bmatrix} f_{1y}^{(1)} \\ m_1^{(1)} \\ f_{2y}^{(1)} \\ m_2^{(1)} \end{Bmatrix} = \underset{4\times4}{[k]} \begin{Bmatrix} v_1 \\ \phi_1 \\ v_2 \\ \phi_2 \end{Bmatrix} - \begin{Bmatrix} -wL/2 \\ -wL^2/12 \\ -wL/2 \\ wL^2/12 \end{Bmatrix} = \begin{Bmatrix} 18{,}750 \text{ lb} \\ 1.35e6 \text{ lb-in} \\ -3{,}750 \text{ lb} \\ 6.75e5 \text{ lb-in} \end{Bmatrix}$$

$$\begin{Bmatrix} f_{2y}^{(2)} \\ m_2^{(2)} \\ f_{3y}^{(2)} \\ m_3^{(2)} \end{Bmatrix} = \underset{4\times4}{[k]} \begin{Bmatrix} v_2 \\ \phi_2 \\ v_3 \\ \phi_3 \end{Bmatrix} - \begin{Bmatrix} -wL/2 \\ -wL^2/12 \\ -wL/2 \\ wL^2/12 \end{Bmatrix} = \begin{Bmatrix} 3{,}750 \text{ lb} \\ -6.75e5 \text{ lb-in} \\ 11{,}250 \text{ lb} \\ 0 \end{Bmatrix}$$

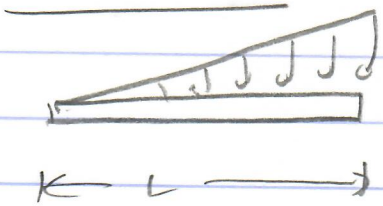
4-21

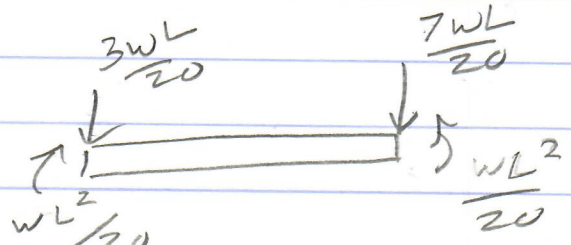


$E = 29e6 \text{ psi}$

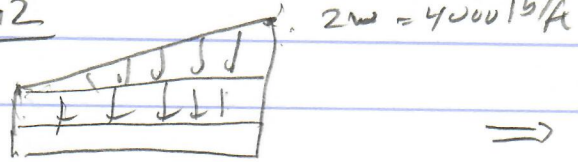
$I = 150 \text{ in}^4$

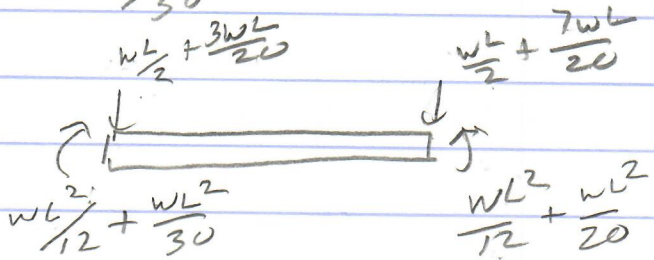
element 1

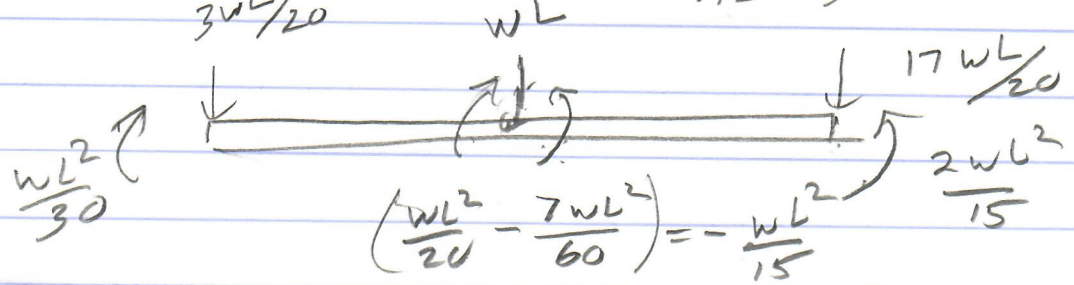




elem2







$$\left(\frac{wL^2}{20} - \frac{7wL^2}{60}\right) = -\frac{wL^2}{15}$$

$$\begin{Bmatrix} F_{1y}^{0} \\ M_1^{0} \\ F_{2y}^{0} \\ M_2^{0} \\ F_{3y}^{0} \\ M_3^{0} \end{Bmatrix} = \begin{Bmatrix} -3wL/20 \\ -wL^2/30 \\ -wL \\ -wL^2/15 \\ -17wL/20 \\ 2wL^2/15 \end{Bmatrix}$$

$\uparrow \{F_0\}$

$$\begin{Bmatrix} F_{1y} \\ M_1 \\ F_{2y} \\ M_2 \\ F_{3y} \\ M_3 \end{Bmatrix} = \begin{bmatrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix} \begin{Bmatrix} v_1 \to 0 \\ \phi_1 \to 0 \\ v_2 \to 0 \\ \phi_2 \\ v_3 \\ \phi_3 \end{Bmatrix}$$

Solve for ϕ_2, v_3, ϕ_3

$$\phi_2 = -.01297 \text{ rad}$$

$$v_3 = -3.277 \text{ in}$$

$$\phi_3 = -.03228 \text{ rad}$$

Reactions,

$$\begin{Bmatrix} F_{1y} \\ M_1 \\ F_{2y} \\ M_2 \\ F_{3y} \\ M_3 \end{Bmatrix} = \begin{Bmatrix} -20,500 & \text{lb} \\ -8.6e5 & \text{lb-in} \\ 60,500 & \text{lb} \\ 0 & \\ 0 & \\ 0 & \end{Bmatrix}$$

$$\begin{Bmatrix} f_{1y}^{(1)} \\ m_1^{(1)} \\ f_{2y}^{(1)} \\ m_2^{(1)} \end{Bmatrix} = \begin{Bmatrix} -20{,}000 \text{ lb} \\ -8.6\text{e}5 \text{ lb-in} \\ 30{,}500 \text{ lb} \\ -2\text{e}6 \text{ lb-in} \end{Bmatrix}$$

$$\begin{Bmatrix} f_{2y}^{(2)} \\ m_2^{(2)} \\ f_{3y}^{(2)} \\ m_3^{(2)} \end{Bmatrix} = \begin{Bmatrix} 30{,}000 \text{ lb} \\ 2.0\text{e}6 \text{ lb-in} \\ 0 \\ 0 \end{Bmatrix}$$

4. a)

$$v(x) = \frac{-w_0 \ell^4}{EI}\left\{\frac{1}{16\ell^2}x^2 - \frac{5}{48\ell^3}x^3 + \frac{1}{24\ell^4}x^4\right\}$$

$$\frac{dv}{dx} = \frac{-w_0 \ell^4}{EI}\left\{\frac{1}{8\ell^2}x - \frac{5}{16\ell^3}x^2 + \frac{1}{6\ell^4}x^3\right\}$$

$$\frac{d^2v}{dx^2} = \frac{-w_0 \ell^4}{EI}\left\{\frac{1}{8\ell^2} - \frac{5}{8\ell^3}x + \frac{1}{2\ell^4}x^2\right\}$$

$$\frac{d^3v}{dx^3} = \frac{-w_0 \ell^4}{EI}\left\{-\frac{5}{8\ell^3} + \frac{1}{\ell^4}x\right\}$$

$$\frac{d^4v}{dx^4} = \frac{-w_0 \ell^4}{EI}\left(\frac{1}{\ell^4}\right)$$

$$\boxed{w_0 = -EI\frac{d^4v}{dx^4}}$$

b) finite element solution for each element

$$v(x) = \underset{1\times 4}{[N]} \begin{Bmatrix} v_1 \\ \phi_1 \\ v_2 \\ \phi_2 \end{Bmatrix}$$

↖ see problem 2

↖ cubic polynomial

```
function hw4
clear; clc; close all; format compact; format short e
%
Problem=input('Enter desired problem (1-4): ');
%
% ---------------------------------------------------------------
%
if Problem==1
    disp('Problem 1')
    E=30e6;
    I=200;
    L=240; % length in inches
    K=k_global(E,I,L)
    % Nodal displacements and rotations:
    F=[-500;0;0]
    Kp=K([1,2,4],[1,2,4])
    d=Kp\F;
    v1=d(1)
    phi1=d(2)
    phi2=d(3)
    % Reactions
    Reactions=K*[v1;phi1;0;phi2;0;0]
    % Element loads
    k=k_element(E,I,L);
    f1=k*[v1;phi1;0;phi2]
    f2=k*[0;phi2;0;0]
    %
    % ---------------------------------------------------------------
    %
elseif Problem==2
    disp('Problem 2')
    E=29e6;
    I=200;
    L=15*12; % length in inches
    K=k_global(E,I,L)
    % Nodal displacements and rotations:
    w=1000/12;  % lb/in
    F=[-w*L;0;w*L^2/12]
    Kp=K([3,4,6],[3,4,6])
    d=Kp\F;
    v2=d(1)
    phi2=d(2)
    phi3=d(3)
    % Reactions
    Reactions=K*[0;0;v2;phi2;0;phi3]-[-w*L/2;-w*L^2/12;-w*L;0;-
w*L/2;w*L^2/12]
    % Element loads
    k=k_element(E,I,L);
    f1=k*[0;0;v2;phi2]-[-w*L/2;-w*L^2/12;-w*L/2;w*L^2/12]
    f2=k*[v2;phi2;0;phi3]-[-w*L/2;-w*L^2/12;-w*L/2;w*L^2/12]
    %
    % ---------------------------------------------------------------
    %
elseif Problem==3
    disp('Problem 3')
    E=29e6;
    I=150;
    L=120; % length in inches
    K=k_global(E,I,L)
    % Nodal displacements and rotations:
    w=2000/12; % lb/in
```

```
    % work equivalent nodal loads
    F1y=-3*w*L/20;
    M1=-w*L^2/30;
    F2y=-w*L;
    M2=-w*L^2/15;
    F3y=-17*w*L/20;
    M3=2*w*L^2/15;
    F=[M2;F3y;M3]
    Kp=K([4:6],[4:6])
    d=Kp\F;
    phi2=d(1)
    v3=d(2)
    phi3=d(3)
    % Reactions
    Reactions=K*[0;0;0;phi2;v3;phi3]-[F1y; M1; F2y; M2; F3y; M3]
    % Element loads
    k=k_element(E,I,L);
    f1=k*[0;0;0;phi2]-[-3*w*L/20;-w*L^2/30;-7*w*L/20;w*L^2/20]
    f2=k*[0;phi2;v3;phi3]-[-13*w*L/20;-7*w*L^2/60;-17*w*L/20;2*w*L^2/15]
    %
    % ---------------------------------------------------------------
    %
else
    disp('Problem 4, part b')
    %
    E=29e6;
    I=200;
    L=15*12;  % length in inches
    % FEA solution
    % element 1
    x=linspace(0,L,101);
    d=[0;0;-1.2569;-3.4914e-03];
    for i=1:101
        xp(i)=x(i);
        N=interp(x(i),L);
        v(i)=N*d;
    end
    plot(xp,v,'b')
    hold on
    % element 2
    d=[-1.2569;-3.4914e-03; 0; .013966];
    for i=1:101
        xp(i)=x(i)+L;
        N=interp(x(i),L);
        v(i)=N*d;
    end
    plot(xp,v,'g')
    % exact
    l=30*12; % length in inches
    w0=1000/12;
    xe=linspace(0,l,101);
    ve=-((w0*l^4)/(E*I))*((1/16)*(xe/l).^2-
(5/48)*(xe/l).^3+(1/24)*(xe/l).^4);
    plot(xe,ve,'r')
    legend('element 1','element 2','exact')
    xlabel('x')
    ylabel('v(x)')
    title('Problem 4')
end
%
%-----------------------------------------------------------------
```

```
%
function K=k_global(E,I,L)
%
k1=zeros(6,6);
k2=zeros(6,6);
k1(1:4,1:4)=k_element(E,I,L);
k2(3:6,3:6)=k_element(E,I,L);
K=k1+k2;
%
%-----------------------------------------------------------------
%
function k=k_element(E,I,L)
%
k=(E*I/L^3)*[12,6*L,-12,6*L;
    6*L,4*L^2,-6*L,2*L^2;
    -12,-6*L,12,-6*L;
    6*L,2*L^2,-6*L,4*L^2];
%
%-----------------------------------------------------------------
%
function N=interp(x,L)
%
N1=(2*x.^3-3*x.^2*L+L^3)/L^3;
N2=(x.^3*L-2*x.^2*L^2+x*L^3)/L^3;
N3=(-2*x.^3+3*x.^2*L)/L^3;
N4=(x.^3*L-x.^2*L^2)/L^3;
N=[N1, N2, N3, N4];
```