Homework #1 Solution

Text Problems A.1, 7, 9, 10; B.3

A.1

(a) $[A] + [B] = \begin{bmatrix} 4 & 0 \\ 1 & 8 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 3 & 12 \end{bmatrix}$

(b) $[A] + [C]$, Nonsense, $[A]$ and $[C]$ not same order

(c) $[A]\,[C]^T$, Nonsense, columns $[A] \neq$ rows $[C]^T$

(d) $[D]\,[E] = \begin{bmatrix} 5 & 2 & 1 \\ 2 & 10 & 0 \\ 1 & 0 & 5 \end{bmatrix} \begin{Bmatrix} 3 \\ 2 \\ 1 \end{Bmatrix}$

$$= \begin{Bmatrix} 5(3) + 2(2) + 1(1) \\ 2(3) + 10(2) + 0(1) \\ 1(3) + 0(2) + 5(1) \end{Bmatrix} = \begin{Bmatrix} 20 \\ 26 \\ 8 \end{Bmatrix}$$

(e) $[D]\,[C]$, Nonsense, columns $[D] \neq$ rows $[C]$

(f) $[C]\,[D] = \begin{bmatrix} 3 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 5 & 2 & 1 \\ 2 & 10 & 0 \\ 1 & 0 & 5 \end{bmatrix}$

$$= \begin{bmatrix} 3(5) + (2)(2) + 0 & 6 + 20 + 0 & 3 + 0 + 0 \\ -5 + 0 + 2 & -2 + 0 + 0 & -1 + 0 + 10 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & 26 & 3 \\ -3 & -2 & 9 \end{bmatrix}$$

A.7 Show that $([A]\,[B])^T = [B]^T\,[A]^T$ by using

$$[A] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad [B] = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}$$

$$([A]\ [B]) = \begin{bmatrix} a_{11}(b_{11}) + a_{12}(b_{21}) & a_{11}(b_{12}) + a_{12}(b_{22}) & a_{11}(b_{13}) + a_{12}(b_{23}) \\ a_{21}(b_{11}) + a_{22}(b_{21}) & a_{21}(b_{12}) + a_{22}(b_{22}) & a_{21}(b_{13}) + a_{22}(b_{23}) \end{bmatrix}$$

$$([A]\ [B])^T = \begin{bmatrix} a_{11}(b_{11}) + a_{12}(b_{21}) & a_{21}(b_{11}) + a_{22}(b_{21}) \\ a_{11}(b_{12}) + a_{12}(b_{22}) & a_{21}(b_{12}) + a_{22}(b_{22}) \\ a_{11}(b_{13}) + a_{12}(b_{23}) & a_{21}(b_{13}) + a_{22}(b_{23}) \end{bmatrix}$$

$$[B]^T = \begin{bmatrix} b_{11} & b_{21} \\ b_{12} & b_{22} \\ b_{13} & b_{23} \end{bmatrix} \qquad [A]^T = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix}$$

$$[B]^T\ [A]^T = \begin{bmatrix} b_{11}(a_{11}) + b_{21}(a_{12}) & b_{11}(a_{21}) + b_{21}(a_{22}) \\ b_{12}(a_{11}) + b_{22}(a_{12}) & b_{12}(a_{21}) + b_{22}(a_{22}) \\ b_{13}(a_{11}) + b_{23}(a_{12}) & b_{13}(a_{21}) + b_{23}(a_{22}) \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}(b_{11}) + a_{12}(b_{21}) & a_{21}(b_{11}) + a_{22}(b_{21}) \\ a_{11}(b_{12}) + a_{12}(b_{22}) & a_{21}(b_{12}) + a_{22}(b_{22}) \\ a_{11}(b_{13}) + a_{12}(b_{23}) & a_{21}(b_{13}) + a_{22}(b_{23}) \end{bmatrix}$$

Answer: $([A]\ [B])^T = [B]^T\ [A]^T$

A.9 Show $\{X\}^T [A] \{X\}$ is symmetric. Given

$$\{X\} = \begin{bmatrix} x & y \\ 1 & x \end{bmatrix},\ [A] = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$$

$$\{X\}^T = \begin{bmatrix} x & 1 \\ y & x \end{bmatrix}$$

$$\{X\}^T [A]\ \{X\} = \begin{bmatrix} x & 1 \\ y & x \end{bmatrix} \begin{bmatrix} a & b \\ b & c \end{bmatrix} \begin{bmatrix} x & y \\ 1 & x \end{bmatrix}$$

$$= \begin{bmatrix} ax + b & bx + c \\ ay + bx & by + cx \end{bmatrix} \begin{bmatrix} x & y \\ 1 & x \end{bmatrix}$$

$$= \begin{bmatrix} ax^2 + bx + bx + c & axy + by + bx^2 + cx \\ axy + bx^2 + by + cx & ay^2 + bxy + bxy + cx^2 \end{bmatrix}$$

as the 1–2 term = 2–1 term $\{X\}^T [A] \{X\}$ is symmetric.

A.10 Evaluate $[K] = \int_0^L [B]^T \ E\,[B]\,dx$, $[B] = \begin{bmatrix} -\frac{1}{L} & \frac{1}{L} \end{bmatrix}$

$$[K] = \int_0^L \begin{Bmatrix} -\frac{1}{L} \\ \frac{1}{L} \end{Bmatrix} E \begin{bmatrix} -\frac{1}{L} & \frac{1}{L} \end{bmatrix} dx$$

$$[K] = \int_0^L \begin{bmatrix} \frac{1}{L^2} & \frac{-1}{L^2} \\ \frac{-1}{L^2} & \frac{1}{L^2} \end{bmatrix} E\,dx$$

$$[K] = E \begin{bmatrix} \frac{1}{L} & \frac{1}{L} \\ \frac{-1}{L} & \frac{1}{L} \end{bmatrix} = \frac{E}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

(Should multiply by A to get actual $[K]$ for a bar)

B.3 Given: $2x_1 - 4x_2 - 5x_3 = 6$

$$2x_2 + 4x_3 = -1 \Rightarrow \qquad \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} \begin{bmatrix} 2 & -4 & -5 \\ 0 & 2 & 4 \\ 1 & -1 & 2 \end{bmatrix} = \begin{Bmatrix} 6 \\ -1 \\ 2 \end{Bmatrix}$$

$$1x_1 - 1x_2 + 2x_3 = 2$$

Find: Solve the system of eq. by Gaussian elimination.

Solution:

Multiply R_1 by -1/2 and add to R_3

$$\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array}\begin{bmatrix} 2 & -4 & -5 \\ 0 & 2 & 4 \\ 0 & 1 & 9/2 \end{bmatrix}\begin{Bmatrix} 6 \\ -1 \\ -1 \end{Bmatrix}$$

Multiply R_2 by -1/2 and add to R_3

$$\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array}\begin{bmatrix} 2 & -4 & -5 \\ 0 & 2 & 4 \\ 0 & 0 & 5/2 \end{bmatrix}\begin{Bmatrix} 6 \\ -1 \\ -1/2 \end{Bmatrix}$$

$$x_3 = (-1/2)(2/5) = -1/5$$

$$x_2 = 1/2(-4(-1/5) - 1) = -1/10$$

$$x_1 = 1/2(4(-1/10) + 5(-1/5) + 6) = 23/10$$

$$x_1 = 23/10$$

$$x_2 = -1/10$$

$$x_3 = -1/5$$