Exam 2 - Solution

Versions

(1) b) For this element, if the nodal displacements are found to be $u_1 = v_1 = u_2 = v_2 = 0$ mm and $u_3 = v_3 = 0.01$ mm, determine the strains $\varepsilon_x, \varepsilon_y$ and γ_{xy} at the centroid of the element.

(2) b) For this element, if the nodal displacements are found to be $u_1 = v_1 = v_2 = u_3 = v_3 = 0$ mm and $u_2 = 0.02$ mm, determine the strains $\varepsilon_x, \varepsilon_y$ and γ_{xy} at the centroid of the element.

(3) b) For this element, if the nodal displacements are found to be $u_1 = v_1 = u_2 = u_3 = v_3 = 0$ mm and $v_2 = 0.04$ mm, determine the strains $\varepsilon_x, \varepsilon_y$ and γ_{xy} at the centroid of the element.

1a)

	Number of rows	Number of columns
[D]	3	3
[B]	3	6
{σ}	3	1

b)

$$\{\epsilon\} = [B]\{d\} = \frac{1}{2A}\begin{bmatrix} \beta_1 & 0 & \beta_2 & 0 & \beta_3 & 0 \\ 0 & \gamma_1 & 0 & \gamma_2 & 0 & \gamma_3 \\ \gamma_1 & \beta_1 & \gamma_2 & \beta_2 & \gamma_3 & \beta_3 \end{bmatrix}\begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix}$$

(1) $\beta_3 = y_1 - y_2 = 0$, $\gamma_3 = x_2 - x_1 = 2$, $2A = 4$

$$\{\epsilon\} = \frac{1}{2A}\begin{Bmatrix} \beta_3 u_3 \\ \gamma_3 v_3 \\ \gamma_3 u_3 + \beta_3 v_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ .005 \\ .005 \end{Bmatrix}$$

(2) $\beta_2 = y_3 - y_1 = 2$, $\gamma_2 = x_1 - x_3 = -1$, $2A = 4$

$$\{\epsilon\} = \frac{1}{2A}\begin{Bmatrix} \beta_2 u_2 \\ 0 \\ \gamma_2 u_2 \end{Bmatrix} = \begin{Bmatrix} .01 \\ 0 \\ -.005 \end{Bmatrix}$$

(3) $\beta_2 = y_3 - y_1 = 2$, $\gamma_2 = x_1 - x_3 = -1$, $2A = 4$

$$\{\epsilon\} = \frac{1}{2A}\begin{Bmatrix} 0 \\ \gamma_2 v_2 \\ \beta_2 v_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -.01 \\ .02 \end{Bmatrix}$$

c)

	Number of rows	Number of columns
[D]	3	3
[B]	3	12
{σ}	3	1

2. Along $x = b$, $N_1 = 0$, $N_2 = 1 - y/h$, $N_3 = y/h$

$$\{f_s\} = t \int_0^h \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1-y/h & 0 \\ 0 & 1-y/h \\ y/h & 0 \\ 0 & y/h \end{bmatrix} \begin{Bmatrix} p_0 y/h \\ 0 \end{Bmatrix} dy = t \int_0^h \begin{Bmatrix} 0 \\ 0 \\ (1-y/h)(p_0 y/h) \\ 0 \\ p_0 y^2/h^2 \\ 0 \end{Bmatrix} dy = \begin{Bmatrix} 0 \\ 0 \\ p_0 t h/6 \\ 0 \\ p_0 t h/3 \\ 0 \end{Bmatrix}$$

(1) $h = 2$ in, $t = 0.1$ in, $p_0 = 2000$ psi

$f_{2x} = 66.7$ lb

$f_{3x} = 133.3$ lb

(2) $h = 3$ in, $t = 0.1$ in, $p_0 = 2000$ psi

$f_{2x} = 100$ lb

$f_{3x} = 200$ lb

(3) $h = 4$ in, $t = 0.1$ in, $p_0 = 2000$ psi

$f_{2x} = 133.3$ lb

$f_{3x} = 267.7$ lb

3. 1x1 $\quad I \approx 4 f(0,0)$

2x2 $\quad I \approx f(-1/\sqrt{3}, -1/\sqrt{3}) + f(1/\sqrt{3}, -1/\sqrt{3}) + f(-1/\sqrt{3}, 1/\sqrt{3}) + f(1/\sqrt{3}, 1/\sqrt{3})$

(1) 1x1 $\quad I \approx 12$

2x2 $\quad I \approx 11.17$ $\quad$ (exact = 11.05)

(2) 1x1 $\quad I \approx 8$

2x2 $\quad I \approx 7.82$ $\quad$ (exact = 7.853)

(3) 1x1 $\quad I \approx 16$

2x2 $\quad I \approx 13.41$ $\quad$ (exact = 13.46)