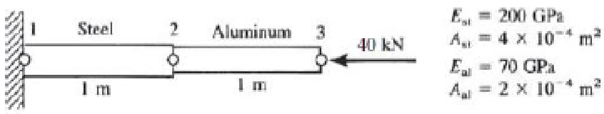
Homework #3 Solution

Text Problems 3.8, 15a, 18a, 23, 27, 56, 58a

3.8



$$[k^{(1)}] = \frac{(4\times10^{-4}\,\text{m}^2)(200\times10^6\,\frac{\text{kN}}{\text{m}^2})}{1\,\text{m}}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[k^{(1)}] = 800\times10^2\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\frac{\text{kN}}{\text{m}}$$

$$[k^{(2)}] = \frac{(2\times10^{-4}\,\text{m}^2)(70\times10^6\,\frac{\text{kN}}{\text{m}^2})}{1\,\text{m}}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[k^{(2)}] = 140\times10^2\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\frac{\text{kN}}{\text{m}}$$

$$\begin{Bmatrix} F_{1x} = 0 \\ F_{2x} \\ F_{3x} = -40\text{ kN} \end{Bmatrix} = 10^2\begin{bmatrix} \cancel{800} & \cancel{-800} & \cancel{0} \\ \cancel{-800} & 940 & -140 \\ \cancel{0} & -140 & 140 \end{bmatrix}\begin{Bmatrix} u_1 = 0 \\ u_2 \\ u_3 \end{Bmatrix}$$

$$\Rightarrow \quad 0 = 10^2\,(940\,u_2 - 140\,u_3) \quad \Rightarrow \quad u_3 = 6.741\,u_2 \qquad (1)$$

$$\Rightarrow -40000 = 10^2\,(-140\,u_2 + 140\,u_3) \qquad (2)$$

Substituting (1) into (2)

$$\Rightarrow \quad -40000 = 10^2\,(-140\,u_2 + 140\,(6.714)\,u_2)$$

$$\Rightarrow \quad u_2 = -0.50\times10^{-3}\text{ m}$$

$$\Rightarrow \quad u_3 = -3.356\times10^{-3}\text{ m}$$

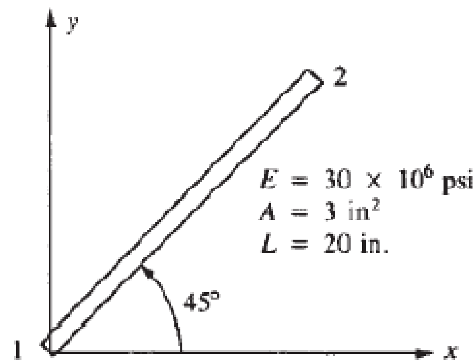
$$F_{1x} = 10^2\,(-800\times(-0.50\times10^{-3}))$$

$$\Rightarrow \quad F_{1x} = 40\text{ kN}$$

$$\begin{Bmatrix} f_{1x} \\ f_{2x} \end{Bmatrix} = 800\times10^2\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} 0 \\ -0.50\times10^{-3} \end{Bmatrix} \Rightarrow \begin{matrix} f_{1x}^{(1)} = 40\text{ kN} \\ f_{2x}^{(1)} = -40\text{ kN} \end{matrix}$$

$$\begin{Bmatrix} f_{2x} \\ f_{3x} \end{Bmatrix} = 140\times10^2\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} -0.50\times10^{-3} \\ -3.356\times10^{-3} \end{Bmatrix} \Rightarrow \begin{matrix} f_{2x}^{(2)} = 40\text{ kN} \\ f_{3x}^{(2)} = -40\text{ kN} \end{matrix}$$

3.15 (a)



$$C = \frac{1}{\sqrt{2}}, S = \frac{1}{\sqrt{2}}$$

$$[K] = \frac{EA}{L}\begin{bmatrix} C^2 & CS & -C^2 & -CS \\ & S^2 & -CS & -S^2 \\ & & C^2 & CS \\ & & & S^2 \end{bmatrix}$$

$$[K] = 2.25 \times 10^6 \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix} \frac{\text{lb}}{\text{in.}}$$

3.18

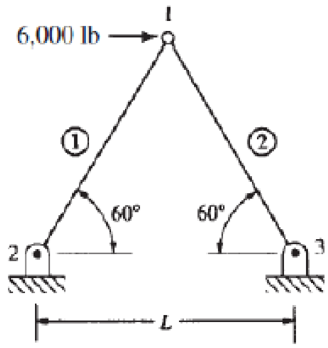
(a) $\sigma = \frac{E}{L}\,[-C \quad -S \quad C \quad S] \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix}, \theta = 45°$

$$C = \frac{\sqrt{2}}{2}, \quad S = \frac{\sqrt{2}}{2}, \quad E = 30 \times 10^6 \text{ psi}, \quad L = 60 \text{ in.}$$

$$\sigma = \frac{30 \times 10^6}{60}\left[-\frac{\sqrt{2}}{2} \quad -\frac{\sqrt{2}}{2} \quad \frac{\sqrt{2}}{2} \quad \frac{\sqrt{2}}{2}\right]\begin{Bmatrix} 0 \\ 0 \\ 0.02 \\ 0.04 \end{Bmatrix}$$

$\Rightarrow \sigma = 21200$ psi

3.23



Element (1)

$$C = \frac{1}{2}; \quad S = \frac{\sqrt{3}}{2}$$

$$[k^{(1)}] = \frac{AE}{L}\left[\begin{array}{cc|cc} \frac{1}{4} & \frac{\sqrt{3}}{4} & \multicolumn{2}{c}{-\lambda} \\ \frac{\sqrt{3}}{4} & \frac{3}{4} & & \\ \hline & & \frac{1}{4} & \frac{\sqrt{3}}{4} \\ \multicolumn{2}{c|}{-\lambda} & \frac{\sqrt{3}}{4} & \frac{3}{4} \end{array}\right]$$

Element (2)

$$[k^{(2)}] = \frac{AE}{L}\left[\begin{array}{cc|cc} \frac{1}{4} & -\frac{\sqrt{3}}{4} & \multicolumn{2}{c}{-\lambda} \\ -\frac{\sqrt{3}}{4} & \frac{3}{4} & & \\ \hline & & \frac{1}{4} & -\frac{\sqrt{3}}{4} \\ \multicolumn{2}{c|}{-\lambda} & -\frac{\sqrt{3}}{4} & \frac{3}{4} \end{array}\right]$$

$$\{F\} = [K]\,\{d\}$$

$$\begin{Bmatrix} 6000 \\ 0 \end{Bmatrix} = \frac{AE}{L}\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{3}{2} \end{bmatrix}\begin{Bmatrix} u_1 \\ v_1 \end{Bmatrix}$$

$$\Rightarrow 6000 = \frac{AE}{L}\frac{u_1}{2}$$

$$\Rightarrow u_1 = \frac{6000 \times 100 \times 2}{1 \times 10 \times 10^6}$$

$$\Rightarrow u_1 = 0.12 \text{ in.}$$

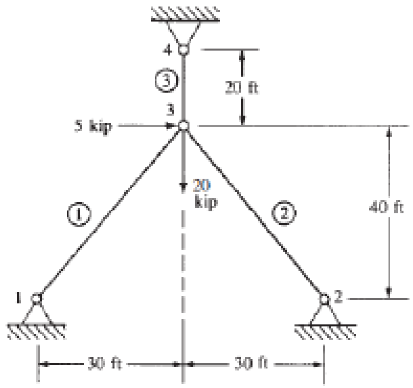
$$v_1 = 0$$

$$\sigma^{(1)} = [C']\{d\} = \frac{E}{L}\begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}\begin{Bmatrix} u_2 = 0 \\ v_2 = 0 \\ u_1 = 0.12 \\ v_1 = 0 \end{Bmatrix}$$

$$\Rightarrow \sigma^{(1)} = \frac{10 \times 10^6}{10^2}\left[\frac{1}{2}(0.12)\right]$$

$$\Rightarrow \sigma^{(1)} = 6000 \text{ psi}$$

3.27



	(1)	(2)	(3)
	1–3	2–3	3–4
L	50 ft	50 ft	20 ft
θ	53.13°	126.87°	90°
$\cos\theta$	0.6	– 0.6	0
$\sin\theta$	0.8	0.8	1

$$[k^{(1)}] = \frac{AE}{50}\begin{array}{c} \begin{array}{cc} \quad(1) & \qquad\qquad(3) \end{array} \\ \left[\begin{array}{cc|cc} 0.36 & 0.48 & -0.36 & -0.48 \\ 0.48 & 0.64 & -0.48 & -0.64 \\ \hline & & & \\ & & & \end{array}\right] \end{array} \begin{array}{c} (1) \\ \\ (3) \end{array}$$

$$[k^{(2)}] = \frac{AE}{50}\begin{array}{c} \begin{array}{cc} \quad(2) & \qquad\qquad(3) \end{array} \\ \left[\begin{array}{cc|cc} 0.36 & -0.48 & -0.36 & 0.48 \\ -0.48 & 0.64 & 0.48 & -0.64 \\ \hline & & & \\ & & & \end{array}\right] \end{array} \begin{array}{c} (2) \\ \\ (3) \end{array}$$

$$[k^{(3)}] = \frac{AE}{20}\begin{array}{c} \begin{array}{cc} (3) & \quad(4) \end{array} \\ \left[\begin{array}{cc|cc} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ \hline & & & \\ & & & \end{array}\right] \end{array} \begin{array}{c} (3) \\ \\ (4) \end{array}$$

Invoking boundary conditions. Therefore, need only 3–3

$$[K] = \begin{bmatrix} \frac{AE}{50}(0.36+0.36)+\frac{AE}{20}(0) & \frac{AE}{50}(0.48-0.48)+\frac{AE}{20}(0) \\ \frac{AE}{50}(0.48-0.48)+\frac{AE}{20}(0) & \frac{AE}{50}(0.64+0.64)+\frac{AE}{20}(1) \end{bmatrix}$$

$$\Rightarrow\ K = \begin{bmatrix} (0.72)\frac{AE}{50} & 0 \\ 0 & (1.28)\frac{AE}{50}+\frac{AE}{20} \end{bmatrix}$$

$$\{F\} = [K]\,\{d\}$$

$$\begin{Bmatrix} F_{3x} = 5\text{ K} \\ F_{3y} = -20\text{ K} \end{Bmatrix} = AE \begin{bmatrix} \frac{0.72}{50} & 0 \\ 0 & \frac{1.28}{50}+\frac{1}{20} \end{bmatrix} \begin{Bmatrix} u_3 \\ v_3 \end{Bmatrix}$$

$$\Rightarrow\quad 5\text{ K} = \frac{(0.72)\,(AE)}{50}\,u_3$$

$$\Rightarrow\ u_3 = \frac{(5000)\times(50)\times(12)}{(0.72)\,(3)\,(30\times10^6)}$$

$$\Rightarrow\ u_3 = 0.0463\text{ in.}$$

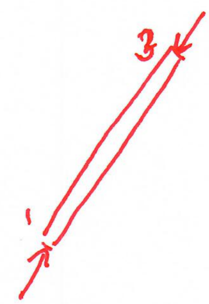
$$\Rightarrow\quad -20\text{ K} = \left[\frac{1.28}{50}+\frac{1}{20}\right] AE\,v_3$$

$$\Rightarrow\ v_3 = -0.0352\text{ in.}$$

Forces on the members ($A = 3$ in.^{2}, $E = 30 \times 10^6$ psi)

Member 1–3 (1)

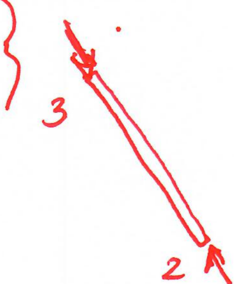
$$\begin{Bmatrix} f'_{1x} \\ f'_{3x} \end{Bmatrix} = \frac{AE}{600}\begin{bmatrix} 0.6 & 0.8 & -0.6 & -0.8 \\ -0.6 & -0.8 & 0.6 & 0.8 \end{bmatrix}\begin{Bmatrix} u_1 = 0 \\ v_1 = 0 \\ u_3 \\ v_3 \end{Bmatrix} = \begin{Bmatrix} 65.4 \\ -65.4 \end{Bmatrix}$$



$$\Rightarrow\ f'^{(1)}_{1x} = \cancel{69.9\text{ lb}}\quad -65.4\text{ lb (compression)}$$

Member 2–3 (2)

$$\begin{Bmatrix} f'_{2x} \\ f'_{3x} \end{Bmatrix} = \frac{AE}{600}\begin{bmatrix} -0.6 & 0.8 & 0.6 & -0.8 \\ 0.6 & -0.8 & -0.6 & 0.8 \end{bmatrix}\begin{Bmatrix} u_2 = 0 \\ v_2 = 0 \\ u_3 \\ v_3 \end{Bmatrix} = \begin{Bmatrix} 8400 \\ -8400 \end{Bmatrix}$$



$$\Rightarrow\ f'^{(2)}_{2x} = \cancel{8400\text{ lb}}\quad -8400\text{ lb (compression)}$$

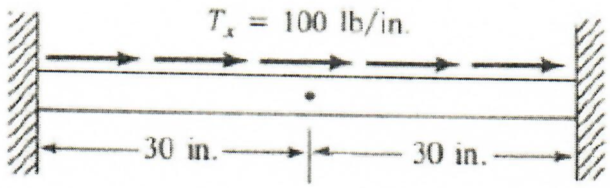
Member 3–4 (3)

$$\begin{Bmatrix} f'_{3x} \\ f'_{4x} \end{Bmatrix} = \frac{AE}{240}\begin{bmatrix} 0 & 1 & 0 & -1 \\ 0 & -1 & 0 & 1 \end{bmatrix}\begin{Bmatrix} u_3 \\ v_3 \\ u_4 = 0 \\ v_4 = 0 \end{Bmatrix} = \begin{Bmatrix} -13{,}230 \\ 13{,}230 \end{Bmatrix}$$

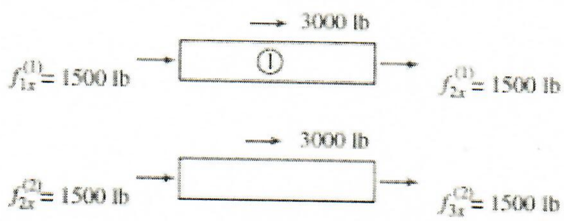
$$\Rightarrow \; f'_{3x} = \frac{AE}{240}(-0.0352)$$

$$\Rightarrow \; f'_{3x} = \cancel{-13240}\text{ lb} \quad +13{,}230 \text{ lb} \quad (\text{tension})$$

3.56



$F = (1000 \frac{\text{lb}}{\text{in.}})(30 \text{ in.}) = 3000$ lb



$$\{F\} = \begin{Bmatrix} F_{1x} + 1500 \text{ lb} \\ 1500 + 1500 \\ F_{3x} + 1500 \end{Bmatrix}$$

$$[k^{(1)}] = \frac{AE}{30}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[k^{(2)}] = \frac{AE}{30}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Global equations

$$\frac{(2)(30\times 10^6)}{30}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 1+1 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 = 0 \\ u_2 \\ u_3 = 0 \end{Bmatrix} = \begin{Bmatrix} F_{1x} + 1500 \\ 3000 \\ F_{3x} + 1500 \end{Bmatrix}$$

Solving Equation (2)

$2 \times 10^6 (2 u_2) = 3000$

$$u_2 = 0.75 \times 10^{-3} \text{ in.}$$

Element stresses

$$\sigma^{(1)} = [C'] \{d\} = \frac{E}{L} [-C \quad -S \quad C \quad S] \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix}$$

$$= \frac{30 \times 10^6}{30} [-1 \quad 0 \quad 1 \quad 0] \begin{Bmatrix} 0 \\ 0 \\ 0.75 \times 10^{-3} \\ 0 \end{Bmatrix}$$

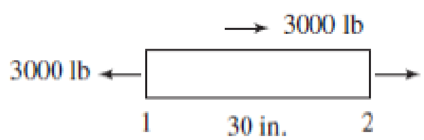
$$\Rightarrow \sigma^{(1)} = 750 \text{ psi (T)}$$

$$F_{1x} + 1500 = 2 \times 10^6 (-1)(0.75 \times 10^{-3})$$

$$\Rightarrow F_{1x} = -3000 \text{ lb} \quad (\leftarrow)$$

and $\qquad F_{3x} = -3000 \text{ lb} \quad (\leftarrow)$

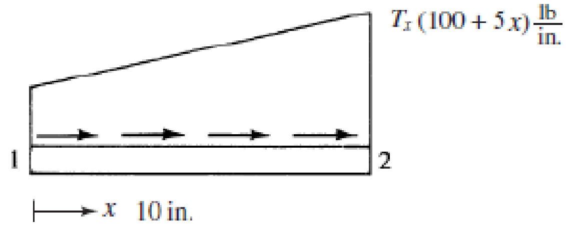
Total applied force = $60 \times 100 = 6000$ lb $(\rightarrow)$



$$\sigma = 0 \text{ (at node 2)}$$

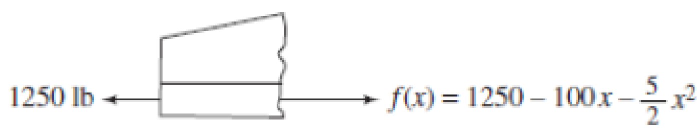
$$\sigma(x = 15'') = \frac{3000 - 1500}{2} = 750 \text{ psi}$$

3.58



$$\text{Total } T_x = 100 \times 10 + \frac{1}{2} \times 5(10)^2 \text{ lb} = 1250 \text{ lb}$$

$$u(x) = (u_2 - u_1)(\frac{x}{L}) + u_1$$



$$f_{1x}\,u_1 + f_{2x}\,u_2 = \int_0^{10} (100 + 5\,x)\left[\left(\frac{u_1 - u_2}{10}\right)x + u_1\right]dx$$

$$= \int_0^{10}\left[10\,(u_2 - u_1)\,x + 100\,u_1 + \frac{(u_2 - u_1)}{2}x^2 + 5u_1 x\right]dx$$

$$= 500\,(u_2 - u_1) + 1000\,u_1 + \frac{(u_2 - u_1)\,1000}{6} + 250\,u_1$$

Let $u_1 = 1$; $u_2 = 0$

$$\therefore \quad f_{1x} = -\,500 + 1000 - \frac{1000}{6} + 250 = 583.33$$

$$\therefore \quad f_{1x} = 583.33 \text{ lb}$$

Let $u_1 = 0$, $u_2 = 1$, $f_{2x} = 500 + 1000/6 = 666.7$ lb