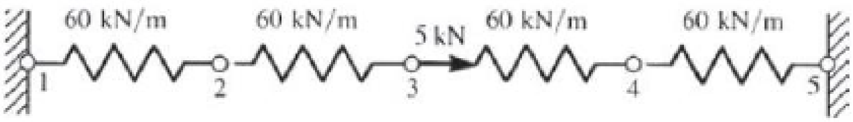
Homework #2 Solution

Text Problems 2.13, 15, 21

2.13



$$[k^{(1)}] = [k^{(2)}] = [k^{(3)}] = [k^{(4)}] = 60\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\{F\} = [K]\,\{d\}$$

$$\begin{Bmatrix} F_{1x} = ? \\ F_{2x} = 0 \\ F_{3x} = 5\text{ kN} \\ F_{4x} = 0 \\ F_{5x} = ? \end{Bmatrix} = 60\begin{Bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{Bmatrix}\begin{Bmatrix} u_1 = 0 \\ u_2 = ? \\ u_3 = ? \\ u_4 = ? \\ u_5 = 0 \end{Bmatrix}$$

$$\left.\begin{array}{l} 0 = 2u_2 - u_3 \Rightarrow u_2 = 0.5\,u_3 \\ 0 = -u_3 + 2u_4 \Rightarrow u_4 = 0.5\,u_3 \end{array}\right\} \Rightarrow u_2 = u_4$$

$$\Rightarrow \quad 5\text{ kN} = -60\,u_2 + 120\,(2\,u_2) - 60\,u_2$$

$$\Rightarrow \quad 5 = 120\,u_2 \Rightarrow u_2 = 0.042\text{ m}$$

$$\Rightarrow \quad u_4 = 0.042\text{ m}$$

$$\Rightarrow \quad u_3 = 2(0.042) \Rightarrow u_3 = 0.084\text{ m}$$

Element (1)

$$\begin{Bmatrix} f_{1x} \\ f_{2x} \end{Bmatrix} = 60\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} 0 \\ 0.042 \end{Bmatrix} \Rightarrow \begin{array}{l} f_{1x}^{(1)} = -2.5\text{ kN} \\ f_{2x}^{(1)} = 2.5\text{ kN} \end{array}$$

Element (2)

$$\begin{Bmatrix} f_{2x} \\ f_{3x} \end{Bmatrix} = 60\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} 0.042 \\ 0.084 \end{Bmatrix} \Rightarrow \begin{array}{l} f_{2x}^{(2)} = -2.5\text{ kN} \\ f_{3x}^{(2)} = 2.5\text{ kN} \end{array}$$

Element (3)

$$\begin{Bmatrix} f_{3x} \\ f_{4x} \end{Bmatrix} = 60\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} 0.084 \\ 0.042 \end{Bmatrix} \Rightarrow \begin{array}{l} f_{3x}^{(3)} = 2.5\text{ kN} \\ f_{4x}^{(3)} = -2.5\text{ kN} \end{array}$$

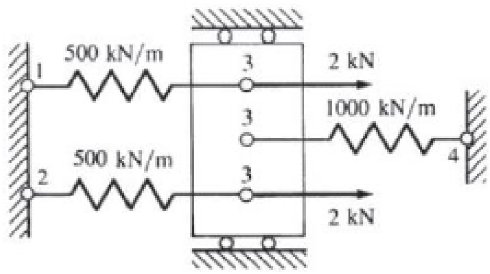
Element (4)

$$\begin{Bmatrix} f_{4x} \\ f_{5x} \end{Bmatrix} = 60 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 0.042 \\ 0 \end{Bmatrix} \Rightarrow \begin{matrix} f_{4x}^{(4)} = 2.5 \text{ kN} \\ f_{5x}^{(4)} = -2.5 \text{ kN} \end{matrix}$$

$$F_{1x} = 60\,[1 \quad -1] \begin{Bmatrix} 0 \\ 0.042 \end{Bmatrix} \Rightarrow F_{1x} = -\,2.5 \text{ kN}$$

$$F_{5x} = 60\,[-1 \quad 1] \begin{Bmatrix} 0.042 \\ 0 \end{Bmatrix} \Rightarrow F_{5x} = -\,2.5 \text{ kN}$$

2.15



$$[k^{(1)}] = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix};\ [k^{(2)}] = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix};\ [k^{(3)}] = \begin{bmatrix} 1000 & -1000 \\ -1000 & 1000 \end{bmatrix}$$

$$\begin{Bmatrix} F_{1x} = ? \\ F_{2x} = ? \\ F_{3x} = 4 \text{ kN} \\ F_{4x} = ? \end{Bmatrix} = \begin{bmatrix} 500 & 0 & -500 & 0 \\ 0 & 500 & -500 & 0 \\ -500 & -500 & 2000 & -1000 \\ 0 & 0 & -1000 & 1000 \end{bmatrix} \begin{Bmatrix} u_1 = 0 \\ u_2 = 0 \\ u_3 = ? \\ u_4 = 0 \end{Bmatrix}$$

$$\Rightarrow \qquad u_3 = 0.002 \text{ m}$$

Reactions

$$F_{1x} = (-\,500)\,(0.002) \Rightarrow F_{1x} = -\,1.0 \text{ kN}$$

$$F_{2x} = (-\,500)\,(0.002) \Rightarrow F_{2x} = -\,1.0 \text{ kN}$$

$$F_{4x} = (-\,1000)\,(0.002) \Rightarrow F_{4x} = -\,2.0 \text{ kN}$$

Element (1)

$$\begin{Bmatrix} f_{1x} \\ f_{3x} \end{Bmatrix} = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.002 \end{Bmatrix} \Rightarrow \begin{Bmatrix} f_{1x} \\ f_{3x} \end{Bmatrix} = \begin{Bmatrix} -\,1.0 \text{ kN} \\ 1.0 \text{ kN} \end{Bmatrix}$$

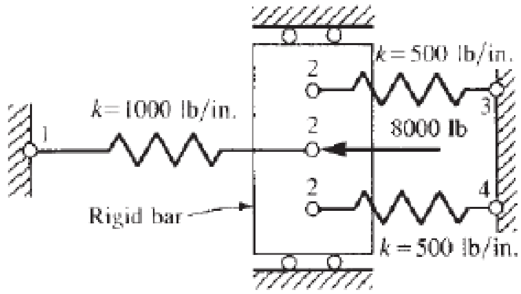
Element (2)

$$\begin{Bmatrix} f_{2x} \\ f_{3x} \end{Bmatrix} = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.002 \end{Bmatrix} \Rightarrow \begin{Bmatrix} f_{2x} \\ f_{3x} \end{Bmatrix} = \begin{Bmatrix} -1.0\text{ kN} \\ 1.0\text{ kN} \end{Bmatrix}$$

Element (3)

$$\begin{Bmatrix} f_{3x} \\ f_{4x} \end{Bmatrix} = \begin{bmatrix} 1000 & -1000 \\ -1000 & 1000 \end{bmatrix} \begin{Bmatrix} 0.002 \\ 0 \end{Bmatrix} \Rightarrow \begin{Bmatrix} f_{3x} \\ f_{4x} \end{Bmatrix} = \begin{Bmatrix} 2.0\text{ kN} \\ -2.0\text{ kN} \end{Bmatrix}$$

2.21 Solve Problem 2.10 using P.E. approach



$$\pi_p = \sum_{e=1}^{3} \pi_p^{(e)} = \frac{1}{2}\, k_1 (u_2 - u_1)^2 + \frac{1}{2}\, k_2 (u_3 - u_2)^2 + \frac{1}{2}\, k_3 (u_4 - u_2)^2$$

$$- f_{1x}^{(1)} u_1 - f_{2x}^{(1)} u_2 - f_{2x}^{(2)} u_2$$

$$- f_{3x}^{(2)} u_3 - f_{2x}^{(3)} u_2 - f_{4x}^{(3)} u_4$$

$$\frac{\partial \pi_p}{\partial u_1} = -k_1 u_2 + k_1 u_1 - f_{1x}^{(1)} = 0 \qquad (1)$$

$$\frac{\partial \pi_p}{\partial u_2} = k_1 u_2 - k_1 u_1 - k_2 u_3 + k_2 u_2 - k_3 u_4$$

$$+ k_3 u_2 - f_{2x}^{(1)} - f_{2x}^{(2)} - f_{2x}^{(3)} = 0 \qquad (2)$$

$$\frac{\partial \pi_p}{\partial u_3} = k_2 u_3 - k_2 u_2 - f_{3x}^{(2)} = 0 \qquad (3)$$

$$\frac{\partial \pi_p}{\partial u_4} = k_3 u_4 - k_3 u_2 - f_{4x}^{(3)} = 0 \qquad (4)$$

In matrix form (1) through (4) become

$$\begin{bmatrix} k_1 & -k_1 & 0 & 0 \\ -k_1 & k_1 + k_2 + k_3 & -k_2 & -k_3 \\ 0 & -k_2 & k_2 & 0 \\ 0 & -k_3 & 0 & k_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} f_{1x}^{(1)} \\ f_{2x}^{(1)} + f_{2x}^{(2)} + f_{2x}^{(3)} \\ f_{3x}^{(2)} \\ f_{4x}^{(3)} \end{Bmatrix} \qquad (5)$$

or using numerical values

$$\begin{bmatrix} 1000 & -1000 & 0 & 0 \\ -1000 & 2000 & -500 & -500 \\ 0 & -500 & 500 & 0 \\ 0 & -500 & 0 & 500 \end{bmatrix} \begin{Bmatrix} u_1 = 0 \\ u_2 \\ u_3 = 0 \\ u_4 = 0 \end{Bmatrix} = \begin{Bmatrix} F_{1x} \\ -8000 \\ F_{3x} \\ F_{4x} \end{Bmatrix} \quad (6)$$

Solution now follows as in Problem 2.10

Solve 2nd of Equations (6) for $u_2 = -4$ in.

For reactions and element forces, see solution to Problem 2.10

From Problem 2.10 solution:

$$\{F\} = [K]\{d\}$$

$$\begin{Bmatrix} F_{1x} = ? \\ F_{2x} = -8000 \\ F_{3x} = ? \\ F_{4x} = ? \end{Bmatrix} = \begin{bmatrix} 1000 & -1000 & 0 & 0 \\ -1000 & 2000 & -500 & -500 \\ 0 & -500 & 500 & 0 \\ 0 & -500 & 0 & 500 \end{bmatrix} \begin{Bmatrix} u_1 = 0 \\ u_2 = ? \\ u_3 = 0 \\ u_4 = 0 \end{Bmatrix}$$

$$\Rightarrow \qquad u_2 = \frac{-8000}{2000} = -4 \text{ in.}$$

Reactions

$$\begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \\ F_{4x} \end{Bmatrix} = \begin{bmatrix} 1000 & -1000 & 0 & 0 \\ -1000 & 2000 & -500 & -500 \\ 0 & -500 & 500 & 0 \\ 0 & -500 & 0 & 500 \end{bmatrix} \begin{Bmatrix} 0 \\ -4 \\ 0 \\ 0 \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \\ F_{4x} \end{Bmatrix} = \begin{Bmatrix} 4000 \\ -8000 \\ 2000 \\ 2000 \end{Bmatrix} \text{lb}$$

Element (1)

$$\begin{Bmatrix} f_{1x}^{(1)} \\ f_{2x}^{(1)} \end{Bmatrix} = \begin{bmatrix} 1000 & -1000 \\ -1000 & 1000 \end{bmatrix} \begin{Bmatrix} 0 \\ -4 \end{Bmatrix} \Rightarrow \begin{Bmatrix} f_{1x}^{(1)} \\ f_{2x}^{(1)} \end{Bmatrix} = \begin{Bmatrix} 4000 \\ -4000 \end{Bmatrix} \text{lb}$$

Element (2)

$$\begin{Bmatrix} f_{2x}{}^{(2)} \\ f_{3x}{}^{(2)} \end{Bmatrix} = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix} \begin{Bmatrix} -4 \\ 0 \end{Bmatrix} \Rightarrow \begin{Bmatrix} f_{2x}{}^{(2)} \\ f_{3x}{}^{(2)} \end{Bmatrix} = \begin{Bmatrix} -2000 \\ 2000 \end{Bmatrix} \text{lb}$$

Element (3)

$$\begin{Bmatrix} f_{2x}{}^{(3)} \\ f_{4x}{}^{(3)} \end{Bmatrix} = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix} \begin{Bmatrix} -4 \\ 0 \end{Bmatrix} \Rightarrow \begin{Bmatrix} f_{2x}{}^{(3)} \\ f_{4x}{}^{(3)} \end{Bmatrix} \begin{Bmatrix} -2000 \\ 2000 \end{Bmatrix} \text{lb}$$