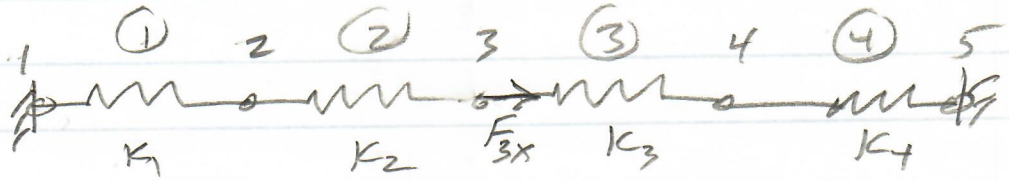
HW 2 2.13, 15, 21

2.13




$K_1 = K_2 = K_3 = K_4 = 60 \ kN/m$

$F_{3x} = 5 \ kN$

$$[k] = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} = (60 \ kN/m) \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

1 2 3 4 5

$$\begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \\ F_{4x} \\ F_{5x} \end{Bmatrix} = 60 \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & (1+1) & -1 & 0 & 0 \\ 0 & -1 & (1+1) & -1 & 0 \\ 0 & 0 & -1 & (1+1) & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix}$$

5×1 5×5 5×1

$$\begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \\ F_{4x} \\ F_{5x} \end{Bmatrix} \begin{matrix} \\ 0 \\ 5 \\ 0 \\ \end{matrix} = 60 \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix} \begin{matrix} 0 \\ \\ \\ \\ 0 \end{matrix}$$

$$\begin{Bmatrix} 0 \\ 5 \\ 0 \end{Bmatrix} = 60 \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix}$$

1st $\quad 0 = 60\,(2u_2 - u_3)$

$$u_2 = u_3/2$$

3rd $\quad 0 = 60\,(-u_3 + 2u_4)$

$$u_4 = u_3/2$$

2nd

$$5 = 60\,(-u_2 + 2u_3 - u_4)$$

$$\frac{1}{12} = -u_3/2 + 2u_3 - u_3/2$$

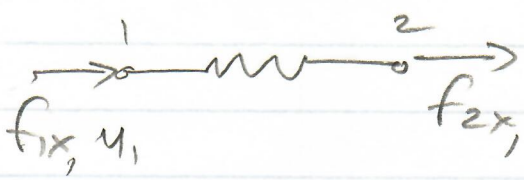
$$u_3 = 1/12 = 0.0833 \text{ m}$$

$$u_2 = u_4 = 0.0417 \text{ m}$$

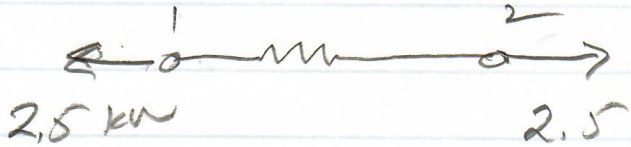
$$\{d\} = \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix} = \begin{Bmatrix} 0 \\ .0417 \\ .0833 \\ .0417 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \\ F_{4x} \\ F_{5x} \end{Bmatrix} = (60) \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ .0417 \\ .0833 \\ .0417 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -2.5 \\ 0 \\ 5 \\ 0 \\ -2.5 \end{Bmatrix} \text{kN}$$

element 1



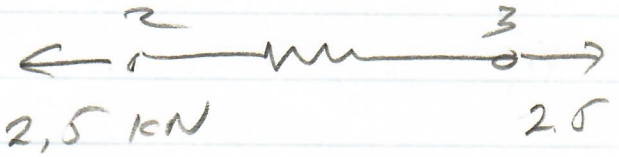

$$\begin{Bmatrix} f_{1x} \\ f_{2x} \end{Bmatrix} = \begin{bmatrix} 60 & -60 \\ -60 & 60 \end{bmatrix} \begin{Bmatrix} 0 \\ .0417 \end{Bmatrix} = \begin{Bmatrix} -2.5 \\ +2.5 \end{Bmatrix} \text{ kN}$$




(Tension)

element 2

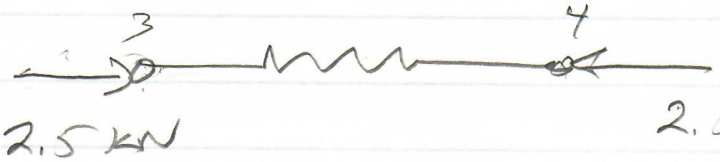
$$\begin{Bmatrix} f_{2x} \\ f_{3x} \end{Bmatrix} = \begin{bmatrix} 60 & -60 \\ -60 & 60 \end{bmatrix} \begin{Bmatrix} .0417 \\ .0833 \end{Bmatrix} = \begin{Bmatrix} -2.5 \\ +2.5 \end{Bmatrix} \text{ kN}$$




(Tension)

element 3

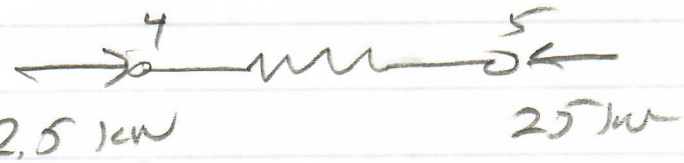
$$\begin{Bmatrix} f_{3x} \\ f_{4x} \end{Bmatrix} = \begin{bmatrix} 60 & -60 \\ -60 & 60 \end{bmatrix} \begin{Bmatrix} .0833 \\ .0417 \end{Bmatrix} = \begin{Bmatrix} +2.5 \\ -2.5 \end{Bmatrix} \text{ kN}$$




(compression)

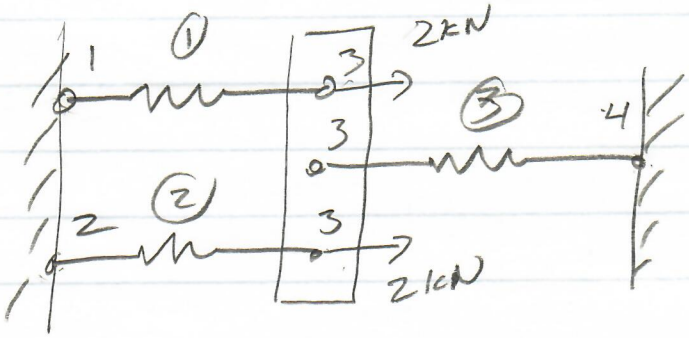
element 4

$$\begin{Bmatrix} f_{4x} \\ f_{5x} \end{Bmatrix} = \begin{bmatrix} 60 & -60 \\ -60 & 60 \end{bmatrix} \begin{Bmatrix} .0417 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 2.5 \\ -2.5 \end{Bmatrix} \text{ kN}$$




(compression)

2.15




$$K_1 = K_2 = 500 \text{ kN/m}$$

$$K_3 = 1000 \text{ kN/m}$$

$$[K_1] = [K_2] = (500) \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[K_3] = 1000 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \\ F_{4x} \end{Bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 500 & 0 & -500 & 0 \\ 0 & 500 & -500 & 0 \\ -500 & -500 & (500+500+1000) & -1000 \\ 0 & 0 & -1000 & 1000 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix}$$

$4 \times 1$

$$\begin{Bmatrix} F_{1x} \\ F_{2x} \\ 4 \\ F_{4x} \end{Bmatrix} = \begin{bmatrix} 500 & 0 & -500 & 0 \\ 0 & 500 & -500 & 0 \\ -500 & -500 & 2000 & -1000 \\ 0 & 0 & -1000 & 1000 \end{bmatrix} \begin{Bmatrix} \cancel{u_1} \\ \cancel{u_2} \\ u_3 \\ \cancel{u_4} \end{Bmatrix} \begin{matrix} 0 \\ 0 \\ \\ 0 \end{matrix}$$

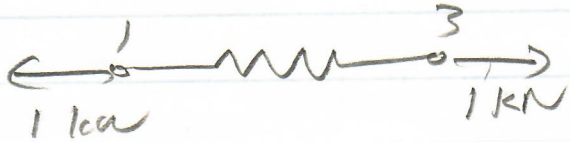
Eq. 3

$$4 = 2000\ u_3 \Rightarrow \boxed{u_3 = .002\ m}$$

$$\begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \\ F_{4x} \end{Bmatrix} = \begin{Bmatrix} -1.0 \\ -1.0 \\ 4.0 \\ -2.0 \end{Bmatrix}$$

element 1

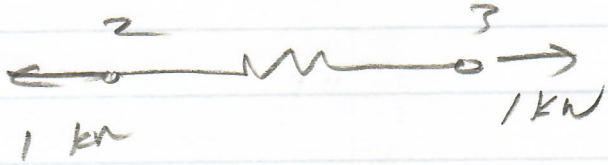
$$\begin{Bmatrix} f_{1x} \\ f_{3x} \end{Bmatrix} = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix} \begin{Bmatrix} 0 \\ .002 \end{Bmatrix} = \begin{Bmatrix} -1.0 \\ 1.0 \end{Bmatrix} \text{ kN}$$




(Tension)

element 2 (same as 1)

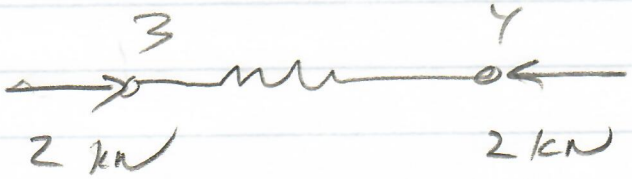
$$\begin{Bmatrix} f_{2x} \\ f_{3x} \end{Bmatrix} = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix} \begin{Bmatrix} 0 \\ .002 \end{Bmatrix} = \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} \text{ kN}$$




(Tension)

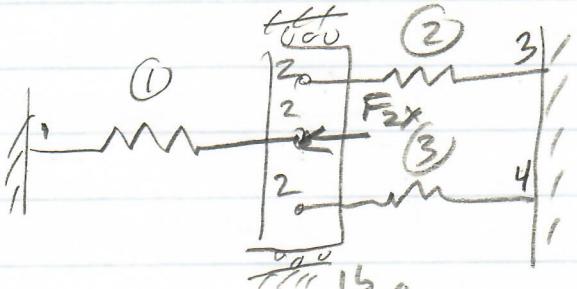
element 3

$$\begin{Bmatrix} f_{3x} \\ f_{4x} \end{Bmatrix} = \begin{bmatrix} 1000 & -1000 \\ -1000 & 1000 \end{bmatrix} \begin{Bmatrix} .002 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 2 \\ -2 \end{Bmatrix} \text{ kN}$$




(compression)

2.21




$k_1 = 1000$ lb/in

$k_2 = k_3 = 500$ lb/in

$F_{2x} = -8000$ lb

$$\Pi_p = \frac{1}{2} K_1 (u_2 - u_1)^2 + \frac{1}{2} K_2 (u_3 - u_2)^2$$

$$+ \frac{1}{2} K_3 (u_4 - u_2)^2 - F_{1x} u_1 - F_{2x} u_2$$

$$- F_{3x} u_3 - F_{4x} u_4$$

TMPE

$$\left.\begin{aligned} \frac{\partial \Pi_p}{\partial u_1} &= 0 \\ \frac{\partial \Pi_p}{\partial u_2} &= 0 \\ \frac{\partial \Pi_p}{\partial u_3} &= 0 \\ \frac{\partial \Pi_p}{\partial u_4} &= 0 \end{aligned}\right\} \Rightarrow \underset{4\times 1}{\{F\}} = \underset{4\times 4}{[K]} \underset{4\times 1}{\{D\}}$$

First, impose B.C. $\Rightarrow u_1 = u_3 = u_4 = 0$

$$\frac{\partial \Pi_p}{\partial u_2} = 0$$

$$\Pi_p = \frac{1}{2} K_1 u_2^2 + \frac{1}{2} K_2 u_2^2 + \frac{1}{2} K_3 u_2^2 - F_{2x} u_2$$

$$= \frac{1}{2} (K_1 + K_2 + K_3) u_2^2 - F_{2x} u_2$$

$$\frac{\partial \Pi_p}{\partial u_2} = (K_1 + K_2 + K_3) u_2 - F_{2x} = 0$$

$$= \left(2000 \tfrac{lb}{in}\right) u_2 - (-8000 lb) = 0$$

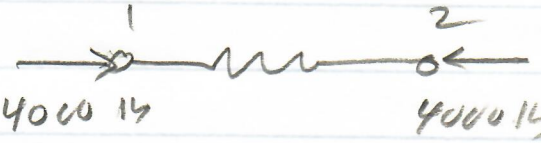
$$\boxed{u_2 = -4 \text{ in}}$$

$$\begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \\ F_{4x} \end{Bmatrix} = \begin{bmatrix} 1000 & -1000 & 0 & 0 \\ -1000 & 2000 \ (1000+500+500) & -500 & -500 \\ 0 & -500 & 500 & 0 \\ 0 & -500 & 0 & 500 \end{bmatrix} \begin{Bmatrix} 0 \\ -4 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \\ F_{4x} \end{Bmatrix} = \begin{Bmatrix} 4000 \\ -8000 \\ 2000 \\ 2000 \end{Bmatrix} \text{ lb}$$

element 1

$$\begin{Bmatrix} f_{1x} \\ f_{2x} \end{Bmatrix} = \begin{bmatrix} 1000 & -1000 \\ -1000 & 1000 \end{bmatrix} \begin{Bmatrix} 0 \\ -4 \end{Bmatrix} = \begin{Bmatrix} 4000 \\ -4000 \end{Bmatrix} \text{ lb}$$




(compression)

element 2

$$\begin{Bmatrix} f_{2x} \\ f_{3x} \end{Bmatrix} = \begin{bmatrix} 500 & -500 \\ -502 & 500 \end{bmatrix} \begin{Bmatrix} -4 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -2000 \\ 2000 \end{Bmatrix} \text{ lb}$$


(tension)

element 3

$$\begin{Bmatrix} f_{2x} \\ f_{4x} \end{Bmatrix} = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix} \begin{Bmatrix} -4 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -2000 \\ 2000 \end{Bmatrix} \text{ lb}$$


(tension)