HW#5 Solution, 4.5, 10, 13, 14, 15, 17

4-5 $\sigma_{ij} = \lambda e_{kk} \delta_{ij} + 2\mu e_{ij}$

$$\sigma_{mm} = \lambda e_{kk} \cancel{\delta_{mm}}^{3} + 2\mu e_{mm}$$

Since $e_{mm} = e_{kk}$ and $\sigma_{mm} = \sigma_{kk}$

$$e_{kk} = \frac{\sigma_{kk}}{(3\lambda + 2\mu)}$$

$$\sigma_{ij} = \left(\frac{\lambda}{3\lambda + 2\mu}\right) \sigma_{kk} \delta_{ij} + 2\mu e_{ij}$$

$$e_{ij} = \frac{1}{2\mu} \sigma_{ij} - \frac{\lambda}{2\mu(3\lambda + 2\mu)} \sigma_{kk} \delta_{ij}$$

Table 4-1

$$\mu = \frac{E}{2(1+\nu)}$$

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$$

$$\frac{1}{2\mu} = \frac{1+\nu}{E}$$

$$\frac{\lambda}{2\mu(3\lambda + 2\mu)} = \frac{\frac{E\nu}{\cancel{(1+\nu)}(1-2\nu)}}{\cancel{\left(\frac{E}{1+\nu}\right)}\left[\frac{3E\nu}{(1+\nu)(1-2\nu)} + \frac{E}{1+\nu}\right]}$$

$$= \frac{\nu / \cancel{1-2\nu}}{\frac{3E\nu + E - 2E\nu}{(1+\nu)\cancel{(1-2\nu)}}} = \frac{\nu}{E\left(\cancel{1+\nu}/\cancel{1+\nu}\right)} = \frac{\nu}{E}$$

Hence

$$\boxed{e_{ij} = \frac{1+\nu}{E} \sigma_{ij} - \frac{\nu}{E} \sigma_{kk} \delta_{ij}} \quad (\text{eq. } 4.2.10)$$

Since

$$\frac{1}{2\mu} = \frac{1+\nu}{E}$$

$$\frac{\nu}{E} = \frac{\lambda}{2\mu(3\lambda+2\mu)}$$

$$\nu = \frac{E\lambda}{2\mu(3\lambda+2\mu)}$$

$$\frac{1}{2\mu} = \frac{1+\left[\frac{E\lambda}{2\mu(3\lambda+2\mu)}\right]}{E}$$

$$E = 2\mu + \frac{E\lambda}{(3\lambda+2\mu)}$$

$$E\left[1 - \frac{\lambda}{3\lambda+2\mu}\right] = 2\mu$$

$$E\left[\frac{\cancel{3}^{2}\lambda + \cancel{2}\mu - \cancel{\lambda}}{3\lambda+2\mu}\right] = \cancel{2}\mu$$

$$E\left[\frac{\lambda+\mu}{3\lambda+2\mu}\right] = \mu$$

$$\boxed{E = \frac{\mu(3\lambda+2\mu)}{\lambda+\mu}}$$

$$\nu = \frac{\left[\frac{\cancel{\mu}\,\cancel{(3\lambda+2\mu)}}{\lambda+\mu}\right]\lambda}{2\cancel{\mu}\,\cancel{(3\lambda+2\mu)}}$$

$$\boxed{\nu = \frac{\lambda}{2(\lambda+\mu)}}$$

4-10 $E>0,\ K>0,\ \mu>0$

Table 4-1

$$E = 3K(1-2\nu) \Rightarrow 1-2\nu>0 \Rightarrow \nu<1/2$$

$$E = 2\mu(1+\nu) \Rightarrow 1+\nu>0 \Rightarrow \nu>-1$$

Hence $\boxed{-1<\nu<1/2}$

If $0<\nu<1/2$

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \Rightarrow \boxed{\lambda>0}$$

4.13.

(a) Aluminum : $E = 68.9GPa, \nu = 0.34, \mu = 25.7GPa, k = 71.8GPa$

Simple Tension : $$e_{ij} = \begin{bmatrix} \frac{\sigma}{E} & 0 & 0 \\ 0 & -\frac{\nu}{E}\sigma & 0 \\ 0 & 0 & -\frac{\nu}{E}\sigma \end{bmatrix} = \begin{bmatrix} 2.17 & 0 & 0 \\ 0 & -0.74 & 0 \\ 0 & 0 & -0.74 \end{bmatrix} \times 10^{-3}$$

Pure Shear : $$e_{ij} = \begin{bmatrix} 0 & \tau/2\mu & 0 \\ \tau/2\mu & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1.46 & 0 \\ 1.46 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times 10^{-3}$$

Hydrostatic Compression : $$e_{ij} = \begin{bmatrix} -\frac{p}{3k} & 0 & 0 \\ 0 & -\frac{p}{3k} & 0 \\ 0 & 0 & -\frac{p}{3k} \end{bmatrix} = \begin{bmatrix} -2.32 & 0 & 0 \\ 0 & -2.32 & 0 \\ 0 & 0 & -2.32 \end{bmatrix} \times 10^{-3}$$

(b) Steel : $E = 207GPa, \nu = 0.29, \mu = 80.2GPa, k = 164GPa$

Simple Tension : $$e_{ij} = \begin{bmatrix} \frac{\sigma}{E} & 0 & 0 \\ 0 & -\frac{\nu}{E}\sigma & 0 \\ 0 & 0 & -\frac{\nu}{E}\sigma \end{bmatrix} = \begin{bmatrix} 1.45 & 0 & 0 \\ 0 & -0.42 & 0 \\ 0 & 0 & -0.42 \end{bmatrix} \times 10^{-3}$$

Pure Shear : $$e_{ij} = \begin{bmatrix} 0 & \tau/2\mu & 0 \\ \tau/2\mu & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0.935 & 0 \\ 0.935 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times 10^{-3}$$

Hydrostatic Compression : $$e_{ij} = \begin{bmatrix} -\frac{p}{3k} & 0 & 0 \\ 0 & -\frac{p}{3k} & 0 \\ 0 & 0 & -\frac{p}{3k} \end{bmatrix} = \begin{bmatrix} -1.02 & 0 & 0 \\ 0 & -1.02 & 0 \\ 0 & 0 & -1.02 \end{bmatrix} \times 10^{-3}$$

(c) Rubber : $E = 0.0019GPa, \nu = 0.499, \mu = 0.000654GPa, k = 0.326GPa$

Simple Tension : $$e_{ij} = \begin{bmatrix} \frac{\sigma}{E} & 0 & 0 \\ 0 & -\frac{\nu}{E}\sigma & 0 \\ 0 & 0 & -\frac{\nu}{E}\sigma \end{bmatrix} = \begin{bmatrix} 7894 & 0 & 0 \\ 0 & -3939 & 0 \\ 0 & 0 & -3939 \end{bmatrix} \times 10^{-3}$$

Pure Shear : $$e_{ij} = \begin{bmatrix} 0 & \tau/2\mu & 0 \\ \tau/2\mu & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 5351 & 0 \\ 5351 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times 10^{-3}$$

Hydrostatic Compression : $$e_{ij} = \begin{bmatrix} -\frac{p}{3k} & 0 & 0 \\ 0 & -\frac{p}{3k} & 0 \\ 0 & 0 & -\frac{p}{3k} \end{bmatrix} = \begin{bmatrix} -511 & 0 & 0 \\ 0 & -511 & 0 \\ 0 & 0 & -511 \end{bmatrix} \times 10^{-3}$$

4-14 $\tilde{\sigma} = \frac{1}{3}\sigma_{kk}\delta_{ij}$, $\tilde{e} = \frac{1}{3} e_{kk}\delta_{ij}$

From problem 4-5

$$e_{kk} = \frac{\sigma_{kk}}{3\lambda+2\mu}$$

$$\tilde{\sigma}_{ij} = \frac{1}{3}(3\lambda+2\mu)\, e_{kk}\,\delta_{ij}$$

$$\tilde{\sigma}_{ij} = (3\lambda+2\mu)\,\tilde{e}_{ij}\,\delta_{ij}$$

From Table 4-1

$$3K = 3\lambda+2\mu$$

$$\boxed{\tilde{\sigma}_{ij} = 3K\,\tilde{e}_{ij}}$$

$\hat{\sigma}_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij}$, $\hat{e}_{ij} = e_{ij} - \frac{1}{3}e_{kk}\delta_{ij}$

$$\hat{\sigma}_{ij} = (\cancel{\lambda e_{kk}\delta_{ij}} + 2\mu e_{ij}) - \frac{1}{3}(\cancel{3\lambda}+2\mu)e_{kk}\delta_{ij}$$

$$= 2\mu\left(e_{ij} - \tfrac{1}{3}e_{kk}\delta_{ij}\right)$$

$$\boxed{\hat{\sigma}_{ij} = 2\mu\,\hat{e}_{ij}}$$

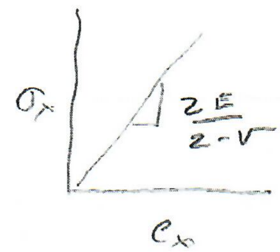
4.15 $\sigma_x = 2\sigma_y$

$$e_x = \frac{1}{E}\left[\sigma_x - \nu\left(\overset{\sigma_x/2}{\cancel{\sigma_y}} + \overset{0}{\cancel{\sigma_z}}\right)\right]$$

$$= \frac{\sigma_x\left(1 - \nu/2\right)}{E}$$

$$e_x = \frac{\sigma_x(2-\nu)}{2E}$$

$$\sigma_x = \frac{2E}{2-\nu}\, e_x$$



4-~~15~~ 17

$$\sigma_{ij} = \begin{bmatrix} 20 & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ MPa}$$

$$e_{ij} = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E}\sigma_{kk}\delta_{ij}$$

$$e_x = \frac{1}{E}\left[(1+\nu)\sigma_x - \nu(\sigma_x+\sigma_y)\right]$$

$$= \frac{1}{207\text{ GPa}}\left[(1.29)(20\text{ MPa}) - (.29)(50\text{ MPa})\right]$$

$$= 5.46\text{ e-}5$$

$$e_y = \frac{1}{E}\left[(1+\nu)\sigma_y - \nu(\sigma_x+\sigma_y)\right]$$

$$= \frac{1}{207\text{ GPa}}\left[(1.29)(30\text{ MPa}) - (.29)(50\text{ MPa})\right]$$

$$= 11.7\text{ e-}5$$

$$e_z = \frac{1}{E}\left[-\nu(\sigma_x+\sigma_y)\right]$$

$$= \frac{-.29}{207\text{ GPa}}(50\text{ MPa})$$

$$= -7.00\text{ e-}5$$

$$\Delta x = e_x l_x = (5.46\text{e-}5)(300\text{ mm}) = \boxed{.0164\text{ mm}}$$

$$\Delta y = e_y l_y = (11.7\text{e-}5)(200\text{ mm}) = \boxed{.0234\text{ mm}}$$

$$\Delta z = e_z l_z = (-7.00\text{e-}5)(4\text{ mm}) = \boxed{-.00028\text{ mm}}$$