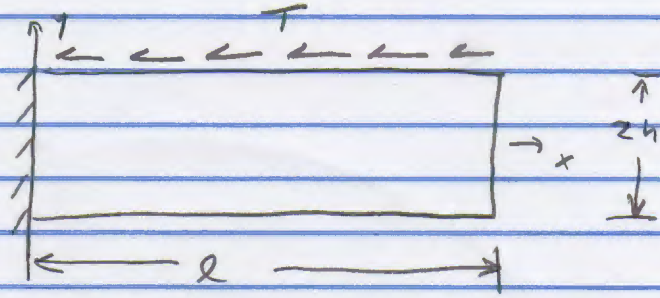
HW # 6

5-1 d)



$$T_x(l,y) = \sigma_x(l,y) = 0$$
$$T_y(l,y) = \tau_{xy}(l,y) = 0$$
$$T_x(x,h) = \tau_{xy}(x,h) = -T$$
$$T_y(x,h) = \sigma_y(x,h) = 0$$
$$T_x(x,-h) = -\tau_{xy}(x,-h) = 0$$
$$T_y(x,-h) = -\sigma_y(x,-h) = 0$$
$$u(0,y) = v(0,y) = 0$$

5-6 d)

$$\int_{-h}^{h} T_x(0,y)\,dy = Tl$$

$$\int_{-h}^{h} T_y(0,y)\,dy = 0$$

$$\int_{-h}^{h} T_x(0,y)\,y\,dy = Tlh$$

5-11

$$\sigma_x = Axy$$
$$\tau_{xy} = B + Cy^2$$
$$\sigma_y = 0$$

equilibrium (2-0), no body forces

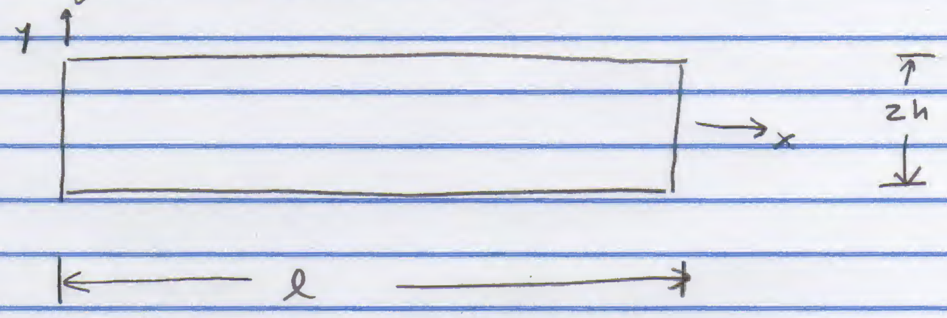
$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 = Ay + 2Cy \Rightarrow C = -A/2$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0 \checkmark$$

Eq 7.2.7

$$\nabla^2(\sigma_x + \sigma_y) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)(Axy + 0) = 0 \ \checkmark$$

Rectangular domain $(\ell >> h)$



left edge $(x = 0, -\frac{h}{2} \le y \le h/2)$

$$\sigma_x(0, y) = 0$$
$$\tau_{xy}(0, y) = B + Cy^2$$

right edge $(x = \ell, -h \le y \le h)$

$$\sigma_x(\ell, y) = A\ell y$$
$$\tau_{xy}(\ell, y) = B + Cy^2$$

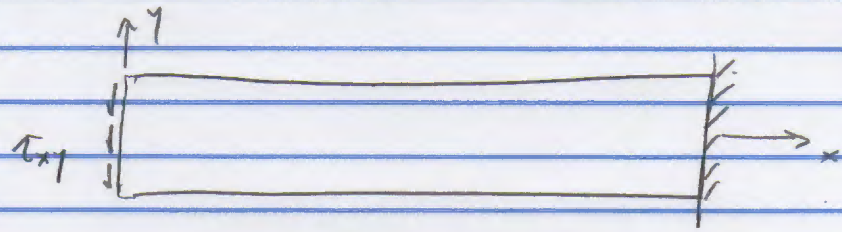
top & bottom edges $(0 \le x \le \ell, y = \pm h)$

$$\sigma_y(x, \pm h) = 0$$
$$\tau_{xy}(x, \pm h) = B + Ch^2$$

Requre $B = -Ch^2 \Rightarrow \tau_{xy}(x, \pm h) = 0$
(traction free top & bottom surfaces)

5-11 (cont.)

Consider cantilever beam



Net load on left edge (z-direction thickness = t)

$$\int_{-h}^{h} \tau_{xy}\, t\, dy = -P$$

$$\int_{-h}^{h} C(y^2 - h^2)\, t\, dy = -P$$

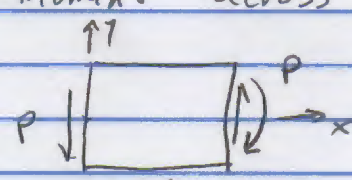
$$Ct\left(\frac{y^3}{3} - h^2 y\right)\Big|_{-h}^{h} = -P$$

$$Ct\left(\frac{4h^3}{3}\right) = -P$$

$$C = -\frac{3P}{4th^3} \qquad \frac{t(2h)^3}{12} = I \qquad = -\frac{P}{2I}$$

$$B = \frac{Ph^2}{2I}$$

net moment across section at any section



$$\Sigma M_0 = Px - M = 0$$

$$M = \int_{-h}^{h} \sigma_x\, y t\, dy = \int_{-h}^{h} A x y^2 t\, dy = \frac{2Axh^3 t}{3}$$

$$P\cancel{x} = \frac{\overset{8}{\cancel{24}}\,A\,\cancel{x}\,\cancel{h^3 t}}{\cancel{3}} \qquad \frac{I}{\frac{\cancel{t(2h)^3}}{\cancel{12}}}$$

$$A = \frac{P}{I}$$

$$\boxed{\begin{aligned} \sigma_x &= \frac{P}{I}\,x\,y \\ \tau_{xy} &= \frac{P}{2I}\left(h^2 - y^2\right) \\ \sigma_y &= 0 \end{aligned}}$$

matches beam theory