Homework Problems #3 - 2.6, 2.7, 2.10, 3.21, 3.24

1. Solve text problem 2.6

2.6 For polar coordinates defined by Figure 1.8, show that the transformation relations can be used to determine the normal and shear strain components e_r, e_θ, and $e_{r\theta}$ in terms of the corresponding Cartesian components

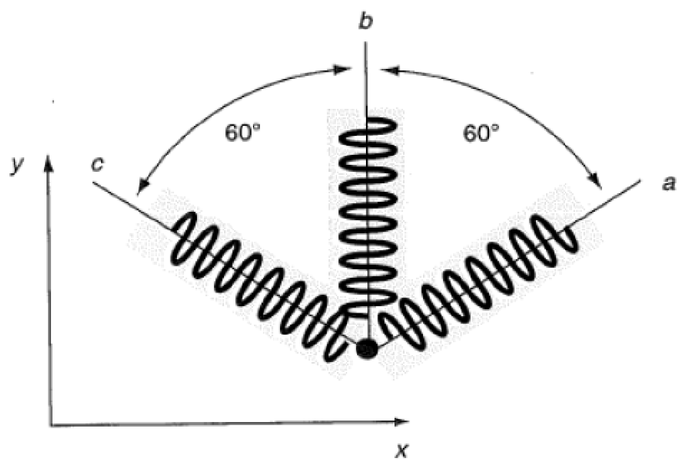
$$e_r = \frac{e_x + e_y}{2} + \frac{e_x - e_y}{2}\cos 2\theta + e_{xy}\sin 2\theta$$

$$e_\theta = \frac{e_x + e_y}{2} - \frac{e_x - e_y}{2}\cos 2\theta - e_{xy}\sin 2\theta$$

$$e_{r\theta} = \frac{e_y - e_x}{2}\sin 2\theta + e_{xy}\cos 2\theta$$

2. Solve text problem 2.7

2.7 A rosette strain gage is an electromechanical device that can measure relative surface elongations in three directions. Bonding such a device to the surface of a structure allows determination of elongational strains in particular directions. A schematic of one such gage is shown in the following figure, and the output of the device will provide data on the strains along the gage arms a, b, and c. During one application, it is found that $e_a = 0.001$, $e_b = 0.002$, and $e_c = 0.004$. Using the two-dimensional strain transformation relations, calculate the surface strain components e_x, e_y, and e_{xy}.



3. Solve text problem 2.10 (see example C.2 in Appendix C)

2.10* Using MATLAB®, determine the principal values and directions of the following state of strain

$$e_{ij} = \begin{bmatrix} 2 & -2 & 0 \\ -2 & -4 & 1 \\ 0 & 1 & 6 \end{bmatrix} \times 10^{-3}$$

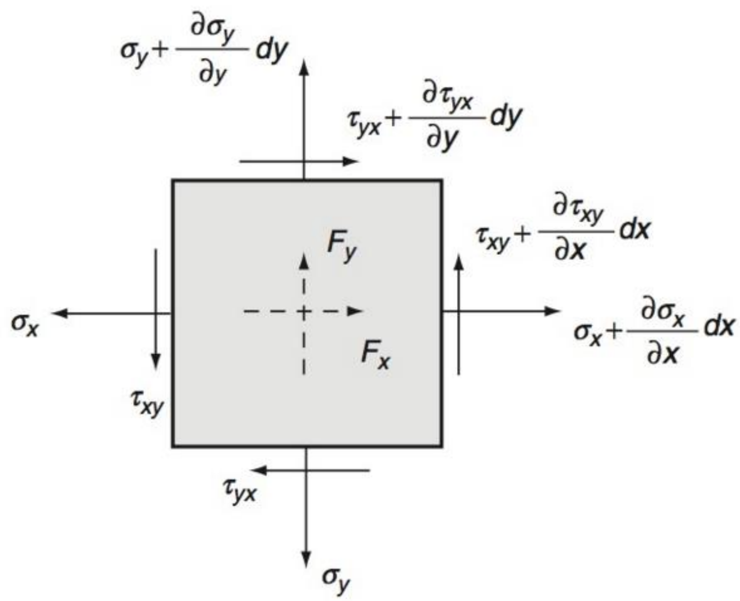
4. Solve text problem 3-21

Consider the equilibrium of a two-dimensional differential element in Cartesian coordinates, as shown in the following figure. Explicitly sum the forces and moments and develop the two-dimensional equilibrium equations

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + F_x = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + F_y = 0$$

$$\tau_{xy} = \tau_{yx}$$



5. Solve text problem 3-24

For a beam of circular cross-section, analysis from *elementary strength of materials theory* yields the following stresses:

$$\sigma_x = -\frac{My}{I}, \ \tau_{xy} = \frac{V(R^2 - y^2)}{3I}, \ \sigma_y = \sigma_z = \tau_{xz} = \tau_{yz} = 0$$

where R is the section radius, $I = \pi R^4/4$, M is the bending moment, V is the shear force, and $dM/dx = V$. Assuming zero body forces, show that these stresses do not satisfy the equilibrium equations. This result is one of many that indicate the approximate nature of strength of materials theory.