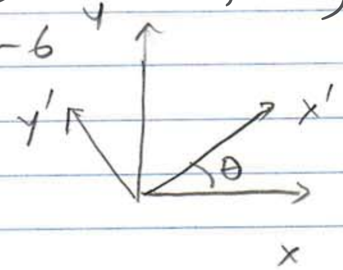
HW 3 - 2.6, 7, 10; 3.21, 24

1) 2-6



$$Q = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$e'_{ij} = \begin{bmatrix} e_r & e_{r\theta} \\ e_{r\theta} & e_\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} e_x & e_{xy} \\ e_{xy} & e_y \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$\vdots$

$$e_r = \frac{e_x+e_y}{2} + \frac{e_x-e_y}{2}\cos 2\theta + e_{xy}\sin 2\theta$$

$$e_\theta = \frac{e_x+e_y}{2} - \frac{e_x-e_y}{2}\cos 2\theta - e_{xy}\sin 2\theta$$

$$e_{r\theta} = \frac{e_y-e_x}{2}\sin 2\theta + e_{xy}\cos 2\theta$$

2) 2-7 For each gage, we can write

$$e_x' = e_x\cos^2\theta + e_y\sin^2\theta + 2e_{xy}\sin\theta\cos\theta$$

Gage 1: $\theta_1 = 30^\circ \Rightarrow \cos\theta_1 = \sqrt{3}/2,\ \sin\theta_1 = 1/2$

Gage 2: $\theta_2 = 90 \Rightarrow \cos\theta_2 = 0,\ \sin\theta_2 = 1$

Gage 3: $\theta_3 = 150^\circ \Rightarrow \cos\theta_3 = -\sqrt{3}/2,\ \sin\theta_3 = 1/2$

Matrix form

$$\begin{Bmatrix} e_a \\ e_b \\ e_c \end{Bmatrix} = \begin{Bmatrix} .001 \\ .002 \\ .004 \end{Bmatrix} = \begin{bmatrix} 3/4 & 1/4 & \sqrt{3}/2 \\ 0 & 1 & 0 \\ 3/4 & 1/4 & \sqrt{3}/2 \end{bmatrix} \begin{Bmatrix} e_x \\ e_y \\ e_{xy} \end{Bmatrix}$$

Solving (Matlab), $e_x = .00267,\ e_y = .002,\ e_{xy} = -.00173$

2-7 (cont.)

Matlab code:

```
format long; clc; clear all
A=[3/4 1/4 sqrt(3)/2; 0 1 0; 3/4 1/4 -sqrt(3)/2]
b=[.001; .002; .004]
x=A\b
```

Command Window output:

```
A =
   0.750000000000000   0.250000000000000   0.866025403784439
                   0   1.000000000000000                   0
   0.750000000000000   0.250000000000000  -0.866025403784439
b =
   0.001000000000000
   0.002000000000000
   0.004000000000000
x =
   0.002666666666667
   0.002000000000000
  -0.001732050807569
```

Problem 2.10

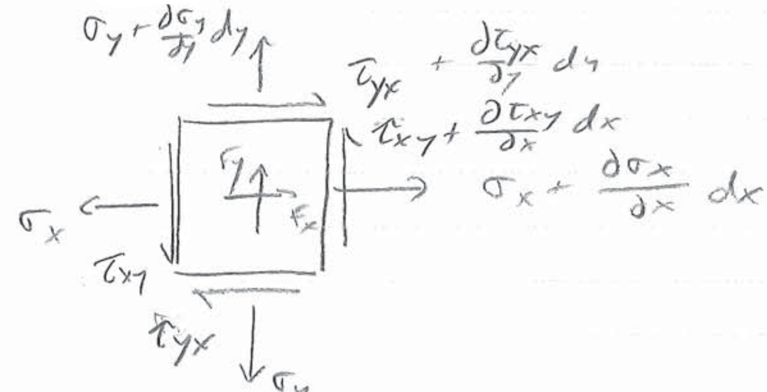
Matlab code

```
clc; clear; format long; format compact
%
e=[ 2 -2 0; -2 -4 1; 0 1 6]*1e-3
%
[evec,eval]=eig(e)
%
eigenvalues=diag(eval)
%
eigenvector_1=evec(:,1)
eigenvector_2=evec(:,2)
eigenvector_3=evec(:,3)
```

Command Window Output

```
e =
   0.002000000000000  -0.002000000000000                   0
  -0.002000000000000  -0.004000000000000   0.001000000000000
                   0   0.001000000000000   0.006000000000000
evec =
  -0.285232971001407   0.956998360782550  -0.052879955684759
  -0.954292956698998  -0.278422838944308   0.108653925602690
   0.089258641309364   0.081454651271142   0.992672168814006
eval =
  -0.004691322909470                   0                   0
                   0   0.002581866908772                   0
                   0                   0   0.006109456000698
eigenvalues =
  -0.004691322909470
   0.002581866908772
   0.006109456000698
eigenvector_1 =
  -0.285232971001407
  -0.954292956698998
   0.089258641309364
eigenvector_2 =
   0.956998360782550
  -0.278422838944308
   0.081454651271142
eigenvector_3 =
  -0.052879955684759
   0.108653925602690
   0.992672168814006
```

4) 3.21



$$\Sigma F_x = \left(\sigma_x + \frac{\partial \sigma_x}{\partial x} dx\right) dy - \sigma_x dy +$$

$$\left(\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y} dy\right) dx - \tau_{yx} dx + F_x \, dx dy = 0$$

$$\left(\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + F_x\right) dx dy = 0$$

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + F_x = 0$$

$$\Sigma F_y = \left(\cancel{\sigma_y} + \frac{\partial \sigma_y}{\partial y} dy\right) dx - \cancel{\sigma_y} dx + \left(\cancel{\tau_{xy}} + \frac{\partial \tau_{xy}}{\partial x} dx\right) dy$$

$$- \cancel{\tau_{xy}} dy + F_y \, dx dy = 0$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + F_y = 0$$

$$\Sigma M_{centroid} = -\left(\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y} dy\right) dx \left(\frac{dy}{2}\right) - \tau_{yx}\, dx \left(\frac{dy}{2}\right)$$

$$+\left(\tau_{xy} + \frac{\partial \tau_{xy}}{\partial y} dy\right)\left(\frac{dx}{2}\right) + \tau_{xy}\, dy \left(\frac{dx}{2}\right) = 0$$

$$= \left[(\tau_{xy} - \tau_{yx}) + \frac{1}{2}\left(\frac{\partial \tau_{xy}}{\partial y} - \frac{\partial \tau_{yx}}{\partial y}\right)\right] dx\, dy$$

$$\Rightarrow \tau_{xy} = \tau_{yx}$$

---

5) 3.24 $\sigma_x = -My/I, \quad \tau_{xy} = \dfrac{V(R^2 - y^2)}{3I}$

$$\sigma_y = \sigma_z = \tau_{xz} = \tau_{yz} = 0$$

$$F_x = F_y = F_z = 0$$

equilibrium

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + F_z$$

$$= -\frac{y}{I}\frac{\partial M}{\partial x} - \frac{2y}{3I} V \neq 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + F_y = \left(\frac{R^2 - y^2}{3I}\right)\frac{\partial V}{\partial x} \neq 0$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + F_z = 0$$

Hence, equilibrium is not satisfied