

$$a_{ij}n_j = \lambda n_i \quad (1.6.1)$$

$$\det[a_{ij} - \lambda \delta_{ij}] = -\lambda^3 + I_a \lambda^2 - II_a \lambda + III_a = 0 \quad (1.6.2)$$

where

$$\begin{aligned} I_a &= a_{ii} = a_{11} + a_{22} + a_{33} \\ II_a &= \frac{1}{2}(a_{ii}a_{jj} - a_{ij}a_{ji}) = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} + \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} \\ III_a &= \det[a_{ij}] \end{aligned} \quad (1.6.3)$$

$$\begin{aligned} \text{Gradient of a Scalar } \nabla \phi &= \phi_{,i} \mathbf{e}_i \\ \text{Gradient of a Vector } \nabla \mathbf{u} &= u_{i,j} \mathbf{e}_i \mathbf{e}_j \\ \text{Laplacian of a Scalar } \nabla^2 \phi &= \nabla \cdot \nabla \phi = \phi_{,ii} \\ \text{Divergence of a Vector } \nabla \cdot \mathbf{u} &= u_{i,i} \\ \text{Curl of a Vector } \nabla \times \mathbf{u} &= \varepsilon_{ijk} u_{k,j} \mathbf{e}_i \\ \text{Laplacian of a Vector } \nabla^2 \mathbf{u} &= u_{i,kk} \mathbf{e}_i \end{aligned} \quad (1.8.4)$$

$$\iint_S \mathbf{u} \cdot \mathbf{n} \, dS = \iiint_V \nabla \cdot \mathbf{u} \, dV \quad (1.8.6)$$

$$\begin{aligned} e_{ij} &= \frac{1}{2}(u_{i,j} + u_{j,i}) \\ \omega_{ij} &= \frac{1}{2}(u_{i,j} - u_{j,i}) \end{aligned} \quad (2.1.7)$$

$$e'_{ij} = Q_{ip} Q_{jq} e_{pq} \quad (2.3.1)$$

$$\begin{aligned} Q_{ij} &= \cos(x'_i, x_j) \\ Q_{ij} &= \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned} \quad (2.3.4)$$

$$\tilde{e}_{ij} = \frac{1}{3} e_{kk} \delta_{ij} = \frac{1}{3} \vartheta \delta_{ij} \quad (2.5.1)$$

$$\hat{e}_{ij} = e_{ij} - \frac{1}{3} e_{kk} \delta_{ij} \quad (2.5.2)$$

$$e_{ij,kl} + e_{kl,ij} - e_{ik,jl} - e_{jl,ik} = 0 \quad (2.6.1)$$

$$\begin{aligned}
\frac{\partial^2 e_x}{\partial y^2} + \frac{\partial^2 e_y}{\partial x^2} &= 2 \frac{\partial^2 e_{xy}}{\partial x \partial y} \\
\frac{\partial^2 e_y}{\partial z^2} + \frac{\partial^2 e_z}{\partial y^2} &= 2 \frac{\partial^2 e_{yz}}{\partial y \partial z} \\
\frac{\partial^2 e_z}{\partial x^2} + \frac{\partial^2 e_x}{\partial z^2} &= 2 \frac{\partial^2 e_{zx}}{\partial z \partial x} \\
\frac{\partial^2 e_x}{\partial y \partial z} &= \frac{\partial}{\partial x} \left(-\frac{\partial e_{yz}}{\partial x} + \frac{\partial e_{zx}}{\partial y} + \frac{\partial e_{xy}}{\partial z} \right) \\
\frac{\partial^2 e_y}{\partial z \partial x} &= \frac{\partial}{\partial y} \left(-\frac{\partial e_{zx}}{\partial y} + \frac{\partial e_{xy}}{\partial z} + \frac{\partial e_{yz}}{\partial x} \right) \\
\frac{\partial^2 e_z}{\partial x \partial y} &= \frac{\partial}{\partial z} \left(-\frac{\partial e_{xy}}{\partial z} + \frac{\partial e_{yz}}{\partial x} + \frac{\partial e_{zx}}{\partial y} \right)
\end{aligned} \tag{2.6.2}$$

$$T_i^n = \sigma_{ji} n_j \tag{3.2.6}$$

$$\tilde{\sigma}_{ij} = \frac{1}{3} \sigma_{kk} \delta_{ij} \tag{3.5.1}$$

$$\hat{\sigma}_{ij} = \sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij} \tag{3.5.2}$$

$$\begin{aligned}
N = \sigma_{oct} &= \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) = \frac{1}{3}\sigma_{kk} = \frac{1}{3}I_1 \\
S = \tau_{oct} &= \frac{1}{3}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2} \\
&= \frac{1}{3}(2I_1^2 - 6I_2)^{1/2}
\end{aligned} \tag{3.5.4}$$

$$\sigma_e = \sigma_{vonMises} = \frac{1}{\sqrt{2}}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2} \tag{3.5.5}$$

$$\sigma_{ij,j} + F_i = 0 \tag{3.6.8}$$

$$\sigma_{ij} = \lambda e_{kk} \delta_{ij} + 2\mu e_{ij} \tag{4.2.7}$$

$$e_{ij} = \frac{1+\nu}{E} \sigma_{ij} - \frac{\nu}{E} \sigma_{kk} \delta_{ij} \tag{4.2.10}$$