8-1, 3 and 4

8.1 Explicitly show that the fourth-order polynomial Airy stress function

$$A_{40}x^4 + A_{22}x^2y^2 + A_{04}y^4$$

will not satisfy the biharmonic equation unless $3A_{40} + A_{22} + 3A_{04} = 0$.

8.3 Determine the displacement field for the beam problem in Exercise 8.2. To determine the rigid-body motion terms, choose fixity conditions

$$u(L,0) = v(L,0) = \frac{\partial v(L,0)}{\partial x} = 0$$

Note that with our approximate Saint—Venant solution, we cannot ensure pointwise conditions all along the built-in end. Finally, for the special case with $N = 0$, compare the elasticity displacement field with the corresponding results from mechanics of materials theory (see Appendix D). *Answer*:

$$N = 0: \quad v_{elasticity}(x,0) = \frac{P}{4Ec^3}\left(x^3 - 3L^2x + 2L^3\right) = v_{MOM}(x)$$

8.4 The solution to the illustrated two-dimensional cantilever beam problem is proposed using the Airy stress function $\phi = C_1x^2 + C_2x^2y + C_3y^3 + C_4y^5 + C_5x^2y^3$, where C_i are constants. First determine requirements on the constants so that ϕ satisfies the governing equation. Next find the values of the remaining constants by applying exact pointwise boundary conditions on the top and bottom of the beam and integrated resultant boundary conditions on the ends $x = 0$ and $x = L$.