9-9, 11, 22, 23

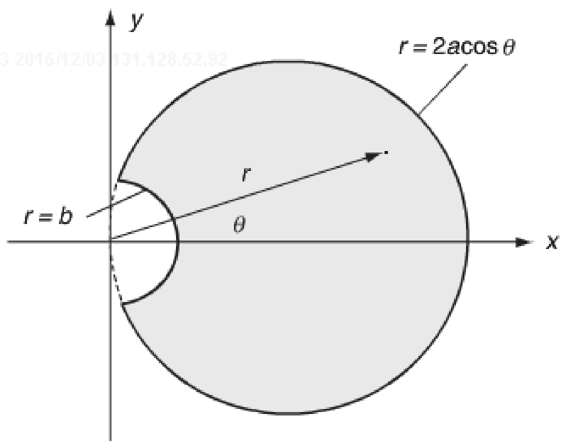
9.9 Using the stress results for the torsion of the elliptical section, formally integrate the strain−displacement relations and develop the displacement solution (9.4.11).

9.11 Develop relation (9.4.14) for the load-carrying torque of an equilateral triangular section.

9.22 A circular shaft with a keyway can be approximated by the section shown in the following figure. The keyway is represented by the boundary equation $r = b$, while the shaft has the boundary relation $r = 2a\cos\theta$. Using the technique of Section 9.4, a trial stress function is suggested of the form

$$\phi = K(b^2 - r^2)\left(1 - \frac{2a\cos\theta}{r}\right)$$

where K is a constant to be determined. Show that this form will solve the problem and determine the constant K. Compute the two shear stress components τ_{xz} and τ_{yz}.



9.23* For the keyway section of Exercise 9.22, show that resultant stresses on the shaft and keyway boundaries are given by

$$\tau_{shaft} = \mu\alpha a\left(\frac{b^2}{4a^2\cos^2\theta} - 1\right), \quad \tau_{keyway} = \mu\alpha(2a\cos\theta - b)$$

Determine the maximum values of these stresses, and show that for $b << a$, the magnitude of the maximum keyway stress is approximately twice that of the shaft stress. Finally, make a plot of the stress concentration factor

$$\frac{(\tau_{\max})_{keyway}}{(\tau_{\max})_{solid\ shaft}}$$

versus the ratio b/a over the range $0 \le b/a \le 1$. Note that $(\tau_{\max})_{solid\ shaft}$ is the maximum shear stress for a solid shaft of circular section and can be determined from Example 9.1 or strength of materials theory. Show that the stress concentration plot gives

$$\frac{(\tau_{\max})_{keyway}}{(\tau_{\max})_{solid\ shaft}} \to 2, \quad \text{as } b/a \to 0$$

thus indicating that a small notch will result in a doubling of the stress in a circular section under torsion.