Homework Problems 1-1b, 2b, and 5

1.1 For the given matrix/vector pairs, compute the following quantities: a_{ii}, $a_{ij}a_{ij}$, $a_{ij}a_{jk}$, $a_{ij}b_j$, $a_{ij}b_ib_j$, b_ib_j, b_ib_i. For each case, point out whether the result is a scalar, vector or matrix. Note that $a_{ij}b_j$ is actually the matrix product $[\boldsymbol{a}]\{\boldsymbol{b}\}$, while $a_{ij}a_{jk}$ is the product $[\boldsymbol{a}][\boldsymbol{a}]$.

$$\text{(b)}\quad a_{ij} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 2 & 1 \\ 0 & 4 & 2 \end{bmatrix}, \quad b_i = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

1.2 Use the decomposition result (1.2.10) to express a_{ij} from Exercise 1.1 in terms of the sum of symmetric and antisymmetric matrices. Verify that $a_{(ij)}$ and $a_{[ij]}$ satisfy the conditions given in the last paragraph of Section 1.2.

1.5 Formally expand the expression (1.3.4) for the determinant and justify that either index notation form yields a result that matches the traditional form for $\det[a_{ij}]$.