Homework Problems #2 – 1-6, 12b, 14b; 2-1b, 3

1.6 Determine the components of the vector b_i and matrix a_{ij} given in Exercise 1.1 in a new coordinate system found through a rotation of 45° ($\pi/4$ radians) about the x_1-axis. The rotation direction follows the positive sense presented in Example 1.2.

1.12 Determine the invariants, and principal values and directions of the following matrices. Use the determined principal directions to establish a principal coordinate system, and following the procedures in Example 1.3, formally transform (rotate) the given matrix into the principal system to arrive at the appropriate diagonal form.

$$\text{(b)} \begin{bmatrix} -2 & 1 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{(Answer : } \lambda_i = -3,\ -1,\ 0)$$

1.14 Calculate the quantities $\nabla \cdot \boldsymbol{u}$, $\nabla \times \boldsymbol{u}$, $\nabla^2 \boldsymbol{u}$, $\nabla \boldsymbol{u}$, $tr(\nabla \boldsymbol{u})$ for the following Cartesian vector fields:

(b) $\boldsymbol{u} = x_1^2 \boldsymbol{e}_1 + 2x_1x_2\boldsymbol{e}_2 + x_3^3\boldsymbol{e}_3$

2.1 Determine the strain and rotation tensors e_{ij} and ω_{ij} for the following displacement fields:

$$\text{(b)} \quad u = Ax^2, \quad v = Bxy, \quad w = Cxyz$$

where A, B, and C are arbitrary constants.

2.3 A two-dimensional problem of a rectangular bar stretched by uniform end loadings results in the following constant strain field

$$e_{ij} = \begin{bmatrix} C_1 & 0 & 0 \\ 0 & -C_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

where C_1 and C_2 are constants. Assuming the field depends only on x and y, integrate the strain–displacement relations to determine the displacement components and identify any rigid-body motion terms.