MACM 316 – Assignment 4

Due Date: October 24th, at 11pm.

You must upload both your code (to Computing Code 4) and your report (to Computing Report 4) in Crowdmark. The assignment is due at 11:00pm. If you submit late, you will be given 0 on the assignment. Your computing report must be exactly 1 page. There will be a penalty given if your report is longer than one page. Your code needs to be printed out from Matlab as a .pdf file in order to upload it to Crowdmark.

- Please read the **Guidelines for Assignments** first.
- Please use the Canvas discussion board and please keep in mind that Canvas discussions are open forums.
- Acknowledge any collaborations and assistance from colleagues/TAs/instructor.

A. Computing Assignment – Finding zeros of Bessel functions

Required submission: 1 page PDF document and scripts/codes uploaded to Canvas.

The Bessel function $J_0(x)$ of the first kind is the solution of Bessel's differential equation

$$x^2 \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + x \frac{\mathrm{d}y}{\mathrm{d}x} + x^2 y = 0,$$

that is finite at the origin $x = 0$ (the other solution, the Bessel function of the second kind $Y_0(x)$ is singular at the origin). In Matlab you can evaluate $J_0(x)$ using the command *besselj(0,x)*. A plot of $J_0(x)$ is given on the next page.

A number of important physical applications require explicit calculation of the zeros of $J_0(x)$. It is known that $J_0(x)$ has infinitely-many zeros in the range $0 < x < \infty$. We shall write these zeros as $0 < x_1 < x_2 < x_3 < \ldots < \infty$.

In this assignment you will use the bisection method to compute the first M of these zeros. It is up to you to choose appropriate parameters a, b and TOL for the bisection method. To aid in choosing a and b, I strongly recommend you plot $J_0(x)$ first.

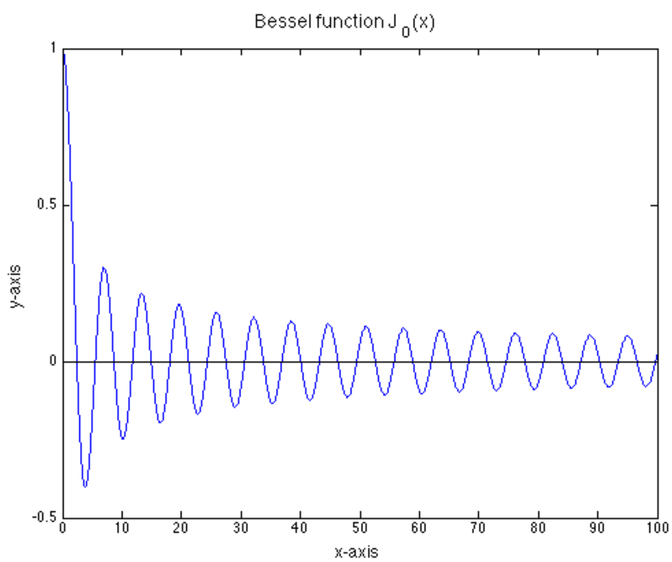
Once you have computed the first M zeros, your goal is to determine the asymptotic behaviour of the M^{th} zero x_M as $M \to \infty$. You should find that x_M obeys an approximate linear relationship for large M. That is to say

$$x_M = \alpha(M + \beta) + \mathcal{O}(1/M), \quad M \to \infty,$$

for certain constants α and β. Using the roots you have calculated, find approximate values of α and β. Make sure to take M large enough to get good approximations for α and β.

Your conclusions should be explained in a one-page report. Make sure to include the following:

- Justification for the values of a, b, TOL and M you chose in terms of the three key concepts of the course: *accuracy*, *efficiency* and *robustness*.
- The values of α and β you computed, and an explanation as to how you found them.
- A hypothesis, based on the values you computed, as to the exact values of α and β.


Bessel function J 0(x)
1
0.5
0
-0.5
y-axis
0
10
20
30
40
50
60
70
80
90
100
x-axis