MACM 316 – Computing Assignment 7

Due Date: Monday November 21st, at 11pm.

You must upload both your code (to Computing Code 7) and your report (to Computing Report 7) in Crowdmark. The assignment is due at 11:00pm. If you submit late, you will be given 0 on the assignment. Your computing report must be exactly 1 page. There will be a penalty given if your report is longer than one page. Your code needs to be printed out from Matlab as a .pdf file in order to upload it to Crowdmark.

- Please read the **Guidelines for Assignments** first.
- Please use the Canvas discussion board and please keep in mind that Canvas discussions are open forums.
- Acknowledge any collaborations and assistance from colleagues/TAs/instructor.

Computing Assignment – Trapezoidal rule

Required submission: **1 page** PDF document and Matlab code uploaded to Crowdmark.

Consider a partition $x_0, \dots, x_N$ of an interval $[a, b]$. For an integrable function f defined in the interval $[a, b]$, the *trapezoidal rule* for the integral of f is

$$\int_a^b f(x)dx \approx \frac{1}{2}\sum_{k=1}^{N} \big(f(x_{k-1}) + f(x_k)\big)(x_k - x_{k-1}).$$

In general, the distance between consecutive points in the partition of $[a, b]$ does not have to be the same. When, this partition consists of equally-spaced points, we have the formula

$$\int_a^b f(x)dx \approx \frac{h}{2}\sum_{k=1}^{N} \big(f(x_{k-1}) + f(x_k)\big), \quad h := x_k - x_{k-1}, \quad \forall\, k.$$

The goal of this assignment is to investigate the rate of convergence of this rule depending on the functions we want to integrate. To do this, first download from Canvas the MATLAB function that implements the trapezoidal rule. It has the following inputs: lower and upper bounds of the interval, number of points in the partition, and the function you want to integrate. To test the function, compute the integral of the function $f(x) := x^3$ over $[0, 1]$ with $N = 100$ points. What is the value you get?

Consider the intervals $I_1 := [0, \frac{\pi}{3}]$ and $I_2 := [0, 2\pi]$. Approximate the integrals over both I_1 and I_2 of the functions $f_1(x) := \sin(\frac{1}{2}x)$, $f_2(x) := |\sin(2x)|$, and $f_3(x) := \cos(x)$ by using the Trapezoidal rule. Then plot the absolute error of the computed integral and its corresponding true value. Note that the true values can be easily computed by hand. Also, find the rate of convergence at which each error is going to zero. Which one is the fastest? Note that one error plot is always close to machine epsilon, what do you think is causing this extreme accuracy? This last question is difficult - you should use an internet search to try and get some information on what is happening and perhaps why.

Your conclusions should be explained in a one-page report. Your report **must** include the following:

(a) Output (value of the integral) of your test case $f(x) = x^3$ on $[0, 1]$ with $N = 100$ points.

(b) Two loglog plots of the absolute errors, one for I_1 and one for I_2.

(c) Rate of convergence of each error (6 in total). Why is one of the plots around machine epsilon for almost any values of N?

(d) Make sure you answer all the questions in the document.