Computing Assignment 2

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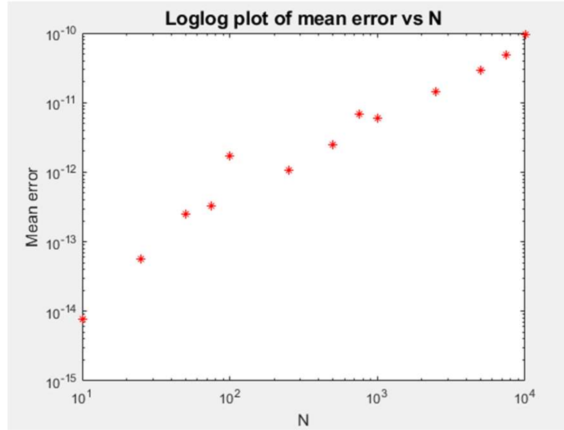
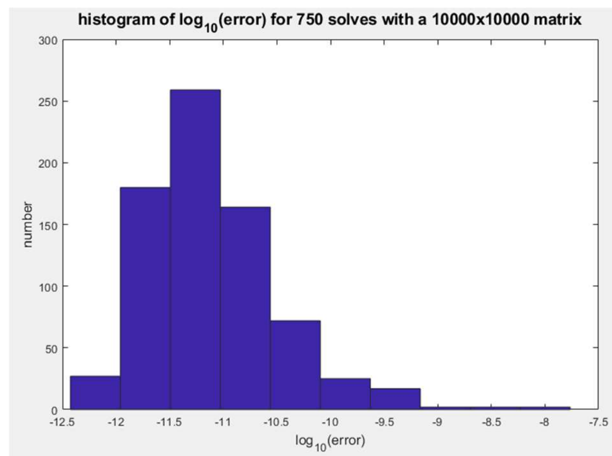
a) Here I plotted the mean error (E_N) vs the size of the matrices (N) on a loglog plot. Where E_N is on the y axis and N is on the x axis. I noticed almost immediately that with a logarithmic scale, a linear relation can be observed. This implies that the error of gaussian elimination increases logarithmically in relation to the size of the matrix being operated on. The sizes of the matrices operated on are 10, 25, 50, 75, 100, 250, 500, 750, 1000, 2500, 5000, 7500 and 10000. And the number of trials (M) I completed per matrix is 750.

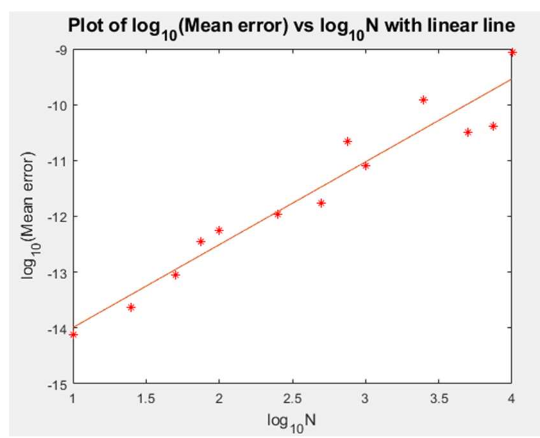


b) Originally, my values of N were 10, 100, 1000, and 10000 as I noticed these values seemed to increase E_N logarithmically. That is, it seemed that as N increased by a factor of 10, so did E_N. I then added 4 evenly spaced values between each factor of 10 to see more detail in the pattern.

I also noticed that, from the histogram generated each time by the algorithm, that the error produced is normally distributed. Since the error produced is normally distributed, more trials yield more accurate data. As such I gradually increased the number of trials for each value of N until the program took too long to run on my PC. This resulted in my value of M being set to 750. A screenshot of the histogram discussed in this paragraph can be seen to the right.



c & d) Now with my values of N and M set, I proceeded to create a best fit linear line through the graph of $\log_{10}(N)$ vs $\log_{10}(E_N)$ to confirm my hypothesis of a logarithmic relation. This graph can be seen to the right and confirmed my theory.



Now with my theory confirmed, I took the equation of the best fit line and asked MATLAB to find the roots of this equation, as $\log_{10}(1) = 0$, meaning that to find 100% error by extrapolation I need to find when the line is equal to 0. The result of this was $\log_{10}(N^*) = 11.3547$. I then took $10^{11.3547}$ or in more simple terms, I took the inverse log of this result. The output of this execution of the program gave me $N^* = 2.2629 \times 10^{11}$ to be the size of a matrix that would yield in 100% mean error for Gaussian Elimination.

To both show the extrapolation process I went through to gain this result, and to confirm that I performed the process correctly, I then took the line produced in the previous graph and extended its range. I exteded its range to reach y = 0, which would show the root of the equation that I gained before, assuming the process was correct. Clicking on the end of the line in the graph that MATLAB produced, I was shown the same result as before ($\log_{10}(N^*) = 11.3547$). Therefore, my final estimation produced for N^* is 2.2629×10^{11}. The graph discussed in this paragraph can be seen to the right.