MACM 316 – Assignment 5

Due Date: October 31st, at 11pm.

You must upload both your code (to Computing Code 4) and your report (to Computing Report 4) in Crowdmark. The assignment is due at 11:00pm. If you submit late, you will be given 0 on the assignment. Your computing report must be exactly 1 page. There will be a penalty given if your report is longer than one page. Your code needs to be printed out from Matlab as a .pdf file in order to upload it to Crowdmark.

- Please read the **Guidelines for Assignments** first.
- Please use the Canvas discussion board and please keep in mind that Canvas discussions are open forums.
- Acknowledge any collaborations and assistance from colleagues/TAs/instructor.

A. Computing Assignment – Polynomial interpolation and node distribution

Required submission: 1 page PDF document and scripts/codes uploaded to Canvas.

The purpose of this assignment is to examine how the locations of the nodes $x_0, x_1, \ldots, x_n$ affect the accuracy and robustness of polynomial interpolation. To compute the interpolating polynomial, you will be using the barycentric form described in lectures. For this, you need to download the matlab functions *baryweights.m* and *baryinterp.m* from the computing assignment page in Canvas. The first computes the weights $w_0, w_1, \ldots, w_n$ of the barycentric form and the second computes the interpolating polynomial.

To begin, consider the equally-spaced nodes on $[-1, 1]$, given by

$$x_i = -1 + \frac{2i}{n}, \quad i = 0, \ldots, n. \tag{1}$$

Write code to compute the error of the interpolating polynomial

$$e_n = \max_{-1 \leq x \leq 1} |P(x) - f(x)|,$$

for some suitable range of n (in practice, you should replace this maximum by the maximum on some sufficiently fine grid) and plot $\log_{10}(e_n)$ versus n for the test functions:

$$f_1(x) = \frac{1}{5 - 4x}, \qquad f_2(x) = \frac{1}{1 + 16x^2}.$$

Using this, comment on the accuracy of polynomial interpolation at equally-spaced nodes.

Next, consider the so-called *Chebyshev nodes* on $[-1, 1]$, given by

$$x_i = \cos(i\pi/n), \quad i = 0, \ldots, n. \tag{2}$$

Repeat the previous experiment with these nodes instead of (1) and test it on the functions above. Note that in this case you should not use the function *baryweights.m* to find the weights, but instead use the known formula

$$w_0 = \frac{1}{2}, \quad w_j = (-1)^j,\ j = 1, \ldots, n-1, \quad w_n = \frac{1}{2}(-1)^n,$$

(if you don't do this, your computation may result in under/overflow). Is polynomial interpolation at Chebyshev nodes accurate? Is it robust?

Finally, use the Chebyshev nodes to approximate the function

$$f_3(x) = \cos(10^4 x).$$

Find the smallest value of n (to within ± 10) such that $e_n \leq 10^{-5}$.