MACM 316 – Computing Assignment 2

Due Date: September 26th, at 11pm.

You must upload both your code (to Computing Code 2) and your report (to Computing Report 2) in Crowdmark. The assignment is due at 11:00pm. If you submit late, you will be given 0 on the assignment. Your computing report must be exactly 1 page. There will be a penalty given if your report is longer than one page. Your code needs to be printed out from Matlab as a .pdf file in order to upload it to Crowdmark.

- Please read the **Guidelines for Assignments** first.
- Please use the Canvas discussion board and please keep in mind that Canvas discussions are open forums..
- Acknowledge any collaborations and assistance from colleagues/TAs/instructor.

Computing Assignment – Gaussian Elimination on a large random matrix

Required submission: 1 page PDF document and Matlab code uploaded to Canvas.

To complete this assignment, I suggest you download in class demo *LSRandom.m* from Canvas (lecutre notes page). You will need to modify this script suitably in order to complete the assignment. Also, you may find the following Matlab commands useful:

1. A=spdiags(rand(N,3), -1:1, N,N); which creates a tridiagonal matrix with random entries along each diagonal.
2. loglog(N_it, Err, 'r*'); which plots the error versus N in a loglog plot.
3. p=polyfit(log10(N_it), log10(Err), 1); which fits a straight line to the loglog data.

In class we saw that finite-precision computations with Gaussian Elimination, implemented via Matlab's backslash command, leads to small errors for small matrices but possibly larger errors for larger matrices. The purpose of this assignment is to quantify the growth of this error for **tridiagonal** matrices.

A tridiagonal matrix is a matrix with non-zero entries only on the main diagonal, as well as the diagonals above and below the main diagonal.

$$A = \begin{bmatrix} a_{1,1} & a_{2,1} & 0 & \cdots & 0 \\ a_{1,2} & a_{2,2} & a_{2,3} & & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & & a_{n-1,n-2} & a_{n-1,n-1} & a_{n-1,n} \\ 0 & \cdots & 0 & a_{n,n-1} & a_{n,n} \end{bmatrix}.$$

Let A be a random $N \times N$ tridiagonal matrix, $\underline{x} = (1, 1, \ldots, 1)^\top$ be an N-vector of ones and $\underline{b} = A\underline{x}$ be the right-hand side vector. As in class, let $\underline{z} = (z_j) \in \mathbb{R}^N$ be the resulting vector

$$\underline{z} = A \backslash \underline{b},$$

computed in finite precision using the backslash command. To measure the error between $\underline{x}$ and $\underline{z}$, we let

$$\delta = \max_{j=1,\ldots,N} |x_j - z_j|,$$

be the maximum componentwise difference between the two vectors. Since A is a matrix with random values, we need to run this calculation a number of times with different realizations of A in order to get a reasonable value for δ. Let M be the number of trials and suppose that for the k^{th} trial the error is $\delta^{(k)}$. We define the mean error as follows:

$$E_N = \frac{1}{M}\left(\delta^{(1)} + \delta^{(2)} + \ldots + \delta^{(M)}\right).$$

The goal of this assignment is to investigate the size of the matrix $N = N^*$ at which the mean error for Gaussian Elimination is $E_N \approx 1$. In other words, the point at which round-off error in Gaussian elimination is of the same magnitude as the vector $\underline{x}$.

In practice, your computer will likely not have the processing power to find N^* exactly. Instead, you should extrapolate your data to find an estimate of N^*. Find E_N for reasonable values of N, make a plot of $\log_{10}(N)$ versus $\log_{10}(E_N)$ and then perform a suitable extrapolation.

Your conclusions should be explained in a one-page report. Your report **must** include the following:

(a) A plot of E_N versus N for the values of N you choose.

(b) Justification for the values of N and M you chose.

(c) Explanation of how you do the extrapolation.

(d) An estimation of the number N^*.