$$f(x) = \frac{\sin\left(\frac{\log(x)}{x}\right)}{x}$$

The zeros of the function $f(x)$ are where

$\frac{\log(x)}{x} = -i\pi$ where $i = 1, 2, 3, \dots$

We want to find all of the zeros in the interval $(0,1)$

① Plot the function using Wolfram alpha so you can see what it looks like.

② I notice that there appears to be a zero at $x=1$ and then increasingly more zeros as x approaches 0

Check if $x=1$ is a zero $\quad f(1) = \frac{\sin\left(\frac{\log(1)}{1}\right)}{1} = \sin(0) = 0$ ✓

There appears to be no zeros after $x=1$. This can be checked by taking the derivative etc.

The question is then, where does

$$\frac{\sin\left(\frac{\log(x)}{x}\right)}{x} = 0 \Rightarrow \sin\left(\frac{\log(x)}{x}\right) = 0 \Rightarrow \frac{\log(x)}{x} = -i\pi$$

$i = 1, 2, 3, \dots$

and how do I efficiently find those using MATLAB.

There are several ways that you could consider using the fzero command to do this.

Here is the way that is outlined in the assignment with more steps to make it easier to follow the algebra. Note that i here is an index and NOT the imaginary constant $\sqrt{-1}$

We want to solve

This is because $\log(a_i) < 0$ when $0 < a_i < 1$ and a_i is positive

$$\frac{\log(a_i)}{a_i} = -i\pi$$

where a_1 is the value that makes this expression true when $i=1$ and a_2 is the value that makes this expression true when $i=2$ etc.

Then $\frac{\log(a_i)}{a_i} + i\pi = 0$ given $e^{\log(a_i)} = a_i$, $e^{-\log(a_i)} = \frac{1}{a^i}$

$$\log(a_i) \cdot e^{-\log(a_i)} + i\pi = 0$$

Let $b_i = -\log(a_i)$ Then $a_i = e^{-b_i}$

And $-b_i \cdot e^{b_i} + i\pi = 0$ or

$$\boxed{b_i e^{b_i} - i\pi = 0 \quad \text{and} \quad a_i = e^{-b_i}}$$