MACM 316 – Assignment 6

Due Date: November 7th, at 11pm.

You must upload both your code (to Computing Code 6) and your report (to Computing Report 6) in Crowdmark. The assignment is due at 11:00pm. If you submit late, you will be given 0 on the assignment. Your computing report must be exactly 1 page. There will be a penalty given if your report is longer than one page. Your code needs to be printed out from Matlab as a .pdf file in order to upload it to Crowdmark.

- Please read the **Guidelines for Assignments** first.
- Please use the Canvas discussion board and please keep in mind that Canvas discussions are open forums.
- Acknowledge any collaborations and assistance from colleagues/TAs/instructor.

A. Computing Assignment – Numerical Integration of an unpleasant function

In this assignment you will be computing the integral

$$I = \int_0^1 x^{-1} \sin(x^{-1} \log(x)) \, \mathrm{d}x.$$

It is known that $I \approx -0.5$. This is a challenging integral to compute, however, because the integrand $f(x) = x^{-1} \sin(x^{-1} \log(x))$ has infinitely-many oscillations in the interval $[0, 1]$ and is also singular at $x = 0$. You may quickly want to check that any standard numerical quadrature (e.g. Trapezoidal, Simpson's, etc) gives you poor approximations to I.

To compute this integral, you will be using a subdivision scheme based on splitting $[0, 1]$ into subintervals defined by the zeros of $f(x)$. Note that $f(x) = 0$ at the points

$$1 > a_1 > a_2 > a_3 > \ldots > 0,$$

where $a_i = \exp(-b_i)$ and b_i is the unique solution to the equation

$$b \exp(b) - i\pi = 0, \quad 1 < b < \infty. \tag{0.1}$$

To compute I, you first need to find these roots. Recalling Part 3 of the course, this can be done in Matlab using the *fzero* command:

```
b=fzero(@(x) x*exp(x)-i*pi,0);
a(i)=exp(-b);
```

Given the $a_1, a_2, \ldots, a_n$, we can now approximate I as follows:

$$I \approx Q_n = \sum_{i=0}^{n} I_i, \qquad I_i = \int_{a_i}^{a_{i+1}} f(x) \, \mathrm{d}x, \qquad a_0 = 1.$$

Write a code that computes Q_n by applying a standard numerical quadrature to each integral I_i. I recommend you use a built-in routine to evaluate each I_i, e.g. Matlab's *integral* or *quad* commands. List your results for $n = 100, 200, 300, \ldots, 1000$. How many digits of I can you accurately compute?

You will hopefully have noticed that Q_n is converging to I rather slowly. Fortunately, there's a way to get a faster converging approximation, known as Aitken's Δ^2 Method (see Burden & Faires, Sec 2.5). Given the sequence $\{Q_n\}_{n=0}^{\infty}$ we define the new sequence $\{\hat{Q}_n\}_{n=0}^{\infty}$ by

$$\hat{Q}_n = Q_n - \frac{(Q_{n+1} - Q_n)^2}{Q_{n+2} - 2Q_{n+1} + Q_n}.$$

Compute this new sequence and use it to get as good an approximation to I as you can (in reasonable computing time). State your answer to as many digits as you believe are correct. Make sure to justify the number of digits you give.

Computing Assignment - Bonus credit

This bonus questions is worth one mark and must be submitted as part of your one-page report. It is therefore possible to get 6 marks for this computing assignment.

Adapt your code to compute the integral

$$J = \int_0^1 x^{-1} \cos(x^{-2} \log(x))\, dx.$$

To get the bonus mark you must (i) explain the changes you made to your code and (ii) get an answer that is correct to at least 10 digits.