Computing Assignment 6

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D100

During this computing assignment I computed an approximation to the integral $I = \int_0^1 x^{-1}\sin(x^{-1}\log(x))\,dx$ using a subdivision scheme, splitting the interval into subintervals defined by the zeros of $f(x) = x^{-1}\sin(x^{-1}\log(x))$, and then computing a sum of the integral of $f(x)$ for each subinterval. I found that I could compute the integral I accurate to 4 decimal spaces using this method. I then improved my approximation by implementing the Aitken's Δ^2 method, and found that for N = 7000, I could get accurate up to 6 decimal spaces. I picked N = 7000 as this is the highest N I can compute for within reasonable time. My estimate for the integral I is $I \approx 0.459382$, based on the output from my code pasted below. I chose to state 6 digits of accuracy, as for the last 5 iterations of my code, the 6th digit remained unchanged, while the 7th digit and beyond changed.

```
Last 5 Q hat for original function:
          0.459383800455442

          0.459383441137745

          0.459383092985969

           0.45938275548299

          0.459382428148905

          0.459382110530889
```

I then modified my code to compute the integral $J = \int_0^1 g(x)dx,\ where\ g(x) = x^{-1}\cos(x^{-2}\log(x))$. I did this by attempting to find the zeros for $g(x)$, which I did by first observing that $g(x) = 0$ when $\cos(x^{-2}\log(x)) = 0$. Knowing that $\cos(x) = 0$ when $x = -\frac{\pi}{2}i\ where\ i = \{1, 2, 3, \ldots\}$, I reasoned that the zeros of $g(x)$ are where $\frac{\log(x)}{x^2} = -i\frac{\pi}{2}$. It then follows that for zeros a_i, $\frac{\log(a_i)}{a_i^2} + i\frac{\pi}{2} = 0$ and as such $\log(a_i)e^{-2\log(a_i)} + i\pi = 0$. Then, letting $b_i = -2\log(a_i)$ we get $a_i = e^{-\frac{bi}{2}}$, $\frac{b_i}{2}e^{b_i} - i\pi = 0$, and $a_i = e^{-\frac{b_i}{2}}$. Then, calculating J for N = 20,000 I get $J = 0.2795060966$, which is accurate up to 10 decimal spaces. The output from my program for the calculation of J is pasted below.

```
Last 5 Q hat for secondary function:
            0.27950609667615

            0.27950609667537

           0.279506096673351

           0.279506096669331

           0.279506096674386

            0.27950609666226
```