MACM 316 – Assignment 8

Due Date: November 28th, at 11pm.

You must upload both your code (to Computing Code 8) and your report (to Computing Report 8) in Crowdmark. The assignment is due at 11:00pm. If you submit late, you will be given 0 on the assignment. Your computing report must be exactly 1 page. There will be a penalty given if your report is longer than one page. Your code needs to be printed out from Matlab as a .pdf file in order to upload it to Crowdmark.

- Please read the **Guidelines for Assignments** first.
- Please use the Canvas discussion board and please keep in mind that Canvas discussions are open forums.
- Acknowledge any collaborations and assistance from colleagues/TAs/instructor.

A. Computing Assignment – Numerical Solution of Kepler's problem

One of science's great achievements was the discovery of Kepler's laws for planetary motion; in particular, that the planets follow closed elliptical orbits around the sun. In this assignment you will compare different numerical methods for solving Kepler's problem for the motion of a simple solar system consisting of two planets (the two-body problem).

For a system of two planets, we may assume one is fixed at the origin with the motion of the other planet being in a 2D plane. Let

$$\mathbf{q}(t) = \begin{pmatrix} q_1(t) \\ q_2(t) \end{pmatrix}, \qquad \mathbf{p}(t) = \begin{pmatrix} p_1(t) \\ p_2(t) \end{pmatrix},$$

be the position and momentum vectors of the moving planet, respectively. Kepler's laws give the following ordinary differential equation for $\mathbf{q}(t)$ and $\mathbf{p}(t)$:

$$\mathbf{q}'(t) = \mathbf{p}(t), \qquad \mathbf{p}'(t) = -\frac{1}{(q_1(t)^2 + q_2(t)^2)^{3/2}}\mathbf{q}(t). \tag{1}$$

1. Write a code that implements Euler's method for this problem for time $0 \leq t \leq T = 200$ and stepsize $h = 0.0005$. Use the initial conditions

$$q_1(0) = 1 - e, \quad q_2(0) = 0, \quad p_1(0) = 0, \quad p_2(0) = \sqrt{\frac{1+e}{1-e}}, \qquad e = 0.6.$$

Plot your output in the q_1-q_2 plane, i.e. plot the approximate position of the moving planet at time t_n for $n = 0, 1, \ldots, N$, where $N = \lceil T/h \rceil$. Briefly describe the *qualitative* behaviour of the numerical solution.

2. The ODE (1) has several conserved quantities, including the angular momentum $A(t)$ and Hamiltonian $H(t)$, defined by

$$A(t) = q_1(t)p_2(t) - q_2(t)p_1(t), \qquad H(t) = \frac{1}{2}(p_1(t)^2 + p_2(t)^2) - \frac{1}{\sqrt{q_1(t)^2 + q_2(t)^2}}.$$

Compute these quantities for your numerical solution. Does your numerical solution also conserve the angular momentum and Hamiltonian? If not, briefly comment on their behaviour for large t.

3. For systems such as (1), an alternative to the standard Euler's method is the so-called symplectic Euler method

$$\mathbf{q}_{n+1} = \mathbf{q}_n + h\mathbf{p}_n, \qquad \mathbf{p}_{n+1} = \mathbf{p}_n - \frac{h}{(q_{n+1,1}^2 + q_{n+1,2}^2)^{3/2}}\mathbf{q}_{n+1}.$$

Here $q_{n+1,1}$ and $q_{n+1,2}$ are the components of the vector $\mathbf{q}_{n+1}$. Implement this method and compare it with the standard Euler's method. Describe the behaviour of the numerical solution, and also the angular momentum and Hamiltonian.