

1. If  $x$  is  $m$ -bits long and  $y$  is  $n$ -bits long, the run time of the function is  $O(m*n)$ . The function is called recursively  $n$  times ( $y/2$  performs a right shift, so  $n$  bits requires  $n$  shifts to complete) and the addition operation  $x + 2z$  requires one operation for every bit in  $x$ . In summary, there are  $n$  recursive calls, each requiring  $m$  operations, for a total of  $O(m*n)$ .

2.  $\text{gcd}(770, 546)$

a.  $770 = 2 * 5 * 7 * 11$

$546 = 2 * 3 * 7 * 13$

$\text{gcd}(770, 546) = 2 * 7 = 14$

b.  $\text{gcd}(a, b) = \text{gcd}(b, a \bmod b)$

$\text{gcd}(770, 546) = \text{gcd}(546, 224) = \text{gcd}(224, 98) = \text{gcd}(98, 28) = \text{gcd}(28, 14) = \text{gcd}(14, 0)$

$\text{gcd}(770, 546) = 14$

c.  $\text{eE}(a, b)$ : //(extendedEuclid)

$x', y', d = \text{eE}(b, a \bmod b)$

return  $y', x' - \text{floor}(a/b)*y', d$

Iteration	$x', y', d$	return value
$\text{eE}(770, 546)$	7, -17, 14	-17, 24, 14
$\text{eE}(546, 224)$	-3, 7, 14	7, -17, 14
$\text{eE}(224, 98)$	1, -3, 14	-3, 7, 14
$\text{eE}(98, 28)$	0, 1, 14	1, -3, 14
$\text{eE}(28, 14)$	1, 0, 14	0, 1, 14
$\text{eE}(14, 0)$		1, 0, 14

$\text{extendedEuclid}(770, 546) = -17, 24, 14$

$a = 770, b = 546, d = 14, x = -17, y = 24$

$-17*770 + 24*546 = 14; \text{gcd}(770, 546) = 14$

3.  $7^{7293} \bmod 342 \equiv (7^3)^{2431} \bmod 342 \equiv (343)^{2431} \bmod 342 \equiv 1^{2431} \bmod 342 \equiv 1$