Name: ____________________

GSI's name: ____________________

Lab section: ____________________

Statistics 153 (Introduction to Time Series) Homework 3

Due on October 16, 2018

Instructions: Homework is due by 3:50pm in lecture on due date. Please staple your homework when you turn it in. For the computing exercises, be sure to attach all relevant code and plots.

Computer/Theoretical Exercise

1. **Simulation of ARMA Processes and Difference Equations**

   Let Z_t be iid Gaussian white noise with mean 0 and variance 1. Let C_t be iid Cauchy white noise with location 0 and scale 1.

   a. For times $t \in [1, 100]$, simulate the ARMA(2,2) process $X_t = .7X_{t-1} - .3X_{t-2} + Z_t + 4Z_{t-2}$ and plot both the process and its sample ACF using the follow steps:

   (1 point)

      1. Generate observations $Z_1, ..., Z_{100}$, where $Z_i \overset{iid}{\sim} N(0, 1)$.
      2. Set $X_1 = X_2 = 0$.
      3. For $3 \leq t \leq 100$, define X_t recursively using the ARMA parametrization.
      4. Plot the process and its sample ACF.

   b. For times $t \in [1, 100]$, simulate the ARMA(2,2) process $X_t = .7X_{t-1} - .3X_{t-2} + C_t + 4C_{t-2}$ and plot both the process and its sample ACF using the following steps:

   (1 point)

      1. Generate observations $C_1, ..., C_{100}$ where $C_i \overset{iid}{\sim} Cauchy(0, 1)$.
      2. Steps 2-4 are the same as above.

   c. Compare the two processes and their sample ACF plots, and comment on their similarities/differences.

   (1 point)

   d. **For process (a) only**: Plot the theoretical ACF for lags $1 \leq h \leq 20$ using the function `ARMAacf`, and also plot its asymptotic 95% confidence bands using Bartlett's formula (you can approximate the variances in Bartlett's formula by just summing up the first 20 terms - use R to save time). Then, plot its sample ACF again for lags $1 \leq h \leq 20$. Compare the two, and comment on their differences.

   (2 points)

   e. **For process (a) only**: To get a more accurate simulation, we should try to sample X_1 and X_2 from their actual distributions. Use the following steps to better simulate process (a) and compare to the original simulation in (a):

   (1 point)

      1. Find the first 20 coefficients of the MA(∞) using the function `ARMAtoMA`. (This gives $\psi_1, ...\psi_{20}$)
      2. Generate 20 new observations $Z_0, ...Z_{-19}$, where $Z_i \overset{iid}{\sim} N(0, 1)$.
      3. Use the coefficients in Step 1 and the Gaussian observations from both Step 2 and (a) to calculate both X_1 and X_2.
      4. Follow Step 3-4 as above to finish the simulation, using the Gaussian observations generated in Step 2 and part (a).
      5. Compare the plots/sample ACF of this new simulation with those in (a), and comment.

f. **For process (a) only**: Use the difference equations method in lecture, calculate $\gamma(h)$ for $0 \leq h \leq 20$ (use R for part of this to save time).

(3 points)

g. **For process (a) only**: From (f), calculate $\rho(h)$ for lags $1 \leq h \leq 20$. Plot these values and the `ARMAacf` values together in the same plot. Also, plot the error in a separate plot. Comment on both.

(1 point)

Theoretical Exercises

2. **Causality and Bartlett's Formula**

Let Z_t be a white noise process with variance σ^2. Consider the ARMA process X_t:

$$X_t = \frac{1}{4}X_{t-2} + Z_t + \frac{1}{2}Z_{t-1} + Z_{t-2} + \frac{1}{2}Z_{t-3}$$

a. Write X_t in its simplest polynomial form, removing any possible common factors.

(1 point)

b. Show that X_t is causal, and give the MA(∞) representation.

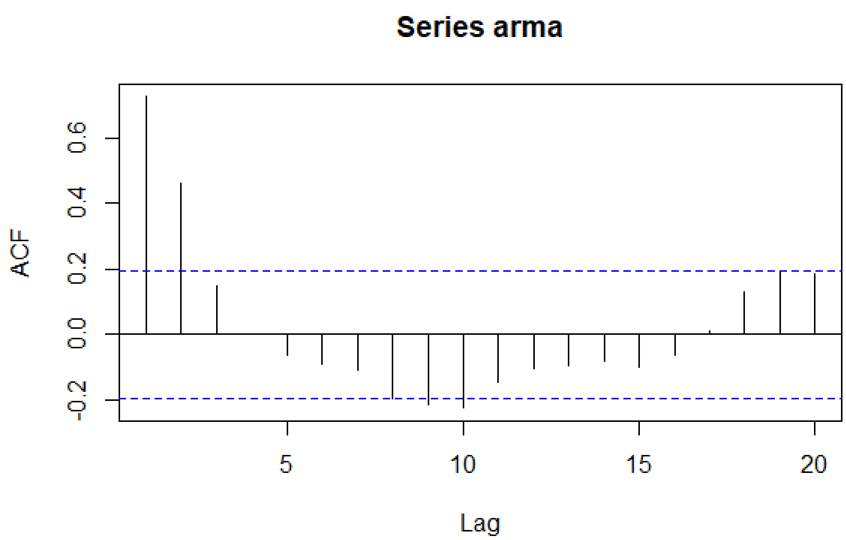
(2 points)

c. Calculate the covariance function $\gamma(h) = Cov(X_t, X_{t+h})$.

(2 points)

d. A simulation in R for $n = 100$ datapoints of X_t gave the following sample ACF plot. Explain why $\hat{r_1}$ exceeds the blue band so much.

(1 point)



3. **Bartlett's Formula for White Noise and MA Processes**

a. Assuming its conditions are met, show that for an ARMA(p, q) process X_t with $p = q = 0$ (ie. X_t is white noise) Bartlett's formula gives the following result:

(2 points)

$$\sqrt{n}\begin{pmatrix}\hat{r_1}\\ \vdots \\ \hat{r_k}\end{pmatrix} \xrightarrow{d} N_k(0, I_k)$$

***This is the asymptotic result for the sample correlations of white noise covered earlier in class*

b. For the MA(2) process $X_t = Z_t + \frac{1}{2}Z_{t-1} + 2Z_{t-2}$, calculate the asymptotic variance of $\sqrt{n}\hat{r_k}$ for $k \geq 3$.

(2 points)