Authority in Complex Organizations*

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Job Market Paper

September 2019
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Abstract

I study organizational design under informational interdependence—the fact that a single piece of information affects many decisions. In my model, a principal must decide on the allocation of authority over two decisions in order to aggregate information dispersed among biased agents. The allocation of decision rights alters the interaction between interdependence and biases, shaping individual incentives for communication. I find the principal prefers to delegate a high-conflict decision if that improves information transmission on the other dimension. When agents have extreme preferences, centralization can discipline decision-specific biases and improve communication. I also analyze how interdependence affects information acquisition. Agents can decide to specialize, which signals commitment not to manipulate information and, hence, enhances credibility. Finally, I show that delegation reduces the initiative to acquire information: it will be lower overall and concentrated on states that are more important for the decision at hand.

Keywords: Multidimensional Cheap Talk, Industrial Organization, Delegation, Organizational Design.
JEL: D21, D83.

*I am grateful to my supervisors Marina Halac and Francesco Squintani for their invaluable support, to Robert Akerlof, Costas Cavounidis, Rick Harbaugh, Heikki Rantakari, Ayush Pant, Federico Trombetta, Horng Wong, and participants at 2019 Midwest Economic Theory Conference (Bloomington, IN), SIOE Annual Conference (Stockholm), Stony Brook International Game Theory Conference (New York), EEA-ESEM Congress (Manchester), and EARIE Annual Conference (Barcelona). All remaining errors are mine. Financial support from the ESRC is gratefully acknowledged.

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1 Introduction

When a principal in charge of many decisions needs to aggregate information from agents, incentives for communication depend on their preferences over decisions and on how information affects these decisions. In some cases, a single piece of information affects many decisions; there is *informational interdependence*. The allocation of authority—who decides what—alters the interaction between interdependence and preferences: the number of decisions a player controls, together with his preferences, shapes other players' communication incentives. Hence, making dispersed information available for decision-making requires an appropriate organizational structure. This paper studies the allocation of authority under informational interdependence.

If decisions were independent, the principal would delegate authority to a better informed, but biased agent only if the informational gains compensate for the loss of control (Aghion and Tirole, 1997; Dessein, 2002). When information is dispersed among many agents, the principal delegates a decision to one of them only if he aggregates more information, and that compensate for the loss of control (Austen-Smith and Riker, 1987; Austen-Smith, 1990; Dewan and Squintani, 2015). But informational interdependence alters this trade-off. When an agent is in charge of one decision, any information transmitted to him affects only that decision. Hence, depending on how interdependence aggregates each agent's preferences, the principal may have higher or lower incentives to delegate than under informational independence. For instance, multinational corporations locate some of their activities close to economic and technological agglomerates in order to gains access to their knowledge (Ghoshal and Bartlett, 1990; Andersson and Forsgren, 2000). But subsidiaries' ability to identify and assimilate new knowledge depends on forging close relationships with local business partners, which can conflict with the organizational goals (Andersson et al., 2005; Ecker et al., 2013). As a consequence, the degree of autonomy over product decisions granted to subsidiaries is correlated to the extent to which knowledge created spills over to sister units (Andersson et al., 2007; Boutellier et al., 2008).[1]

In this paper I construct a model of allocation of authority under informational interdependence. A principal in charge of two decisions needs information about two state variables. Information is dispersed among n biased agents, each of whom observes noisy signals about each state. Before any communication takes place, the principal allocates decision rights among all members of the organization (including herself). Incentives for communication are shaped by the degree of preference misalignment (conflict of interest) between each agent and each decision maker, depending on the decision(s) the latter controls. When authority is centralized, information about each state affects both

[1] Andersson et al. (2007) identify that "*one the one hand, a high degree of external embeddedness is important for the development of a subsidiary's competence, with some competence spilling over to other subsidiaries; and, on the other, a high degree of external embeddedness indicates that the subsidiary is largely involved in long-term business interactions with resulting in issues external to the MNC being prioritized.*"

decisions and, thus, informational interdependence aggregates decision-specific interests. The model features linear interdependence, which allows me to study its consequences in a parsimonious way.

The optimal organizational structure resolves a trade-off between informational gains and loss of control. Informational gains can be of two types. Direct gains refer to the additional information a decision maker is expected to receive in equilibrium when the principal delegates authority to him. These gains reflect the traditional argument under which the principal delegates to an agent with sufficiently large 'informational advantage' (see Dessein, 2002; Dewan et al., 2015).[2] Indirect gains, on the contrary, refer to the additional information the principal expects to receive when she delegates one decision and retains authority on the other (partial delegation). These gains arise when some agents reveal more information under partial delegation than under centralization; which occurs when the conflict of interest is high in one decision and low in the other. Delegation of the high-conflict decision allows the principal to restrict agents' communication with her to the low-conflict decision.

Interdependence can lead to informational spillovers. An agent's preferences feature negative informational spillovers when high conflict in one dimension prevents communication under centralization, despite the conflict in the other is very low (Levy and Razin, 2007). There can also be positive spillovers in this context. When an agent's preferences are extreme in both dimensions but interdependence aggregates them in such a way that dimension-specific conflict of interests counteract each other—say, revealing information leads to utility gains in one dimension and losses in the other (see Chakraborty and Harbaugh, 2007, 2010). The notion of positive spillovers relates to ideas such that new public information triggers institutional change in policy implementation (Baumgartner and Jones, 2009), and that institutional arrangements affect the extent to which inssues (and interest) can be linked in international trade deals, influencing their outcomes (Morgan, 1990; Davis, 2004).[3]

I find that organizational design plays a crucial role in cases of informational spillovers. Proposition 2 shows negative spillovers require delegation of high-conflict decisions and retaining authority over low-conflict ones. The optimal organizational structure is thus partial delegation. On the contrary, agents whose preferences are affected by positive informational spillovers reveal more information to the principal under centralization. Hence, if the number such agents is sufficiently large, the principal prefers to centralize decision making. The mechanism behind this result is similar to 'mutual discipline' in public communication with multiple audiences (Farrell and Gibbons, 1989; Goltsman and Pavlov, 2011), and 'persuasive cheap talk' in multidimensional communication (Chakraborty and Harbaugh, 2010): information affects decisions in a way that decision-specific conflict of interest compensate each

[2]Informational advantage conducive to delegation arises ex-post and on path, as in Dewan et al. (2015).

[3]The intuition of positive spillovers exists in international relations since "an agreement leading to the peaceful resolution of an international crisis often becomes possible when an issue, not originally in contention, is brought into the bargaining for linkage purposes" (Morgan, 1990).

other.

Informational interdependence not only affects communication, it also affects information acquisition. In the second part of the paper I introduce endogenous information acquisition. After the allocation of decision rights, each agent decides on the information he wants to observe (if any) at a given cost. Similar to Di Pei (2015), no agent acquires information he is not willing to reveal on the equilibrium path. This investment is worth making only if it is cost-effective, i.e. the expected utility gains from revealing the acquired information must compensate its cost. Because agents can only acquire signals with a fixed amount of noise, expected utility gains are decreasing in the amount of information the decision maker receives on path.[4] This imposes an upper bound on the number of agents who become informed in any equilibrium.

Agents decide on information acquisition about two states variables, so they can specialize. I show that if an agent acquires information about one state, truthful revelation is incentive compatible for a larger set of biases. When an agent observes information about both states, there are some realizations for which one part of the information favours him and the other part does not. In such a case, he has incentives to follow the favourable information, which involves deviations from truth-telling. These incentives impose a penalty on credibility: incentive compatibility constraints are tighter than if he observes only one signal. But, then, these perverse incentives are absent when he specializes, a sort of commitment not to manipulate information. This result has implications on organizational practices about subunits' access to information.

Finally, I study how the organizational structure affects incentives to acquire information. I find that, under delegation, the expected investment in information is both lower overall and more concentrated on the state that is more important for the decision at hand. The first of these results—lower overall investment—stems from the fact that agents typically influence one decision under delegation. Hence, the expected marginal utility from any piece of information is smaller than when it affects both decisions. The second result—unbalanced investment—relates to the fact that information about each state affects decisions differently. Under delegation, then, agents expect to derive higher marginal utility when they acquire information about the state that is most important for each decision. These results represent a qualification on the informational benefits from delegation (see Aghion and Tirole, 1997; Dessein, 2002), and could be interpreted as part of the 'long rung informational' effects from organizational design.

[4] I assume signals have a fixed amount of noise and agents can acquire at most one of such signals per state. As a consequence, individual incentives for communication are decreasing in the amount of information the decision maker is expected to have in equilibrium (see Morgan and Stocken, 2008; Galeotti et al., 2013).

Related Literature. The paper builds the literature on multidimensional cheap talk. In a seminal paper, Battaglini (2002) shows that multidimensionality allows a receiver to extract all the information from perfectly-informed senders, by restricting the influence of each of them on the dimension of common interest. When there are as many senders as decisions, the receiver can commit to ignore part of the information provided by each sender because it is provided (in equilibrium) by the others. Note that, as a consequence, the receiver does not need delegation. Dewan and Hortala-Vallve (2011) apply this result to explain the design of jurisdictions in parliamentary cabinets. They argue that a Prime Minister uses her prerogatives on ministerial appointments and allocation of portfolios to limit each minister's influence to decisions for which preferences are somewhat aligned. This way there is no need for effective delegation of authority. But the argument above relies on the assumption that ministers are perfectly informed. When this is not the case and decisions are interdependent, the receiver loses her equilibrium commitment power. Levy and Razin (2007) show this leads to communication breakdown if the conflict of interest in one dimension is sufficiently large. [5] In such a scenario, senders' incentives depend on how information affects decisions and, thus, their interests. My paper shows delegation substitutes the principal's ability to ignore information in Battaglini (2002) and, more generally, presents how the allocation of decision rights helps her to cope with informational spillovers (as defined in Levy and Razin, 2007).

The main contribution of the paper is on organizational design, in which strategic communication has important consequences on the allocation of decision rights. In unidimensional decision problems with a perfectly informed sender, delegation involves a trade-off between informational gains and loss of control (biased decision). Dessein (2002) shows that delegation dominates cheap talk communication for all conflicts of interest for which there is transmission of information (under communication). The principal's willingness to delegate, however, is decreasing in the sender's informational advantage (measured by the ex-ante variance of the state variable). The same intuitions underlie the organization of legislative institutions (Austen-Smith and Riker, 1987; Austen-Smith, 1990), policy-making cabinets (Dewan and Hortala-Vallve, 2011; Dewan et al., 2015), political parties (Dewan and Squintani, 2015), and multi-divisional firms (Alonso et al., 2008; Rantakari, 2008). Introducing informational interdependence represents a step towards a more realistic understanding of the underlying forces.

Multi-divisional firms trade off the need for adaptation to local shocks with the need for coordination between divisions. Communication frictions may lead to inefficiencies in terms of giving up benefits from specialization of production (Dessein and Santos, 2006) or the need for coordination through scheduled task instead of using communication (Dessein et al., 2016). When divisional

[5] In a complementary paper, Chakraborty and Harbaugh (2007) show that, in two-player games, the sender can credibly communicate any ranking of the decision dimensions that reflects (at least partially) the order of the realization of the states across dimensions.

managers' information is not verifiable, the allocation of decision rights—along with non-separability of preferences and divisional conflict of interest—shapes incentives for communication (Alonso et al., 2008, 2015; Rantakari, 2008). But the trade-off between adaptation and coordination is not the relevant problem in many applications. In multinational corporations, for instance, "*Control and coordination needed for the sake of internal consistency seems to apply little to research-based organization. Bureaucratic and hierarchical control as well as social control does not work as much as scientists feel more affiliated with their profession than to their employers*" (Boutellier et al., 2008). My framework isolates the effects of informational interdependence on communication, and shows how organization can be designed to produce and aggregate information effectively.

The allocation of decision rights also affects incentives to acquire information. Aghion and Tirole (1997) show that delegation of real authority motivates an agent to acquire information, resulting in a loss of control for the principal. But delegation may discourage information acquisition in this context, either because the agent prefers to put more effort on information that benefits him personally (Rantakari, 2012), or because he no longer has to convince a principal with divergent opinion about the best course of action (Che and Kartik, 2009). When the agent has access to imperfect (non-verifiable) information, the principal induces him to over-invest in information gathering if punishments upon deviations are credible (Argenziano et al., 2016); in such cases, centralization dominates. My paper shows that informational interdependence has meaningful consequences for incentives to acquire information. By allowing information acquisition about multiple states, my framework also captures different drivers of specialization.

More recently, Deimen and Szalay (2019) have also integrated the ideas of information acquisition and strategic communication. In their framework, a principal and an agent face an uni-dimensional problem that depends on information about two states variables. Players disagree on the state upon which the decision has to be calibrated, such that the conflict of interest is decreasing in the correlation between states. The principal's benefits from delegation are thus increasing in the correlation. In my paper, informational interdependence (correlation) affects communication in a non-monotonic way because decision-specific conflict of interest are state independent (see Crawford and Sobel, 1982). Besides, interdependence interacts with the multidimensionality of the decision problem to affect incentives for information acquisition under different organizational structures.

The next section presents the baseline model with no information acquisition and the results on optimal allocation of decision-rights. In section 4 I integrate the allocation of decision-rights with endogenous information acquisition. Section 5 discusses some extensions and conclude.

2 Baseline Model

Players and preferences. An organization consists of a principal, P, and n agents.[6] There are two decisions to be made, $\mathbf{y} \in \Re^2$, but the outcome of each of them depends on two state variables θ_1 and θ_2 (to be defined later). Optimal decisions thus require information about θ_1 and θ_2. The decision-specific uncertainty is represented by two composite states $\delta_1(\theta_1, \theta_2)$ and $\delta_2(\theta_1, \theta_2)$. Preferences for player $i = \{P, 1, \ldots, n\}$ are given by:

$$U^i(\boldsymbol{\theta}, \mathbf{y}, \mathbf{b}^i) = -\left(y_1 - \delta_1(\theta_1, \theta_2) - b_1^i\right)^2 - \left(y_1 - \delta_2(\theta_1, \theta_2) - b_2^i\right)^2$$

Where $\mathbf{b}^i \in \Re$ represents i's bias vector, which is normalized to $\mathbf{b}^P = (0, 0)$ for the principal.

Information structure. Pay-off relevant states, θ_1 and θ_2, are uniformly distributed with support in the interval $[0, 1]$, and $\theta_1 \perp \theta_2$. Information about each of these states affects both decisions, that is, there is *informational interdependence.* In the example of multinational corporations, the states can be interpreted as different technological attributes relevant for many products the firm produces. Decisions would represent the different product lines using these technologies; arguably, each technology is a salient attribute for a different product. If the example consists of policy implementation, states can be interpreted as the different goals of a policy intervention, while decisions represent the different policy instruments that address those goals; arguably, different instruments address goals with different degree of success. The composite states δ_1 and δ_2 capture informational interdependence:

$$\begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} \equiv \begin{bmatrix} w_{11}\,\theta_1 + w_{12}\,\theta_2 \\ w_{21}\,\theta_1 + w_{22}\,\theta_2 \end{bmatrix}$$

The uniform distribution of states represents the canonical example in Crawford and Sobel (1982) and has been extensively used in the cheap talk literature. Assuming independent states allows me to isolate the effect of informational interdependence through $\boldsymbol{\delta}$. The elements of the weighting matrix W are indexed by w_{dr}, for $y_d = \{y_1, y_2\}$ and $\theta_r = \{\theta_1, \theta_2\}$ (d represents decisions and r represents states). All the weights are weakly positive and I normalize them as $w_{d1} + w_{d2} = 1$. Without loss, I also take $w_{11}, w_{22} > \frac{1}{2}$, so that the index corresponding to the state also reflects which decision that state is more important for. As a consequence, the informational interdependence between decisions is linear, and captured by the ex-ante correlation between the composite states.

$$\text{Corr}(\delta_1, \delta_2) = \frac{(w_{11}w_{12} + w_{21}w_{22})}{[(w_{11}^2 + w_{21}^2)(w_{12}^2 + w_{22}^2)]}$$

[6] As usual in this literature I use female pronouns to refer to the principal, and male pronouns for each agent.

Agents' signals and communication. Agents have access to noisy, non-verifiable information about both states. Each agent observes one signal associated to each state (two in total). Let $\mathbf{S}^i = (S_1^i, S_2^i) \in \mathcal{S} \equiv \{0,1\}^2$ be i's signals, and $\tilde{\mathbf{S}} \in \mathcal{S}$ be the vector of realizations. Signals are independent across players, conditionally on $\boldsymbol{\theta}$. The prior probability distribution for each signal is characterized by $\Pr(\tilde{S}_1^i = 1) = \theta_1$ and $\Pr(\tilde{S}_2^i = 1) = \theta_2$.[7]

Each agent sends private, non-verifiable (cheap talk) messages to decision maker $j = \{P, 1, .., n\}$. Let $\mathbf{m}_j^i\left(\mathbf{S}^i\right) \in \{0,1\}^2$ denote agent i's message to decision maker j, in charge of $y_d = \{y_1, y_2\}$.Note that i's message strategies associated to each signal can take one of two forms (up to relabelling messages): the truthful one, $m_j^i(S_r^i) = \tilde{S}_r^i$ for all S_r^i, and the babbling one, $m_j^i(\tilde{S}_r^i = 0) = m_j^i(\tilde{S}_r^i = 1)$. Besides, the set of message strategies is not just based on babbling or revealing information on separate dimensions independently. An agent could, for instance, truthfully reveal both signals for some realizations and send the babbling message for others. Such message strategies arise because states are orthogonal and information about one state does not reveal information about the other. I call these strategies 'dimensional non-separable' (DNS).

Let $\mathbf{m}_j = \{..., \mathbf{m}_j^i, ...\}$ denote the matrix containing all the messages decision maker j receives from agents (including himself if applicable). The updated expectation and variance for each state depend on the number of agents revealing the corresponding signal truthfully, $k_r(j) \leq n$, and the the number of those agents who report a 1, $\ell_r(j)$, for $\theta_r = \{\theta_1, \theta_2\}$, as follows.

$$E\left(\theta_r|\mathbf{m}_j\right) = \frac{(\ell_r(j)+1)}{(k_r(j)+2)} \qquad \text{Var}\left(\theta_r|\mathbf{m}_j\right) = \frac{(\ell_r(j)+1)(k_r(j)-\ell_r(j)+1)}{(k_r(j)+2)^2(k_r(j)+3)}$$

Allocation of decision rights. Decision-rights are allocated before each agent learns his information. Formally, the principal decides on a set of assignments that grants decision making authority over the set of decisions. The assignment grants complete jurisdiction over the delegated decision, such that authority over each decision is granted to a unique individual. Decision makers are also granted the possibility of private communication with each agent. Different allocations of decision-rights lead to different organizational structures. I group these structures into three categories: under *centralization* the principal decides on both issues; under *full delegation* the principal grants authority to two different agents, each of them assigned to a different decision; under *partial delegation* the principal retains authority over one issue and delegates the other to an agent.

At this point two clarifications are necessary. First, I am not considering the case of delegation of both decisions to a single agent. This is without loss since agents' incentives for communication will use the same measure of conflict of interest as in centralization, and optimality of such allocation would

[7] A similar information structure, for unidimensional problems with one state variable, has been used in Austen-Smith and Riker (1987); Morgan and Stocken (2008); Galeotti et al. (2013) among others.

depend on the degree of centrality of preferences. As I shall show later, the two forms of delegation considered in the paper involve different measures of conflict of interest. The second clarification relates to the distinction between delegation of decision authority and decentralization on the access to information. In my framework, authority can be centralized or decentralized, the latter is called 'delegation' throughout the paper. Information, on the contrary, is always decentralized because it is dispersed among agents.

Equilibrium concept The equilibrium concept is pure-strategies Perfect Bayesian Equilibria (equilibrium, henceforth). A full characterization including mixed strategies is cumbersome and does not provide much intuitions beyond the pure-strategies case.[8]

In the following section I analyse optimal organizational design when agents observe one signal associated to each state (do not decide on information acquisition).

3 Organizational Design and Information Transmission

In this section I characterize the role of informational interdependence in organizational design. To do that, I first describe incentives for communication under each organizational structure (the formal analysis is left to the appendix). I then characterize the cases in which the principal prefers to delegate a decision to an agent. The figure below describe the timing of the game.

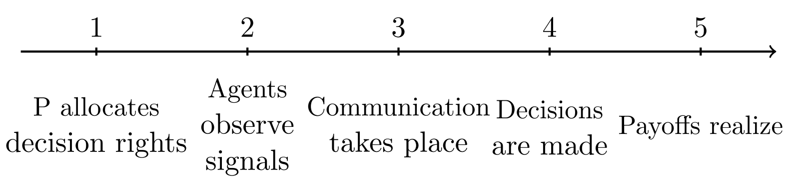


Figure 1: Timing of the Organizational Structure Game

I introduce some notation before describing the incentives for communication under different organizational structures. Let $k_r^*(j) \equiv k_r(\mathbf{m}_j^*)$ and $\ell_r^*(j)$ denote the number of truthful messages and 'ones' decision maker j receives in equilibrium. Also, let k_r^j be agent i's conjecture about the number of agents other than him who reveal information about θ_r to j (on path).[9] To keep track of who decides what, let $j', j'' \in \{P, 1, ..., n\}$ be the decision makers for y_1 and y_2, respectively (note that the number of apostrophes coincide with the index of the decision allocated to each player).

A equilibrium is characterized by the triple $\left(\mathbf{y}^*, \mathbf{m}_{j'}^*, \mathbf{m}_{j''}^*\right)$, representing the vector of decisions

[8]For an analysis of mixed-strategies in a similar framework see Habermacher (2018).

[9]Note that i's conjecture will be correct in equilibrium, and whenever his message strategy involves revealing the corresponding signal then $k_r^*(j) = k_r(j) + 1$, otherwise $k_r^*(j) = k_r(j)$.

and the vectors of message strategies to j' and j'', respectively. Optimal actions satisfy:

$$y_1^* = w_{11}E(\theta_1|\mathbf{m}_{j'}^*) + w_{12}E(\theta_2|\mathbf{m}_{j'}^*) + b_1' \qquad y_2^* = w_{21}E(\theta_1|\mathbf{m}_{j''}^*) + w_{22}E(\theta_2|\mathbf{m}_{j''}^*) + b_2''$$

Where b_1' (b_2'') represents the bias of decision maker j' (j'') with respect to y_1 (y_2). Note that centralization means $j' = j'' = P$ and the associated biases are equal to zero. From the principal's perspective, delegation of decision-rights has two payoff-relevant consequences. On the one hand, it implies a biased agent decides on behalf of the principal, resulting in a biased decision. On the other hand, individual incentives for communication depend on the conflict of interest between each decision maker and the agent in question. Different organizational structures, and decision makers, then result in different communication incentives. Agent i's optimal message strategy to decision maker j solves:

$$\mathbf{m}_j^{i*}(\mathbf{S}^i, \mathbf{b}^i, b_1', b_2'') = \arg\max_{\mathbf{m}_j^i} \left\{ E\Big[-\left(y_1\left(m_{j'}^i, \mathbf{m}_{j'}^{-i}\right) - \delta_1 - b_1^i\right)^2 - \left(y_2\left(m_{j''}^i, \mathbf{m}_{j''}^{-i}\right) - \delta_2 - b_2^i\right)^2 \Big] \Big| \mathbf{S}^i \right\}$$

Before the analysis of communication incentives, I clarify the equilibrium-selection criterion and introduce some notational conventions. Communication equilibria is selected on the basis of the decision maker's ex-ante expected utility, a natural extension of the notion of 'most informative equilibrium' used in the literature.In order to simplify notation, let $k_r^{\mathrm{C}} \equiv \{k_r(j)|\, j' = j'' = P\}$ denote the number of truthful messages about $\theta_r = \{\theta_1, \theta_2\}$ the principal receives under centralization; let $k_r^{\mathrm{P}1} \equiv \{k_r(P)|\, j' = P\}$ the number of messages she receives when decides on y_1 only, and $k_r^{\mathrm{P}2} \equiv \{k_r(P)|\, j'' = P\}$ when she decides on y_2 only. For when P does not decide at all, let $k_r' \equiv k_r^*(j')$ and $k_r'' \equiv k_r^*(j'')$ refer to the number of truthful messages for decision makers of y_1 and y_2, respectively, while keep $k_r^j \equiv k_r^*(j)$ for a generic decision maker.

Note that the principal's expected utility from different allocations of decision rights depends on the amount of information the different decision makers are expected to receive on the equilibrium path (and their biases). I first analyse incentives for communication for a typical agent under different organizational structures; then, I proceed to study the role of informational interdependence on the principal-optimal structure.

Incentives for communication under delegation. I first describe agent i's incentives to reveal information to decision maker j in charge of y_d. Because communication between i and j is private, incentives depend on the associated conflict of interest between them, b_d^i and b_d^j. But since i is imperfectly informed, communication also depend on how many other agents are expected to be truthful to j on the equilibrium path. These two determinants constitute the traditional mechanism determining communication of imperfect information via cheap talk (Austen-Smith and Riker, 1987; Morgan and

Stocken, 2008; Galeotti et al., 2013). In my framework, agents observe signals about two independent states, introducing a third determinant.

Due to informational interdependence, information about a state affects both decisions. But states are orthogonal, and there are some signal realizations for which i *believes* decisions should move in different directions. In such cases, one of the signals moves y_d towards i's bias (with respect to j, that is, $b_d^i - b_d^j$). As a consequence, i has higher incentives to follow the 'most favourable' signal of the two, for all possible message strategies. These additional deviation incentives lead to a *credibility loss* in the form of tighter IC constraints for all message strategies. This credibility loss arises because of what i believes according to his information, which implies his incentives for communication now depend on how much information *about both states* other agents are expected to reveal in equilibrium, even when i reveals information about one of them.

Lemma 5 and Proposition 9 in Appendix A characterize the equilibrium communication between agent i and decision maker j. Revealing information means that i contributes to increase the precision of j's decision. This benefits i because prevents j from making big mistakes that would cause high disutility to the agent. But if j receives information from many agents other than i, the decision will be 'too close' to j's ideal, and away from i's. This is the trade-off governing incentives for communication.

When, for instance, i is expected to reveal information about θ_1 in equilibrium, whatever he says about θ_2 is not influential on path. If his information about θ_1 is unfavourable and that about θ_2 is favourable, his incentives to lie about the former are larger because *he believes* the overall influence on y_1 should be smaller than what his information about θ_1 conveys. This leads to the credibility loss, which in Lemma 5 takes the form of a term depending on k_2^j. Indeed, i's credibility loss is decreasing k_2^j because it depends on i's expectation about the influence S_2^i would have on y_1 (which is decreasing in k_2). In a companion paper I discuss how such incentives can lead to congestion effects that benefit the decision maker (see Habermacher, 2018).

Incentives are somewhat similar if i is expected to reveal both signals. When i's signals realizations coincide, $\mathbf{S}^i = \{(0,0),(1,1)\}$, the influence of each signal reinforce the other and, as a consequence, the overall influence is larger than revealing only one of such signals. This would, in principle, provide higher incentives for communication. But when his signals do not coincide, $\mathbf{S}^i = \{(0,1),(1,0)\}$, each signal counteracts the influence of the other, for which communication will be incentive compatible for a smaller set of biases. Because of this difference in influence, types $\mathbf{S}^i = \{(0,1),(1,0)\}$ have incentives to deviate from truth-telling, following the most favourable signals (depending on i's preferences)—leading to the credibility loss. As a consequence of the latter, full revelation requires that types $\mathbf{S}^i = \{(0,1),(1,0)\}$ find incentive compatible to reveal both signals. Also, the credibility loss affects the incentives in the same way as for the case of revealing one signal, such that the IC constraints for both strategies are equivalent. Proposition 9 in Appendix A shows that the equilibrium involves only two message strategies: full revelation, $\mathbf{m}^{i*} = \{\{(0,0)\}; \{(1,1)\}; \{(1,0)\}; \{(0,1)\}\}$, and babbling, $\mathbf{m}^{i*} = \{\{(0,0); (1,1); (1,0); (0,1)\}\}$.

Incentives for communication under centralization. Information transmitted to the principal under centralization affects both decisions due to the informational interdependence (governed by $\mathbf{W}$). But the influence on decisions also depends on the amount of information revealed by agent i. To see this, note that revealing only S_1^i has a larger influence on y_1 and, thus, the bias on the first dimension weighs more heavily in determining i's incentives. If he reveals both signals, on the contrary, the overall influence is more balanced and so the weights of b_1^i and b_2^i on the IC constraints. This leads to different measures of conflict of interest depending on the information revealed by different message strategies.

The fact that the conflict of interest depends on the information revealed leads to two main differences with respect to delegation. First, revealing only one signal is incentive-compatible for non-empty set of $\mathbf{b}^i$, as shown in Lemma 6 in the appendix. Indeed, Proposition 10 in the same appendix shows that such strategies arise in equilibrium for some $\mathbf{b}^i$. The overall influence of revealing information about a single state depends on how it affects decisions, which is governed by $\mathbf{W}$. Because i is imperfectly informed about both states, the influence also depends on the information the principal receives from other agents on the equilibrium path, which follows the same intuition of the delegation case.The relevant measure of conflict of interest for this strategy aggregates the decision-specific biases according to their weight on each decision—$\beta_1 = b_1^i w_{11} + b_2^i w_{21}$ and $\beta_2 = b_1^i w_{12} + b_2^i w_{22}$.

The second difference with delegation relates to the effects of ambiguous information. Under centralization, revealing both signals has a balanced overall influence on decisions. Indeed, truthful revelation of $\mathbf{S}^i = \{(0,1), (1,0)\}$ moves in opposite directions; for instance, if i credibly announces $\mathbf{m}_P^i = (0,1)$, then updated y_1 would be closer to 0 and y_2 closer to 1. Because the influence determines how decision-specific biases aggregate, it can lead to dimensional non-separable strategies in which i credibly reveals ambiguous signals and sends babbling messages otherwise. Proposition 10 in the appendix shows the most informative message strategies in equilibrium, which involve: full revelation, revelation of information about one state, full revelation of some signal realization and nothing otherwise, and babbling. The complete characterization of the most-informative equilibrium under centralization can be found in Appendix A. I now focus on the analysis of the optimal organizational structure.[10]

[10]For a more thorough discussion of communication incentives under centralization see Habermacher (2018).

Optimal Organizational Structure. In this subsection I characterize the optimal organizational structure and present the main result of the communication game: the optimal allocation of decision-rights depends on the existence and the extent of informational spillovers.

The profile of preferences, $\mathbf{B} = \{\mathbf{b}^1, ..., \mathbf{b}^n\}$, determines the amount of equilibrium information for each possible organizational structure. By allocating decision rights the principal affects agents' incentives for communication (and the resulting equilibrium); effectively, she chooses among the different equilibria induced by $\mathbf{B}$. The principal's preferred allocation takes into account two main elements: the amount of information revealed to each decision maker and their biases. The definition below formalizes these two elements on the principal's ex-ante expected utility, for two generic decision makers j' and j''.

Definition 1 (principal's ex-ante expected utility)**.** *Consider an equilibrium* $(\mathbf{y}, \mathbf{m})$ *in which* $j', j'' = \{P, 1, ..., n\}$ *decide on* y_1 *and* y_2*, respectively. The equilibrium is characterized by the number of agents reporting truthfully to each decision maker,* k_1'*,* k_2'*,* k_1''*, and* k_2''*. Then, the principal ex-ante expected utility is given by:*

$$\hat{U}^P(\mathbf{b}, j', j'') = -\left[(b_1')^2 + \frac{(w_{11})^2}{6\big(k_1' + 2\big)} + \frac{(w_{12})^2}{6\big(k_2' + 2\big)}\right] - \left[(b_2'')^2 + \frac{(w_{21})^2}{6\big(k_1'' + 2\big)} + \frac{(w_{22})^2}{6\big(k_2'' + 2\big)}\right] \tag{1}$$

The derivation of equation (1) is in Appendix B.2. The first term in square brackets represents the principal's ex-ante expected utility associated to delegation of y_1 to decision maker j'. Because his decision will be biased, the principal's utility is decreasing in b_1'. But her utility also depends on equilibrium posterior beliefs about θ_1 and θ_2, determined by k_1' and k_2'. The principal thus would delegate authority to a player whose preferences are close to those of other agents on the associated decision. This is the trade-off the literature focuses on: informational gains ($k'1$ and k_2') must compensate the loss of control (b_1').

But informational interdependence can lead to a different source of informational gains for the principal. Because delegation breaks the interdependence, there may be agents willing to reveal information under delegation but not under centralization. This happens when, for instance, agents' biases are very large on one dimension and small on the other. If the principal delegates the high-conflict decision and retains authority over the low-conflict decision, those agents will reveal information to her. I call these informational gains *indirect*, since they do not arise because the new decision maker has more central preferences, but as a by-product of delegation. Indeed, indirect informational gains arise because some agents are affected by *negative informational spillovers*—under centralization, a high-conflict dimension impedes communication in a dimension of low-conflict of interest (Levy and Razin, 2007). The proposition below defines both types of informational gains arising in this game, and characterized the necessary conditions for each of them.

Proposition 1. *Consider the equilibrium under centralization, characterized by* $\mathbf{k}^{\mathrm{C}} = \{k_1^{\mathrm{C}}, k_2^{\mathrm{C}}\}$*; and the equilibrium in which the principal delegates* y_1 *to agent* j' *and retain authority on* y_2*, characterized by* $\mathbf{k}'$ *and* $\mathbf{k}^{\mathrm{P}2}$*. Utility gains from delegation (if any) consist of:*

- ***Direct Informational Gains*** *if:*

$$DIG_{j'}(y_1) \equiv \frac{(w_{11})^2}{6}\left[\frac{1}{\left(k_1^{\mathrm{C}}+2\right)} - \frac{1}{\left(k_1'+2\right)}\right] + \frac{(w_{12})^2}{6}\left[\frac{1}{\left(k_2^{\mathrm{C}}+2\right)} - \frac{1}{\left(k_2'+2\right)}\right] \geq (b_1')^2$$

Moreover, such direct informational gains require that there exists an agent i *whose preferences satisfy:*

$$|b_1^i - b_1'| < \left|b_1^i + b_2^i\,\frac{w_{21}}{w_{11}}\right| \tag{2}$$

- ***Indirect Informational Gains*** *if:*

$$IIG(y_2) \equiv \frac{(w_{21})^2}{6}\left[\frac{1}{\left(k_1^{\mathrm{C}}+2\right)} - \frac{1}{\left(k_1^{\mathrm{P}2}+2\right)}\right] + \frac{(w_{22})^2}{6}\left[\frac{1}{\left(k_2^{\mathrm{C}}+2\right)} - \frac{1}{\left(k_2^{\mathrm{P}2}+2\right)}\right] \geq 0$$

Moreover, such indirect informational gains require that there exists an agent i *whose preferences satisfy:*

$$|b_2^i| < \left|b_1^i\,\frac{w_{12}}{w_{22}} + b_2^i\right| \tag{3}$$

Moreover, whenever the optimal allocation of decision rights involves delegation of y_1 *to* j'*, then either* $DIG_{j'}(y_1) > b_1'$*, or* $IIG(y_2) > 0$*, or both.*

Proof. The expressions for the informational gains arises from comparing equation (1) for different allocations of decision rights. The last part of the lemma results from the complete characterization of the optimal organizational structure of Proposition 8 in Appendix A. For the necessary conditions, see Appendix B.5 □

Direct informational gains arise when more agents transmit information to the new decision maker than to the principal under centralization. A immediate implication of this is such direct gains arise only if j' has more central preferences on y_1; that is, if there is at least one agent who is willing to reveal information to j' but not to the principal under centralization. Equation (2) reflects this condition, it shows that the relevant conflict of interest between i and j' (left-hand side) is lower than that between i and the principal under centralization (right-hand side). As mentioned earlier, delegation to agents with more central preferences constitutes the 'traditional' argument in the literature of organizational design (see Aghion and Tirole, 1997; Dessein, 2002; Dewan et al., 2015). The presence of informational interdependencies, however, can lead to other mechanisms.

Indirect informational gains arise when more agents transmit information to the principal when she delegates one decision, as compared to centralization. As a necessary condition then there must be at least an agent who reveals more information to the principal when she decides on y_2 only. Equation (3) shows that the conflict of interest with the principal under partial delegation (left-hand side) is lower than the aggregate conflict of interest. In particular, since b_2^i is in both sides of the equation, the inequality requires that the bias on the first dimension is so large that communication under centralization is less informative than when the principal only decides on y_2. This is the notion of *negative informational spillovers*,[11] which are a necessary condition for indirect informational gains.

In Proposition 8 I show that full delegation is optimal when there are two different agents with central preferences and no informational spillovers associated to retaining authority. The following observation summarizes the intuition behind Full Delegation.

Observation. *When the principal delegates both decisions, it is due to direct informational gains in both —that is, $k_1^{'} > k_1^{\mathrm{C}}$ and $k_2^{''} > k_2^{\mathrm{C}}$.*

The allocation of decision rights depends then on the different communication equilibria induced by the profile of preferences. The previous discussion made clear that the argument reduces to whether the direct and/or indirect informational gains are sufficiently large, where sufficiency involves the loss of control from delegation. Proposition 8 in Appendix A.2 provides a precise formulation of this argument, serving as a formal counterpart of the previous discussion.

The role of informational spillovers on organizational design. I now present the main result of this section: how the presence of informational spillovers lead to specific organizational structures. Proposition 1 showed that delegation is optimal if the additional information the decision makers expect to receive on path compensates the loss of control due to biased decisions. Such informational gains can be direct, when the new decision maker's preferences are close to other agents in the delegated dimension, or indirect, when the principal's preferences are sufficiently close to the agents' in the retained dimension. I showed that indirect gains result from delegating a decision for which many agents have extreme biases (with respect to the principal), and retaining authority over a decision for which those agents' preferences are aligned with the principal. The presence of negative informational spillovers is thus a necessary condition for indirect gains, which in turn leads to partial delegation.

[11] Levy and Razin describe negative informational spillovers as the case when i's bias with respect to y_1 is so large that he would not reveal any information under centralization, even though $|b_2^i|$ is very small. Their analysis stops at the negative effects of such informational spillovers on communication.

But informational spillovers are not limited to be negative in this context. Positive spillovers occur when i's decision-specific biases are both large, but the aggregate conflict of interest (associated to the information he reveals in equilibrium) is small. When i is expected to reveal information about θ_1 only, for instance, his incentives are maximal when $b_1^i = -\frac{w_{12}}{w_{11}} b_2^i$. Intuitively, if there were many agents whose preferences are affected by positive spillovers the principal would prefer centralization to any other organizational structure. The proposition below presents the optimal organizational structure under informational spillovers.

Proposition 2. *Let the triple* $(\mathbf{k}^{\mathrm{C}}, \mathbf{k}', \mathbf{k}'')$ *characterize the optimal organizational structure under the profile of biases* $\mathbf{B}^n = (\mathbf{b}^1, ..., \mathbf{b}^n)$*. Suppose that* $k_1' = k_2'$*,* $k_1'' = k_2''$*, and* $k_1^{\mathrm{C}} = k_2^{\mathrm{C}}$*. Now, consider the associated game consisting of* $n+m$ *agents, with the profile of biases for the first* n *being* $\mathbf{B}^n$*, such that* $\mathbf{B}^{n+m} = (\mathbf{b}^1, ..., \mathbf{b}^n, \mathbf{b}^{n+1}, ..., \mathbf{b}^{n+m})$*. For a sufficiently large* $\mathfrak{b} \in \Re_+$*, it is true that:*

1. *For* $\mathbf{b}^{n+1} = ... = \mathbf{b}^{n+m} = (\mathfrak{b}, 0)$*; then, there exists a sufficiently large* m *such that the optimal organizational structure in the game with* $n+m$ *agents is partial delegation of* y_1 *only.*
2. *For* $\mathbf{b}^{n+1} = ... = \mathbf{b}^{\frac{2n+m+1}{2}} = (-\mathfrak{b}, \mathfrak{b})$ *and* $\mathbf{b}^{\frac{2n+m+1}{2}} = ... = \mathbf{b}^{n+m} = (\mathfrak{b}, \mathfrak{b})$*; then, there exists a sufficiently large* m *such that the optimal organizational structure in the game with* $n+m$ *agents is centralization.*

Proof. See Appendix B.6 □

Informational spillovers in the above result are captured by the preferences of the 'additional' agents, such that m reflects the intensity of the spillovers. The proposition shows that sufficiently large negative spillovers lead to partial delegation, because the principal finds optimal to delegate the controversial decision in order to induce the additional agents to reveal the information they have. Sufficiently large spillovers means the additional information the principal expects to receive brings her a higher expected utility than the allocation of decision rights with the original profile of preferences.

Similarly, the second case in Proposition 2 reflects the case of positive informational spillovers. The additional agents in both groups of preferences are willing to play dimensional non-separable strategies under centralization. Senders in the first group fully reveal their signals when $\mathbf{S}^i = \{(0,0);(1,1)\}$ and announce the babbling message for the other possible realizations; while senders in the second group fully reveal their signals when $\mathbf{S}^i = \{(0,1);(1,0)\}$ and announce the babbling message otherwise. As in the previous case, the additional information the principal expects to receive from these senders bring her a higher expected utility than the optimal allocation under the original profile of preferences.

Because the choice among organizational structure depends on the trade-off between informational gains and loss of control, in the following subsection I study it in more depth.

3.1 Relationship between informational gains and loss of control

Since agents are imperfectly informed about the two states, the principal prefers delegation if sufficiently many agents reveal their information to at least one decision maker. Sufficiency in this case means that informational gains must compensate the principal for the bias on the associated decision, defining a relationship between informational gains and loss of control. Proposition 3 below presents this relationship, after the following notational convention.

If player j decides on y_d, the ex-ante expected variance associated to state θ_r on the equilibrium path is given by $\text{Var}(\theta_r|\mathbf{m}_j) \equiv \frac{1}{6(k_r^j+2)}$. For any profile of biases, denote by $\mathbf{m}_{\text{C}}$ the vector of messages the principal receives in the equilibrium under centralization, and by $\mathbf{m}_j = \{\mathbf{m}_{j'}, \mathbf{m}_{j''}\}$ the equilibrium messages for j' and j'' in the equilibrium under delegation, for $j', j'' = \{P, 1, 2, ...\}$.

Proposition 3 (Maximum Admissible Loss of Control)**.** *Let* $\mathbf{b}^{\text{D}} = (b_1', b_2'')$ *be the biases of decision makers for* y_1 *and* y_2*, respectively. Then, the maximum* $\mathbf{b}^{\text{D}}$ *for which the principal is willing to delegate at least one decision is given by:*

$$||\mathbf{b}^{\text{D}}|| \equiv \left[\sum_{y_d}\left[w_{d1}^2\big(\mathit{Var}(\theta_1|\mathbf{m}_{\text{C}}) - \mathit{Var}(\theta_1|\mathbf{m}_j)\big) + w_{d2}^2\big(\mathit{Var}(\theta_2|\mathbf{m}_{\text{C}}) - \mathit{Var}(\theta_2|\mathbf{m}_j)\big)\right]\right]^{\frac{1}{2}}$$

Proof. See Appendix B.7 □

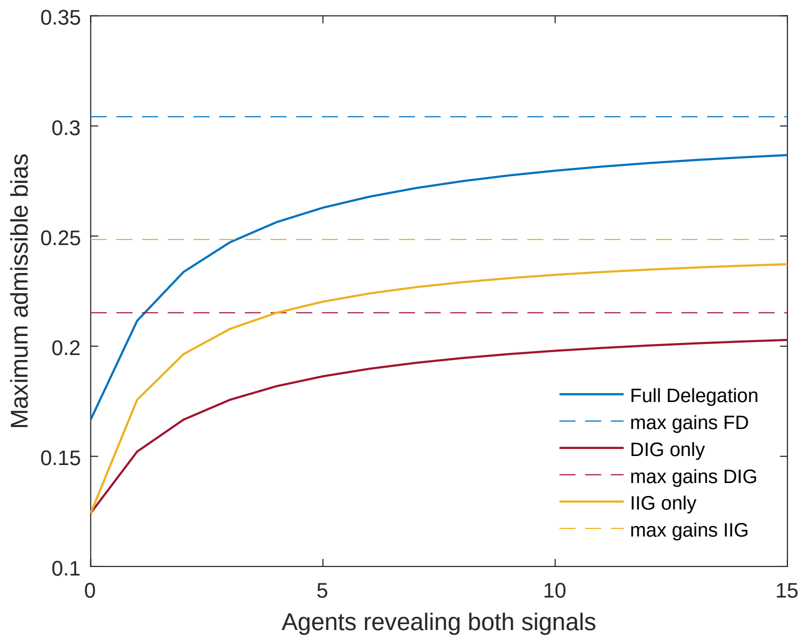
There is a positive relationship between informational gains from delegation and the 'loss of control' the principal tolerates. The expression $||\mathbf{b}^{\text{D}}||$ represents the relevant measure for the loss of control: how far are both decisions to the principal's (state-dependent) ideal. Note that under partial delegation one of its components is zero, marking the maximum bias the principal will tolerate in a single decision. The right-hand side of the equation above represents how delegation increases decisions precision in equilibrium, due to the informational gains.

Figure 2 helps to visualize the relationship between informational gains and loss of control. It shows the maximum loss of control the principal will tolerate as a function of the number of agents revealing both signals, in three cases.[12] First, the overall maximum loss of control the principal is willing to tolerate when she delegates both decision to different agents (blue lines). Minimum informational gains in this case mean each decision maker decides with his own signals, but no other agent reveals any additional information. The concavity of the curve represents the decreasing marginal utility of additional information, which comes from quadratic preferences and the updating process.[13] Maxi-

[12] To make the comparison clear, I assume $k_1^{\text{C}} = k_2^{\text{C}} = 0$.

[13] The marginal influence of an additional signal on the associated posterior belief is decreasing (see Morgan and Stocken, 2008; Galeotti et al., 2013).

Figure 2: Admissible loss of control as a function of informational gains



Note: $w_{11} = w_{22} = \frac{2}{3}$, $k_1^C = k_2^C = 0$

mum informational gains are represented with the dashed line, showing the maximum bias vector the principal tolerates when she expects each decision maker *to become perfectly informed in equilibrium.*

Secondly, Figure 2 shows the loss of control the principal tolerates when informational gains are direct only (red lines). To isolate the effects of direct gains, I assume the principal retains authority on the other decision and does not receive any additional information. Minimum informational gains thus mean the agent deciding on the delegated decision uses his own information, and the principal decides with none. As a consequence, the maximum bias the principal tolerates in this case is lower than the full delegation case, and this holds true for all possible informational gains. The red dashed lines shows the maximum (decision-specific) bias the principal tolerates when delegating a decision, since it involves the decision maker becoming perfectly informed as a result of the communication stage.

In the third place, the figure also shows the relationship between indirect informational gains and loss of control (yellow lines); that is, partial delegation leading to more information received by the principal. In this case, the agent deciding on the delegated decision observes his signals, which in part explains why the principal is willing to tolerate a higher bias than when informational gains are only direct. Because of the quadratic prefrences, these additional signals on the decision not affected by informational gains have a high marginal for the principal. This is evident when comparing the admissible loss of control when informational gains are maximal (yellow and red dashed lines), with the same different at the origin: they are equal.

The intuitions just described assume constant information under centralization ($k_1^{\mathrm{C}} = k_2^{\mathrm{C}}$). I now show how the maximum admissible loss of control depends on k_1^{C} and k_2^{C}. In summary, the principal's marginal expected utility from an additional signal is decreasing in the amount of information she expects to receive under centralization.

Corollary 1. *The 'marginal value' of an additional signal is decreasing in the amount on information the principal receives under centralization. Let* $\Delta\, Var(\theta_r|k_r^{\mathrm{C}}) = Var(\theta_r|\mathbf{m}_{\mathrm{C}}) - Var(\theta_r|\mathbf{m}_j, k_r^j = k_r^{\mathrm{C}} + 1)$, *then:*

$$\frac{\partial \Delta\, Var(\theta_r|k_r^{\mathrm{C}})}{\partial k_r^{\mathrm{C}}} = \frac{1}{6}\left[\frac{1}{(k_r^{\mathrm{C}}+3)^2} - \frac{1}{(k_r^{\mathrm{C}}+2)^2}\right] < 0 \tag{4}$$

Equation (4) means that an additional signal represents a small informational gain when the principal expects to be well informed under centralization. The maximum bias she is willing to tolerate thus decreases as the profile of biases allows more agents to truthfully reveal information under centralization. The result extends Corollary 1 in Dessein (2002) to the case of imperfectly informed senders.[14] The residual variance here depends on the amount of information received by the principal under centralization.

4 Endogenous Information Acquisition

In this section I analyse agents' incentives to acquire information before the communication stage, but after decision-rights have been allocated. I first present the extended model. Secondly, I derive the two incentive-compatibility constraints involved in information acquisition decisions—communication IC and cost-effectiveness— and show how information costs impose restrictions on informational gains from delegation. I then characterize the equilibrium strategies for a generic agent, showing the cases in which he decides to specialize. Finally, I characterize how the optimal organizational structure of Proposition 8 changes with the cost of information, and show that delegation typically leads to *ex-post specialization*.

Baseline model with endogenous information acquisition. Each agent has access to one binary trial per state and decides which realizations to observe (if any).[15] Formally, let $\mathfrak{s}^i \in \big\{\{\emptyset\}, \{\tilde{S}_1\}, \{\tilde{S}_2\}, \{\tilde{S}_1, \tilde{S}_2\}\big\}$

[14] Dessein finds that incentives to delegate depend on the magnitude of the residual variance. In his paper there is a single, perfectly informed sender who advices the principal on a one-dimensional decision problem. He analyses incentives for delegation as a function of the ex-ante variance associated to the state, against the babbling equilibrium. Delegation dominates communication for every bias in which the sender would reveal any information to the principal (Proposition 2).

[15] In principle, agents could have the choice on how much information about each state to observe, involving information acquisition at the intensive and the extensive margins. Here, however, I focus on the extensive margin, meaning that

be agent i's information-acquisition decision. With this formulation i's type is given by the realizations of both signals, but he decides the extent to which he observes his type.

Definition 2. *The information structure for agent i in the Information Acquisition game consists of the following elements: $\mathbf{S}^i = (S_1^i, S_2^i)$ are the signals available to him, $\tilde{\mathbf{S}}^i = (\tilde{S}_1^i, \tilde{S}_2^i)$ the realization of the corresponding signals (his type), and $\mathfrak{s}^i \in \big\{\{\emptyset\}, \{\tilde{S}_1\}, \{\tilde{S}_2\}, \{\tilde{S}_1, \tilde{S}_2\}\big\}$ the information he actually decides to observe.*

The costs of different information structures are captured by the function $C(\mathfrak{s})$, which satisfies $C\left(\{\tilde{S}_1, \tilde{S}_2\}\right) > C\left(\{\tilde{S}_1\}\right) = C\left(\{\tilde{S}_2\}\right) > C\left(\emptyset\right) = 0$. The principal has no direct access to information. The preferences of agent $i = \{1, \ldots, n\}$ are given by:[16]

$$U^i(\boldsymbol{\theta}, \mathbf{x}, \mathbf{b}^i, \mathfrak{s}^i) = -\sum_{y_d = \{y_1, y_2\}} \left(y_d - \delta_d(\theta_1, \theta_2) - b_d^i\right)^2 - C(\mathfrak{s}^i)$$

Figure 3 shows the timing of the game. The allocation of decision rights is observed by all agents. Knowing who decides what, each agent chooses the information he will observe (if any). I assume *overt information acquisition*—individual decisions (but not information) is common knowledge. In Section 5 I discuss the implications of relaxing this assumption.

The communication stage is similar to the previous section, so I keep the notation. Let i be a generic agent and j a generic decision maker, let $j', j'' \in \{P, 1, ..., n\}$ be the decision makers for y_1 and y_2, respectively, such that $i = \{1, ...n\}$ and $j = \{j', j''\}$. Let $k_r^j \equiv k_r^*(\mathbf{m}_j^*(\mathfrak{s}^*))$ be the number of truthful messages decision maker j receives in equilibrium, and $k_r' \equiv k_r^*(j')$ and $k_r'' \equiv k_r^*(j'')$ refer to the number of truthful messages to decision makers j' and j'' (respectively).

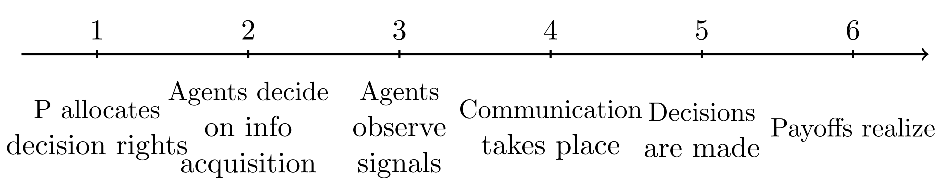


Figure 3: Timing of the Org. Structure / Info Acquisition game.

An equilibrium in this game is then characterized by the decision vector, $\mathbf{y}_d^*$, and collections of message and information acquisition strategies for each agent and decision maker j, $\mathbf{m}_j^* = \{\ldots, \mathbf{m}_j^{i*}, \ldots\}$ and $\mathfrak{s}^* = \{\ldots, \mathfrak{s}^{i*}, \ldots\}$. The expressions for optimal actions and messages are similar to those of the previous section, noting that $k_r^*(\mathbf{m}^*(\mathfrak{s}^*))$, $y_d^*(\mathbf{m}_j^*(\mathfrak{s}^*))$, and $\mathbf{m}_j^{*i}\left(\mathfrak{s}^i, \mathbf{m}^{-i}(\mathfrak{s}^{-i})\right)$. Agent i's information

each agent decides whether to observe at most one binary signal per state. In section 5, I discuss some implications of allowing agents to acquire information on the intensive margin.

[16] The principal's preferences are captured by $U^P(\boldsymbol{\theta}, \mathbf{x}) = -\left(y_1 - \delta_1(\theta_1, \theta_2)\right)^2 - \left(y_2 - \delta_2(\theta_1, \theta_2)\right)^2$.

acquisition strategy is given by:

$$\mathfrak{s}^{i*} = \arg\max_{\mathfrak{s}^i} \left\{ E\Big[-\left(y_1\left(\mathbf{m}^i_{j'}(\mathfrak{s}^i), \mathbf{m}^{-i}_{j'}\right) - \delta_1 - b^i_1\right)^2 - \left(y_2\left(\mathbf{m}^i_{j''}(\mathfrak{s}^i), \mathbf{m}^{-i}_{j''}\right) - \delta_2 - b^i_2\right)^2 \Big] - C(\mathfrak{s}^i) \right\}$$

The expectation is based on equilibrium beliefs. Agent i's equilibrium message strategy depends on the information he acquired in an earlier stage of the game. Both his message and information acquisition decisions depend on beliefs about other agents' strategies. The fact that information acquisition is common knowledge at the communication stage (overt game) simplifies the beliefs space. In other words, each agent forms a conjecture about other agents' acquisition strategies when deciding about his own, and also a conjecture about other agents' message strategies, the latter depending on i's knowledge of the information they have.

At the communication stage, incentive compatibility depends on the signals agent i has previously acquired. When he acquires information about both states, the IC constraints for communication are the same as in Lemmas 5 and 6. When he acquires information about one state, however, the IC constraints change dramatically because his incentives to reveal information are not affected by beliefs about the other state. This kills the credibility loss and truthful communication is incentive compatible for a broader set of bias vectors. Now, let me focus on the details of these arguments.

Incentives to acquire information. For an agent to acquire a piece of information the expected utility gains must compensate its costs. First, costly information acquisition means that each agent will invest in a signal only if he expects to benefit from it. Having more signals *per se* do not affect other agents' communication incentives, but only when the agent in question is willing to reveal that signal on the equilibrium path.[17] Hence, in equilibrium i only acquires signals he is willing to reveal; for if he fails to reveal any piece of information (off path), no other agent will change his on-path message strategy. The number of truthful messages does not change when i acquires a signal he does not reveal, but he still bears the costs. The lemma below formalizes this intuition—incentive compatibility at the information acquisition stage requires incentive compatibility at the communication stage.

Lemma 1. *Let $(\mathbf{y}^*, \mathbf{m}^*, \mathfrak{s}^*)$ be equilibrium strategy profiles for the principal and all agents. The equilibrium is characterized by the number of truthful messages decision makers receive, $k^j_1\left(\mathbf{m}^*_j(\mathfrak{s}^*)\right)$ and $k^j_2\left(\mathbf{m}^*_j(\mathfrak{s}^{i*})\right)$, for $j = \{j', j''\}$. Then, agent i equilibrium information acquisition strategy, $\mathfrak{s}^{i*}$, satisfies:*

- *$S_r \in \mathfrak{s}^{i*}$ only if truthful revelation to j is incentive compatible, given $k^j_r\left(\mathbf{m}^*_j(\mathfrak{s}^*)\right)$;*

[17] Formally, i's incentives for communication depend on having acquired the signal, $\mathbf{b}^i$, and on his conjecture about k^j_1 and k^j_2. Then, for i acquiring S^i_r *off-path* to change another agent h's conjecture about k^j_r, b^i should be such he is willing to reveal that signal. In such a case, h (off-path) conjecture for k^j_r should be larger than the equilibrium value, but then i would be willing to reveal S^i_r in equilibrium and would have acquired it.

- $\{S_1, S_2\} \in \mathfrak{s}^{i*}$ *only if full revelation to* j *is incentive compatible, given* $k_r^j\left(\mathbf{m}_j^*(\mathfrak{s}^*)\right)$.

Proof. See Appendix B.8. □

The main implication of Lemma 1 is that the choice of organizational structure will affect agents' incentives for information acquisition, because it determines the relevant IC constraints at the communication stage. Incentives to acquire information depends on the possibility of being influential. But credibility hinges on both the conflict of interest and the number of other agents expected to reveal similar information on path. Agents thus acquire information they expect to reveal on the equilibrium path, given the allocation of decision rights. The conclusion is similar to Di Pei (2015): information structures available to agents and the cost function satisfy assumptions 1 and 2 of his paper ("Richness" and "Monotonicity"). Both seem rather natural assumptions in my framework; for a given information structure a 'coarser' alternative means investing in less signals, which will also be cheaper than the original choice.

The second element of incentive compatibility of information acquisition relates to its costs. Utility gains from revealing any piece of information are decreasing in the number of other agents revealing the same information (k_r^j). Given costs are strictly positive, there is a maximum number of agents for whom the utility gains of revealing that piece of information compensate the costs. The following lemma presents the *cost-effectiveness* condition, which captures this idea.

Lemma 2. *Let* k_r^j *denote* i*'s conjecture about other agents revealing* S_r *truthfully to* j*.*

Centralization: *acquiring signal* S_r^i *is cost-effective for* i *under centralization if:*

$$C(S_r^i) \leq \frac{(w_{1r})^2 + (w_{2r})^2}{6(k_r^{\mathrm{C}} + 2)(k_r^{\mathrm{C}} + 3)} \tag{5}$$

When willing to play DNS message strategies, cost-effectiveness requires:

$$2\,C(S_1^i, S_2^i) \leq \frac{(w_{11})^2 + (w_{21})^2}{6(k_1^* + 2)(k_1^* + 3)} + \frac{(w_{12})^2 + (w_{22})^2}{6(k_2^* + 2)(k_2^* + 3)} \tag{6}$$

Delegation: *acquiring signal* S_r^i *is incentive compatible for agent* i *if for at least one decision,* y_d*, with the corresponding decision maker* j *is true that:*

$$C(S_r^i) \leq \frac{(w_{dr})^2}{6(k_r^j + 2)(k_r^j + 3)} \tag{7}$$

Proof. See Appendix B.10. □

An agent acquires a signal if its expected influence on decision(s) is sufficiently large. The influence of revealing a signal depends on how many other agents are expected to reveal the same information, which in turn depends on the organizational structure. Under centralization, revealing a given signal influences both decisions. Under delegation, the influence depends on whether i reveals the signal to one or both decision makers—affects one or both decisions. The ex-ante expected utility gains of acquiring (and revealing) a given signal is thus weakly lower under delegation.

Dimensional non-separable message strategies face a more restrictive cost-effectiveness condition, because i expects to reveal information for half of the possible signal realizations. The costs of acquiring both signals must then be sufficiently low for such a strategy to be cost-effective. The latter does not hold if revealing one signal is also incentive compatible for i, in which case the most-informative equilibrium consists of acquiring and revealing information about one state.

Because the expected influence of truthful revelation is decreasing in the number of other agents revealing the same information, there exists a maximum number of agents for whom cost-effectiveness hold with respect to any signal.

Corollary 2. *The maximum number of agents acquiring S_r in any equilibrium under centralization is given by:*

$$K_r^{\mathrm{C}} = \left\lfloor \left[\frac{1}{4} + \frac{[(w_{1r})^2 + (w_{2r})^2]}{6\,C(S_r^i)} \right]^{1/2} - \frac{5}{2} \right\rfloor + 1 \tag{8}$$

And under delegation, the maximum number of agents acquiring S_r in any equilibrium is:

$$K_r^{\mathrm{D}} \in \left[\left\lfloor \left[\frac{1}{4} + \frac{(\hat{w}_{dr})^2}{6\,C(S_r)} \right]^{1/2} - \frac{5}{2} \right\rfloor + 1\,;\, K_r^{\mathrm{C}} \right] \tag{9}$$

Where $\hat{w}_{dr} \equiv \min\{w_{1r}, w_{2r}\}$

Expression (8) and (9) represent the maximum number of agents, other than i, for whom investing in a signal is cost-effective under centralization (K_r^{C}) and delegation (K_r^{D}), respectively. These numbers depend on whether agents influence both or only one decision. Note that under centralization any influential agent affects both decisions; while the same is true under delegation for agents revealing information to both decision makers. Is only when there are sufficiently many of such agents under delegation that $K_r^{\mathrm{C}} = K_r^{\mathrm{D}}$. Typically, however, many agents under delegation will reveal information to one decision maker, which would make $K_r^{\mathrm{C}} > K_r^{\mathrm{D}}$. Proposition 6 below shows the conditions for this to be true.

I now discuss the equilibrium information acquisition and message strategies and how they are affected by the allocation of decision rights. I also present the analysis on specialization.

Equilibrium information acquisition and specialization

Equilibrium information acquisition strategies depend on the organizational structure and the cost of information. The formal characterization can be found Appendix A, but Lemmas 1 and 2 summarize the main intuitions. It is worth noting that the number of agents revealing the same piece of information is an important determinant of cost-effectiveness under any organization structure. It can lead some agents to acquire less information than what is IC at the communication stage, resulting in either specialization or non-investment. Also, dimensional non-separable strategies under centralization can only emerge for sufficiently low information costs, and the principal is expected to receive similar amounts of information about both states (otherwise, the agent will acquire information about the state the principal is expected to be less informed).

Equilibrium information acquisition and communication under different allocation of decision rights depend on the profile of biases. As in section 3, it also determines the degree to which the principal can extract informational gains from delegation. But with endogenous information acquisition, her ability to do that also depends on information costs; in particular, when information costs are zero, she is only limited by incentive compatibility at the communication stage, when cost are sufficiently large, centralization is always optimal. As the cost of a signal increases, the principal's ability to extract informational gains that compensate the corresponding loss of control decrease.

The fact that agents decide on information acquisition about both states allows for specialization. In the following paragraphs I show how specialization enhances information transmission, as well as the different situations in which an agent decides to acquire information about one state only.

Specialization. Individual decisions on information acquisition are equivalent to choosing between message strategies given other agents' equilibrium behaviour. In particular, when agent i acquires information about one state only, his incentives for communication do not depend on information about the other state (because he does not observe any). This eliminates the credibility loss due to ambiguous information, thus enlarging the set of biases for which revealing that signal is incentive compatible. The proposition below shows the result.

Proposition 4. *Let $S_r \in \mathfrak{s}^{i*}$ and $S_{\tilde{r}} \notin \mathfrak{s}^{i*}$ for $\theta_r \neq \theta_{\tilde{r}}$, and $k_r^j\left(\mathbf{m}_j^*(\mathfrak{s}^*)\right)$ be i's conjecture about other agents revealing their information about θ_r to decision maker j. Then, agent i's IC constraint for revealing S_r^i is:*

- *When $j = P$ decides on both issues (centralization),*

$$|\beta_r^i| \leq \frac{(w_{1r})^2 + (w_{2r})^2}{2\left(k_r^{\mathrm{C}} + 3\right)} \tag{10}$$

- *When j decides on y_d only,*

$$|b_d^i - b_d^j| \leq \frac{w_{dr}}{2\big(k_r^j + 3\big)} \tag{11}$$

Proof. See Appendix B.9 □

Acquiring information about one state only eliminates the possibility of ambiguous information. Hence, agent i is not tempted to lie when information about the other state is more favourable; which implies that revealing information about that state is incentive compatible for a larger set of biases. In other words, *specialization* acts as a commitment device because the agent does not know when an unfavourable signal produces an excessive update against his preferences.

Another implication of Proposition 4 relates to the optimal amount of information acquired when costs are zero, as shown in the following corollary.

Corollary 3. *Let $C(S_1^i) = C(S_2^i) = 0$. If agent i's preferences satisfy:*

$$|\beta_1^i| \in \left(\frac{(w_{11})^2 + (w_{21})^2}{2} \left[\frac{1}{(k_1^{\mathrm{C}} + 3)} - \frac{C_1}{(k_2^{\mathrm{C}} + 3)} \right] \; ; \; \frac{(w_{11})^2 + (w_{21})^2}{2\,(k_1^{\mathrm{C}} + 3)} \right]$$

Then, in the most informative equilibrium under centralization he acquires and truthfully reveals information about θ_1 only.

For some range of preferences, even if *i has access to costless information*, both he and the principal prefer that he specializes in information about θ_1. In other words, the principal prefers a less informed agent because it guarantees he will not be tempted to manipulate information. Note that the two results above hinge on the assumption that the information acquisition decisions are observable. In Section 5, I discuss the implications of relaxing it, and show that specialization still increases credibility when the costs of information is not too low. Both results may have interesting implications about how firms organize subunits' access to information, because raising the costs of acquiring some type of information may raise the credibility on information they already have low costs.

I now analyse the different reasons that induce an agent to specialize, with an example with 2 agents. Let assume $w_{11} = w_{22} = w > \frac{1}{2}$ and $C(\mathfrak{s}^i) = c \times (\#\,\mathfrak{s}^i)$, and let denote the agents by A^1 and A^2. I focus on the centralization equilibrium in which A^1 acquires information about θ_1 and A^2 acquires information about θ_2, $\mathfrak{s}^{1*} = \{\tilde{S}_1^1\}$ and $\mathfrak{s}^{2*} = \{\tilde{S}_2^2\}$. In the equilibrium under consideration, the principal is (ex-post) more informed than each of the agents since $k_1^* = 1$ and $k_2^* = 1$.[18] The Corollary below formalizes the result and panel (a) in Figure 4 illustrates the set of biases for which $\mathfrak{s}^{1*} = \{\tilde{S}_1^1\}$ under centralization.

[18] The paper by Alonso et al. (2015) analyses a similar situation in the form of generalist-specialist information structure, where each agent specializes in a different piece of information and fully transmit it to the principal.

Proposition 5 (Specialization under centralization)**.** *Suppose that there are only two agents, A^1 and A^2, and the marginal cost of each signal is linear and equal to c. There exist two cost thresholds, $\underline{c} < \bar{c}$, such that the most-informative equilibrium under centralization, $(\mathbf{y}^*, \mathbf{m}^*, \mathfrak{s}^*)$, consists in A^1 acquiring and revealing information on θ_1 only, and A^2 acquiring and revealing information about θ_2 only, in the following cases:*[19]

1. *For $c \leq \underline{c}$, if and only if revealing S_1^1 is IC for A^1, revealing S_2^2 is IC for A^2, and revealing both signals is not IC for any of them; or*

2. *For $\underline{c} < c \leq \bar{c}$, if and only if revealing S_1^1 is IC for A^1 and revealing S_2^2 is IC for A^2.*

Where $\underline{c} = \frac{w^2+(1-w)^2}{72}$ *and* $\bar{c} = fracw^2 + (1-w)^2 36$.

Proof. See Appendix B.12 □

When the cost of a signal is close to zero, information acquisition does not impose restrictions on communication. Agents will then acquire any information they are willing to reveal and, thus, they specialize on different signals only if these are the only IC message strategies. The stripped region in panel (a) of Figure 4 shows specialization *driven by preferences*.

The cross-hatched region in panel (a) represents the set of biases for which A^1 is willing to reveal information about any state individually, but not if he acquired both signals. On the equilibrium path he expects A^2 to acquire and reveal information about θ_2, such that he expects larger utility gains from specialization on θ_1. This specialization decision is *driven by expected larger influence* on the principal's beliefs, given A^2's equilibrium strategy.[20]

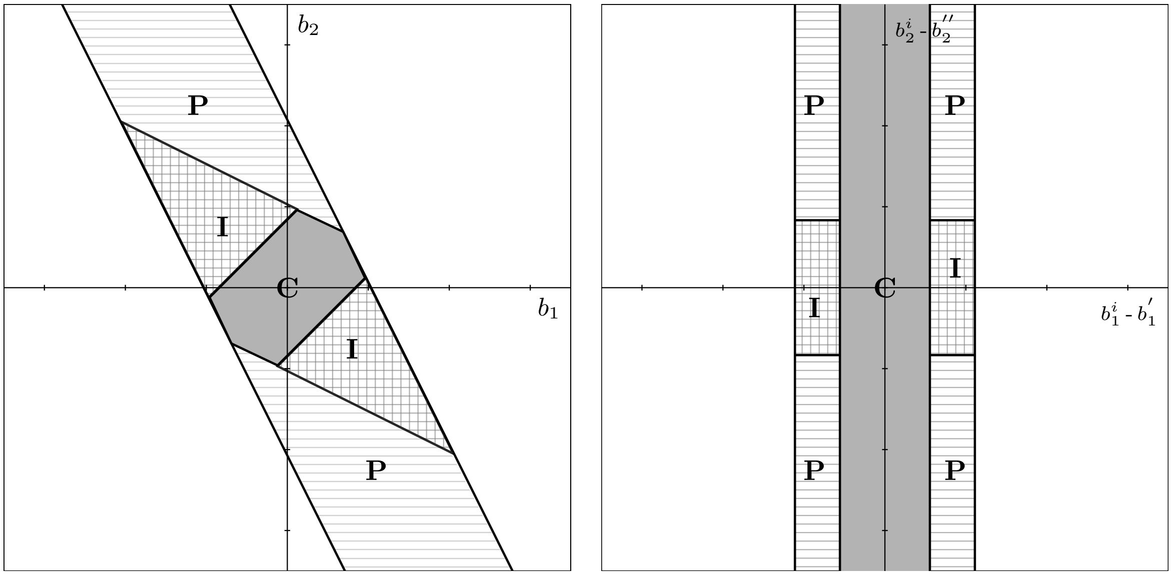
When A^1 is willing to reveal information about both states, specialization only emerges if acquiring such information is too costly. Whether he acquires information about θ_1 or θ_2 depends on what A^2 is expected to do: A^1 acquires information about the state the principal is expected to be less informed on path. The solid gray region in panel (a) illustrates the case of specialization *driven by costs*.

Specialization under delegation follows the same intuitions, with a different cost-effectivess condition (Appendix B.12). Here I describe these intuitions using Panel (b) in Figure 4. Given $w_{11} = w_{22} > 1/2$ and A^1 in equilibrium acquires information about θ_1, his incentives for communication are stronger on the first dimension. Indeed, whenever his preferences are close to the decision maker of the second dimension, j'', A^1 prefers to acquire information about θ_2 (so he becomes specialized on

[19] Formally, the equilibrium consists in $\mathfrak{s}^{1*} = \{\tilde{S}_1^1\}$ and $m^{1*} = \big\{\{(0,0),(0,1)\},\{(1,0),(1,1)\}\big\}$ for A^1, and $\mathfrak{s}^{2*} = \{\tilde{S}_2^2\}$ and $m^{2*} = \big\{\{(0,0),(1,0)\},\{(0,1),(1,1)\}\big\}$ for A^2.

[20] An alternative equilibrium exists when both agents' bias vectors lie on cross-hatched regions. The strategies $\mathfrak{s}^{1*} = \{\tilde{S}_2^1\}$ and $\mathfrak{s}^{2*} = \{\tilde{S}_1^2\}$ can also be sustained; agents thus face a coordination problem for which there is no clear selection criterion—the principal is ex-ante (and ex-post) indifferent between any of these. Both equilibria involve specialization mainly because no agent is willing to reveal both signals.

Figure 4: Specialization in the 2-agents model — Driven by preferences (**P**), influence (**I**), and costs (**C**).



(a) A^1 specializes on θ_1 – Centralization (b) A^1 specializes on θ_1 – Delegation

Notes: $w = \frac{2}{3}$ and $k_1^* = k_2^* = 1$

this information). The intuitions for the different drivers of specialization—preferences, influence, and costs— are the same as for centralization, and are illustrated in the different regions of panel (b).

In the following section, I analyse the effects of different organizational structures on incentives to acquire information. As I will show, delegation affects both the absolute and relative expected investment in information.

4.1 Organizational Design on Incentives to Acquire Information

The optimal organizational structure depends on the profile of biases and the costs of information. As showed in the previous section, the principal can materialize the informational gains from delegation when costs are sufficiently low, and results of Section 3 apply—indeed, specialization enhances information transmission in such cases (Proposition 4). For sufficiently large costs, however, centralization is always optimal, since in the limit no agent can aquire any signal.

In this section, I study how the allocation of decision rights affects incentives to acquire information *independently from the profile of biases.* I aim to capture the idea that institutions may have persistent informational effects on decisions outcomes. In policy-making, for instance, the allocation of authority over a set of issues to a governmental agency effectively grants policy influence to interest groups linked to it (Baumgartner and Jones, 2009). Part of that influence is about 'feeding' the agency with information on states likely to be more favourable to the groups' preferences. In such cases, the policy becomes too responsive to these issues but insensitive to other issues it should also take into account.

I first analyse whether delegation affects the *expected absolute investment in information*, by focusing on the maximum number of agents for whom acquiring a signal is cost-effective. Second, I analyse whether delegation affects the *expected relative investment in information*, by assuming the bias vectors are randomly allocated within a fixed conflict of interest with the principal.

Absolute Investment in Information As discussed in Section 3, the principal delegates a decision if the informational gains compensate the loss of control. When information is costly for agents, there is a limit on the informational gains the principal can obtain from delegation (Corollary 2). Since this limit is decreasing in the cost of each signal, the allocation of decision rights that was optimal with no information acquisition is, in principle, still optimal when costs are sufficiently low. In many cases, however, these costs affect the amount of information different organizational structures are expected to aggregate. The following result shows that information costs impose stronger restrictions on delegation than centralization.

Proposition 6. *Let κ be the maximum number of agents willing to reveal information about $\theta_r = \{\theta_1, \theta_2\}$ to both decision makers under delegation. Then, for every $\kappa < n$, there exist costs values for which the maximum number of agents willing to acquire (and reveal) information about θ_r is strictly lower under delegation than under centralization. Formally,*

$$C(S_r) \in \left(\frac{(\hat{w}_{dr})^2}{6(\kappa+2)(\kappa+3)}, \frac{(w_{1r})^2 + (w_{2r})^2}{6(\kappa+3)(\kappa+4)} \right] \;\Rightarrow\; K_r^{\mathrm{C}} > K_r^{\mathrm{D}}$$

Where $\hat{w}_{dr} = \max\{w_{1r}, w_{2r}\}$.

Proof. See Appendix B.13. □

When information is costly, the maximum informational gains under delegation are weakly lower than under centralization. Typically, agents reveal their information to one decision maker under delegation (probably the reason why authority was given to that agent). Under centralization, any information transmitted affects both decisions and agents willing to reveal any signal have larger overall influence; the expected utility gains for such agents are thus larger under centralization and there will typically be more of them willing to invest in information.

Proposition 6 does not mean that centralization is always optimal. For a non-empty set of cost values, there exist profiles of biases, $\mathbf{b}$, for which delegation of some sort is preferred to centralization ($K_r^{\mathrm{C}} > K_r^{\mathrm{D}}$ is not binding). But for sufficiently large costs for which centralization always dominates (see Corollary 5 in Appendix A). This is the case when cost of each signal is so high that no agent acquires information under delegation.

In addition, Proposition 6 relates to the relationship between loss of control and informational gains in Proposition 3. In the previous section we learned that $||\mathbf{b}^{\mathrm{D}}||$ is increasing in the informational gains from delegation. In the present section I showed that information costs impose limits on informational gains; now I will show how $||\mathbf{b}^{\mathrm{D}}||$ is affected by costs. The maximum bias as a function of K^{D} (maximum informational gains) is given by:

$$||\mathbf{b}^{\mathrm{D}}|| = \left[\frac{[w^2 + (1-w)^2]}{6}\left[1 - \frac{2}{(K^{\mathrm{D}}+2)}\right]\right]^{\frac{1}{2}}$$

Which together with equation (9) in Corollary 2 leads to the following result.

Corollary 4. *The effect of information costs on* $||\mathbf{b}^{\mathrm{D}}||$ *is given by:*

$$\frac{\partial ||\mathbf{b}^{\mathrm{D}}||}{\partial C(S_r)} < 0$$

The maximum admissible bias the principal tolerates decreases as the cost of information increases. Increasing costs reduces the number of agents willing to acquire any signal, decreasing the informational gains that can be achieved through delegation. This is relevant when information costs are large and the principal is not sure about the exact profile of biases at the moment of 'designing' the organization. In such a case, she can expect the benefits from delegation to be low relative to centralization. This relationship between the information costs and distributional loss could be related to information acquisition on the intensive-margin—the higher the costs, the lower the amount of information a single agent is expected to acquire and, thus, the lower his 'informational advantage' with respect to the principal. I explore this intuition in Section 5.

I now analyse different consequences of delegation with costly information acquisition.

Relative Investment in Information I now analyse whether the allocation of decision rights affects the relative investment in information—the amount of information about each state decision makers are expected to receive. In order to isolate the mechanism through with the allocation of decision rights affects relative investment, I assume the principal does not observe the exact distribution of biases when deciding on the allocation of decision rights, and cost of information is zero. The second assumption (zero cost of information) helps to focus the analysis on the relative investment, because agents' information acquisition decision are determined by incentives for communication—which depend on the organizational structure.

First, consider the case of centralization. Let $\varepsilon \in \Re_+$ and $N_\varepsilon = \{1, 2, ..., n_\varepsilon\}$ be a group of agents whose preferences satisfy: for all $i \in N_\varepsilon$ then $||\mathbf{b}^i|| = \varepsilon$. Now, let $\lambda_r \equiv \left\{\mathbf{z} \in \Re^2 | \ \mathbf{z}' \, \mathbf{W}_r = 0\right\}$ be the locus with slope $-\frac{w_{1r}}{w_{2r}}$ related to θ_r. This locus represents maximal incentives to reveal information about $\theta_r = \{\theta_1, \theta_2\}$ to the principal ($\beta_r^i = 0$). The IC constraint for revealing one signal under centralization –equation (10)– can be expressed as:

$$||\mathbf{b}^i - \text{Proj}_{\lambda_r}(\mathbf{b}^i)|| \leq \frac{\left[(w_{1r})^2 + (w_{2r})^2\right]^{\frac{1}{2}}}{2\,(k_r^{\text{C}} + 3)}$$

Where $\text{Proj}_{\lambda_r}(\mathbf{b}^i)$ is the projection of i's bias vector onto the locus λ_r. Note that agents with small conflict of interest reveal both signals. The same conclusion applies to dimensional non-separable message strategies; agents with the corresponding preferences reveal both signals for some realizations and reveal nothing otherwise. Hence, the principal expects to receive more information about one of the states when many agents acquire and reveal the associated signal and few agents acquire and reveal the other. The result below shows whether this is the case under centralization.

Lemma 3. *Let $w \equiv w_{11} = w_{22}$. Given $\varepsilon \in \Re_+$ and arbitrarily large n_ε, then for every integer κ and $i \in N_\varepsilon$ with $||\mathbf{b}^i - Proj_{\lambda_1}(\mathbf{b}^i)|| \leq \frac{\left[(w)^2+(1-w)^2\right]^{\frac{1}{2}}}{2\,(\kappa+3)}$, there exists a $j \in N_\varepsilon$ with $||\mathbf{b}^j - Proj_{\lambda_2}(\mathbf{b}^j)|| \leq \frac{\left[(1-w)^2+(w)^2\right]^{\frac{1}{2}}}{2\,(\kappa+3)}$.*

Proof. See appendix B.14 □

Under centralization, for any agent who is willing to reveal information about one state, the principal expects to find another agent with the same conflict of interest who is willing to reveal information about the other state.[21] If the conflict of interest is sufficiently large ($\gamma\varepsilon$, with $\gamma > 1$) influencial agents either reveal information about one state or play a dimensional non-separable strategy.[22] Agents playing the latter are effectively revealing both signals for half of the possible realization, so it will not 'unbalance' the principal's information. On the contrary, agents' incentives to reveal one signal depend on the distance between the bias vector and the loci λ_1 and λ_2. Lemma 3 points out that the IC constraints associated to revealing information about each state have the same 'width', when number of agents revealing each signal is fixed. In other words, for any given k_1^{C} the number of agents still willing to reveal information about θ_1 is always the same as those willing to reveal information about θ_2, for $k_1^{\text{C}} = k_2^{\text{C}}$. Panel (a) in Figure 5 illustrates this.

Now consider the case of delegation. Let $\lambda_r^d \equiv \{\mathbf{z} \in \Re^2 | \ \mathbf{z}' \, \mathbf{I}_d = 0\}$ be the locus of maximal incentives to reveal about $\theta_r = \{\theta_1, \theta_2\}$ when deciding on $y_d = \{y_1, y_2\}$.[23] The locus λ_r^d captures the

[21]Throughout this analysis I assume states are equally important across decisions –i.e. $w_{11} + w_{21} = w_{12} + w_{22}$.

[22]If the fixed conflict of interest (ε) is sufficiently small there is a finite number of agents for which revealing both signals is incentive compatible, as shown in the inner circle in Figure 5.a.

[23]Where $\mathbf{I}$ is the 2-by-2 identity matrix, and $\mathbf{I}_d$ is its dth column, which matches the index of the decision under

fact that communication depends only on the conflict of interest associated to y_d—coincides with either the vertical or the horizontal axis, for λ_r^1. and λ_r^2 (respectively). Condition (11) for communication with the principal can be expressed as:

$$||\mathbf{b}^i - \text{Proj}_{\lambda_r^d}(\mathbf{b}^i)|| \leq \frac{w_{dr}}{2\big(k_r^{\text{P}d} + 3\big)}$$

Then, the lemma belows shows when the principal can expect to receive more information about one state under (partial) delegation.

Lemma 4. *Let $w \equiv w_{11} = w_{22}$. Given $\varepsilon \in \Re_+$ and an arbitrarily large n_ε, there exists an integer κ and $i \in N_\varepsilon$ with $||\mathbf{b}^i - Proj_{\lambda_r^d}(\mathbf{b}^i)|| \leq \frac{w_{dr}}{2(\kappa+3)}$ such that $||\mathbf{b}^i - Proj_{\lambda_{\tilde{r}}^d}(\mathbf{b}^i)|| > \frac{w_{d\tilde{r}}}{2(\kappa+3)}$.*

Moreover, this is true for the state associated to w_{dr} because $w_{dr} > w_{d\tilde{r}}$

Proof. Given that $\lambda_r^d = \lambda_{\tilde{r}}^d$ and $w_{dr} > w_{d\tilde{r}}$, the result holds for any $b_d^i \in \left(\frac{w_{d\tilde{r}}}{2(\kappa+3)}; \frac{w_{dr}}{2(\kappa+3)}\right]$ ☐

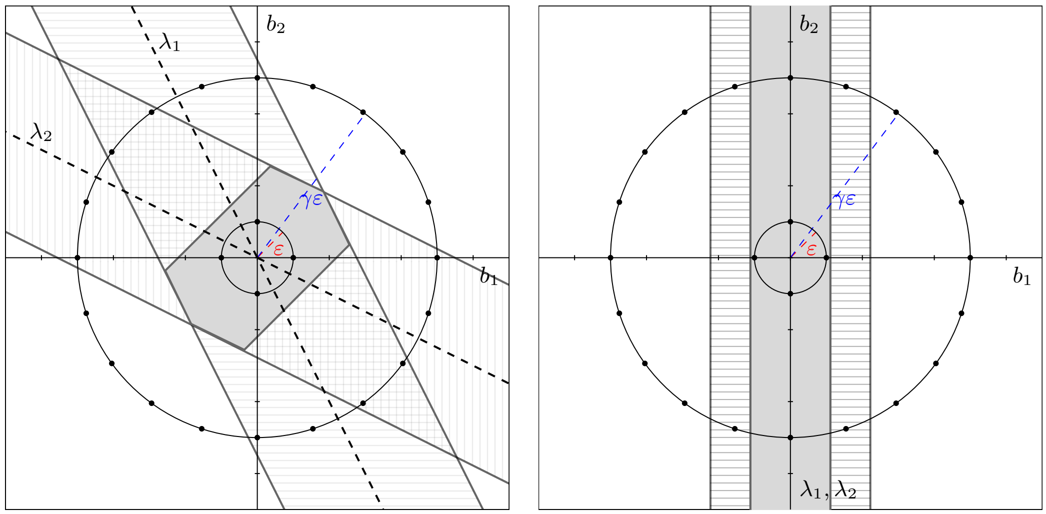
When the principal decides over y_1 only, agents with sufficiently large $|b_1|$ only reveal information about θ_1. In other words, she can expect to have more information about the state that is more relevant for the decision she is in charge of. The intuition relates to the fact that each state is more relevant for a different decision. Under delegation, agents have a relatively larger influence by revealing information about the more relevant state, such that there will be more agents who find revealing the associated signal incentive compatible. The principal can expect to have a larger proportion of advisers *specialized* in the more relevant state.

Panel (b) in Figure 5 illustrates the information flows when the principal decides on y_1 only. Agents' incentives for communication then depend on b_1^i and, given $w_{11} > 1/2$, information about θ_1 has a higher influence (for given $k_1^{\text{P}d} = k_2^{\text{P}d}$). As a consequence, the IC constraints for revealing S_1^i only will hold for a larger sets of biases than that of revealing both signals; which in turn implies there will be more agents willing to reveal S_1^i than those willing to reveal both signals. The principal then expects to have more information about θ_1 when she delegates y_2.

Lemmas 3 and 4 have important implications for the benefits of delegation when the principal does not know the exact profile of biases. In Section 3 I showed that negative informational spillovers can provide a just foundation for partial delegation, but this argument requires the principal to know the profile of biases of informed agents, $\mathbf{B}$. When she does not observe $\mathbf{B}$, however, delegation may lead to losing control over payoff-relevant information (beyond the distributional loss). In particular, each decision maker under delegation is expected to receive more information on the salient state, becoming an *ex-post specialist*. I formalize this argument in the proposition below.

consideration.

Figure 5: Information transmission under different organizational structures



(a) Centralization

(b) Delegation of y_2

Full Revelation S_1 only S_2 only Babbling
$(0,0)$ and $(1,1)$ only $(0,1)$ and $(1,0)$ only

Note: $w_{11} = w_{22} = \frac{2}{3}$

Proposition 7. *Let $w_{11} = w_{22} = w$. For a sufficiently large number of agents, n; for any conflict of interest $\varepsilon \in \Re_+$, such that the profile of biases, $\mathbf{B} = \{\mathbf{b}^1, ..., \mathbf{b}^n\}$, is uniformly distributed between $[0, \varepsilon]$ for agents $i = \{1, .., n\}$; for any integer κ. Then, the principal expects to receive more balanced information under centralization than when she delegates any decision to agent j; that is,*

$$|E\left[Var(\theta_1|\mathbf{m}^*_C)\right] - E\left[Var(\theta_2|\mathbf{m}^*_C)\right]| < \left|E\left[Var(\theta_1|\mathbf{m}^*_j)\right] - E\left[Var(\theta_2|\mathbf{m}^*_j)\right]\right|$$

Proof. See Appendix B.15 □

There are more potentially negative consequences of delegation besides distributional losses. If the principal does not know the exact distribution of biases of informed agents, she can expect delegation to lead to specialization on the issues that are more relevatn for each decision. Proposition 7 shows this specialization in the form of higher expected precision of beliefs about the most important state for each decision. This specialization could, in principle, be compensated by a smaller absolute variance on the salient state under delegation, i.e. $E\left[\text{Var}(y_d|\mathbf{m}^*_C)\right] > E\left[\text{Var}(y_d|\mathbf{m}^*_j)\right]$. But this seems not to be the case according to Proposition 6 and Corollary 5. In addition, Figure 5 helps to estimate the expected number of agents willing to reveal each signal under different allocation of decision rights. Within a given conflict of interest (circle) the mass of points that lies in any IC constraint represents the ex-ante expected 'mass' of agents willing to reveal the corresponding information (for a given κ). It can be seen that the areas around loci λ_1 and λ_2 in panel (a) are larger than the area around the vertical axis in panel (b),[24] suggesting that the amount of information about each state the principal expects to receive under centralization is larger than that for the most salient state under delegation. The intuition behind that relates to the smaller overall influence of delegation (which determines the 'width' of the IC constraints).

5 Discussion and Conclusion

In this section I extend the baseline model with endogenous information acquisition in two directions. First, I relax the assumption that decisions on information acquisition are common knowledge, showing that results are robust for a non-empty set of cost values. Secondly, I discuss the possiblity that agents observe more than one binary signal per state, offering some insights on how the problem could be approached. Finally, I conclude.

Covert Information Acquisition

Let assume that no player other than i observes which signals he acquired, and this is true for all agents. I focus on the pure strategies at the information acquisition and communication stages, under centralization. Lemma 9 in Appendix C shows that it is without loss to focus on equilibria in which messages do not convey information about the acquisition decision (Lemma 3 in Argenziano et al., 2016). As a result, agents cannot change the decision maker's beliefs about individual investment on information at the communication stage. Another implication of Lemma 9 is that any deviation at the information acquisition stage results in a deviation (from truthtelling) at the communication stage (see Lemma 10 in Appendix C).

In the appendix I derive the IC constraints for a typical agent i in the covert information acquisition game. The intuitions relate to the two different deviations available to i: acquiring fewer or more signals than on the equilibrium path. First, when he deviates by *acquiring fewer signals* than expected on the equilibrium path, he saves on information costs but he induces lower-than-optimal precision on beliefs associated to state(s) for which information has not been acquired. Suppose i is expected to acquire information about both states on path; if, instead, he decides to acquire information about θ_1 only, his message corresponding to θ_2 will induce wrong beliefs for half of its possible realizations. Because his (off-path) message strategy will be to announce the most favourable realization of S_2^i, the deviation is equivalent to lying on this signal. Incentive compatibility, hence, requires these utility

[24]Check the right-hand sides of the IC constraints in Lemmas 6 and 5

gains—lying towards his bias *plus* saving on information costs— are lower than expected utility losses from inducing 'more variance' on decisions. Incentive compatibility constraints in the covert game are thus more restrictive than in the overt game.[25]

The second deviation consists of *acquiring more signals* than those specified on the equilibrium path. When i is expected to acquire and reveal information about θ_1, he can profitably deviate by also acquiring information about θ_2. Since, on path, he will only be influential on the message associated to first state, the expected utility gains from such deviation have to do with ambiguous information.This means he conveys favourable information about θ_2 when that for θ_1 is unfavourable. If information costs are sufficiently low, i will not be able to credibly acquire information about θ_1 only. The result below shows sufficient conditions that guarantee i can credibly commit to acquire information about θ_1 only.

Proposition 8. *Let* $(\mathfrak{s}^*, \mathbf{m}^*, \mathbf{y}^*)$ *characterize an equilibrium in the covert game under centralization, and let* k_r^{C} *be agent i's equilibrium conjecture about other agents truthfully revealing information about state* $\theta_r = \{\theta_1, \theta_2\}$. *Agent i's equilibrium strategies consist of acquiring and revealing information about* θ_1 *only,* $\mathfrak{s}^{i*} = \{\tilde{S}_1^i\}$ *and* $m_1^{i*} = \{\tilde{S}_1^i\}$ *if and only if:*

$$\frac{|\beta_1^i|}{(k^{\mathrm{C}}+3)} \leq \frac{(w_{11}^2 + w_{21}^2)}{2(k_1^{\mathrm{C}}+3)^2} - \max\left\{C(S_1^i)\,;\, \frac{(w_{11}w_{12} + w_{21}w_{22})}{2(k_1^{\mathrm{C}}+3)(k_2^{\mathrm{C}}+3)} - 2C(S_2^i)\right\} \tag{12}$$

Proof. See Appendix C. □

If i does not acquire the signal he is expected on path (S_1^i) he would announce the message that favours his bias, but also save on information costs. This deviation is tempting when the first term in brackets is larger. The second deviation consists in acquiring more information and lying when the signal i is supposed to reveal represents unfavourable information. This deviation is captured by the second term in bracket, which is tied to the credibility loss due to ambiguous information. Note that such deviation depends on the presence of informational interdependence; in other words, the second term in brackets represents an additional constraint informational interdependence imposes on communication in the covert game.

More than one binary signal per state

Throughout the paper I assumed each agent's information consists of one binary signal associated to each state, two in total. Here I discuss the how this assumption could be relaxed, in principle, by allowing each agent to observe many binary trials associated to each state. I proceed in two steps. First, I show what happens with communication incentives when *a single agent* observes κ signals about

[25] See equations (25), (26), and (27) in Appendix C, and Lemma 1.

a single state in an *unidimensional* cheap talk problem, based on Förster (2019). I then argue that, under the notion of informational interdependence used in this paper, incentives for communication of *perfectly informed* specialists are characterized by similar same measures of conflict of interests.

Based on a recent paper by Förster, restricting attention to binary signals represents the 'most conservative' estimation of agents' incentives for communication. Förster (2019) studies a sender's incentives for communication to a receiver in charge of one decision, when the former observes $\kappa \geq 1$ binary signals that are independent conditional on the state $\theta \in \Theta = [0, 1]$. In the most informative equilibrium, the sender plays an influential message strategy if his bias is below a threshold $\bar{b}(\kappa)$, which involves full revelation of his information for sufficiently low bias, $b \leq \underline{b}(\kappa) < \bar{b}(\kappa)$.[26] Interestingly, the threshold for influential equilibria is increasing in the number of binary signals the sender observes; in particular, when the state is uniformly distributed, $\bar{b}(\kappa) = \frac{(\kappa+1)}{4(\kappa+2)}$ (see Example 2 in that paper). This suggests that a better informed sender has more flexibility in deciding his message strategy and, thus, finds optimal to transmit some information for larger biases (note that $\lim_{\kappa \to \infty} \bar{b}(\kappa) = 1/4$). Back to the present paper, these results imply that restricting attention to one binary signal represent the 'most conservative' estimation of each agent's incentives to transmit any information.[27]

Now, what happens when agents observe more than one binary signal in the presence of informational interdependence? In a companion paper I analyse the case of 2 specialist, each of whom is perfectly informed about a single (different) state and observe no information about the other. There I show that each sender's message strategy consists of increasing partitions of the state space, and individual IC constraints are isomorphic to those in Crawford and Sobel (1982), where sender i's bias is represented by $\frac{\beta_r^i}{w_{1r}^2 + w_{2r}^2}$. These IC constraints could be interpreted as the maximum bias for which an agent reveals *some* information about one state.

A last question that may arise relates to asymmetries on agents' information. In the paper I showed that individual incentives to reveal information depends on how much information the decision maker is expected to have in equilibrium.[28] If one agent, A^1, observes more than one binary signals in my framework, his incentives should follow a more complex form of Förster's analysis, i.e. partitional communication equilibria. If agents other than A^1 still observe binary signals about each state, their incentives will depend on conjectures about the information the decision maker receives in equilibrium—including the ex-ante expected residual variance from A^1's equilibrium message strategy. Indeed, the fact that incentives for direct communication decrease as the number of binary signals increases (Remark 3 in Förster, 2019) implies the principal (weakly) prefers to delegate any decision

[26] See Propositions 2 and 3 in Förster (2019).

[27] In appendix A I show that the principal strictly prefer to have a single, perfectly informed sender than infinitely many senders each observing single binary signals.

[28] In Krishna and Morgan (2001), for example, two senders are perfectly informed and individual incentives for communication depends on the other sender's bias because it predicts how much information he reveals on path.

to A^1 rather than another agent (A^2) with the same conflict of interest, even if he willing to fully reveal his signal to him.[29] This suggests that, *ceteris paribus*, the principal prefers to delegate decisions to more informed agents. Deepening these intuitions is a subject for future work.

Concluding remarks

Most organizations operate in complex environments, they face multi-causal problems and solutions involve many interrelated courses of action. Decision-relevant information is dispersed among its members, who communicate it strategically. This paper has studied how information is acquired and aggregated under such complexity. I showed that the allocation of decision rights constitutes a key tool to govern the conflict of interests in an organization. In particular, I found a principal may want to delegate controversial decisions if that improves transmission of information on other, less controversial ones. When preferences over all decisions are extreme, centralization can 'discipline' these conflict of interests such that more information is transmitted. I have shown that complexity affects incentives to acquire information under different organizational structures. Under delegation, expected investment in information is not only lower overall but also more concentrated on issues that are salient for the corresponding decision. The analysis presented here has broad applications to the organization of policy-making bodies, advisory committees, knowledge creation in multinational corporations, and other settings where information needs to be obtained and communicated in complex environments.

[29] If the conflict of interest between A^1 and A^2 is greater than $|\underline{b}(\kappa_1)|$, communication is less than fully revealing.

Appendix A Complementary results

A.1 Equilibrium communication under delegation

Lemma 5 (Incentive Compatibility of Communication on y_d.)**.** *Consider an equilibrium* $(\mathbf{y}^*, \mathbf{m}^*)$ *in which the principal delegates* y_d *to agent* j *(and he does not decide on the other decision).*[30] *Then, revealing any information (either* S_r^i *or both signals) is incentive compatible for* i *if:*

$$|b_d^i - b_d^j| \leq \frac{1}{2}\left|\frac{w_{d1}}{(k_1^j+3)} - \frac{w_{d2}}{(k_2^j+3)}\right| \tag{13}$$

And revealing both signals when $\tilde{\mathbf{S}}^i\{(0,0);(1,1)\}$ *and announcing the babbling message for other realizations if:*

$$|b_d^i - b_d^j| \leq \frac{1}{4}\left[\frac{w_{d1}}{(k_1^j+3)} + \frac{w_{d2}}{(k_2^j+3)}\right] \tag{14}$$

Where $y_d = \{y_1, y_2\}$*.*

Proof. See Appendix B.3 □

The proposition below summarizes the equilibrium communication in the case of delegation.

Proposition 9 (Equilibrium Communication for y_d)**.** *Let agent* j *be the decision maker of* y_d*. In the most-informative equilibrium* $(\mathbf{y}^*, \mathbf{m}^*_{j'}, \mathbf{m}^*_{j''})$ *between agents* i *and* j*,* i *fully reveals his information if and only if condition* (13) *hold. If the right-hand side of* (14) *is larger than that of* (13)*, then agents with* $|b_d^i - b_d^j|$ *within these two values reveal both signals when* $(\tilde{S}_1^i, \tilde{S}_2^i) = \{(0,0),(1,1)\}$ *and send the corresponding babbling message otherwise. For other values of* $\mathbf{b}^i$*,* i *always send the babbling message consistent with his bias.*

Proof. See appendix B.4. □

Equilibrium communication in the case of one decision is characterized by the IC constraint in Lemma 5. As already discussed, full revelation dominates message strategies in which i reveals one signal because the IC constraints are the same and the decision maker always prefer the former. The same happens with dimensional non-separable message strategies for most parameter values, so they can be overlooked in this case with little loss. Now is time to analyse communication between i and the principal when she retains authority over both decisions.

A.2 Equilibrium communication under centralization

Lemma 6 (Incentive Compatibility of Communication under Centralization)**.** *Consider an equilibrium* $(\mathbf{y}^*, \mathbf{m}^*)$*, truthful communication is be incentive compatible for agent* i *in the following cases:*

- *Revealing* S_r^i*, if:*

$$|\beta_r^i| \leq \frac{(w_{1r})^2 + (w_{2r})^2}{2}\left[\frac{1}{(k_r^{\mathrm{C}}+3)} - \frac{C_r}{(k_{\tilde{r}}^{\mathrm{C}}+3)}\right] \tag{15}$$

[30] By assumption, communication between decision makers only involves own signals (not information transmitted by other agents). For an analysis on hierarchies as information intermediation see Migrow (2017).

- *Revealing both when* $\tilde{\mathbf{S}}^i = \{(0,0);(1,1)\}$, *if:*

$$\frac{(w_{11})^2+(w_{21})^2}{(k_1^{\mathrm{C}}+3)}\left[\frac{1}{(k_1^{\mathrm{C}}+3)}+\frac{C_1}{(k_2^{\mathrm{C}}+3)}\pm 2\beta_1\right]+\frac{(w_{12})^2+(w_{22})^2}{(k_2^{\mathrm{C}}+3)}\left[\frac{1}{(k_2^{\mathrm{C}}+3)}+\frac{C_2}{(k_1^{\mathrm{C}}+3)}\pm 2\beta_2\right]\geq 0 \tag{16}$$

- *Revealing both when* $\tilde{\mathbf{S}}^i = \{(0,1);(1,0)\}$, *if:*

$$\frac{(w_{11})^2+(w_{21})^2}{(k_1^{\mathrm{C}}+3)}\left[\frac{1}{(k_1^{\mathrm{C}}+3)}-\frac{C_1}{(k_2^{\mathrm{C}}+3)}\pm 2\beta_1\right]+\frac{(w_{12})^2+(w_{22})^2}{(k_2^{\mathrm{C}}+3)}\left[\frac{1}{(k_2^{\mathrm{C}}+3)}-\frac{C_2}{(k_1^{\mathrm{C}}+3)}\mp 2\beta_2\right]\geq 0 \tag{17}$$

Where $\beta_r = b_1^i w_{1r} + b_2^i w_{2r}$, $C_r = \frac{w_{11}w_{12}+w_{21}w_{22}}{w_{1r}^2+w_{2r}^2} \in [0,1]$, *and* $\pm$ *means that the condition must hold for the most restrictive of these operations, given the sign of the corresponding* β.

Proof. See Habermacher (2018). □

Lemma 7 (IC constraints for dimensional non-separable strategies under centralization)**.** *In any equilibrium* $(\mathbf{y}^*, \mathbf{m}^*)$ *in which* $\mathbf{m}^{i*}$ *includes a babbling strategy; then, i's incentives to reveal both signals against this deviation are characterized by:*

- *For* $\mathbf{S}^i = \{0,0\}$ *and* $\mathbf{S}^i = \{1,1\}$, *if:*

$$\left|\frac{\beta_1^i}{(k_1+3)}+\frac{\beta_2^i}{(k_2+3)}\right| \leq \frac{1}{4}\left[\frac{(w_{11})^2+(w_{21})^2}{(k_1+3)^2}+\frac{(w_{12})^2+(w_{22})^2}{(k_2+3)^2}+\frac{2\left[w_{11}\,w_{12}+w_{21}\,w_{22}\right]}{(k_1+3)(k_2+3)}\right] \tag{18}$$

- *For* $\mathbf{S}^i = \{0,1\}$ *and* $\mathbf{S}^i = \{1,0\}$, *if:*

$$\left|\frac{\beta_1^i}{(k_1+3)}-\frac{\beta_2^i}{(k_2+3)}\right| \leq \frac{1}{4}\left[\frac{(w_{11})^2+(w_{21})^2}{(k_1+3)^2}+\frac{(w_{12})^2+(w_{22})^2}{(k_2+3)^2}-\frac{2\left[w_{11}\,w_{12}+w_{21}\,w_{22}\right]}{(k_1+3)(k_2+3)}\right] \tag{19}$$

Proof. See Habermacher (2018) □

Proposition 10 (Characterization of P-Optimal equilibrium under centralization —Proposition 2 in Habermacher, 2018)**.** *The P-optimal Perfect Bayesian Equilibrium for sender i consists of the following strategies:*

1. ***Revealing both signals****, if* $\mathbf{b}^i$ *satisfies conditions* (16), (17) *and* (10) *with respect to both states.*
2. ***Revealing one signal only****, if* $\mathbf{b}^i$ *satisfies condition* (15) *for* S_r^i *only.*
3. ***Dimensional non-separable message strategies*** *in the following cases:*

(a) *Fully revealing* $\mathbf{S}^i = \{(0,0);(1,1)\}$ *if* $\mathbf{b}^i$ *satisfies condition* (18) *only;*

(b) *Fully revealing* $\mathbf{S}^i = \{(0,1);(1,0)\}$ *if* $\mathbf{b}^i$ *satisfies condition* (19) *only.*

4. ***No communication*** *(babbling strategy), if none of the above holds.*[31]

Proof. See Habermacher (2018). □

Lemma 8 (Optimal Organizational Structure)**.** *Given the vector of preferences,* $\mathbf{B} = \{\mathbf{b}_1, \ldots, \mathbf{b}_n\}$*, and generic agents* i*,* j'*, and* j''*; the organizational structure that maximizes the principal's ex-ante welfare is:*

Full Delegation. *That is, agents* j' *and* j'' *decide on* y_1 *and* y_2 *(resp.) if and only if:*

1. $DIG_{j'}(y_1) - (b'_1)^2 > \max\left\{DIG_i(y_1) - (b^i_1)^2\,,\; IIG(y_1)\,,\; -\{DIG_{j''}(y_2) - (b''_2)^2\}\right\}$ *for any* $i \neq j'$*; and*

2. $DIG_{j''}(y_2) - (b''_2)^2 > \max\left\{DIG_i(y_2) - (b^i_2)^2\,,\; IIG(y_2)\,,\; -\{DIG_{j'}(y_1) - (b'_1)^2\}\right\}$ *for any* $i \neq j''$*.*

Partial Delegation. *That is, agent* j *decides on* y_d *and the principal retains decision authority over* $y_{\tilde{d}}$*; if and only if there exist both Direct and Indirect informational gains such that:*

1. $DIG_j(y_d) - (b^j_d)^2 > \max\left\{DIG_i(y_d) - (b^i_d)^2\,,\; IIG(y_d)\,,\; -IIG(y_{\tilde{d}})\right\}$ *for any* $i \neq j$*; and*

2. $IIG(y_{\tilde{d}}) > \max\left\{DIG_i(y_{\tilde{d}}) - (b^i_{\tilde{d}})^2\,,\; -\{DIG_j(y_d) - (b^j_d)^2\}\right\}$ *for any* $i \neq j$*.*

Centralization. *That is, the principal decides on both issues, if and only if there are no agent* i *and* j *such that:*

1. $DIG_j(y_d) - (b^j_d)^2 + IIG(y_{\tilde{d}}) > 0$*; nor*

2. $DIG_{j'}(y_1) - (b'_1)^2 + DIG_{j''}(y_2) - (b''_2)^2 > 0$

Proof. The proof is constructive. The optimal organizational structure maximizes the principal's ex-ante expected utility. Optimality of delegation implies some informational gains, otherwise she can retain authority over both issues and decide with the information transmitted under centralization. Full Delegation is then optimal if there are two agents j' and j'' who decide on y_1 and y_2, respectively; such that the corresponding informational gains more than compensate each decision maker's bias. These gains must be maximal among all agents, and strictly larger than if the principal retained any single decision (IIG).

Partial Delegation is optimal in either of two cases (non-exclusive). First, when direct informational gains from delegation are possible only on one decision, the principal prefers to retain authority on

[31] Where Full Revelation corresponds to the equilibrium message strategy $\mathbf{m}^i = \Big\{\{(0,0)\};\{(0,1)\};\{(1,0)\};\{(1,1)\}\Big\}$; revealing S^i_1 or S^i_2 only correspond to $\mathbf{m}^i = \Big\{\{(0,0);(0,1)\};\{(1,0);(1,1)\}\Big\}$ and $\mathbf{m}^i = \Big\{\{(0,0);(1,0)\};\{(0,1);(1,1)\}\Big\}$; DNS message strategies correspond to $\mathbf{m}^i = \Big\{\{(0,0)\};\{(1,1)\};\{(0,1);(1,0)\}\Big\}$ when $(0,0)$ and $(1,1)$ fully reveal their types, and $\mathbf{m}^i = \Big\{\{(0,0);(1,1)\};\{(0,1)\};\{(1,0)\}\Big\}$ the case in which $(0,0)$ and $(1,1)$ do so; and finally the babbling strategy $\mathbf{m}^i = \Big\{\{(0,0);(1,1);(0,1);(1,0)\}\Big\}$.

the other. If the DIG are sufficiently large, she may be willing to tolerate some informational losses on the retained decision; that is, receiving less information than under centralization.

The second and most interesting case is when indirect informational gains are large. From Proposition 2 we know that the presence of negative informational spillovers under centralization is a necessary condition. Delegating y_d thus breaks the interdependence between decisions and allows communication on the low-conflict dimension. This may hold even if there are no informational gains in the delegated decision, as long as the indirect ones are sufficiently large.

Finally, Centralization is optimal when any potential informational gain is small, such that it does not compensate the loss of control on the delegated decision(s). □

Corollary 5. *Let $w \equiv w_{11} = w_{22}$, $\kappa = 1$, and $\frac{\hat{w}_{dr}^2}{72} < C(S_r) \leq \frac{[(w_{1r})^2+(w_{2r})^2]}{72}$ for both θ_r. If there are no agents j' and j'' whose preferences represent a conflict of interest within $||\mathbf{b}^{\text{D}}|| \leq 1-2w(1-w)$, then the principal strictly prefers centralization over any form of delegation. If there where such agents, the principal still prefers centralization (strictly) as long as there is at least one agent i who fully reveals his information —i.e. $\mathbf{b}^i$ satisfy conditions (15) and (17) with respect to both signals.*

Proof. When $\kappa = 1$, decision makers have the stronger incentives to acquire both signals under delegation. Noting that $6(\kappa+2)(\kappa+3) = 72$, from Proposition 6 I get that $\frac{\hat{w}_{dr}^2}{72} < C(S_r) \leq \frac{[(w_{1r})^2+(w_{2r})^2]}{72}$ implies $K_r^{\text{C}} > K_r^{\text{D}} = 1$. In addition, from Proposition 3 we know that, for $w_{11} = w_{22} = w$ and $k_r^j = 1$:

$$||\hat{\mathbf{b}}^{\text{D}}|| = \left[\frac{w^2}{6}\left(1-\frac{2}{3}\right) + \frac{(1-w)^2}{6}\left(1-\frac{2}{3}\right)\right] = 1-2w(1-w)$$

If there are no agents j' and j'' such that $||\mathbf{b}^{\text{D}}|| \leq ||\hat{\mathbf{b}}^{\text{D}}||$, then the distributional loss from delegation is never compensated by the maximal informational gain; as a consequence, centralization yields higher ex-ante expected utility to the principal. On the other hand, if there were agents j' and j'' such that $(b_1', b_2'') \in ||\hat{\mathbf{b}}^{\text{D}}||$ the principal prefers delegation when there is no agent i whose preferences satisfy conditions (15) and (17). For if there were such an agent, he reveals both signals under centralization and, thus, delegation yields no informational gains.

Under the parameters of Corollary 5, costs are so high that the maximum number of truthful messages under delegation is zero. Having the chance to influence both decisions, i's acquisition of one signal is cost-effective (provided communication is IC). The result also illustrates the restrictions imposed by information costs on the optimality of different organizational structures (Proposition 8). For sufficiently high costs the principal always prefer to retain authority over both issues, but restrictions weaken as the costs of acquiring a signal decreases. □

A.3 Agents' equilibrium strategies.

In this subsection I combine the results of Lemma 1 and Lemma 2 to characterize agent i's equilibrium information acquisition and message strategies. I start with the case of centralization ($j' = j'' = P$) and then proceed to the case of delegated decisions. The following result summarizes the intuitions developed in the previous discussion, presenting the equilibrium acquisition and message strategies for a typical agent under centralization.

Proposition 11 (Equilibrium under Centralization)**.** *In the most informative equilibrium under centralization $(\mathbf{y}^*, \mathbf{m}^*, \mathfrak{s}^*)$, agent i only acquires signals that are cost-effective and incentive compatible. In particular, i's equilibrium strategies are given by:*

Acquiring and revealing both signals: *if and only if conditions (5) and (10) hold for both signals, and (17) hold.*

Acquiring both signals and playing a dimensional non-separable strategy: *if condition* (6) *hold for both signals and* (10) *does not at all, in the following cases:*

- *Fully revealing both signals when they coincide and babbling otherwise, if condition* (18) *holds;*
- *Fully revealing both signals when they do not coincide and babbling otherwise, if condition* (19) *holds.*

Acquiring and revealing one signal only. *Agent i acquires and reveals S_1^i if* (10) *and* (5) *hold with respect to θ_1 and one of the following is true:*

- *Revealing S_2^i is not IC —i.e.* (10) *does not hold for θ_2; or*
- *Acquiring S_2^i is not CE —i.e.* (5) *does not hold for θ_2; or*
- *Acquiring S_2^i is CE and revealing it is IC, but revealing both signals is not IC —i.e.*(15) *and* (5) *hold for both signals, but* (17) *does not and $\frac{(w_{1r})^2+(w_{2r})^2}{(k_r^*+2)(k_r^*+3)} \geq \frac{(w_{1\tilde{r}})^2+(w_{2\tilde{r}})^2}{(k_{\tilde{r}}^*+2)(k_{\tilde{r}}^*+3)}$*

For $r \neq \tilde{r}$.

Acquiring no signal, *if only if any of the statements below is true:*

- *No signal is CE to acquire —i.e. condition* (5) *does not holds for any signal; and/or*
- *No signals is IC to reveal —i.e. condition* (10) *does not hold for any signal, nor* (17) *holds.*

Proof. See Appendix B.11 □

As discussed earlier, the number of agents revealing the same piece of information is an important determinant of cost-effectiveness. It could lead some agents to acquire less information than what each is willing to reveal, resulting in either specialization or non-investment. The possibility of abstaining to acquire a given signal can enhance incentives for communication (for the other signal) because it kills the effects of ambiguous information.

Dimensional non-separable message strategies can arise under centralization. As in the pure communication game, these strategies take the form of full revelation for some realizations and babbling for the rest. Because any of these involves acquiring both signals and revealing them half of the time, they arise when costs are sufficiently low and only if revealing one signal is not IC. Then, agent i typically prefers to reveal a single signal than play a DNS strategy, provided both are incentive compatible and cost-effective.[32]

Similar intuitions apply to the case of delegation. The result below characterizes equilibrium strategies for agent i and decisions y_d and $y_{\tilde{d}}$.

Proposition 12 (Equilibrium under Delegation)**.** *When the organizational structure involves more than one decision maker, agent i only acquires signals that are cost-effective and for which communication is incentive compatible. In the most informative equilibrium ,$(\mathbf{y}^*, \mathbf{m}^*, \mathfrak{s}^*)$, i's equilibrium strategies are:*

Acquiring and revealing both signals: *if and only if conditions* (13) *and* (7) *hold for both signals and at least one decision maker –and the associated decision.*

Acquiring and revealing S_1 only, *if acquiring this signal is both cost-effective and incentive compatible for agent i in the following cases:*

1. *Revealing S_2 is not IC for any decision —i.e. condition* (11) *does not hold for S_2^i; or*

[32] For a formal discussion see Appendix B.10.

2. *Acquiring S_2 is not CE for any decision —i.e. condition* (7) *does not hold for S_2^i for any decision; or*

3. *Both S_1 and S_2 are CE and IC, but revealing both is not IC with respect to any decision maker —i.e. conditions* (11) *and* (7) *hold for both signals and at least one decision maker, but* (13) *does not hold for any of them and $\frac{(w_{dr})^2}{(k_r^*+2)(k_r^*+3)} \geq \frac{(w_{d\tilde{r}})^2}{(k_{\tilde{r}}^*+2)(k_{\tilde{r}}^*+3)}$*

For $r \neq \tilde{r}$.

Acquiring no signal *if only if any of the statements below are true:*

1. *Condition* (11) *does not hold for any signal and any decision, nor* (17) *hold; and/or*

2. *Condition* (7) *does not holds for any signal, any decision.*

Proof. See Appendix B.11. □

In presence of two decision makers agents acquire a signal if it is cost-effective and incentive compatible to reveal it to at least one of them. As discussed in Proposition 4, the possibility of not acquiring one of the signals makes credible the strategy of revealing only one signal. When such a strategy is incentive compatible it always refers to the state the decision maker is less informed in equilibrium. In other words, the agent will acquire the signal for which the influence is larger, which in turn implies that dimensional non-separable strategies are dominated by this strategy. As a consequence, no DNS strategy can emerge in equilibrium under delegation.

Appendix B Proofs

B.1 Generic IC constraints for communication

Proof. Let $j', j'' \in \{P, 1, ..., n\}$ be the decision makers for y_1 and y_2, respectively; and $i \in \{1, ..., n\}$ be a generic sender. Let $\mathbf{m}_j^{i*}$ denote i's equilibrium message strategy with respect to j, and $\hat{\mathbf{m}}_j^i$ an alternative message strategy (deviations to be considered in each case). Then, $y_d(\mathbf{m}_j^i, \mathbf{m}_j^{-i})$ represents the action j takes when i is expected to play $\mathbf{m}_j^{i*}$ and other senders are playing $\mathbf{m}_j^{-i}$. Given i's conjectures about others' strategies are correct in equilibrium, I can simplify notation in the following way: $y_d(\mathbf{m}_j^{i*}(\mathbf{S}^i), \mathbf{m}_j^{-i}) = y_d(\mathbf{m}_j^{i*})$ and $y_d(\hat{\mathbf{m}}_j^i(\mathbf{S}^i), \mathbf{m}_j^{-i}) = y_d(\hat{\mathbf{m}}_j^i)$.

Message strategy $\mathbf{m}^{i*} = \{m_{j'}^{i*}, m_{j''}^{i*}\}$ is then incentive compatible for sender i if and only if for any alternative $\hat{\mathbf{m}}^i$:

$$-\int_0^1 \int_0^1 \Bigg[\Big[\left(\delta_1 + b_1^i - y_1(\mathbf{m}_{j'}^{i*})\right)^2 + \left(\delta_2 + b_2^i - y_2(\mathbf{m}_{j''}^{i*})\right)^2 \Big] - \\ - \Big[\left(\delta_1 + b_1^i - y_1(\hat{\mathbf{m}}_{j'}^{i})\right)^2 + \left(\delta_2 + b_2^i - y_2(\hat{\mathbf{m}}_{j''}^{i})\right)^2 \Big] \Bigg] f(\theta_1, \mathbf{m}^{-i}|\mathbf{S}^i)\, f(\theta_2, \mathbf{m}^{-i}|\mathbf{S}^i)\, d\theta_1\, d\theta_2 \geq 0$$

By operating inside the square brackets with the identity $a^2 - b^2 = (a+b)(a-b)$, by definition of optimal decisions, $y_d^* = E(\delta_d|\mathbf{m}_j) + b_d^j$, and by denoting:

$$\Delta(\delta_1) = E(\delta_1|\mathbf{m}_{j'}^{i*}, \mathbf{m}_{j'}^{-i}) - E(\delta_1|\hat{\mathbf{m}}_{j'}^{i}, \mathbf{m}_{j'}^{-i}) \qquad \Delta(\delta_2) = E(\delta_2|\mathbf{m}_{j''}^{i*}, \mathbf{m}_{j''}^{-i}) - E(\delta_2|\hat{\mathbf{m}}_{j''}^{i}, \mathbf{m}_{j''}^{-i})$$

I get:[33]

[33] Note that $f(\theta_1, \mathbf{m}^{-i}|\mathbf{S}^i) = f(\theta_1|\mathbf{m}^{-i}, S_1^i)\, P(\mathbf{m}^{-i}|S_1^i)$ and that $f(\theta_2, \mathbf{m}^{-i}|\mathbf{S}^i) = f(\theta_2|\mathbf{m}^{-i}, S_2^i)\, P(\mathbf{m}^{-i}|S_2^i)$

$$
\begin{aligned}
-\int_0^1\int_0^1 \Bigg[& \Big[\frac{E(\delta_1|\mathbf{m}^{i*}_{j'},\mathbf{m}^{\text{-}i}_{j'})+E(\delta_1|\hat{\mathbf{m}}^{i}_{j'},\mathbf{m}^{\text{-}i}_{j'})}{2}-\delta_1-(b^i_1-b'_1)\Big]\Delta(\delta_1)+ \\
&+\Big[\frac{E(\delta_2|\mathbf{m}^{i*}_{j''},\mathbf{m}^{\text{-}i}_{j''})+E(\delta_2|\hat{\mathbf{m}}^{i}_{j''},\mathbf{m}^{\text{-}i}_{j''})}{2}-\delta_2-(b^i_2-b''_2)\Big]\Delta(\delta_2)\Bigg] \\
& f(\theta_1|\mathbf{m}^{\text{-}i},S^i_1)\,f(\theta_2|\mathbf{m}^{\text{-}i},S^i_2)\,P(\mathbf{m}^{\text{-}i}_{j'}|S^i_1)\,P(\mathbf{m}^{\text{-}i}_{j'}|S^i_2)\,P(\mathbf{m}^{\text{-}i}_{j''}|S^i_1)\,P(\mathbf{m}^{\text{-}i}_{j''}|S^i_2)\,d\theta_1\,d\theta_2\geq 0
\end{aligned}
$$

Given that the equilibrium message strategies for players other than i, $\mathbf{m}^{\text{-}i}$, are independent of i's actual signal realizations, the expressions $P(\mathbf{m}^{\text{-}i}_j|S^i_1)$ and $P(\mathbf{m}^{\text{-}i}_j|S^i_2)$ can be taken out the double-integral.

I denote the receiver's updated beliefs with respect to δ_d from i's perspective as:

$$
\nu^{i*}_d = E(\delta_d|\mathbf{m}^{i*}_j,\mathbf{m}^{\text{-}i}_j) \qquad \hat{\nu}^i_d = E(\delta_d|\hat{\mathbf{m}}^{i}_j,\mathbf{m}^{\text{-}i}_j) \qquad \nu^i_d = E(\delta_d|\mathbf{S}^i_j,\mathbf{m}^{\text{-}i}_j)
$$

Such that $\Delta(\delta_d)=\nu^{i*}_d-\hat{\nu}^i_d$. The generic IC constraint is then given by:

$$
-\Bigg[\Big[(\nu^{i*}_1+\hat{\nu}^i_1)-2(\nu^i_1+b'_1-b^i_1)\Big]\,\Delta(\delta_1)+\Big[(\nu^{i*}_2+\hat{\nu}^i_2)-2(\nu^i_2+b''_2-b^i_2)\Big]\,\Delta(\delta_2)\Bigg]\,P(\mathbf{m}^{\text{-}i}|S_1)\,P(\mathbf{m}^{\text{-}i}|S_2)\geq 0 \quad (20)
$$

Note that under Centralization $b'_1=b''_2=0$ and we are back to the IC constraints in Habermacher (2018). More importantly, when i reveals one signal only ν^{i*}_d and $\hat{\nu}^i_d$ are different from ν^i. Sender i's strategies in equilibrium and in the deviation under analysis do not transmit all the information he has, and the beliefs he induces on j are different from what he believes are the optimal decisions (in the equilibrium under consideration). As I show later, this generates credibility losses for i because of the possibility of ambiguous information —i.e. signals that move decisions in opposite directions if fully revealed. □

B.2 Proof of equation (1)

Proof. The principal's ex-ante expected utility in the equilibrium characterized by by $\{\mathbf{y},\mathbf{m}_{j'},\mathbf{m}_{j''}\}$ is given by:

$$
E\left[U^{\mathrm{P}}(\boldsymbol{\delta},\mathbf{b});\mathbf{m}\right]=-E\left[(y_1-\delta_1)^2;\mathbf{m}_{j'}\right]-E\left[(y_2-\delta_2)^2;\mathbf{m}_{j''}\right]
$$

Which, by definitions of equilibrium y_d and δ_d yield:[34]

$$
E\left[U^{\mathrm{P}}(\boldsymbol{\delta},\mathbf{b});\mathbf{m}\right]=-(b'_1)^2-(b''_2)^2-\sum_{y_d=\{y_1,y_2\}}E\left[\Big(w_{d1}\big(E(\theta_1|\mathbf{m}_j)-\theta_1\big)+w_{d2}\big(E(\theta_2|\mathbf{m}_j)-\theta_2\big)\Big)^2\right]
$$

With some rearrangement and given $\theta_1\perp\theta_2$, I have:

$$
E\left[U^{\mathrm{P}}(\boldsymbol{\delta},\mathbf{b});\mathbf{m}\right]=-\left[(b'_1)^2+(b''_2)^2\right]-\sum_{y_d=\{y_1,y_2\}}\sum_{\theta_r=\{\theta_1,\theta_2\}}\left[(w_{dr})^2\,E\left[\big(E(\theta_r|\mathbf{m}_j)-\theta_r\big)^2;\mathbf{m}_j\right]\right] \quad (21)
$$

[34]Note that the terms $E\left(E(\delta_d|\mathbf{m}_j)-\delta_d\right)b^j_d=0$.

Now, the expectation of the squared deviation for each state is given by:

$$E\left[\left(E(\theta_r|\mathbf{m}_j)-\theta_r\right)^2;\mathbf{m}_j\right] = \int_0^1 \sum_{\ell_r=0}^{k_r^j}\left(E(\theta_r|\mathbf{m}_j)-\theta_r\right)^2 f(\ell_r^j|k_r^j,\theta_r)\,d\theta_r$$

$$= \int_0^1 \sum_{\ell_r=0}^{k_r^j}\left(E(\theta_r|\mathbf{m}_j)-\theta_r\right)^2 \frac{h(\theta_r|\ell_r^j,k_r^j)}{(k_r^j+1)}\,d\theta_r$$

$$= \frac{1}{(k_r^j+1)}\sum_{\ell_r^j=0}^{k_r^j}\int_0^1 \left(E(\theta_r|\mathbf{m}_j)-\theta_r\right)^2 h(\theta_r|\ell_r^j,k_r^j)\,d\theta_r$$

$$= \frac{1}{(k_r^j+1)}\sum_{\ell_r^j=0}^{k_r^j}\mathrm{Var}(\theta_r|\ell_r^j,k_r^j)$$

$$= \frac{1}{(k_r^j+1)}\sum_{\ell_r^j=0}^{k_r^j}\frac{(\ell_r^j+1)(k_r^j-\ell_r^j+1)}{(k_r^j+2)^2(k_r^j+3)}$$

Solving the sum and plugging the above into (21) yields:

$$\hat{U}^{\mathrm{P}}(\mathbf{b},\mathbf{m}_{j'},\mathbf{m}_{j''}) = -\left[(b_1')^2+\frac{(w_{11})^2}{6\left(k_1'+2\right)}+\frac{(w_{12})^2}{6\left(k_2'+2\right)}\right]-\left[(b_2'')^2+\frac{(w_{21})^2}{6\left(k_1''+2\right)}+\frac{(w_{22})^2}{6\left(k_2''+2\right)}\right]$$

Let denote by $\mathrm{Var}(\theta_r|\mathbf{m}_j) \equiv E\left[\left(E(\theta_r|\mathbf{m}_j)-\theta_r\right)^2;\mathbf{m}_j\right]$. Now, the expected decision-specific uncertainty in the communication equilibrium in which j decides over y_d is given by $\mathrm{Var}(\delta_d|\mathbf{m}_j) = (w_{d1})^2\,\mathrm{Var}(\theta_1|\mathbf{m}_j)+(w_{d2})^2\,\mathrm{Var}(\theta_2|\mathbf{m}_j)$, and the above expression can thus be written as:

$$\hat{U}^{\mathrm{P}}(\mathbf{b},\mathbf{m}_{j'},\mathbf{m}_{j''}) = -\left[(b_1')^2+\mathrm{Var}(\delta_1|\mathbf{m}_{j'})\right]-\left[(b_2'')^2+\mathrm{Var}(\delta_2|\mathbf{m}_{j''})\right] \tag{22}$$

□

B.3 Proof of Lemma 5

Proof. Consider the equilibrium $(\mathbf{y}^*,\mathbf{m}^*_{\mathbf{j'}},\mathbf{m}^*_{\mathbf{j''}})$ in which the principal delegates y_d to agent j, who does not decide on the other decision. By the assumption on private information with each decision-maker, i's messages to j only affect y_d. The IC constraint in (20) then becomes:

$$-\left[(\nu_d^{i*}+\hat{\nu}_d^i)-2(\nu_d^i-b_d^i)\right]\Delta(\delta_d)\geq 0$$

I denote by $\nu_{dr}^{i*}=E(\theta_r|\mathbf{m}_j)$ and $\hat{\nu}_{dr}^i=E(\theta_r|\hat{\mathbf{m}}_j)$ sender i's expectations of j's posterior beliefs about θ_r in equilibrium when he plays strategies m_j^i and $\hat{m}_j$ (eqm and deviation), respectively. In addition, denote by $\nu_{dr}^i=E(\theta_r|S_r^i,\mathbf{m}_j^{\text{-}i})$ sender i's expectation of j's posterior beliefs about θ_r if j knew i's information about that state.

The IC constraint then becomes:

$$-\left[w_{d1}(\nu_{d1}^{i*}-\hat{\nu}_{d1}^i)+w_{d2}(\nu_{d2}^{i*}-\hat{\nu}_{d2}^i)\right]\left[w_{d1}(\nu_{d1}^{i*}+\hat{\nu}_{d1}^i)+w_{d2}(\nu_{d2}^{i*}+\hat{\nu}_{d2}^i)-2[w_{d1}\nu_{d1}^i+w_{d2}\nu_{d2}^i-(b_d^i-b_d^j)]\right]\geq 0$$

When he reveals only one signal, however, the previous is true for the state associated to the signal he reveals truthfully but not for the other state. To see this recall that he is not being influential with respect to the latter, so his expectations about j's beliefs in equilibrium are different from his conjectures including his own information.[35] It is easy to check that:

$$\nu^i_{dr} = E\left(\theta_r|\tilde{S}^i_r, \mathbf{m}^{-i}_j\right) = \frac{(\ell^j_r + 1 + \tilde{S}_r)}{(k^j_r + 3)}$$
$$= \frac{\left(k^j_r + 3 - (-1)^{\tilde{S}^i_r}\right)}{2(k^j_r + 3)}$$

The second equality follows from taking expectation over the realization of others' signals (by the Law of Iterated Expectations); whereas $E(\theta_r|m^{-i}_j) = \nu^{-i}_{dr} = 1/2$ —i.e. what i expects j's beliefs on θ_r are if only considers other senders' truthful messages.

Consider the equilibrium in which i reveals S^i_1 only, the IC constraint becomes:

$$-\left[w_{d1}(\nu^{i*}_{d1} - \hat{\nu}^i_{d1})\right]\left[w_{d1}(\hat{\nu}^i_{d1} - \nu^{i*}_{d1}) + 2w_{d2}(\nu^{i*}_{d2} - \nu^i_{d2}) + 2[(b^i_d - b^j_d)]\right] \geq 0$$

When i's type is given by $\tilde{\mathbf{S}}^i = (0,0)$, then $\nu^{i*}_{d1} = \nu^i_{d1} = \frac{(k^j_1+2)}{2(k^j_1+3)}$, $\hat{\nu}^i_{d1} = \frac{(k^j_1+4)}{2(k^j_1+3)}$; and $\nu^{i*}_{d2} = \hat{\nu}^i_{d2} = 1/2$, while $\nu^i_{d2} = \frac{(k^j_1+2)}{2(k^j_1+3)}$. Replacing these values on the above IC constraint I get:

$$2(b^i_1 - b^j_1) \leq \frac{w_{d1}}{(k^j_1 + 3)} + \frac{w_{d2}}{(k^j_2 + 3)}$$

The case of $\tilde{\mathbf{S}}^i = (1,1)$ is analogous, with the LHS having negative signs. For the case of $\tilde{\mathbf{S}}^i = (0,1)$, all i's conjectures about j's posteriors are the same as the previous case (i reveals the same realization of the same signal), except for $\nu^i_{d2} = \frac{(k^j_1+4)}{2(k^j_1+3)}$. It can be easily checked that this leads to:

$$2(b^i_1 - b^j_1) \leq \frac{w_{d1}}{(k^j_1 + 3)} - \frac{w_{d2}}{(k^j_2 + 3)}$$

Whereas for $\tilde{\mathbf{S}}^i = (1,0)$ the LHS above has a negative sign.

Because all these IC constraints have to be satisfied in order for i to be credible in the equilibrium under consideration, and given that all of them hold for the same measure of conflict of interest between i and j, the following is a necessary and sufficient conditions for i not having incentives to lie on S^i_1:

$$|b^i_1 - b^j_1| \leq \frac{1}{2}\left[\frac{w_{d1}}{(k^j_1 + 3)} - \frac{w_{d2}}{(k^j_2 + 3)}\right]$$

Now, in equilibria in which i reveal both signals truthfully $\nu^{i*}_{dr} = \nu^i_{dr}$; that is, what he expects j's beliefs to be in equilibrium is the same as what he conjectures the optimal decisions will be according to his information. The IC constraint then becomes:

$$-\left[w_{d1}(\nu^{i*}_{d1} - \hat{\nu}^i_{d1}) + w_{d2}(\nu^{i*}_{d2} - \hat{\nu}^i_{d2})\right]\left[w_{d1}(\nu^{i*}_{d1} - \hat{\nu}^i_{d1}) + w_{d2}(\nu^{i*}_{d2} - \hat{\nu}^i_{d2}) - 2(b^i_d - b^j_d)\right] \geq 0$$

[35] In other words, if i is expected to reveal S^i_1, then $\nu^{i*}_{d2} = \hat{\nu}^i_{d2} = E(\theta_s|\mathbf{m}^{-i}_j)$ because i is not influential with respect to θ_2, and $\nu^{i*}_{d2} \neq \nu^i_{d2}$.

Incentive compatibility in this case means i prefers truthful revelation of both signals to any deviation, taking into account that any message he sends is believed to be truthful. So, for each type $\tilde{\mathbf{S}}^i = \{(0,0);(0,1);(1,0);(1,1)\}$ I consider deviations to announce a message different from his own. This leads to three generic deviations: lying in both signals,[36] lying on S_1^i,[37] and lying on S_2^i.[38] A substantial amount of algebra shows that the IC constraints for not lying in one signal are similar to those for revealing one signal, but without the negative term on the RHS (see Habermacher, 2018 for a detailed derivation of these IC). Incentives not to lie on both signals depend on whether the signals coincide or not, enthusiast readers can check that replacing the values for ν_{dr}^{i*} and $\hat{\nu}_{dr}^i$ for each type and deviation leads to the following IC constraints.

$$\text{For } \tilde{S}^i = \{(0,0);(1,1)\}: \qquad |b_1^i - b_1^j| \leq \frac{1}{2}\left[\frac{w_{d1}}{(k_1^j+3)} + \frac{w_{d2}}{(k_2^j+3)}\right]$$

$$\text{For } \tilde{S}^i = \{(0,1);(1,0)\}: \qquad |b_1^i - b_1^j| \leq \frac{1}{2}\left|\frac{w_{d1}}{(k_1^j+3)} - \frac{w_{d2}}{(k_2^j+3)}\right|$$

The last of the above is the necessary condition for full revelation given it is more restrictive (RHS is smaller). Note moreover that the expression is similar to that for revealing one signal only, which mean that whenever 'both' hold the decision-maker will prefer full revelation and, thus, will be the strategy arising in equilibrium.

Finally, I analyse the existence of Dimensional Non-separable (DNS) message strategies. I consider two of them:[39] revealing both signals when they coincide and no information otherwise, $m_j^{i*} = \{\{(0,0)\},\{(1,1)\},\{(0,1),(1,0)\}\}$, and full revelation when they do not coincide and nothing otherwise, $m_j^{i*} = \{\{(0,1)\},\{(1,0)\},\{(0,0),(1,1)\}\}$. Deviation incentives between the two influential messages translate into the IC constrains derived above. Note that this rules out the DNS in which the agent fully reveals ambiguous information because the IC constraint will be similar to that for Full Revelation.

Now I show that the strategy $m_j^{i*} = \{\{(0,0)\},\{(1,1)\},\{(0,1),(1,0)\}\}$ cannot arise in equilibrium. The argument relies on the incentives of the 'non-influential types' to announce influential messages according to their bias. When $\mathbf{S}^i = (0,1)$ does not have incentives to announce $m_j^i = \{(1,1)\}$, $\nu_{1r}^{i*} = 1/2$ and $\hat{\nu}_{1r}^i = \frac{(k_r^j+4)}{2(k_r^j+3)}$ for both signals; whereas $\nu_{11}^i = \frac{(k_1^j+4)}{2(k_1^j+3)}$ and $\nu_{12}^i = \frac{(k_2^j+2)}{2(k_2^j+3)}$. Then, solving the IC constraint I get the following:

$$(b_1^i - b_1^j) \leq \frac{1}{4}\left[\frac{w_{11}}{(k_1^j+3)} - \frac{w_{12}}{(k_2^j+3)}\right]$$

Which, solving for all the relevant cases, leads to:

$$|b_1^i - b_1^j| \leq \frac{1}{4}\left|\frac{w_{11}}{(k_1^j+3)} - \frac{w_{12}}{(k_2^j+3)}\right|$$

Note that this IC constraint is more restrictive than that for Full Revelation, and the decision

[36] Meaning that type $(0,0)$ announces $(1,1)$ or vice-versa, and type $(0,1)$ announces $(1,0)$ or vice-versa.

[37] Meaning that type $(0,0)$ announces $(1,0)$ or vice-versa, and type $(0,1)$ announces $(1,1)$ or vice-versa.

[38] Meaning that type $(0,0)$ announces $(0,1)$ or vice-versa, and type $(1,0)$ announces $(1,1)$ or vice-versa.

[39] In the companion paper I show that full revelation of two types and babbling for the other two are the only two DNS message strategies arising in any equilibrium. The argument is based on the equilibrium selection criterion given the similarity of IC constraints, and applies to the case of one decision as well.

maker will prefer the latter. As a consequence, no DNS strategy arises under delegation. □

B.4 Proof of Proposition 9

Proof. Having derived the necessary conditions for communication (Lemma 5), getting the receiver-optimal (R-optimal) equilibrium consist of finding the system of beliefs consistent with each message strategy (if those exist). Let denote by $\mu_j^*(\mathbf{S}^i|m_j^{i*})$ the beliefs of decision maker j about i's information (her type) upon receiving message m_j^{i*}.

A Fully Revealing message strategy means any of i's messages is taken at face value, that is:

$$\mu^*\left((0,0)|m_j^i=\{(0,0)\}\right)=1 \qquad \mu^*\left((1,0)|m_j^i=\{(1,0)\}\right)=1$$
$$\mu^*\left((0,1)|m_j^i=\{(0,1)\}\right)=1 \qquad \mu^*\left((1,1)|m_j^i=\{(1,1)\}\right)=1$$

From Lemma (5) we know that if i's preferences satisfy condition (13), then he truthfully announces his type and the beliefs above are consistent with that strategy in equilibrium. Because j also knows that, the system of beliefs defined above are implemented only if i's preferences satisfy condition (13); otherwise, there is always a deviation for which the beliefs are not consistent. Now, because the IC constraints for revealing one signal are the same and given I focus on R-optimal equilibria, fully revealing dominates.

For DNS, the Proof of Lemma 5 showed the only of such strategies emerging in equilibrium is $m_j^{i*}=\{\{(0,0)\},\{(1,1)\},\{(0,1),(1,0)\}\}$. Beliefs consistent with such a strategy are given by:

$$\mu^*\left((0,0)|m_j^i=\{(0,0)\}\right)=1 \qquad \mu^*\left((1,1)|m_j^i=\{(1,1)\}\right)=1$$
$$\mu^*\left((0,1)|m_j^i=\{(0,1)\}\right)=\mu^*\left((1,0)|m_j^i=\{(0,1)\}\right)=1/2$$
$$\mu^*\left((0,1)|m_j^i=\{(1,0)\}\right)=\mu^*\left((1,0)|m_j^i=\{(1,0)\}\right)=1/2$$

By the same argument as above, these are equilibrium beliefs if and only if i's preferences satisfy (14). □

B.5 Proof of Proposition 2

Proof. Informational gains arise when more agents reveal information to al least one decision maker, as compared to those revealing to the principal under centralization.

Consider direct informational gains first. For every agent who would reveal information under centralization, there must exist an agent revealing at least the same amount of information to the new decision maker under delegation. Strict gains require that there also exist at least one agent revealing strictly more information to the new decision maker. Let j' decides on y_1 and the principal on y_2. For every other agent $h\in N$ such that β_1^h satisfies (15) there must exist a $i\in N$ such that b_1^i satisfies (13); otherwise, k_1' will be lower than k_1^{C}.[40] At this point, there must also exist an agent such that:

$$|b_1^i w_{11}+b_2^i w_{21}|>\frac{1}{2}\left[\frac{(w_{11})^2+(w_{21})^2}{(k_1^{\mathrm{C}}+3)}-\frac{w_{11}w_{12}+w_{21}w_{22}}{(k_2^{\mathrm{C}}+3)}\right]$$
$$|b_1^i-b_1^j|\leq\frac{1}{2}\left[\frac{w_{11}}{(k_1^j+3)}-\frac{w_{12}}{(k_2^j+3)}\right]$$

[40] Indeed, any of the agents revealing under delegation are revealing both signals.

Multiplying the last inequality by w_{1r}, its right-hand side is strictly lower than that of the expression above. I thus get the condition that $|b_1^i - b_1^j| < |b_1^i + b_2^i \frac{w_{21}}{w_{11}}|$ for direct info gains from delegation.

Indirect informational gains mean that in equilibrium $k_2^{\mathrm{P}2} > k_2^{\mathrm{C}}$; the above equations must hold for y_2. An argument similar to the previous leads to: for every $h \in N$ such that β_2^h satisfies (15), there must exist a $i \in N$ such that b_2^i satisfies (13) and, in addition, there exist exists an agent i such that:

$$|b_1^i w_{12} + b_2^i w_{22}| > \frac{1}{2}\left[\frac{(w_{12})^2 + (w_{22})^2}{(k_2^{\mathrm{C}}+3)} - \frac{w_{11}w_{12} + w_{21}w_{22}}{(k_1^{\mathrm{C}}+3)}\right]$$
$$|b_2^i| \leq \frac{1}{2}\left[\frac{w_{22}}{(k_2^j+3)} - \frac{w_{21}}{(k_1^j+3)}\right]$$

Again, multiplying the last inequality be w_{22} evidences that its RHS is larger than that of the first one. This reduces the necessary condition for IIG to $w_{22}|b_2^i| < |b_1^i w_{12} + b_2^i w_{22}|$. It follows that the previous is only possible when $|b_1^i|$ is sufficiently large. □

B.6 Proof of Proposition 2

Proof. Let $\mathbf{B}^n = (\mathbf{b}^1, ..., \mathbf{b}^n)$ denote a given profile of biases for n informed agents. The optimal organizational structure is characterized by the number of truthful messages decision makers receive in equilibrium, $\mathbf{k}'$, $\mathbf{k}''$, and the number of truthful messages the principal receives under centralization, $\mathbf{k}^{\mathrm{C}}$, as per Proposition 8. Suppose that $k_1' = k_2'$, $k_1'' = k_2''$, and $k_1^{\mathrm{C}} = k_2^{\mathrm{C}}$. Now consider the associated game consisting of $n+m$ agents, with the profile of biases for the first n agents being $\mathbf{B}^n$, that is $\mathbf{B}^{n+m} = (\mathbf{b}^1, ..., \mathbf{b}^n, \mathbf{b}^{n+1}, ..., \mathbf{b}^{n+m})$.

Now, consider the following cases for the preferences of agents $n+1$ to $n+m$:

1. $\mathbf{b}^{n+1} = ... = \mathbf{b}^{n+m} = (\infty, 0)$.

   Suppose first that the optimal organizational structure under $\mathbf{B}^n$ was full delegation. Under the profile $\mathbf{B}^{n+m}$, the principal prefers to retain authority over y_2 if and only if $IIG(y_2) \geq DIG_{j''}(y_2) - (b_2'')^2$; which by Lemma 1 and Proposition 8 translates into:

$$\frac{w_{21}^2}{6}\left[\frac{1}{(k_1''+2)} - \frac{1}{(k_1^{\mathrm{P}2}+m+2)}\right] + \frac{w_{22}^2}{6}\left[\frac{1}{(k_2''+2)} - \frac{1}{(k_2^{\mathrm{P}2}+m+2)}\right] \geq -(b_2'')^2$$

   Then, for any $k_1'', k_2'', k_1^{\mathrm{P}2}, k_2^{\mathrm{P}2} \leq n$, there exists a m for which the above holds.

   Now, suppose the optimal organizational structure under $\mathbf{B}^n$ is centralization. Then the additional information she receives with partial delegation of y_1 under $\mathbf{B}^{n+m}$ must also compensate the distributional loss on the delegated decision; which means:

$$\frac{w_{21}^2}{6}\left[\frac{2}{(k_1^{\mathrm{C}}+2)} - \frac{1}{(k_1^{\mathrm{P}2}+m+2)}\right] + \frac{w_{22}^2}{6}\left[\frac{1}{(k_2^{\mathrm{C}}+2)} - \frac{1}{(k_2^{\mathrm{P}2}+m+2)}\right] \geq$$
$$\geq \frac{w_{11}^2}{6}\left[\frac{1}{(k_1'+2)} - \frac{1}{(k_1^{\mathrm{C}}+2)}\right] + \frac{w_{12}^2}{6}\left[\frac{1}{(k_2'+2)} - \frac{1}{(k_2^{\mathrm{C}}+2)}\right] + b_1'$$

   Note that the RHS of the expression above is non-negative because of the optimality of centralization under $\mathbf{B}^n$. A sufficient condition for optimality of partial delegation is that there exists an agent j' whose bias satisfies:[41]

$$(b_1')^2 \leq \frac{w_{11}^2 + w_{21}^2}{6(n+2)} + \frac{w_{12}^2 + w_{22}^2}{6(n+2)} - \frac{w_{11}^2}{18} - \frac{w_{12}^2}{18}$$

Then, there exists a finite m such that Partial Delegation (of y_1) is preferred by the principal over centralization.

2. $\mathbf{b}^{n+1} = ... = \mathbf{b}^{\frac{2n+m+1}{2}} = (-\infty, \infty)$ and $\mathbf{b}^{\frac{2n+m+1}{2}} = ... = \mathbf{b}^{n+m} = (\infty, \infty)$.

   Lemma 7 implies that, for $k_1^{\mathrm{C}} = k_2^{\mathrm{C}}$, senders with these preferences will have maximal incentives to play equilibrium DNS strategies. In particular, those in the first group satisfy:

$$\left|\frac{\beta_1^i}{(k_1^{\mathrm{C}}+3)} + \frac{\beta_2^i}{(k_2^{\mathrm{C}}+3)}\right| = \left|\frac{w_{11}b_1^i + w_{12}b_2^i + w_{21}b_1^i + w_{22}b_2^i}{(k^{\mathrm{C}}+3)}\right| = \left|\frac{b_1^i + b_2^i}{(k^{\mathrm{C}}+3)}\right| = 0$$

   And those in the second group:

$$\left|\frac{\beta_1^i}{(k_1^{\mathrm{C}}+3)} - \frac{\beta_2^i}{(k_2^{\mathrm{C}}+3)}\right| = \left|\frac{w_{11}b_1^i + w_{12}b_2^i - w_{21}b_1^i - w_{22}b_2^i}{(k^{\mathrm{C}}+3)}\right| = \left|\frac{b_1^i - b_2^i}{(k^{\mathrm{C}}+3)}\right| = 0$$

   Then, $m = 2n$ is a sufficient condition for the principal to prefer centralization over any other organizational structure that was optimal with the original profile of biases.

□

B.7 Proof of Proposition 3

Proof. Suppose the equilibrium organizational structure involve some form of delegation, and let $j', j'' \in \{P, 1, 2, ...\}$ be decision makers for y_1 and y_2, respectively. The principal's ex-ante utility gain with respect to centralization is given by:

$$\begin{aligned}(b_1')^2 + (b_2'')^2 \leq & \frac{(w_{11})^2}{6}\left[\frac{1}{(k_1^{\mathrm{C}}+2)} - \frac{1}{(k_1'+2)}\right] + \frac{(w_{12})^2}{6}\left[\frac{1}{(k_2^{\mathrm{C}}+2)} - \frac{1}{(k_2'+2)}\right] + \\ & \frac{(w_{21})^2}{6}\left[\frac{1}{(k_1^{\mathrm{C}}+2)} - \frac{1}{(k_1''+2)}\right] + \frac{(w_{22})^2}{6}\left[\frac{1}{(k_2^{\mathrm{C}}+2)} - \frac{1}{(k_2''+2)}\right]\end{aligned}$$

According to expression (22), and denoting by $\mathbf{m}_{\mathrm{C}}$ the messages sent to the principal under centralization, the above can be expressed as:

$$\left[(b_1')^2 + (b_2'')^2\right]^{\frac{1}{2}} \leq \left[\sum_{y_d}\left[w_{d1}^2\big(\mathrm{Var}(\theta_1|\mathbf{m}_{\mathrm{C}}) - \mathrm{Var}(\theta_1|\mathbf{m}_j)\big) + w_{d2}^2\big(\mathrm{Var}(\theta_2|\mathbf{m}_{\mathrm{C}}) - \mathrm{Var}(\theta_2|\mathbf{m}_j)\big)\right]\right]^{\frac{1}{2}}$$

Denote by $\hat{\mathbf{b}}^{\mathrm{D}}$ the vector whose component satisfy the above expression with equality. Because the LHS above represents the euclidean distance of a vector with components b_1' and b_2'' to the origin, then $\hat{\mathbf{b}}^{\mathrm{D}}$ represents the maximum conflict of interest the principal will tolerate as a function of the informational gains from delegation of the corresponding decision(s).

[41]The expression reflects the case in which all n agents fully reveal their signals under centralization ($k_r^{\mathrm{C}} = n$), j' does not receive any signals from other agents in equilibrium ($k_r' = 1$), and the indirect informational gains are maximal ($m = \infty$).

For the second part of the proof let assume that $k_r^j = k^{\mathrm{C}} + 1$, and denote the reduction in variance under delegation by $\Delta\mathrm{Var}(\theta_r|\mathbf{m}_j) = \mathrm{Var}(\theta_r|\mathbf{m}_{\mathrm{C}}) - \mathrm{Var}(\theta_r|\mathbf{m}_j, k_r^j = k_r^{\mathrm{C}} + 1)$, which then is given by:

$$\Delta\mathrm{Var}(\theta_r|\mathbf{m}_j) = \frac{1}{6}\left[\frac{1}{(k_r^{\mathrm{C}}+2)} - \frac{1}{(k_r^{\mathrm{C}}+3)}\right]$$

Then, taking the derivative with respect to k_r^{C} gives expression (4). □

B.8 Proof of Lemma 1

Proof. The proof proceed by contradiction. I focus on the centralization case, since decentralization follows the same logic. Let $(\{\mathbf{y}^*\}, \{\mathbf{m}^*, \mathfrak{s}^*\})$ be the equilibrium strategy profiles for the receiver and all agents, respectively. Recall the equilibrium is characterized by k_1^* and k_2^*.

Acquisition of S_1. Suppose that i's equilibrium info acquisition strategy has $S_1 \in \mathfrak{s}^{i*}$ but condition (15) does not hold for S_1. In such a case revealing information about θ_1 is not incentive compatible for i despite he acquired information about it. Other agents base their message strategies on conjectures about k_1^*, but i is not included among agents revealing S_1 truthfully. At the information acquisition stage, i's expected payoff of $\mathfrak{s}^{i*}$ is thus given by:

$$E\left[U^i\left(\mathbf{y}^*(\mathbf{m}^*(\mathfrak{s}^*)), \delta, b^i\right)\right] = -E\left[\left(\mathbf{y}_1\left(m^{i*}(\mathfrak{s}^{i*}), \mathbf{m}^{\text{-}i*}\right) - \delta_1 - b_1^i\right)^2 + \left(\mathbf{y}_2\left(m^{i*}(\mathfrak{s}^{i*}), \mathbf{m}^{\text{-}i*}\right) - \delta_2 - b_2^i\right)^2\right] - C(\mathfrak{s}^{i*})$$

Now, consider the following deviation: $\hat{\mathfrak{s}}^i = \mathfrak{s}^{i*} \backslash \{S_1\}$. Note that this deviation does not affect k_1^* or k_2^*, and i's overall influence on j's decision(s) is thus unaltered—i.e. $y_d\left(m^i(\hat{\mathfrak{s}}^i), \mathbf{m}_j^{\text{-}i}\right) = y_d\left(m^i(\mathfrak{s}^{i*}), \mathbf{m}_j^{\text{-}i}\right)$. Note also that $C(\mathfrak{s}^{i*}) > C(\hat{\mathfrak{s}}^i)$, given $\#\mathfrak{s}^{i*} > \#\hat{\mathfrak{s}}^i$. Consequently,

$$E\left[U^i\left(\mathbf{y}^*(\mathbf{m}^*(\mathfrak{s}^*)), \delta, b^i\right)\right] - E\left[U^i\left(\mathbf{y}\left(m^i(\hat{\mathfrak{s}}^i), \mathbf{m}^{\text{-}i*}\right), \delta, b^i\right)\right] = -C(\mathfrak{s}^{i*}) + C(\hat{\mathfrak{s}}^i) < 0$$

So, $\hat{\mathfrak{s}}^i$ is a profitable deviation from $\mathfrak{s}^{i*}$. $\Rightarrow\Leftarrow$

Acquisition of both signals. The proof is similar to the previous, with i's proposed equilibrium strategy being $\mathfrak{s}^{i*} = \{S_1, S_2\}$ but conditions for Full Revelation do not hold. Then, a profitable deviation for i will be to acquire the signal he is willing to reveal on the equilibrium path (if any). □

B.9 Proof of Proposition 4

Proof. I analyse the case of delegation, as centralization follows the same argument but requires more algebra (see Habermacher, 2018 for a reference). Suppose agent i's acquires only S_1^i; then strategy $\mathbf{m}^{i*} = \{m_{j'}^{i*}, m_{j''}^{i*}\}$ is preferred to any alternative $\hat{\mathbf{m}}$ iff (IC constraint in section B.3):

$$-\left[w_{d1}(\nu_{d1}^{i*} - \hat{\nu}_{d1}^i) + w_{d2}(\nu_{d2}^{i*} - \hat{\nu}_{d2}^i)\right]\left[w_{d1}(\nu_{d1}^{i*} + \hat{\nu}_{d1}^i) + w_{d2}(\nu_{d2}^{i*} + \hat{\nu}_{d2}^i) - 2[w_{d1}\nu_{d1}^i + w_{d2}\nu_{d2}^i - (b_d^i - b_d^j)]\right] \geq 0$$

But since i has information about θ_1 only, then $E(\theta_2|S_1^i, \mathbf{m}_j^{\text{-}i}) = \nu_{d2}^i = \nu_{d2}^{i*} = \hat{\nu}_{d2}^i$. Moreover, the strategy space when i has only one signal is degenerated, such that he can only reveal it or lie. Revealing S_1^i is thus IC iff:

$$-\left[w_{d1}(\nu_{d1}^{i*} - \hat{\nu}_{d1}^i)\right]\left[w_{d1}(\nu_{d1}^{i*} - \hat{\nu}_{d1}^i) - 2(b_d^j - b_d^i)\right] \geq 0$$

Following the same steps as in section B.3 it is easy to note that the above expression becomes:

$$\text{For } \tilde{S}_1^i = 0: \qquad 2(b_d^i - b_d^j) \leq \frac{w_{d1}}{(k_1^j + 3)}$$
$$\text{For } \tilde{S}_1^i = 1: \quad -2(b_d^i - b_d^j) \leq \frac{w_{d1}}{(k_1^j + 3)}$$

Which together imply equation (11). □

B.10 Equilibrium Information Acquisition and Cost-effectiveness condition

Before the proof of Lemma 2 I derive the information-acquisition IC constraint.

Observation. *Let* $k_r^{j*} \equiv k_r^j\left(\mathbf{m}_j^i(\mathfrak{s}^{i*}, \mathfrak{s}^{-i}), \mathbf{m}_j^{-i}(\mathfrak{s}^{i*}, \mathfrak{s}^{-i})\right)$ *and* $\hat{k}_r^j \equiv k_r^j\left(\mathbf{m}_j^i(\hat{\mathfrak{s}}^i, \mathfrak{s}^{-i}), \mathbf{m}_j^{-i}(\mathfrak{s}^{i*}, \mathfrak{s}^{-i})\right)$ *for* $\theta_r = \{\theta_1, \theta_2\}$*. Let* $\mathfrak{s}^{i*}$ *denote* i*'s information acquisition strategy in an equilibrium characterized by* $(\mathbf{y}^*, \mathbf{m}^*, \mathfrak{s}^*)$*. Then,* i*'s ex-ante expected utility from* $\mathfrak{s}^{i*}$ *is given by:*

$$E\left[U^i\left(\mathbf{m}^*, \mathfrak{s}^{i*}, \mathfrak{s}^{-i}, \boldsymbol{\delta}, \mathbf{b}^i\right)\right] = -\left[(b_1^i)^2 + (b_2^i)^2\right] - \sum_{y_d=\{y_1,y_2\}}\left[\frac{(w_{d1})^2}{6\left(k_1^{j*}+2\right)} + \frac{(w_{d2})^2}{6\left(k_2^{j*}+2\right)}\right]$$

Now, let $(\mathbf{y}^*, \mathbf{m}^*, \mathfrak{s}^*)$ *be equilibrium strategy profiles. Then,* $\mathfrak{s}^{i*}$ *is incentive compatible for agent* i *if and only if, for every alternative* $\hat{\mathfrak{s}}^i$*:*

$$\sum_{y_d=\{y_1,y_2\}}\sum_{\theta_r=\{\theta_1,\theta_2\}}\frac{(w_{dr})^2}{6}\left[\frac{1}{\left(\hat{k}_r^j+2\right)} - \frac{1}{\left(k_r^{j*}+2\right)}\right] \geq \left[C(\mathfrak{s}^{i*}) - C(\hat{\mathfrak{s}}^i)\right] \tag{23}$$

Proof of Lemma 2

Proof. I first derive the cost-effectiveness condition (5) and then the maximum number of agents for which acquiring a given piece of information is cost-effective –condition (8). In order to derive cost-effectiveness (CE), I consider each possible info acquisition strategy in equilibrium.

The number of agents revealing truthfully their signals in equilibrium, k_r^*, includes i's message strategy when he acquires (and reveals) it in equilibrium.[42] Here I need to make two clarifications. Firstly, if there were agents who acquired information on θ_1 but were not willing to reveal it when $\hat{k}_1 = k_1^* + 1$, then only one of them changes his message strategy because when one of these agents stop revealing, then $\hat{k}_1 = k_1^*$ again. As a consequence, $\hat{k}_1 = \{k_1^*, k_1^* + 1\}$. I take the most conservative of these approaches by making $\hat{k}_1 = k_1^* + 1$ whenever i acquires S_1 off-path.

The second clarification relates to what happens when i acquires a signal and does not reveal it. Since other agents' message strategies will depend on conjectures about k_r, i not revealing the signal acquired off-path does not affect their equilibrium behaviour at the communication stage. In other words, Lemma 1 holds: i gains nothing from acquiring a signal he will not reveal.

Centralization. Let first consider the acquisition of both signals in equilibrium; that is $\mathfrak{s}^{i*} = \{S_1^i, S_2^i\}$. Expression (23) for each possible alternative strategy becomes:

[42] In equilibria in which i does not acquire S_1^i, k_1^* does not count him; but in any deviation in which he does acquire it, then $\hat{k}_1 = k_1^* + 1$.

1) $\tilde{\mathfrak{s}}^i = \{\emptyset\}$

$$\sum_{\theta_r} \frac{(w_{1r})^2 + (w_{2r})^2}{6(k_r^* + 2)(k_r^* + 3)} \geq C(S_1^i, S_2^i)$$

2) $\tilde{\mathfrak{s}}^i = \{S_r^i\}$

$$\frac{(w_{1\tilde{r}})^2 + (w_{2\tilde{r}})^2}{6(k_{\tilde{r}}^* + 2)(k_{\tilde{r}}^* + 3)} \geq C(S_{\tilde{r}}^i)$$

Now, when the equilibrium strategy consists of one signal only, $\mathfrak{s}^{i*} = \{\tilde{S}_r^i\}$, the IC constraints become:

3) $\tilde{\mathfrak{s}}^i = \{\emptyset\}$

$$\frac{(w_{1r})^2 + (w_{2r})^2}{6(k_r^* + 2)(k_r^* + 3)} \geq C(S_r^i)$$

4) $\tilde{\mathfrak{s}}^i = \{S_{\tilde{r}}^i\}$

$$\frac{(w_{1r})^2 + (w_{2r})^2}{6(k_r^* + 2)(k_r^* + 3)} \geq \frac{(w_{1\tilde{r}})^2 + (w_{2\tilde{r}})^2}{6(k_{\tilde{r}}^* + 2)(k_{\tilde{r}}^* + 3)}$$

5) $\tilde{\mathfrak{s}}^i = \{S_1^i, S_2^i\}$

$$\frac{(w_{1\tilde{r}})^2 + (w_{2\tilde{r}})^2}{6(k_{\tilde{r}}^* + 2)(k_{\tilde{r}}^* + 3)} < C(S_{\tilde{r}}^i)$$

Case 3) represents the necessary condition to acquire any individual signal S_r^i, since it implies case 1) (in which it holds for both signals) and case 4) (in which the agent acquires the signal that would have the highest influence). This case corresponds to equation (5).

Now I work out the expression for the maximum number of agents to acquire a given signal under centralization. According to the equation (5) as k_r increases the ex-ante expected utility of acquiring S_r decreases. So, the maximum number of agents who will acquire that signal is given by the largest k^* for which the cost-effectiveness condition hold. Re-arranging this condition I get the following polynomial:

$$-(k_r^*)^2 - 5k_r^* - \left[6 - \frac{(w_{1r})^2 + (w_{2r})^2}{6\,C(S_r^i)}\right] \geq 0$$

Then, solving for the highest positive root I get K_r^{C} in (8).

Delegation. As before, i is not willing to acquire signals he is not willing to reveal on-path (Lemma 1). But in this case there are two decision makers and IC can refer to any of them (or both). From (23) we know that acquiring S_r^i requires that i is willing to reveal it to at least one decision maker, say:

$$C(S_r^i) \leq \frac{(w_{dr})^2}{6(k_r^j + 2)(k_r^j + 3)}$$

For at least one y_d. Consider the case of acquiring both signals, which is cost-effective in two generic cases. First, when i is willing to reveal at least one signal to a different decision maker, the CE condition above should hold for each decision maker. Second, when i is willing to reveal both signals to a single j the RHS of the above expression becomes larger. As a consequence, equation (7) is a necessary condition for investing in any individual signal.

To get the expression for the maximum number of agents to invest in S_r I need to analyse also two cases. The minimal incentives to reveal are given by the case in which all agents are willing to reveal

S_r to decide on the dimension it is less important. This will define the minimum upper-bound, since the CE condition becomes

$$C(S_r^i) \leq \min_{w_{dr}} \left\{ \frac{(w_{dr})^2}{6(k_r^j+2)(k_r^j+3)} \right\}$$

Now consider the case in which all agents are willing to reveal *both signals* to *both decision makers*. This is the maximum upper-bound. In such a case, the CE condition will be just like the centralization case; that is:

$$C(S_r^i) \leq \frac{(w_{1r})^2 + (w_{2r})^2}{6(k_r^j+2)(k_r^j+3)}$$

Then, following the same steps as the proof of Lemma 2 (Appendix B.10) I have the first and the second expressions in square bracket in equation (9), respectively. □

Proof of equation (6) *(DNS under Centralization).* For any dimensional non-separable strategy in which i fully reveals some realizations and play babbling on the others, the expected payoff in equation (23) becomes.

$$\frac{[(w_{11})^2 + (w_{21})^2]}{6} \left[\frac{1}{(\hat{k}_1+2)} - \frac{1}{2(k_1^*+2)} - \frac{1}{2(k_1^*+3)} \right] +$$
$$+ \frac{[(w_{12})^2 + (w_{22})^2]}{6} \left[\frac{1}{(\hat{k}_2+2)} - \frac{1}{2(k_2^*+2)} - \frac{1}{2(k_2^*+3)} \right] \geq \left[C(\mathfrak{s}^{i*}) - C(\hat{\mathfrak{s}}^i) \right]$$

Agent i fully reveals his signals half of the time (in expectation), where k_r^* indicates the equilibrium number of agents revealing information about θ_r apart from i. Then, solving for deviations as in the previous result, I get that acquiring both signals to play a DNS message strategy is cost effective if:

$$\frac{(w_{11})^2 + (w_{21})^2}{6(k_1^*+2)(k_1^*+3)} + \frac{(w_{12})^2 + (w_{22})^2}{6(k_2^*+2)(k_2^*+3)} \geq 2C(S_1^i, S_2^i)$$

Now, i would prefer to acquire both signals and play DNS strategy to acquire only S_r and reveal it for sure if:

$$\frac{(w_{1\tilde{r}})^2 + (w_{2\tilde{r}})^2}{6(k_{\tilde{r}}^*+2)(k_{\tilde{r}}^*+3)} - \frac{(w_{1r})^2 + (w_{2r})^2}{6(k_r^*+2)(k_r^*+3)} \geq 2C(S_{\tilde{r}}^i)$$

Which is easily shown that never holds when $w_{11} = w_{22}$ and $w_{d1} + w_{d2} = 1$. □

B.11 Proof of Proposition 11

Proof. We know from Lemmas 1 and 2 that any equilibrium information acquisition strategy must be CE and IC at the communication stage. Recall that the principal observes agents' choices of information, for which she knows the relevant message space for each agent. Equilibrium communication is then characterized as in Proposition 9 and its equivalent under centralization (Proposition 2 in Habermacher, 2018).

But cost-effectiveness can impose restrictions on equilibrium communication; for instance, when i cannot afford to acquire all the information he is willing to reveal on-path. Consider the case in which i revealing both signals is IC but acquiring only one of them is CE. Revelation of each signal individually is a necessary condition for Full Revelation, so i acquires one signal if CE and reveal it in equilibrium. Now, which of those signals he actually acquires depend on the ex-ante expected utility –see case 4) in the previous proof.

Similar argument applies to any equilibria in which i is willing to reveal S_r^i only. If acquiring is CE, then he reveals that information in equilibrium; if not, he does not reveal any information –indeed, he has no message to send and the receiver observes that.

Finally, no information is transmitted when i's preferences are such that he is not willing to reveal any information, or when acquiring any signal is not cost-effective.

For the case of delegation the same argument applies, with Lemmas 5 and 1, and Corollary 2. □

B.12 Proof of Proposition 5

Proof. When costs do not impose restrictions on information acquisition, incentive compatibility at the communication stage dictates agents' equilibrium strategies (expressions in Lemma 6). In the case of two agents and linear costs, CE does not restrict agents' communication strategies if the cost of any signal is lower than the ex-ante expected utility in the equilibrium when $k_r^* = 1$; that is:

$$\begin{aligned} C(S_r^i) \leq & \frac{(w_{1r})^2 + (w_{2r})^2}{6(k_r^* + 2)(k_r^* + 3)} \\ c \leq & \frac{(w)^2 + (1-w)^2}{72} \end{aligned}$$

Agent A^1 acquires and reveals S_1^1 in equilibrium if his preferences satisfy expression (10) associated to θ_1 and with $k_1^* = 1$ (see Section B.4 for the supporting system of beliefs on communication). But his preferences should not be such that full revelation is IC, otherwise the principal would be better-off under this equilibrium.[43] A similar argument applies for A^2 with respect to S_2^2.

When $\frac{(w)^2+(1-w)^2}{72} < c \leq \frac{(w)^2+(1-w)^2}{36}$ it is not CE to acquire both signals, so agents acquire the signal each of them is willing to reveal. In other words, the necessary and sufficient condition for specialization in this case is that (10) holds for different signals for each agent. □

B.13 Proof of Proposition 6

Proof. Let $(\mathfrak{s}, \mathbf{m}, \mathbf{y})$ denote a generic equilibrium in which $\kappa < n$ is the maximum number of agents willing to reveal S_r to both decision makers (suppose $\kappa > 0$). For any of such agents cost-effectiveness under delegation –condition (7)– is given by:

$$C(S_r^i) \leq \frac{(w_{1r})^2 + (w_{2r})^2}{6(k_r^j + 2)(k_r^j + 3)}$$

But for any other agent, the CE condition is at most:

$$C(S_r^i) \leq \frac{(\hat{w}_{dr})^2}{6(k_r^j + 2)(k_r^j + 3)}$$

Then, $C(S_r^i) \leq \frac{(w_{1r})^2+(w_{2r})^2}{6(\kappa+2)(\kappa+3)}$ implies acquiring information about θ_r is CE for agents willing to reveal it to both decision makers. But if at the same time $C(S_r^i) > \frac{(\hat{w}_{dr})^2}{6(\kappa+2)(\kappa+3)}$, agents willing to reveal S_r to at most one decision maker do not acquire this signal because it is not CE.

On the other hand, the first of the above equations (8) determines the maximum number of agents for which acquiring S_r is CE under centralization. But since $C(S_r^i) \leq \frac{(w_{1r})^2+(w_{2r})^2}{6(\kappa+3)(\kappa+4)}$, it should be greater

[43] Note that if condition (10) holds for S_2^1 (but still not those for full revelation) he will still find optimal to acquire S_1^1 only, *given A^2 is acquiring the other signal in equilibrium.*

or equal to $\kappa+1$. Then, $K_r^{\text{D}} = \kappa < K_r^{\text{C}}$ ∎

B.14 Proof of Lemma 3

Proof. Let $w \equiv w_{11} = w_{22}$, let κ be an non-negative integer, and let $\varepsilon \in \Re_+$ with associated integer n_ε. Also, let $i \in N_\varepsilon$ be an agent whose preferences satisfy equation (10) with respect to θ_r for $k_r = \kappa$. Note that $\boldsymbol{\lambda}_r = \left(1, -\frac{w_{1r}}{w_{2r}}\right)$, the associated unit vector is $\hat{\boldsymbol{\lambda}}_r = \frac{\boldsymbol{\lambda}_r}{||\boldsymbol{\lambda}_r||} = \left(\frac{w_{1r}}{(w_{1r}^2+w_{2r}^2)^{1/2}}, \frac{w_{2r}}{(w_{1r}^2+w_{2r}^2)^{1/2}}\right)$, and that $\text{Proj}_{\boldsymbol{\lambda}_r}(\mathbf{b}^i) = (\mathbf{b}^i \cdot \hat{\boldsymbol{\lambda}}_r)\boldsymbol{\lambda}_r$. Then, careful algebra leads to condition (10) expressed as:

$$||\mathbf{b}^i - \text{Proj}_{\lambda_1}(\mathbf{b}^i)|| \le \frac{\left[(w)^2 + (1-w)^2\right]^{\frac{1}{2}}}{2\,(\kappa+3)}$$

Now consider an agent j with the following preferences: $b_1^j = b_2^i$ and $b_2^j = b_1^i$. I need to show 1) $j \in N_\varepsilon$, and 2) $\mathbf{b}^j$ satisfies equation (10) for θ_2. Proving the first claim is straightforward, since j's preference vector is just i's with its components swapped. This says that both i and j agents have exactly the same conflict of interest with the principal.

The second part of the proof requires work out $||\mathbf{b}^j - \text{Proj}_{\lambda_2}(\mathbf{b}^j)||$, which yield:

$$\begin{aligned} ||\mathbf{b}^j - \text{Proj}_{\lambda_2}(\mathbf{b}^j)|| &= |b_1^j\,(1-w) + b_2^j\,w|\ \left[(w)^2 + (1-w)^2\right]^{-\frac{1}{2}} \\ &= |b_2^i\,(1-w) + b_1^i\,w|\ \left[(w)^2 + (1-w)^2\right]^{-\frac{1}{2}} \\ &= ||\mathbf{b}^i - \text{Proj}_{\lambda_1}(\mathbf{b}^i)|| \end{aligned}$$

∎

B.15 Proof of Proposition 7

Proof. Note first that $\text{Var}(y_d|\mathbf{m}^*) = w_{d1}^2\text{Var}(\theta_1|\mathbf{m}^*) + w_{d2}^2\text{Var}(\theta_2|\mathbf{m})$ which, given $w_{11} = w_{22} = w$, implies that $\text{Var}(y_1|\mathbf{m}^*) - \text{Var}(y_2|\mathbf{m}^*) = w^2(1-w)^2\left[\text{Var}(\theta_1|\mathbf{m}^*) - \text{Var}(\theta_2|\mathbf{m}^*)\right] = w^2(1-w)^2\left[\frac{1}{6(\tilde{k}_1^*+2)} - \frac{1}{6(\tilde{k}_2^*+2)}\right]$.

Under centralization, the principal's expectation about her update beliefs on each state—and, thus, the residual variance— depends on how much information she expects to receive from agents on the equilibrium path. Since biases are uniformly distributed between $[0, \varepsilon]$, Lemma 3 implies that for any agent she expects to be revealing information about θ_1, she expects another agent with the same conflict of interest who will be willing to reveal information about θ_2. Then, in expectation, the number of agents revealing each signal coincide, that is $\tilde{k}_1^{\text{C}} = \tilde{k}^{\text{C}} = 2$, which implies that $E\left[\text{Var}(\theta_1|\mathbf{m}^*) - \text{Var}(\theta_2|\mathbf{m}^*)\right] = 0$.

Now suppose the principal delegates y_2 to agent $i = 0$. Again, given the uniform distribution of biases, by Lemma 4 she expects more agents willing to reveal information about θ_1 than θ_2—and the opposite pattern for $i = 0$. Then, her ex-ante expectations about equilibrium information on each state satisfy: $\tilde{k}_1^{\text{P1}} > \tilde{k}_2^{\text{P1}}$ and $\tilde{k}_1^{0} < \tilde{k}_2^{0}$; as a consequence, $E\left[\text{Var}(\theta_1|\mathbf{m}^{\text{P1}})\right] < E\left[\text{Var}(\theta_2|\mathbf{m}^{\text{P1}})\right]$ and $E\left[\text{Var}(\theta_1|\mathbf{m}^{0})\right] > E\left[\text{Var}(\theta_2|\mathbf{m}^{0})\right]$. ∎

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Appendix C (Online Appendix)

C.1 Covert Information Acquisition Game

In the covert game the decision maker does not observe agents' information acquisition decisions. This implies that a Perfect Bayesian Equilibrium must also specify the decision maker's beliefs about agents' investments in information. I will focus on pure strategy equilibria at the information acquisition stage. In principle, the agent in question may try to convey information about which signals he acquired to the decision maker by means of his cheap talk message. However, a result from Argenziano et al. (2016) allows me to restrict attention to equilibria in which agents do not signal how much information each has acquired, and this is without loss of generality. Below I present the result.

Lemma 9 (Argenziano et al., 2016)**.** *Any outcome supported in a Perfect Bayesian Equilibrium of the covert game in which an agent follows a pure strategy in the choice of information can be supported in a Perfect Bayesian Equilibrium in which the decision makers beliefs about his information acquisition decision do not vary with the agents message.*

There would be two classes of deviations available to agents if the decision maker's beliefs about information acquisition decisions could be affected by the choice of messages. First, an agent could acquire an off-path amount of information but still send the message corresponding to the equilibrium amount of information. Secondly, the agent could acquire an off-path amount information and send a message corresponding to an off-path information acquisition choice, which in turn may no be true. The lemma says that any equilibrium outcome under the second class of deviations can be supported as an equilibrium in which the agent cannot change the decision maker's beliefs about his information acquisition decision.

When an agent acquires an off-path amount of information he can choose among the equilibrium messages according to his preferences. As a consequence, any deviation at the information acquisition stage implies a deviation at the communication stage. The result below summarizes this.

Lemma 10. *When agent i acquires fewer signals than what is expected on the equilibrium path, the messages used under the deviation are a strict subset of the equilibrium messages available. When i acquires more signals than expected on path, he uses the additional information to deviate from truth-telling for some signal realizations.*

The argument of the above lemma is straightforward. When i acquires fewer signals, he will not be able to condition his message on the information that has not been observed. As a consequence, the messages effectively used under the deviation will be fewer than those available on the equilibrium path, which implies that i is inducing beliefs that do not reflect his signal realizations. When i acquires more signals he cannot transmit that additional information on the equilibrium path (there is no way of signalling he acquired more information). Because additional information implies additional costs, i must be obtaining some utility gains with respect to equilibrium communication—by inducing beliefs according to his preferences for some signals realizations. This clearly implies that he will deviate from truth-telling when he observes the corresponding realizations.

Let $(\mathfrak{s}^*, \mathbf{m}^*(\mathfrak{s}^*), \mathbf{y}^*(\mathbf{m}^*(\mathfrak{s}^*)))$ be the equilibrium information acquisition decisions, message strategies, and decisions (respectively). Then, agent i's IC constraint at the information acquisition stage must consider any possible deviation $\hat{\mathfrak{s}}^i$ and the corresponding message strategy $\hat{m}^i(\hat{\mathfrak{s}}^i)$; that is,

$$E\Bigg[\int_0^1\int_0^1 -\sum_{y_d}^{\{y_1,y_2\}}\left(y_d\left(m^{i*},\mathbf{m}^{\text{-}i*}\right)-\delta_d-b_d^i\right)^2 f(\theta_1|\mathfrak{s}^{i*},\mathbf{m}^{\text{-}i*})f(\theta_2|\mathfrak{s}^{i*},\mathbf{m}^{\text{-}i*})+$$
$$+\int_0^1\int_0^1\sum_{y_d}^{\{y_1,y_2\}}\left(y_d\left(\hat{m}^{i},\mathbf{m}^{\text{-}i*}\right)-\delta_d-b_d^i\right)^2 f(\theta_1|\hat{\mathfrak{s}}^{i},\mathbf{m}^{\text{-}i*})f(\theta_2|\hat{\mathfrak{s}}^{i},\mathbf{m}^{\text{-}i*})\Bigg]\geq C(\mathfrak{s}^{i*})-C(\hat{\mathfrak{s}}^i) \qquad (24)$$

Then, given that deviations at the info acquisition stage do not affect the set of influential messages (Lemma 9) and that this deviations necessarily imply deviations at the communication stage (Lemma 10), the above expression can be solved by computing the expectation over all possible signals realizations and the corresponding messages on- and off-path. In particular, the utility gains from deviations will be given by the realizations in which the messages on- and off- path are different. Formally, let $\tilde{\mathbf{S}}^i$ represent the realization of the signals corresponding to agent i, independent of which of these he observes (determined by $\mathfrak{s}^i$). I can then express and compare message strategies on- and off-path as functions of i's type and the information he observes. In other words, *before deciding on information acquisition* and given the equilibrium under play, he can assess the utility gains from any info acquisition strategy and the corresponding messages he expects to send conditional on each possible pair of signal realizations. Equation 24 then becomes:

$$\sum_{\tilde{\mathbf{S}}\in\mathcal{S}}\Pr(\tilde{\mathbf{S}}^i)\times\int_0^1\int_0^1 -\sum_{y_d}^{\{y_1,y_2\}}\left[\left(y_d\left(m^i(\mathfrak{s}^{i*},\tilde{\mathbf{S}}^i)\right)-\delta_d-b_d^i\right)^2-\left(y_d\left(m^i(\hat{\mathfrak{s}}^{i},\tilde{\mathbf{S}}^i)\right)-\delta_d-b_d^i\right)^2\right]\times$$
$$\times f(\theta_1|\tilde{S}_1^i,\mathbf{m}^{\text{-}i*})f(\theta_2|\tilde{S}_2^i,\mathbf{m}^{\text{-}i*})\geq C(\mathfrak{s}^{i*})-C(\hat{\mathfrak{s}}^i)$$

Now, I proceed to analyse deviations from different equilibrium info acquisition strategies.

Agent i acquires both signals in equilibrium ($\mathfrak{s}^{i*}=\mathbf{S}^i$)

Let denote by $\nu_r^{i*}(\tilde{\mathbf{S}}^i)=E\left(\theta_r|m^i(\mathfrak{s}^{i*},\tilde{\mathbf{S}}^i),\mathbf{m}^{\text{-}i*}\right)$ the beliefs about $\theta_r=\{\theta_1,\theta_2\}$ induced by i under the equilibrium information acquisition strategies and the message corresponding to the realizations given by $\tilde{\mathbf{S}}\in\mathcal{S}$. Equivalently, denote by $\hat{\nu}_r^{i}(\tilde{\mathbf{S}}^i)=E\left(\theta_r|m^i(\hat{\mathfrak{s}}^{i},\tilde{\mathbf{S}}^i),\mathbf{m}^{\text{-}i*}\right)$ be the beliefs induced under the deviation at the information acquisition stage (for the same signals realizations). Then, the IC constraint at the information acquisition stage for agent i becomes:

$$\sum_{\tilde{\mathbf{S}}^i\in\mathcal{S}}\Pr(\tilde{\mathbf{S}}^i)\Bigg[-\sum_{y_d}\left[w_{d1}\left(\nu_1^{i*}(\tilde{\mathbf{S}}^i)-\hat{\nu}_1^{i}(\tilde{\mathbf{S}}^i)\right)+w_{d2}\left(\nu_2^{i*}(\tilde{\mathbf{S}}^i)-\hat{\nu}_2^{i}(\tilde{\mathbf{S}}^i)\right)\right]\times$$
$$\left[-w_{d1}\left(\nu_1^{i*}(\tilde{\mathbf{S}}^i)-\hat{\nu}_1^{i}(\tilde{\mathbf{S}}^i)\right)-w_{d2}\left(\nu_2^{i*}(\tilde{\mathbf{S}}^i)-\hat{\nu}_2^{i}(\tilde{\mathbf{S}}^i)\right)-2b_d^i\right]\Bigg]\geq C(\mathfrak{s}^{i*})-C(\hat{\mathfrak{s}}^i)$$

First consider the deviation in which i only acquires information about θ_1; that is, $\hat{\mathfrak{s}}^i=\{S_1^i\}$. It is straightforward to note that this deviation *per se* does not imply any difference in induced beliefs with respect to θ_1, formally $\nu_1^{i*}(\tilde{\mathbf{S}}^i)=\hat{\nu}_1^{i}(\tilde{\mathbf{S}}^i)$ for all $\tilde{\mathbf{S}}^i\in\mathcal{S}$. Now, i's message associated with S_2^i does not depend on the signal's realization, but depends on $\mathbf{b}^i$ and may also depend on S_1^i.

Let consider the case in which $\hat{m}^i=\{\tilde{S}_1^i,1\}$, i.e. i truthfully reveals his information about θ_1 and announces always a 1 for θ_2. Then, $\hat{\nu}_1^{i}(\tilde{\mathbf{S}}^i)=\frac{(k_2+4)}{2(k_2+3)}$ and it is different from $\nu_1^{i*}(\tilde{\mathbf{S}}^i)$ only when $\tilde{S}_2^i=\{0\}$

which, in turn, happens for $\tilde{\mathbf{S}}^i = \{(0,0);(1,0)\}$. The IC constraint in such a case is:

$$\Pr\left(\tilde{\mathbf{S}} = \{(0,0)\}\right)\left[\sum_{y_d}\left[w_{d2}\left[\frac{(k_2+2)}{2(k_2+3)} - \frac{(k_2+4)}{2(k_2+3)}\right]\right]\left[w_{d2}\left[\frac{(k_2+2)}{2(k_2+3)} - \frac{(k_2+4)}{2(k_2+3)} - 2b_d^i\right]\right]\right] +$$
$$\Pr\left(\tilde{\mathbf{S}} = \{(1,0)\}\right)\left[\sum_{y_d}\left[w_{d2}\left[\frac{(k_2+2)}{2(k_2+3)} - \frac{(k_2+4)}{2(k_2+3)}\right]\right]\left[w_{d2}\left[\frac{(k_2+2)}{2(k_2+3)} - \frac{(k_2+4)}{2(k_2+3)} - 2b_d^i\right]\right]\right] \geq C(S_2^i)$$

Given that $\Pr\left(\tilde{\mathbf{S}} = \{(0,0)\}\right) = \Pr\left(\tilde{\mathbf{S}} = \{(1,0)\}\right) = 1/4$, the IC constraint becomes.

$$\frac{1}{(k_2+3)}\left[\frac{(w_{12}^2 + w_{22}^2)}{2(k_2+3)} - \beta_2^i\right] \geq C(S_2^i)$$

That is, the expected utility gains of inducing the correct beliefs about θ_2 should be greater than the extra utility from saving in the costs of becoming informed about that state. It is easy to show that the case of $\hat{m}^i = \{\tilde{S}_1^i, 0\}$ has the sign of β_2^i reversed, for which the generic IC constraint for not acquiring signal S_r^i becomes:

$$\frac{1}{(k_r+3)}\left[\frac{(w_{1r}^2 + w_{2r}^2)}{2(k_r+3)} - |\beta_r^i|\right] \geq C(S_r^i) \tag{25}$$

For the deviation involving no information acquisition, $\hat{\mathfrak{s}}^i = \{\emptyset\}$, the expression of the IC constraint will depend on the message i decides to announce at the communication stage. On the one hand, when the message is $\hat{m)}^i = \{(0,0),(1,1)\}$, the induced beliefs will coincide with the equilibrium strategy when $\tilde{\mathbf{S}}^i = (1,1)$, but also partially for other realizations. Formally $\nu_r^{i*}(1,1) = \hat{\nu}_r^i(1,1)$ for $\theta_r = \{\theta_1, \theta_2\}$, $\nu_1^{i*}(1,0) = \hat{\nu}_1^i(1,1)$, and $\nu_2^{i*}(0,1) = \hat{\nu}_r^i(1,1)$. Following the characterization of equilibrium communication under centralization, the IC constraint becomes:

$$\left[\frac{1}{(k_1+3)}\left[\frac{(w_{11}^2 + w_{21}^2)}{2(k_1+3)} + \frac{(w_{11}w_{12} + w_{21}w_{22})}{(k_2+3)} - |\beta_1^i|\right] + \right.$$
$$\left. + \frac{1}{(k_2+3)}\left[\frac{(w_{12}^2 + w_{22}^2)}{2(k_2+3)} + \frac{(w_{11}w_{12} + w_{21}w_{22})}{(k_1+3)} - |\beta_2^i|\right]\right] \geq 2C(S_1^i, S_2^i) + \frac{(w_{11}w_{12} + w_{21}w_{22})}{(k_1+3)(k_2+3)} \tag{26}$$

Which basically is a more strict version of the IC constraint for full revelation when signals coincide (under centralization).

Similarly, when the deviation involves announcing $\hat{m)}^i = \{(0,1),(1,0)\}$ the IC constraint becomes:

$$\left[\frac{1}{(k_1+3)}\left[\frac{(w_{11}^2 + w_{21}^2)}{2(k_1+3)} - \frac{(w_{11}w_{12} + w_{21}w_{22})}{(k_2+3)} - |\beta_1^i|\right] + \right.$$
$$\left. + \frac{1}{(k_2+3)}\left[\frac{(w_{12}^2 + w_{22}^2)}{2(k_2+3)} - \frac{(w_{11}w_{12} + w_{21}w_{22})}{(k_1+3)} - |\beta_2^i|\right]\right] \geq 2C(S_1^i, S_2^i) + \frac{(w_{11}w_{12} + w_{21}w_{22})}{(k_1+3)(k_2+3)} \tag{27}$$

Agent i acquires one signal on path ($\mathfrak{s}^{i*} = \{S_1^i\}$).

When i acquires only one signal on path, his assessment of the consequences of any deviation still conditions on each possible pair of signal realizations. As I show in this section, this becomes particularly important for deviation involving acquisition of more signals. Not is necessary to distinguish between the induced beliefs on- and off-path, and the actual information i has access to. Thus, in addition to the previously defined $\nu_r^{i*}(\tilde{\mathbf{S}}^i)$ and $\hat{\nu}_r^i(\tilde{\mathbf{S}}^i)$, I now denote by $\nu_r^i(\tilde{\mathbf{S}}^i) = E\left(\theta_r|\tilde{\mathbf{S}}^i, \mathbf{m}^{\text{-}i*}\right)$ the beliefs about θ_r that would result from the decision maker observing the signals available to agent i (independent of his information acquisition strategy). Then, i's IC constraint at the information acquisition stage becomes:

$$\sum_{\tilde{\mathbf{S}}^i \in \mathcal{S}} \Pr(\tilde{\mathbf{S}}^i)\Bigg[-\sum_{y_d}\Big[w_{d1}\left(\nu_1^{i*}(\tilde{\mathbf{S}}^i)-\hat{\nu}_1^i(\tilde{\mathbf{S}}^i)\right)+w_{d2}\left(\nu_2^{i*}(\tilde{\mathbf{S}}^i)-\hat{\nu}_2^i(\tilde{\mathbf{S}}^i)\right)\Big]\times$$
$$\Big[w_{d1}\left(\nu_1^{i*}(\tilde{\mathbf{S}}^i)+\hat{\nu}_1^i(\tilde{\mathbf{S}}^i)-2\nu_1^i(\tilde{\mathbf{S}}^i)\right)+w_{d2}\left(\nu_2^{i*}(\tilde{\mathbf{S}}^i)+\hat{\nu}_2^i(\tilde{\mathbf{S}}^i)-2\nu_2^i(\tilde{\mathbf{S}}^i)\right)-2b_d^i\Big]\Bigg] \geq C(S_1^i)-C(\hat{\mathfrak{s}}^i)$$

When i considers the deviation of not acquiring any signals and decides to announce $\hat{m}_1^i = \{1\}$, he induces incorrect beliefs as compared to the equilibrium in two cases, namely $\tilde{\mathbf{S}} = \{(0,0);(0,1)\}$. The ex-ante expected utility losses of such strategy depends on the signal realizations, as can be noted in the expression for the IC constraint below:

$$\Pr\left(\tilde{\mathbf{S}}^i=(0,0)\right)\Bigg[-\sum_{y_d}\left[w_{d1}\left(\frac{(k_1+2)}{2(k_1+3)}-\frac{(k_1+4)}{2(k_1+3)}\right)\right]\times$$
$$\left[w_{d1}\left(\frac{(k_1+2)}{2(k_1+3)}+\frac{(k_1+4)}{2(k_1+3)}-\frac{2(k_1+2)}{2(k_1+3)}\right)+w_{d2}\left(\frac{1}{2}+\frac{1}{2}-\frac{2(k_2+2)}{2(k_2+3)}\right)-2b_d^i\right]\Bigg]+$$
$$\Pr\left(\tilde{\mathbf{S}}^i=(0,1)\right)\Bigg[-\sum_{y_d}\left[w_{d1}\left(\frac{(k_1+2)}{2(k_1+3)}-\frac{(k_1+4)}{2(k_1+3)}\right)\right]\times$$
$$\left[w_{d1}\left(\frac{(k_1+2)}{2(k_1+3)}+\frac{(k_1+4)}{2(k_1+3)}-\frac{2(k_1+2)}{2(k_1+3)}\right)+w_{d2}\left(\frac{1}{2}+\frac{1}{2}-\frac{2(k_2+4)}{2(k_2+3)}\right)-2b_d^i\right]\Bigg] \geq C(S_1^i)$$

Which, after some algebra gives:

$$\frac{1}{(k_1+3)}\left[\frac{(w_{11}^2+w_{21}^2)}{2(k_1+3)}-\beta_1^i\right] \geq C(S_1^i) \tag{28}$$

It is straightforward to note that the generic IC constraint involves the absolute value of β_r^i.

Deviations involving the acquisition of more information have the issue that i cannot signal this to the decision maker. This additional information will thus be used to identify situations (i.e. signal realizations) under which i will deliberately lie to the decision maker. Such deviations are thus related to the credibility loss, because i would like to induce beliefs about the signal he is not expected to acquire on path by means of messages on the signal he is believed on path.

As analysed in the communication game, the credibility loss takes place when signals do not coincide, $\tilde{\mathbf{S}}^i = \{(0,1);(1,0)\}$, so any deviation at the communication stage will take place in one of these cases. Moreover, given that β_1^i is typically not zero, i's incentives to lie will always be in a single direction, that is either when $\tilde{\mathbf{S}}^i = (0,1)$ or when $\tilde{\mathbf{S}}^i = (1,0)$ but not in both. The IC constraint for the deviation of acquiring both signals and announcing $\hat{m}_1^i = 0$ when $\tilde{\mathbf{S}}^i = (1,0)$ will be given by:

$$-\Pr\left(\tilde{\mathbf{S}}^i = (1,0)\right) \sum_{y_d} \left[\left[w_{d1} \left(\frac{(k_1+4)}{2(k_1+3)} - \frac{(k_1+2)}{2(k_1+3)} \right) \right] \times \right.$$
$$\left. \left[w_{d1} \left(\frac{(k_1+4)}{2(k_1+3)} + \frac{(k_1+2)}{2(k_1+3)} - \frac{(k_1+4)}{(k_1+3)} \right) + w_{d2} \left(1 - \frac{(k_2+2)}{(k_2+3)} \right) - 2b_d^i \right] \right] \geq C(S_2^i)$$

Which yields:

$$\left[\frac{(w_{11}^2 + w_{21}^2)}{2(k_1+3)^2} - \frac{(w_{11}w_{12} + w_{21}w_{22})}{2(k_1+3)(k_2+3)} - \beta_1^i \right] \geq -2C(S_2^i) \tag{29}$$

Which is equivalent to say that the cost of acquiring the second signal is too large with respect to the utility gain from deviating under ambiguous information.

Incentive compatibility then depends on $|\beta_1^i|$ being within the limits imposed by equations (10), (28), and (29). Note that equation (28) implies (10), meaning that if i is willing to acquire $\tilde{S}_1^i$ instead of acquiring no signal, then he will certainly reveal it. Incentive compatibility thus is captured by equations (28), and (29), which lead to:

$$\frac{|\beta_1^i|}{(k^{\mathrm{C}}+3)} \leq \min \left\{ \frac{(w_{11}^2 + w_{21}^2)}{2(k_1^{\mathrm{C}}+3)^2} - C(S_1^i)\,;\, \frac{(w_{11}^2 + w_{21}^2)}{2(k_1^{\mathrm{C}}+3)^2} - \frac{(w_{11}w_{12} + w_{21}w_{22})}{2(k_1+3)(k_2+3)} + 2C(S_2^i) \right\}$$

C.2 Information acquisition on the intensive margin

This subsection tries to convey a sense of how restrictive is the assumption that sender can acquire at most one signal associated to each state. Despite the question relates to the model with information acquisition (Section 4), throughout this analysis I will 'endow' agents with different amount of information and study their communication incentives. The analysis with information acquisition on the intensive margin, and its consequences for optimal organizational design will be subject of future work.

I focus in a one-decision, one-state cheap talk model between a sender and a receiver, much in the spirit of the uniform-quadratic example in Crawford and Sobel (1982). I perform two exercises. First, I compare the uniform-quadratic case with the beta-binomial model in which infinitely many senders with the same bias receive a binary signal. Since each of these virtual senders will truthfully reveal his signal when the expected precision of the decision is sufficiently low, the exercise allows me to understand how much the *receiver* gains from allowing a single sender to partition the message space. I show that the receiver strictly prefers to have a single, perfectly-informed sender than infinitely many of them having noisy information, for each possible bias.

In order to gain further intuition, in the second analysis I parametrize the sender's information as a finite number of iid binary trials correlated to the state. I borrow this information structure from Argenziano et al. (2016), but I extend their work by characterizing the maximum number of partitions as a function of the number of signals available to the sender (and his bias). I show the maximum number of partitions in any influential equilibria converges to the perfectly informed case in Crawford and Sobel (1982) when the number of binary experiments goes to infinity. So far I cannot compare the receiver's ex-ante expected utility under centralization and delegation as a function of the sender's information, due to not being able to find a closed form solution for the residual variance under communication.

Binary signals vs perfect information. Let $i = \{1, 2, ..., N\}$ be senders, each of which observes an iid signal $S^i \in \{0, 1\}$ correlated with the state $\theta \sim U[0, 1]$ such that $\Pr(S^i = 1) = \tilde{\theta}$. The receiver has to decide on $y \in \Re$ and her preferences are given by $U_R = -(y - \theta)^2$; while that for a typical sender is given by $U_i = -(y - \theta - b^i)^2$. Let assume that $b^i = b \in \Re_+$ for all $i = \{1, 2, ..., N\}$. As has been the case so far, the senders first observe their signals, send cheap talk messages to the receiver, and then she decides on y.

I denote by k the number of sender revealing truthfully their signals to the receivers, and ℓ those that report a 1. Following Galeotti et al. (2013), a sender reveals his signal truthfully to the receiver if and only if:

$$b \leq \frac{1}{2(k + 3)}$$

And the maximum number of senders willing to reveal their signals truthfully will be given by

$$k = \left\lfloor \frac{1}{2b} - 3 \right\rfloor$$

The receiver's ex-ante expected variance as a function of the number of agents truthfully revealing their binary signals is, then:

$$E[\text{Var}(\theta|\ell, k)] = \frac{1}{6(k + 2)} \simeq \frac{b}{3(1 - 2b)}$$

Now, the ex-ante expected variance when there is only one sender who perfectly observes θ and has the same bias b as before—the uniform-quadratic case in Crawford and Sobel (1982)— is given by:

$$E[\text{Var}(\theta|N(b))] = \frac{1}{12N^2} + \frac{b^2(N - 1)}{3}$$

Where the maximum number of partitions of the state space, $N(b)$, is given by:

$$N(b) = \left\lceil -\frac{1}{2} + \frac{1}{2}\left(1 + \frac{2}{b}\right)^{1/2} \right\rceil$$

Figure **??** below shows the comparison between the residual variance in both cases. The receiver derives higher ex-ante expected utility—expected variance is lower—when the sender is perfectly informed rather than when many senders fully reveal their information. The intuition behind this observation relates to the possibility of partitioning the state space when the sender is perfectly informed. In order to see that I now analyse a sender-receiver game in which the former observes k binary signals.