Information Aggregation in Multidimensional Cheap Talk*

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Abstract

I examine a cheap talk game with two decisions, two payoff-relevant states, and two senders. The model features interdependence because information about each state affects both decisions. Senders are imperfectly informed, and communication depends on the nature of their information. I first analyse the case in which each sender observes a signal that fully reveals one of the states. I show that communication depends on how the interdependence aggregates decision-specific biases. This aggregation can lead to positive or negative informational spillovers, which affect communication. Secondly, I analyse the case in which each sender observes noisy signals about both states. Because some realizations influence decisions in different directions, a sender is tempted to follow the most favourable of them. In equilibrium, this leads to a loss of credibility that harms communication. Finally, I show how this credibility loss leads to beneficial congestion effects.

Keywords: Information Economics, Industrial Organization, Cheap Talk, Multidimensional Communication.
JEL: D21, D83.

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1 Introduction

Complex problems are typically defined in terms of multiple determinants, and solutions comprise multiple courses of action. Each of these actions addresses the different causes with different degree of success, which in turn determines its effectiveness relative to other actions. Such multi-causal problems with multi-dimensional solutions are present in the implementation of public policies. Energy policy in developed countries, for instance, has two main determinants: the costs and stability of energy supply on the one hand, and its environmental impact on the other. Policy instruments broadly consist of governmental support for initiatives on renewable and non-renewable sources, along with measures to increase consumption efficiency.[1] From the policy-maker's perspective, the optimal amount of public funds on each source depends on how it addresses the different determinants, and their relative salience at any given point in time.[2] More often than not, relevant information about the determinants lies in hands of agents other than the policy-maker; interested political actors whose communication incentives depend on how policy decisions will be affected. This paper studies communication under informational interdependence—information mostly relevant for one decision also affects other decisions. Such interdependence arises in many real-world scenarios including policy making, product development in multinational corporations, and diplomatic negotiations.

Information transmission in such environments poses a challenge due to three main factors. First, payoff-relevant information is dispersed among many interested agents, rather than being directly accessible to the decision-maker. Second, much of the information agents observe is non-verifiable ("soft") and cannot be transmitted as hard facts, raising the issue of credibility. Third, different players can have conflicting preferences over decisions and, thus, communication is strategic. Informational interdependence means that information about any determinant affects multiple decisions; as a consequence, any agent's incentives for communication depend on how the interdependence aggregates his preferences over decisions.

Returning to the example on energy policy, relevant information is generally dispersed among government agencies, legislative committees, and special interest groups including environmental associations and the fossil fuels lobby. Each of these actors has some degree of access to policy-makers,[3] such that communication —say, in the form of assessments about the reliability of new technologies on renewables, or the risk of disruptions in future oil prices—is relatively easy, but providing verifiable

[1]According to a recent article in The Guardian, "[s]witching just some of the huge subsidies supporting fossil fuels to renewables would unleash a runaway clean energy revolution, [...] significantly cutting the carbon emissions that are driving the climate crisis" (Carrington, 2019).

[2]For instance, technological innovation increasing efficiency of renewables would increase subsidies on these sources in the short run, but in the long run it will also reduce incentives to invest in certain non-renewable sources. Indeed, the aforementioned article also observes that "[...]ending [fossil fuel subsidies] could cause short-term price rises and political difficulties, as the benefits of lower costs in the future and reduced air pollution are less obvious" (Carrington, 2019).

[3]See, for instance Grossman and Helpman (2001); Baumgartner and Jones (2009).

evidence is much harder. Political actors thus use that information strategically in their own interest; meaning that they have preferences over decisions and take into account how information is expected to affect each decision. Indeed, this interaction between preferences and interdependence can lead to 'informational spillovers.'

To illustrate these spillovers, consider the creation of the Department of Energy and Climate Change (DECC) by the British Government in October 2008. The creation of DECC involved merging energy divisions of the Department for Trade and Industry (DTI) with the Office for Climate Change from Environment Food and Rural Affairs (DEFRA); with the aim of 'bringing together energy and climate change policies to respond to the challenges of de-carbonising the economy and ensuring energy security' (White and Dunleavy, 2010). Despite the clear rationale, the two radically different halves of the new department faced both a cultural clash and strong conflict of interests. White and Dunleavy cite anecdotal evidence of such problems; for instance, the fact that DECC slogan was 'Committed to sustainability' affected credibility of former DTI staff with 'hard-headed business stakeholders' (White and Dunleavy, 2010, pp. 73).[4] The department was disbanded and its functions transferred to Business, Energy, and Industrial Strategy in July 2016, having showed little success regarding its policy goals.

The example illustrates how communication incentives and credibility change when the same information affects a large set of decisions. In the case of DECC, information transmission was negatively affected by informational interdependence—a situation I denote 'negative informational spillovers.' Similar effects have been analysed by Levy and Razin (2007), who find that such negative spillovers can impede communication when senders are imperfectly informed. But informational interdependence can lead to positive spillovers as well. Depending on how the interdependence aggregates decision-specific preferences, bundling decisions can enhance communication incentives as compared to separate single-decision problems. The intuition happens to be important on negotiations in international relations, and is illustrated by the notion that "an agreement leading to the peaceful resolution of an international crisis often becomes possible when an issue, not originally in contention, is brought into the bargaining for linkage purposes" (Morgan, 1990).[5]

This paper constructs a model of multidimensional decision-making under informational interdependence. A receiver faces a problem defined in terms of two state variables, and the available solution comprises two decisions. Calibration of each decision requires information about both states or, equivalently, information about any state affects both decisions. The receiver is in charge of decisions, while

[4] A former senior civil servant from the Cabinet Office declared that *"The downside [of the merge] is... that when you have two areas of policy with are so much in conflict, it's much better to externalise the argument than to internalise it.. And that's exactly what has happened [with DECC], nobody else gets a look in"* (White and Dunleavy, 2010).

[5] The literature on international relations has recently started to use cheap talk to model credibility in international negotiations (see Ramsay, 2011; Trager, 2011, 2017; Lindsey, 2017).

two senders observe private signals about the payoff-relevant states and can send cheap talk messages about each signal. Incentives for communication depend on how information affects decisions, meaning that informational interdependence aggregates decision-specific preferences. The model features linear interdependence, which allows me to capture in a parsimonious way the observation that information about any state affects both decisions. To isolate the effect of interdependence I assume preferences are additively separable on decisions. [6]

My main result is that, besides interdependence and decision-specific biases, communication depends on the nature of senders' information. By analysing different information structures I characterize two effects of informational interdependence. First, because information affects both decisions, communication depends on the *aggregate conflict of interest* between each sender and the receiver. This is the unique determinant of communication when each sender observes perfectly informative signals about one state and has no information about the other—i.e. senders are specialists. Such specialization guarantees that individual strategies do not depend on the other sender's strategy, due to the orthogonality of states. The problem is then similar to the uniform-quadratic case in Crawford and Sobel (1982), where the bias consists of the 'aggregate conflict of interest' given by interdependence. I exploit the apparent simplicity of each senders' incentive compatibility constraint to characterize informational spillovers.

Incentives for communication change significantly when senders have information about both states, leading to the second effect of interdependence on communication. Imperfect information makes individual message strategies dependent on each other,[7] but the novel intuition is given the fact that senders have information about both states. Senders may observe information that, if revealed, moves decisions in different directions. Fully revealing that information has a smaller overall influence than revealing each signal individually because signals compensate each other; that is, information is somehow 'ambiguous'. In such a case, the sender has more incentives to deviate from truth-telling when one of his signals favours his preferences and the other goes against them. In other words, he has incentives to convey the information carried by the most favourable signal. Having information about both states then leads to a *credibility loss* due to the possibility of ambiguous information.[8]

Both effects, aggregate conflict of interest and credibility loss, are non-monotonic in the degree of informational interdependence. Higher interdependence means that state-specific information affects

[6] Chapter 9 in Baumgartner and Jones (2009) provides an extensive discussion on how different issue areas are home to different configuration of policy biases. Returning to the energy policy example, pro-environment groups may have strong preferences for renewables and against fossil fuels, with little willingness to trade between these. Corporations related to fossil fuels have recently started to invest in some renewable sources because they consider them as complement of their main business (see, for instance, **?**).

[7] See Austen-Smith, 1990; Galeotti et al., 2013.

[8] Effects of information about multiple states have been analysed for different communication protocols (see Milgrom, 2008; Dziuda, 2011).

decisions more similarly. Decision-specific preferences thus have more similar weights on the relevant measure of conflict of interest. But as interdependence increases, incentives to manipulate ambiguous information also increase and, hence, the associated credibility loss. As a consequence, strategies involving ambiguous information become less credible and communication tends to concentrate on information that reflects that interdependence.

Finally, I study the possibility of beneficial congestion effects. In the canonical unidimensional problem with imperfectly informed senders, individual communication incentives are decreasing in the information transmitted by other senders on path (see Galeotti et al., 2013). When senders observe information about both states, however, more senders revealing information on path alleviates the credibility loss, which benefits communication in two cases. First, when a sender has extreme preferences about one decision and the other sender reveals more information about the associated state, the influence of information about both states is lower on the high-conflict decision—but still relevant for the low-conflict one— and the first sender is now willing to fully reveal his information. Second, when a sender is expected to reveal information about a single state and the second sender reveals more information about *the other state*, the first sender's opportunity cost of revealing favourable information decreases. The first sender is then more willing to reveal unfavourable information about the state he is believed on path. These results therefore show that congestion can have positive effects, an insight that may be relevant when thinking about the optimal composition of government cabinets, legislative committees, and board of directors in firms.

Related literature. The paper contributes to the theoretical literature on multidimensional cheap talk. When each sender observes both states perfectly, the receiver can extract all the information by restricting influence to the dimension of common interests (Battaglini, 2002).[9] In other words, the receiver (in equilibrium) can commit to ignore part of the information each sender provides because it will be provided by the other sender on path. Levy and Razin (2007) show that the receiver loses such (equilibrium) commitment power when senders are imperfectly informed and decisions are interdependent. Senders' incentives thus depend on how information affects both decisions, which can impede communication when the conflict of interest in one dimension is sufficiently large. My paper builds on Levy and Razin (2007) and presents a framework to fully characterize incentives for communication under linear interdependence.

Equilibrium communication in my model shares key elements with prior work on multidimensional cheap talk. The very notion of aggregate conflict of interest is somewhat similar to 'mutual discipline' in Goltsman and Pavlov (2011) since decision-specific conflict of interest compensate each other —i.e.

[9] Battaglini (2004) shows that this results is robust to imperfect signals when states are orthogonal. Ambrus and Takahashi (2008) show that it is not robust to restricted state spaces for large conflict of interests.

revealing information leads to utility gains in one dimension that compensate the utility losses in the other (see also Farrell and Gibbons, 1989; Chakraborty and Harbaugh, 2010). In addition, my framework features a class of strategies consisting in full revelation of some signal realizations and non-influential messages otherwise. A sender fully revealing ambiguous information (and nothing otherwise) is somewhat equivalent to providing rankings of the different attributes associated to decisions, as in Chakraborty and Harbaugh (2007).

The paper also contributes to the literature on organizational design. Strategic communication has important consequences on the organization of legislative institutions (Austen-Smith and Riker, 1987; Austen-Smith, 1990), decision-making cabinets (Dewan et al., 2015), and political parties (Dewan and Squintani, 2015). Most of the existing literature restricts attention to uni-dimensional decision problems. My focus on multi-causal problems is thus a step forward to understand incentives in such complex environments. A very similar notion of complexity has been used by Baumgartner and Jones (2009) to study the effects of both problem prioritization and information on agenda setting and institutional evolution.

The notion of interdependent decisions is also important among firms, but the focus of this strand of literature has been on the trade-off between coordination and adaptation (Alonso et al., 2008; Rantakari, 2008). Incentives for communication in such contexts are characterized by non-separability of preferences (need for coordination), senders' informational advantage (need for adaptation), and the difference in issue salience among players (biases). But the nature of firms may lead to different trade-offs. For instance, product design in multi-product firms relies on consumers' preferences over different attributes, with different products having different combinations of those attributes. Such firms face the need to gather information about consumer preferences, technological innovations, and government regulations related to each attribute, potentially leading to informational spillovers that will affect the organization of project development teams.[10] My framework captures these issues and can be used to study how organizational structures respond to the effects of informational interdependencies.

The rest of the paper proceeds as follows. Section 2 presents the basic set up. In section 3 I analyse the case of senders as specialists, characterize the equilibrium and define informational spillovers. Section 4 presents the analysis of senders as local representatives, shows the effects of interdependence in communication, and defines the cases in which congestion can be beneficial for the receiver. Finally, section 5 concludes.

[10] See, for example, Andersson and Forsgren (2000); Gassmann and von Zedtwitz (1999); Boutellier et al. (2008).

2 Baseline Model

A receiver has to decide on two issues, $\mathbf{y} = (y_1, y_2) \in \Re^2$, for which she needs information in hands of two senders. Each decision is affected by two states, θ_1, θ_2, such that information about any of them *affects both decisions.* If decisions were part of a public policy, states would represent policy goals and decisions would be the courses of actions available to address these goals. Each of these actions addresses the goals with different degrees of success and, thus, the receiver needs the information to calibrate them. I represent the (state-dependent) 'optimal calibration' of decisions as composite states $\delta = (\delta_1, \delta_2)$. Player j's payoff is thus defined in terms of decisions, composite states, and biases as follows (for $j = \{R, 1, 2\}$):

$$U^j(\mathbf{y}, \mathbf{b}^j) = -(y_1 - \delta_1 - b_1^j)^2 - (y_2 - \delta_2 - b_2^j)^2$$

The vector $\mathbf{b}^j = (b_1^j, b_2^j) \in \Re^2$ represents j's bias in the two dimensions. I normalize $\mathbf{b}^R = 0$, such that $\mathbf{b}^i$ represents the conflict of interest between sender i (he) and the receiver (she). Optimal actions depend on the realization of states, as follows:

$$\begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} \equiv \begin{bmatrix} w_{11}\, \theta_1 + w_{12}\, \theta_2 \\ w_{21}\, \theta_1 + w_{22}\, \theta_2 \end{bmatrix}$$

States are uniformly distributed in the unit square, $\theta_1, \theta_2 \sim U[0, 1]$. This assumption is not without loss of generality, but is one that has been used extensively in the cheap talk literature (e.g. uniform-quadratic case in Crawford and Sobel, 1982). The weighting matrix W characterizes the informational interdependence, where all weights are weakly positive and $w_{11}, w_{22} > 1/2$, so that the index corresponding to the state also reflects which decision that state is more important for. As a consequence of these normalizations, informational interdependence is linear and positive, the latter meaning that information about any state affects decisions in the same direction. In particular, informational interdependence is present despite states being themselves independent; the interdependence (given the normalizations) is characterized by:

$$\mathrm{Corr}(\delta_1, \delta_2) = \frac{(w_{11}\, w_{21}\ + w_{12}\, w_{22})}{\left[(w_{11}^2 + w_{12}^2)(w_{21}^2 + w_{22}^2)\right]^{\frac{1}{2}}}$$

The receiver obtains information about the policy determinants through private, cheap talk communication with the two senders. Each sender observes a signal associated to each state, $\mathbf{S}^i = (S_1^i, S_2^i) \in \mathcal{S}$, with their precision given by σ_1^i and σ_2^i, respectively. I analyse two different information structures. In the first, the signal and state spaces coincide, $\mathcal{S} = [0, 1]^2$ and, for each sender, one of the signals is perfectly informative while the other is a random draw. In particular, each sender observes a perfectly informative signal associated to a different state, so the senders are *specialist*. Which state each sender specialises on is common knowledge. This information structure has been used in the organizational economics literature (see Alonso et al., 2008, 2015; Rantakari, 2008). In section 3 I characterize how informational interdependence affects communication when senders are specialists.

Secondly, I analyse the case of senders observing noisy signals about both states. Formally, the signal space is given by $\mathcal{S} = \{0,1\}^2$ and the precision of a any given signal is such that $Pr(S_1^i = 1|\theta_1) = \theta_1$ and $Pr(S_2^i = 1|\theta_2) = \theta_2$. This information structure is commonly used to study policy debate (see Austen-Smith, 1990; Galeotti et al., 2013; Dewan et al., 2015) in which each legislator/committee member has information about the likely effects of decisions on his/her constituency. I thus refer to senders in this case as *local representatives*.

The game proceeds as follows: first, senders privately observe their information and send simultaneous cheap talk messages to the receiver; second, the receiver chooses $\mathbf{y} = (y_1, y_2)$ and payoffs realize. The message space under each information structure coincides with the signal space, that is $\mathbf{m}^i \in \mathcal{M} = \mathcal{S}$.[11] I focus on pure-strategy Perfect Bayesian Equilibria (PBE). Communication incentives will depend on the information each sender has and, in some cases, on the other sender's equilibrium message strategy. Denote by $\mathbf{m}^i(\mathbf{S}^i) \in M$ the message strategy of sender i, where $M \in \mathcal{S}$ represents the message space. A PBE of this game is characterized by a decision vector, $\mathbf{y}^*$, and a collection of message strategies, $\mathbf{m}^* = \{\mathbf{m}^{1*}, \mathbf{m}^{2*}\}$, such that:

- The receiver's decisions satisfy:

$$\mathbf{y}^* = \mathrm{W}\, E(\boldsymbol{\theta}|\mathbf{m}^*)$$

- Sender i's message strategy satisfies:

$$\mathbf{m}^{i*}\left(\mathbf{S}^i\right) \in \arg\max_{\mathbf{m}^i} \left\{ E\left[-\left(\mathbf{y}^*(\mathbf{m}^i, \mathbf{m}^{-i}) - \mathrm{W}\,\boldsymbol{\theta} - \mathbf{b}^i\right)' \left(\mathbf{y}^*(\mathbf{m}^i, \mathbf{m}^{-i}) - \mathrm{W}\,\boldsymbol{\theta} - \mathbf{b}^i\right) \left|\mathbf{S}^i\right.\right]\right\}$$

In the next section I analyse the case of senders as specialists.

3 Senders as Specialists

In this section each sender is perfectly informed about one of the states and observes no additional information about the other. This information structure is commonly used in applications of multidimensional cheap talk to organizations (Alonso et al., 2008; Rantakari, 2008). Additionally, this

[11] This assumption is without loss since the type of information each sender observes is common knowledge. In the case of binary signals, any message between 0 and 1 could, in principle, reflect a mixed strategy, but I restrict the analysis to pure strategies

information structure provides a useful benchmark to extend Levy and Razin (2007) characterization of informational spillovers and analyse its implications for communication incentives. It will also serve as benchmark to illustrate how incentives change when senders have information about both states (Section 4).

A a notational convention, let sender 1 receive a perfectly informative signal about θ_1 and sender 2 about θ_2. Each sender's problem becomes a Crawford-Sobel (CS) problem in which his message affects two decisions according to the interdependence structure. Following CS, let $a^i_{\tilde{r}}$ denote sender i's generic 'boundary type' $\tilde{\theta}_r$. Also, let $\bar{y}_d(a^i_{\tilde{r}}, a^i_{\tilde{r}+1}) = \frac{a^i_{\tilde{r}+1} - a^i_{\tilde{r}}}{2}$ be the expected action on dimension $d = \{1, 2\}$ when reporting on the upper interval, and similarly with respect to $a^i_{\tilde{r}-1}$.

Lemma 1. *Suppose sender $i \in \{1, 2\}$ observes a perfectly informative signal about θ_i. Sender i's equilibrium message strategy involves noisy messages unless there is no conflict of interest with the receiver. Moreover, the set of actions induced in equilibrium is finite.*

The incentive-compatibility constraint leads to the following arbitrage condition:

$$a^i_{\tilde{r}+1} = 2a^i_{\tilde{r}} - a^i_{\tilde{r}-1} + 4\frac{(b^i_1 w_{1r} + b^i_2 w_{2r})}{(w^2_{1r} + w^2_{2r})} \tag{A}$$

Proof. See Appendix A.1 □

The corollary below characterizes the more immediate effect of informational interdependence on communication incentives, which I denote by 'aggregate conflict of interest'.

Corollary 1. *Condition (A) coincides with the arbitrage condition in Crawford and Sobel (1982) when the bias equal to:*

$$B^i_r \equiv \frac{(b^i_1 w_{1r} + b^i_2 w_{2r})}{(w^2_{1r} + w^2_{2r})}$$

Information about θ_1 affects both decisions and the magnitude (and direction) of that influence is given by w_{11} and w_{12}. When there is no informational interdependence ($w_{11} = 1$ and $w_{12} = 0$), communication is characterized as in CS. As long as information about θ_1 affects y_2 in addition to y_1, incentives for communication must take into account the bias on both dimensions. The aggregate conflict of interest (B^i_r) can be decomposed into two elements. First, the interdependence aggregates decision-specific biases (b^i_1 and b^i_2). Because of linearity, the aggregate conflict of interest is a weighted average of the decision-specific biases (b^i_1, b^i_2), where the weights are given by the matrix $\mathbf{W}$. Second, $\mathbf{W}$ also governs the overall influence of the information at the sender's disposal. Changing the weights affects how much any message strategy reduces variance; in particular, as w_{1i} and w_{2i} become equal, the ability of sender i to influence the receiver's decisions decreases. As a consequence, communication is negatively affected. I now proceed to characterize equilibrium communication for sender i.

Proposition 1. *Suppose sender $i \in \{1,2\}$ observes a perfectly informative signal about θ_i. Any equilibrium message strategy consists of finite partitions of the state space unless $B_r^i = 0$. Moreover, the intervals are characterized by:*

$$a_{\tilde{r}}^i = a_1^i \tilde{r} + 2\tilde{r}(\tilde{r}-1)B_i^i$$

And the maximum number of intervals is given by:

$$N(B_i^i) = \left\lceil -\frac{1}{2} + \frac{1}{2}\left(1 + \frac{2}{B_i^i}\right)^{1/2} \right\rceil$$

Proof. Follows from Crawford and Sobel (1982) when $b = B_r^i$. □

Proposition 1 characterizes communication for each sender when they are specialists. The expressions concide with those in Crawford and Sobel (1982) when $b = B_r^i$. Note that the information at i's disposal is orthogonal to that of the other sender and, thus, he cannot infer the counterpart's message strategy (beyond the prior). This is why i's message strategy does not depend on the other sender's strategy.

Maximal Incentives for Communication. In order to get a graphical intuition of communication incentives and informational spillovers, I define the maximal incentives to reveal information as a function of the interdependence.

Definition 1. *Let $\lambda_r \equiv \left\{\mathbf{z} \in \Re^2 |\ \mathbf{z}'\,\mathbf{W}_r = 0\right\}$ be the locus with slope $-\frac{w_{1r}}{w_{2r}}$ related to θ_r. This locus represents the bias vectors for which incentives to reveal information about θ_r are maximal.*

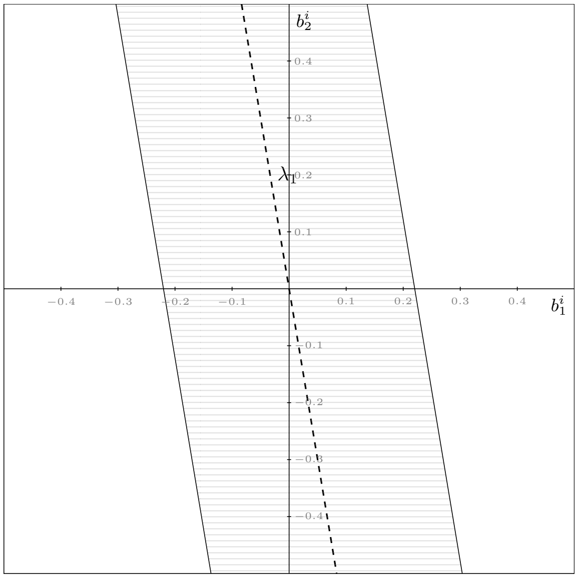
Note that $\mathbf{b}^i \in \lambda_i$ implies that $B_i^i = 0$, meaning that i fully reveals his information to the receiver. Definition 1 helps characterize the set of bias vectors for which there is information transmission (influential messages) from any individual sender. Lemma 1 and Proposition 1 imply that equilibria with influential messages exist only if $B_i^i \leq 1/4$, which can be re-expressed as:

$$||\mathbf{b}^i - \text{Proj}_{\lambda_i}(\mathbf{b}^i)|| \leq \frac{\left[(w_{1i})^2 + (w_{2i})^2\right]^{\frac{1}{2}}}{4}$$

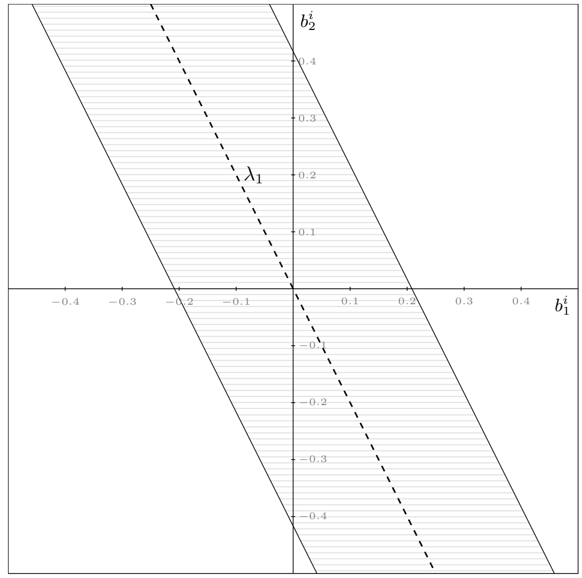
Here $\text{Proj}_{\lambda_r}(\mathbf{b}^i)$ is the projection of i's bias vector onto the locus λ_r.[12] Figure 1 shows the set of $\mathbf{b}^1$ for which an influential equilibrium exists. It also shows the maximal incentives to reveal information (λ_1) for different levels of informational interdependence. Note that informational interdependence allows some compensation among decision-specific biases. The overall width of the region for which

[12]More precisely, the LHS of the expression above constitutes the module of the rejection of $\mathbf{b}^i$ on λ_i.

Figure 1: Bias vectors for which an influential equilibrium for Sender 1 exists ($B_1 \leq 1/4$)



(a) $\mathrm{Corr}(\delta_1, \delta_2) = 0.3$



(b) $\mathrm{Corr}(\delta_1, \delta_2) = 0.8$

Notes: Shaded areas indicate bias vectors for which $B_1 \leq 1/4$.

$B_1 \leq 1/4$ is decreasing in interdependence, which reflecting the denominator of B_r^i: as interdependence increases, the overall influence of each sender is split between two decisions.

Informational Spillovers. Levy and Razin show how informational spillovers can impede communication when signals are imperfect. In their argument, large bias in one dimension is a sufficient condition for communication breakdown even when preferences of the sender and the receiver are perfectly aligned in the other dimension. Unlike the Fully Revealing Equilibrium in Battaglini (2002), the receiver cannot commit (in equilibrium) to ignore information across dimensions because senders are imperfectly informed. This communication breakdown constitutes a negative informational spillover. In the next lemma, I show that there can also be positive informational spillovers in my framework. I define these spillovers by comparing the two-dimensional decision problem with two separate unidimensional problems with the same preferences and information structure.

When senders are specialists, I can define informational spillovers by comparing the two-dimensional problem with those associated to the decisions for which each sender's information is more important. The lemma below characterizes i's incentives for communication in the one-dimensional problem for which his information has the highest influence ($w_{dr} = w_{ii} \geq 1/2$), and determines the necessary conditions for negative and positive spillovers.

Lemma 2. *When sender $i = \{1, 2\}$ observes θ_i, the one-dimensional problem associated to decision*

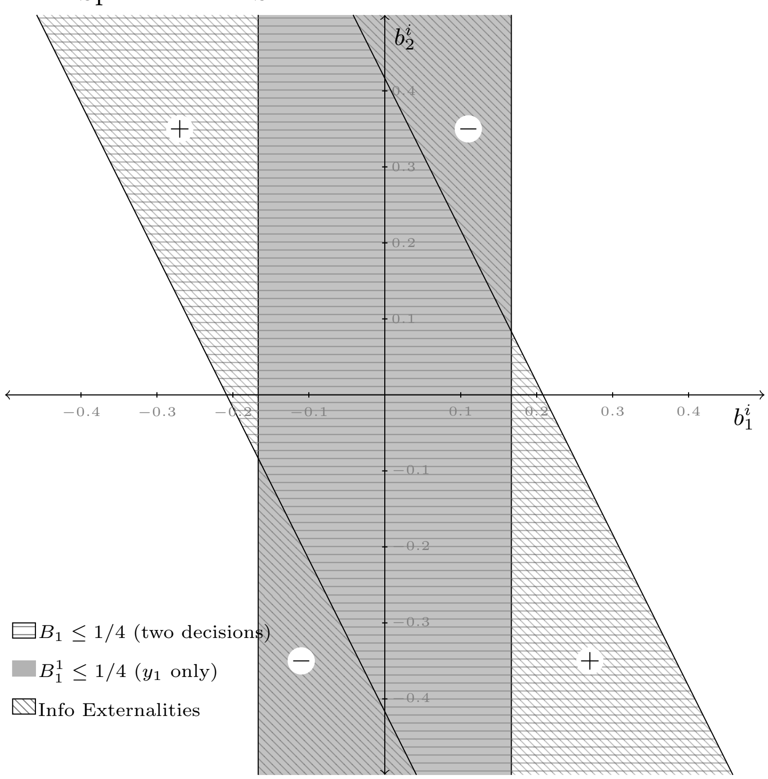
$d = \{1, 2\}$ *is characterized by:*

$$B^i_{di} \equiv \frac{b^i_d}{w_{di}}$$

Let $j = \{1, 2\}$ *denote the decision for which sender* i*'s information is less important* ($w_{ji} < 1/2$)*. A necessary condition for negative informational spillovers is* $B^i_{ii} < B^i_{ji}$*; whereas, a necessary condition for positive informational spillovers is* $B^i_{ii} > B^i_{ji}$*.*

Consider the case of sender 1, who is perfectly informed about θ_1. Negative spillovers affect him when there is no information transmission in the equilibrium of the two-dimensional problem, but he would have transmitted some information if y_1 were decided in isolation. Such negative effects increase when b^1_2 is large relative to how his information affects y_2 (w_{21}). Positive spillovers, on the other hand, arise when his incentives to reveal information are stronger in the two-decisions as compared to the one-decision problem. This takes place when b^1_2 is low relative to b^i_1. As a consequence, 'adding' this second decision dilutes the conflict of interest in the first dimension. Both type of spillovers are depicted in Figure 2, which superposes the set of $\mathbf{b}^1$ for which there exists information transmission in the single-decision problem (in grey), and in the two-decisions problem that I analysed throughout the section (horizontal stripes).

Figure 2: Informational Spillovers for Sender 1 – Two decisions vs one-dimensional problem (y_1)



Note: Corr$(\delta_1, \delta_2) = 0.8$. + (−) indicates positive (negative) Info Spillovers.

4 Senders have noisy information about both states

In this section, I study the case in which each sender observes two noisy signals, one associated to each state. This can be interpreted as the senders observing information from local constituencies in the policy-making application, or from different areas within the firm, or different subsidiaries in the multi-national corporations application. In terms of the model, each sender observes one binary signal associated to each state, $\{S_1^i, S_2^i\} \in \mathcal{S} = \{0,1\}^2$, such that $\Pr(S_1^i = 1) = \theta_1$ and $\Pr(S_2^i = 1) = \theta_2$. Recall that the message and signal spaces coincide: $(m_1^i, m_2^i) \in \mathcal{M} = \{0,1\}^2$. After messages have been sent simultaneously the receiver updates beliefs according to a Beta-binomial process. Let $k_r \leq 2$ be the number of senders truthfully revealing their signals and ℓ_r the number of those signals that equals 1, where $r = \{1,2\}$ indexes the states. Then, the updated expectations for each state are given by:[13]

$$E(\theta_1|k_1,\ell_1) = \frac{(\ell_1+1)}{(k_1+2)} \qquad\qquad E(\theta_2|k_2,\ell_2) = \frac{(\ell_2+1)}{(k_2+2)}$$

Let k_1^* and k_2^* denote the number of truthful messages the receiver has in equilibrium; also, let ℓ_1^* and ℓ_2^* denote the number of "ones" reported in equilibrium. From now on, k and ℓ (no superscript) denote i's *conjecture* about other senders revealing truthfully and reporting ones on the equilibrium path.[14]

Because of the possibility of multiple equilibria, I need to specify the equilibrium-selection criterion. I focus on the receiver-optimal equilibrium as most papers in the cheap talk literature. In the analysis of the previous section, this criterion chooses the equilibrium with the maximum number of partitions given B_r^i. In the current analysis it is defined as the message strategy that maximizes the receiver's ex-ante expected utility, since higher number of messages not always lead to better communication. For a given $\mathbf{m}^*$ the receiver's equilibrium actions are given by:

$$y_1^* = w_{11}\frac{(\ell_1^*+1)}{(k_1^*+2)} + w_{12}\frac{(\ell_2^*+1)}{(k_2^*+2)} \qquad\qquad y_2^* = w_{21}\frac{(\ell_1^*+1)}{(k_1^*+2)} + w_{22}\frac{(\ell_2^*+1)}{(k_2^*+2)}$$

The above expressions show how a sender's information, if revealed, affects the receiver's beliefs and decisions. Revealing one signal affects decisions according to the importance of the associated state, governed by $\mathbf{W}$. Just as in the specialists case, incentives for communication depend on the way

[13] The conditional pdf and the receiver's posterior on state θ_r are, respectively:

$$f(\ell_r|\theta_r,k_r) = \frac{k_r!}{\ell_r!(k_r-\ell_r)!}\theta_r^{\ell_r}(1-\theta_r)^{k_r-\ell_r} \qquad\qquad h(\theta_r|\ell_r,k_r) = \frac{(k_r+1)!}{\ell_r!(k_r-\ell_r)!}\theta_r^{\ell_r}(1-\theta_r)^{k_r-\ell_r}$$

[14] Sender i's conjecture will be correct on path, and whenever his equilibrium message strategy involves revealing the corresponding signal then $k_r^* = k_r + 1$, while $k_r^* = k_r$ otherwise.

the interdependence aggregates decision-specific biases. But having information about both states makes each sender's incentives depend on the information the other sender is expected to reveal in equilibrium.[15] Moreover, independence of states can produce signals that, if fully revealed would move decisions *against* the informational interdependence. The possibility of such information raises some additional incentives to deviate for each sender, leading to a loss of credibility that harms communication. This 'credibility loss' is another important effect of informational interdependence.

In order to build up intuition about equilibrium communication I start with the analysis of i's incentives to reveal one signal only and then derive incentives for full revelation. Finally, I show that equilibrium communication is not only based on revealing information on separate dimensions independently but there are equilibrium strategies in which the amount of information a sender transmits depends on his signals realizations. The section finalises with the characterization of the receiver-optimal communication equilibrium for sender i in Proposition 2.

Incentives to reveal one signal only. Revealing any single signal amounts to revealing information about one state only, which would move both decisions in the same direction due to the positive informational interdependence. As in the previous section, this means that sender i's incentives depend on the aggregate conflict of interest. But having information about both states alters these incentives: when i reveals information about one state only he cannot, by construction, transmit information about the other state. This is problematic when information about the latter state is more favourable than the associated to the state he is expected to reveal. In other words, i has higher incentives to lie when the signal he is expected to reveal moves decisions against his bias, while the other signal would move decision towards his bias if revealed. The possibility of such 'ambiguous information' harms credibility.

In order to see the effects of ambiguous information, consider the case in which i is expected to reveal information about θ_1 only, his biases are $b_1^i, b_2^i > 0$, and his signals $(\tilde{S}_1^i, \tilde{S}_2^i) = (0, 1)$. Note that revealing information about θ_1 moves both decisions against i's biases but, if he could reveal S_2^i, the influence would be smaller because it counteracts that of S_1^i. Sender i would then be better off if he could reveal both signals, which is not possible in the equilibrium under consideration. As a consequence, incentives for communication are negatively affected as compared to the case i observes information about one state only. Below I present the incentive compatibility (IC) constraint for this message strategy.

Lemma 3. *Consider an equilibrium* $(\mathbf{y}^*, \mathbf{m}^*)$ *in which* $\{S_r^i\} \in \mathbf{m}^{i*}$*. Revealing information about state*

[15] Indeed, this is caused by the fact that both senders observe information about the same states (see Krishna and Morgan, 2001) and that information is noisy (see Austen-Smith, 1993; Galeotti et al., 2013).

θ_r *is incentive compatible for sender* i *if:*

$$|B_r^i| \leq \frac{1}{2}\left[\frac{1}{(k_r+3)} - \frac{C_r}{(k_{\tilde{r}}+3)}\right] \tag{1}$$

where $C_r = \frac{(w_{11}w_{12}+w_{21}w_{22})}{(w_{1r}^2+w_{2r}^2)} \in [0,1]$*; such that when* $w_{11} = w_{22} = w$*, then* $C_1 = C_2 = Corr(\delta_1, \delta_2)$*.*

Equation (1) is i's IC constraint for revealing one signal only. His incentives for communication depends on the other sender's strategy, for two reasons. First, i's information is noisy and this imposes an upper bound on how much 'additional precision' he can induce by revealing his signal. Since both senders have binary signals, i can predict the marginal effect of revealing his signal on the corresponding posterior, captured by the presence of k_r in the IC constraint. [16] The first term in square brackets shows how i's incentives depend on whether the other sender (j) reveals information about the *same state*, which is similar to the incentives arising on unidimensional decision problems with imperfectly informed senders (see Austen-Smith, 1990; Galeotti et al., 2013).

The second effect shaping communication incentives relates to the fact that senders have information about both states. I have already described how i experiences increased incentives to lie when information about one state is favourable and that about the other is unfavourable. Recall that revealing information about one state moves decisions in the same direction (positive interdependence) and, thus, ambiguous information arises when signal realizations are $(S_1^i, S_2^i) = \{(0,1),(1,0)\}$. Indeed, i's incentives to lie in such cases are lower when the other sender is revealing information about the other state.

To see this consider the previous example in which i is expected to reveal information about θ_1 only, his biases are $b_1^i, b_2^i > 0$, and his signals are $(S_1^i, S_2^i) = (0,1)$. We have seen that i's impediment about revealing information on θ_2 in the equilibrium under consideration harms his incentives for communication, leading to a credibility loss. Now, when the other sender is not expected to reveal information about θ_2 on path, i knows that the overall influence of $S_1^i = 0$ on both decisions would be greatly compensated by $S_2^i = 1$ if he were believed on this signal too – this would be the receiver's only signal about θ_2. His incentives to lie on S_1^i are thus maximal in this case. On the contrary, if j is expected to reveal S_2^j (on path), i's marginal influence were he able to convey information about θ_2 would be smaller than the previous case. He thus has less incentives to lie, which reduces the credibility loss. In section 4.2 I show that this can lead to beneficial congestion.

Note also that the credibility loss is increasing in the degree of informational interdependence. The second term in square brackets in expression (1) is increasing in the informational interdependence,

[16] In particular, if the other sender is not expected to reveal S_r^j on path the set of possible posteriors (depending on i's information) is given by $E(\theta_r|m_1^i = S_1^i) = \{1/3, 2/3\}$; while if he is expected to reveal, the set becomes $E(\theta_r|m_1^i = S_1^i, m_1^j = S_1^j) = \{1/4, 1/2, 3/4\}$.

which is captured by C_r (for given k_1 and k_2).[17] Now, having characterized the incentives to reveal one signal and introduced the main effects of informational interdependence, I proceed to analyse incentives to reveal both signals.

Incentives to reveal both signals. The influence of i's signals on decisions depends on their realization; because of positive informational interdependence, $(S_1^i, S_2^i) = \{(0,0); (1,1)\}$ move decisions in the same direction, whereas $(S_1^i, S_2^i) = \{(0,1); (1,0)\}$ move decisions in opposite directions. As a consequence, i's incentives to fully reveal his signals will also depend on their realizations. The two cases just described determine both how b_1^i and b_2^i aggregate and the extent of the credibility loss. Below I present i's incentives to reveal both signals:

Lemma 4. *Consider the equilibrium $(\mathbf{y}^*, \mathbf{m}^*)$ in which $\{S_1^i, S_2^i\} \in \mathbf{m}^{i*}$. Sender i finds full revelation incentive compatibility if:*

- *Sender i does not lie on any signal individually; that is, for $\theta_r = \{\theta_1, \theta_2\}$:*

$$|B_r^i| \leq \frac{1}{2\,(k_r + 3)} \tag{2}$$

- *Sender i does not lie on both signals when $\mathbf{S}^i = \{(0,0); (1,1)\}$:*

$$0 \leq \frac{1}{(k_1+3)} \left[\frac{1}{(k_1+3)} + \frac{C_1}{(k_2+3)} \pm 2\,B_1^i \right] + \frac{1}{(k_2+3)} \left[\frac{1}{(k_2+3)} + \frac{C_2}{(k_1+3)} \pm 2\,B_2^i \right] \tag{3}$$

- *Sender i does not lie on both signals when $\mathbf{S}^i = \{(0,1); (1,0)\}$:*

$$0 \leq \frac{1}{(k_1+3)} \left[\frac{1}{(k_1+3)} - \frac{C_1}{(k_2+3)} \pm 2\,B_1^i \right] + \frac{1}{(k_2+3)} \left[\frac{1}{(k_2+3)} - \frac{C_2}{(k_1+3)} \mp 2\,B_2^i \right] \tag{4}$$

Proof. See Appendix B.3. □

The operators $\pm$ in (3) mean that the expression must hold when B_1^i and B_2^i are both added and when they are subtracted, keeping their signs. Incentive compatibility thus requires the RHS to be non-negative for both operations, which in turn indicates the way in which decision-specific biases aggregate. Similarly, the operators $\pm$ and $\mp$ in (4) mean that the expression must hold when B_1^i is added and B_2^i subtracted, and vice-versa. The latter means that this IC constraint is slack when decision-specific biases (b_1^i and b_2^i) have the same sign, as opposed to expressions (1), (2), and (3) that are slack when b_1^i and b_2^i have different signs.

[17] Alternatively, note that when $\mathrm{Corr}(\delta_1, \delta_2) = 0$, i's information about θ_2 does not affect y_1.

The IC constraints in Lemma 4 show that the influence of revealing both signals depends on both the interdependence and the signals' realizations. Fully revealing $(S^i_1, S^i_2) = (0, 0)$ or $(S^i_1, S^i_2) = (1, 1)$ move both decisions in the same direction. Revealing each signal is thus reinforcing the effect of revealing the other, which is represented in the terms C_1 and C_2 in (3). Terms in square brackets in this equation are related to the incentives to reveal each signal individually (see appendix B.2), in this case weighted by the equilibrium variance reduction from revealing each signal. To see this consider the case in which j is expected to reveal information about θ_1 but not on θ_2. On the equilibrium path i correctly conjectures j's strategy; then, i's influence from revealing both signals will be lower on y_1 than on y_2 because the receiver is expected to be better informed about θ_1 (from j equilibrium message strategy). As a consequence, i's IC constraints will weight B^i_2 more heavily than B^i_1. The following corollary summarizes this intuition:

Corollary 2. *When sender i reveals both signals, the larger (smaller) k_1 relative to k_2 the smaller (larger) the influence on y_1 relative to y_2.*

Incentives to fully reveal signals $(S^i_1, S^i_2) = \{(0, 1); (1, 0)\}$ are somewhat different; not only decision moves in different directions, but also the net influence on each decision is smaller than the influence of revealing any of the signals individually. Moving decisions in opposite directions leads to a different aggregation of decision-specific biases (for details see Appendix B.3). As a consequence, the RHS of equation (4) is less restrictive when when B^i_1 and B^i_2 have the same sign. The smaller overall influence makes this IC constraint relatively more restrictive as compared to the others in Lemma 4. Finally, Corollary 2 also applies to this IC constraint, meaning that the relative importance of each piece of information depends on whether the other sender is revealing the same information or not.

Dimensional non-separable message strategies. The equilibrium characterization in this game is not just based on communication on separate dimensions independently. In fact, it is possible that senders credibly transmit information for some signals realizations but not for others. This is the case when, for example, i's biases are $b^i_1 > 0$ and $b^i_2 < 0$, and they are large but similar in magnitude ($|b^i_1| \sim |b^i_2|$). If such a sender truthfully announces $\mathbf{m}^i = \{(1, 1)\}$, the receiver should believe him because his utility gains from increasing y_1 compensate the utility losses from decreasing y_2 —i.e. y_1 is moved towards b^i_1 while y_2 against b^i_2. But announcing $\mathbf{m}^i = \{\{(0, 1)\}\}$ or $\mathbf{m}^i = \{\{(1, 0)\}\}$ does not have the same compensation across dimensions; if these messages were believed, i would *always* announce $\mathbf{m}^i = \{(1, 0)\}$, which cannot be an equilibrium. As a result, a sender with such biases fully reveals signals that coincide and transmits no information when they do not (meaning that he plays the corresponding babbling strategy). I call such message strategies dimensional non-separable (DNS, henceforth). In Appendix C I show that the only strategies of this class arising in the receiver-optimal equilibrium has the structure of the example: full revelation for two combinations of signal realizations and babbling for the other two. The lemma below characterizes communication incentives when the message strategy includes both babbling and influential messages, for the class of DNS strategies arising in equilibrium:

Lemma 5. *Consider an equilibrium $(\mathbf{y}^*, \mathbf{m}^*)$ in which $\mathbf{m}^{i*}$ includes a babbling strategy and full revelation of some signal realizations. Then, full revelation is incentive compatible for sender i when:*

- *Not lying on both signals when $\mathbf{S}^i = \{(0,0);(1,1)\}$:*

$$\frac{1}{(k_1+3)}\left[\frac{1}{(k_1+3)}+\frac{C_1}{(k_2+3)}\pm 4\,B_1^i\right]+\frac{1}{(k_2+3)}\left[\frac{1}{(k_2+3)}+\frac{C_2}{(k_1+3)}\pm 4\,B_2^i\right]\geq 0 \tag{5}$$

- *Not lying on both signals when $\mathbf{S}^i = \{(0,1);(1,0)\}$:*

$$\frac{1}{(k_1+3)}\left[\frac{1}{(k_1+3)}-\frac{C_1}{(k_2+3)}\pm 4\,B_1^i\right]+\frac{1}{(k_2+3)}\left[\frac{1}{(k_2+3)}-\frac{C_2}{(k_1+3)}\mp 4\,B_2^i\right]\geq 0 \tag{6}$$

Proof. See Appendix B.4. □

The similarities between conditions in Lemmas 4 and 5 reflect the fact that the influential messages fully reveal the sender's information. The main difference between the two is how the aggregate conflict of interests weigh; DNS message strategies put more weight on B_1^i and B_2^i, meaning that conditions (5) and (6) hold for a smaller set of biases. This is due to the influence of deviations to non-influential messages. When any of these messages are announced the receiver does not update decisions, but if a sender with any of such signals realizations announces an influential message both decisions move away from the prior. This represent a deviation with a lower overall influence as compared to lying on both signals. Because of this lower influence, non-influential types[18] are more tempted to deviate and, thus, credibility becomes harder.

Having described incentive compatibility for the different communication strategies, I now characterize the receiver-optimal equilibrium of this game.

Proposition 2. *The receiver-optimal Perfect Bayesian Equilibrium for sender i consists of the following message strategies:*

1. ***Revealing both signals**, if $\mathbf{b}^i$ satisfies conditions in Lemma 4 with respect to both states.*
2. ***Revealing one signal only**, if $\mathbf{b}^i$ satisfies conditions in Lemma 3 with respect to one state only and does not satisfy those in Lemma 4.*

[18] A sender's type is given by his signals realization. By non-influential type I mean a sender whose signals lead to babbling message strategies on path.

3. ***Dimensional non-separable message strategies*** *in the following cases:*

   (a) *Fully revealing* $(S_1^i, S_2^i) = \{(0,0); (1,1)\}$ *if* $\mathbf{b}^i$ *satisfies condition* (5) *only;*

   (b) *Fully revealing* $(S_1^i, S_2^i) = \{(0,1); (1,0)\}$ *if* $\mathbf{b}^i$ *satisfies condition* (6) *only.*

4. ***No communication*** *(babbling strategy), if none of the above holds.*[19]

Proof. For the full characterization and proofs see Appendix C. □

First note that the set of strategies that constitute an equilibrium is a strict subset of the strategy space. Proposition 2 implies that equilibrium communication depends on the profile of biases and the informational interdependences. The system of beliefs that supports each strategy is characterized in Appendix C. Intuitively, when the strategy involves some sort of pooling between types (Parts 2 and 3), there are at least two messages that induce the same actions and, upon hearing any of such messages, the receiver assigns equal probability to the types involved. Note also that the non-influential messages in DNS strategies provide no information at all to the receiver. This is due to the fact that the signals realizations involved perfectly compensate each other (and the receiver puts equal probability on each pair). Figure 3 illustrates the set of i's biases for which the different strategies arise in equilibrium, for a given set of parameter values.

The Fully Separating Equilibrium arises only if the aggregate conflict of interest between i and the receiver is sufficiently small. The shape of the different correspondences reflects how each message strategy affects decisions. Recall that λ_1 and λ_2 represent the maximal incentives to reveal information about each state. Positive informational interdependence then leads to the negative slopes of λ_1 and λ_2[20] because incentives to reveal any single signal are greater when decision-specific biases have different signs. Condition (1) can thus be re-expressed as follows:

$$||\mathbf{b}^i - \text{Proj}_{\lambda_r}(\mathbf{b}^i)|| \leq \frac{[w_{1r}^2 + w_{2r}^2]^{\frac{1}{2}}}{2} \left[\frac{1}{(k_r+3)} - \frac{C_r}{(k_{\tilde{r}}+3)}\right]$$

This formulation allows the comparison with the specialist case. The main difference relates to the effects of ambiguous information on incentives for communication: the set of biases for which there is any information transmission is strictly smaller than when senders are specialists.[21]

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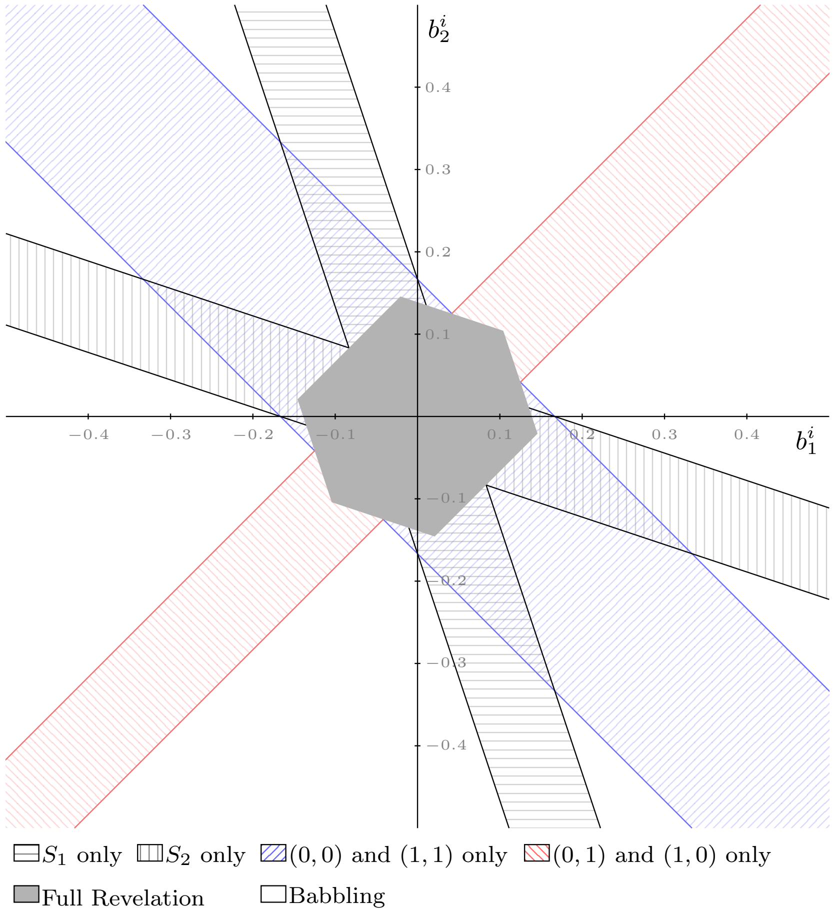
[19] Formally, the message strategy in (1) is $\mathbf{m}^i = \Big\{\{(0,0)\}; \{(0,1)\}; \{(1,0)\}; \{(1,1)\}\Big\}$; in (2) when revealing S_1^i is $\mathbf{m}^i = \Big\{\{(0,0); (0,1)\}; \{(1,0); (1,1)\}\Big\}$, and when revealing S_2^i is $\mathbf{m}^i = \Big\{\{(0,0); (1,0)\}; \{(0,1); (1,1)\}\Big\}$; in (3).$a$ is $\mathbf{m}^i = \Big\{\{(0,0)\}; \{(1,1)\}; \{(0,1); (1,0)\}\Big\}$, and in (3).$b$ is $\mathbf{m}^i = \Big\{\{(0,0); (1,1)\}; \{(0,1)\}; \{(1,0)\}\Big\}$; and in (4) is $\mathbf{m}^i = \Big\{\{(0,0); (1,1); (0,1); (1,0)\}\Big\}$.

[20] The different slopes relate to the fact that incentives to reveal information about θ_1 (θ_2) weighs more the bias on y_1 (y_2).

[21] Here I only refer to whether there is communication or not, leaving aside the *amount* of information transmitted in each case.

Figure 3: Equilibria in Proposition 2



Note: $w_{11} = w_{22} = 3/4$ ($\mathrm{Corr}(\delta_1, \delta_2) = 0.6$) and $k_1 = k_2 = 0$

The blue and red regions in Figure 3 correspond to DNS message strategies. Note that the blue region consists mainly of biases in the II and IV quadrants, meaning that b_1^i and b_2^i have different signs. When i's biases lie on this region and his signals are $(S_1^i, S_2^i) = \{(0,0); (1,1)\}$, fully revealing them moves both decisions in the same direction and, as a consequence, i's utility gains associated to one decision compensate the utility losses associated to the other. This interaction between interdependence, biases, and messages makes the aggregate conflict of interest relatively small, and the message credible. On the contrary, if the receiver were to believe any of the messages given by $\mathbf{m}^i = \{\{(0,1)\}; \{(1,0)\}\}$, i will announce the message that moves decisions according to the biases. Similar intuition applies when biases are in the red region (quadrants I and III), for full revelation of signals $(S_1^i, S_2^i) = \{(0,1); (1,0)\}$ —recall that in this case decisions move in different directions. The next section presents a formal discussion on the effects of informational interdependence on communication incentives.

4.1 The effect of informational interdependence on communication

In this section I analyse how incentives for communication change with informational interdependence captured by $\mathrm{Corr}(\delta_1, \delta_2) \geq 0$. Senders having information about both states led to two main effects on incentives, which I summarized as the 'aggregate conflict of interest' and the 'credibility loss' associated to ambiguous information.

To provide a more intuitive interpretation of the results, I assume symmetry on the decision-state weights; i.e. $w_{11} = w_{22} = w$ and $w > \frac{1}{2}$. I analyse how sender i's IC constraints change when w changes, for a given message strategy of the other sender (fixed k_1 and k_2). Note that a higher w means less informational interdependence; in the limit of $w = 1$, information about θ_1 (θ_2) only affects y_1 (y_2). I now show the effects of informational interdependence on communication incentives.

Proposition 3. *Let $1/2 \leq w \leq 1$. Increasing interdependence affects incentives for communication through two mechanisms: the aggregate conflict of interest and the credibility loss. In particular, the effect through the credibility loss associated to state $\theta_r = \{\theta_1, \theta_2\}$ is given by:*

$$CL_r = \frac{\partial C_r}{\partial w} < 0$$

The effect through the aggregate conflict of interest associated to state $\theta_r = \{\theta_1, \theta_2\}$ is given by:

$$ACI_r = \frac{\partial |B_r^i|}{\partial w}$$

Proof. For the complete characterization see Appendix D.1 □

There are two effects on communication incentives when interdependence increases. On the one hand, interdependence changes the extent to which the credibility loss affects communication incentives. Incentives for communication through this channel are affected negatively; that is, lower w increases the credibility loss. To see the intuition, consider the case in which i is expected to reveal information on θ_1. When interdependence increases, the opportunity costs of not revealing favourable information about θ_2 increases as well, because revealing that signal would have a relatively higher influence on both decisions. In the case of revealing both signals, the intuition relates to the relatively lower influence on the signal that favour i, such that announcing both signals are favourable would secure a much higher influence to agent i (if believed).

On the other hand, interdependence also affects the aggregate conflict of interest. The corollary below gives some intuitions on this channel.

Lemma 6. *A necessary condition for interdependence to reduce the aggregate conflict of interest is $sign(B_1^i) = sign(b_1^i - b_2^i)$, which implies that:*

1. *if* $sign(b_1^i) = sign(b_2^i)$*, the necessary condition becomes:* $|b_1^i| > |b_2^i|$*;*

2. *if* $sign(b_1^i) \neq sign(b_2^i)$*, the necessary condition becomes:* $w|b_1^i| > (1-w)|b_2^i|$*.*

Increasing interdependence reduces the aggregate conflict of interest when i's bias on the 'secondary decision' affected by θ_1 (b_2^i) is sufficiently small relative to b_1^i. In other words, information in hands of agent i has a higher influence on a decision of smaller conflict of interest between him and the principal.

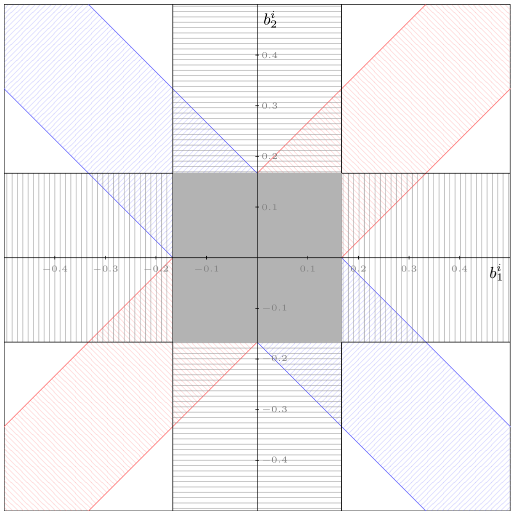
Figure 4 depicts the intuitions behind Proposition 3 and Lemma 6, showing three concurrent effects. The first of these consists in the rotation of λ_1 and λ_2 as a result of changes in the aggregate conflict of interest. Recall that the loci represent the maximal incentives to reveal information about θ_1 and θ_2 (respectively). Because higher interdependence means that information about θ_1 (θ_2) affects both decisions more similarly, then the weights of b_1^i and b_2^i on the corresponding IC constraint will be more similar; that is, λ_1 (λ_2) rotate towards the -45^o degree line.

The second effect relates to the credibility loss. In particular, stronger informational interdependence exacerbates the loss of credibility due to ambiguous information. Revealing one signal is thus incentive compatible for a smaller set of bias vectors, represented in the first term of (20) and as how the IC constraints for revealing one signal narrow in Figure 4. When information about θ_1 only affects y_1 ($w = 1$), i's incentives to reveal S_1^i are not affected by what he knows about θ_2 —the utility he derives from revealing S_1^i is orthogonal to that from revealing S_2^i. When there is informational interdependence ($w < 1$), though, signals are imperfect substitutes in terms of the utility gains from communication because both states affect both decisions; this generates incentives to misrepresent or manipulate information. In the limit when interdependence is perfect ($w = 1/2$), each state has the same influence on decisions and, thus, if i observes ambiguous information he would announce only the favourable signal.[22] This kills credibility of any message except for $\mathbf{m}^i = \{\{(0,0)\}; \{(1,1)\}\}$, unless decision-specific biases compensate perfectly, as panel (d) in Figure 4 shows.

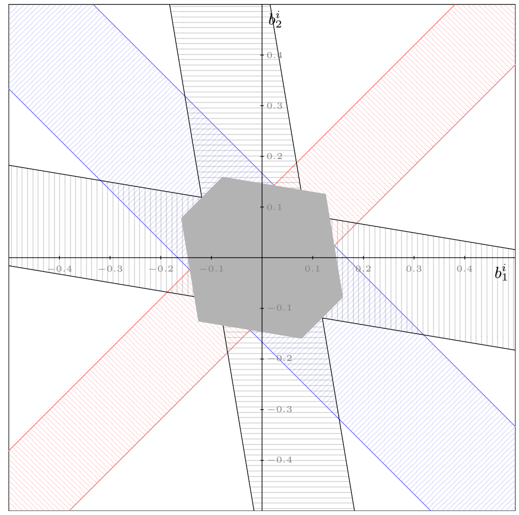
This effect on credibility is also present for the DNS strategy involving full revelation of ambiguous information. Recall that when i fully reveals $(S_1^i, S_2^i) = \{(0,1),(1,0)\}$ the overall influence on decisions is smaller than revealing each signal individually. Higher interdependence means that the overall influence becomes smaller, since they compensate the influence of the other on each decision This lower influence is equivalent to a lower opportunity costs of lying when i is believed to be fully revealing ambiguous information. Figure 4 shows this effect as the IC constraint for this strategy (in red) shrinks as Corr $= (\delta_1, \delta_2)$ increases. The effect tends to be weaker relative to revealing one signal only because it depends on the receiver's equilibrium information about both states (k_1 and k_2).

[22] The signal that moves decisions in the direction of his (aggregate) bias.

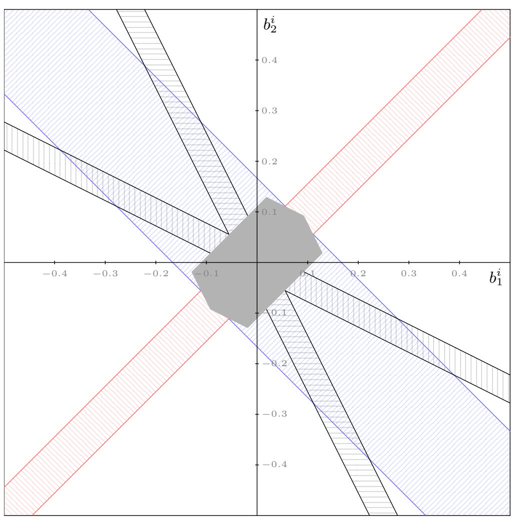
Figure 4: Equilibria in Proposition 2 for different levels of interdependence



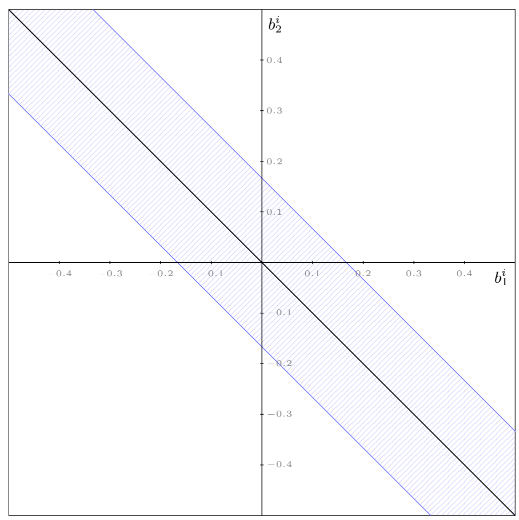
(a) Corr(δ_1, δ_2) = 0



(b) Corr(δ_1, δ_2) = 0.3



(c) Corr(δ_1, δ_2) = 0.8



(d) Corr(δ_1, δ_2) = 1

Notes: $k_1 = k_2 = 0$

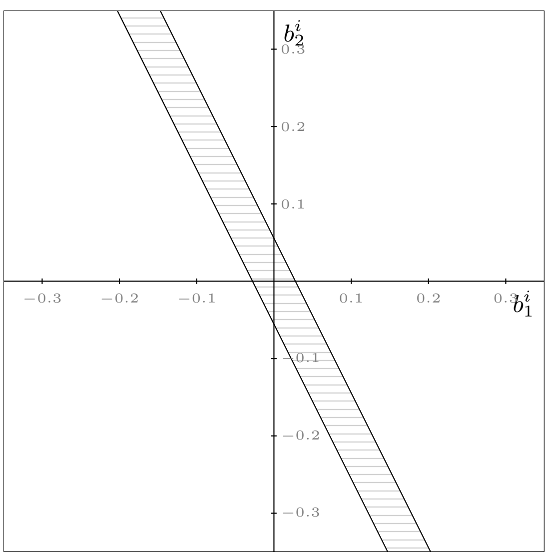
The third effect of interdependence relates to the aggregate conflict of interest on DNS strategies. As the last two terms in equations (21) and (22) show, the 'aggregate bias' effect depends on which signal the other sender is expected to reveal in equilibrium (k_1 and k_2). When i reveals both signals, the relative influence of each is lower when the other sender reveals the same information. For instance, if $k_1 > k_2$ the influence of S_2^i is relatively larger and full revelation will have a greater influence on y_2 from i's perspective; his IC constraint thus puts more weight on b_2^i. Any of the IC constraints involving DNS strategies would then be closer to the horizontal axis as compared to the case of $k_1 = k_2$. Finally, DNS strategy involving full revelation of $(S_1^i, S_2^i) = \{(0, 0); (1, 1)\}$ is not affected by the credibility effect. The influence of revealing each signal reinforces that of the other and, thus, i has little room for manoeuvre to manipulate his information. His incentives to deviate from truth-telling are driven only by his bias and not his information.

In the next section I explore the possibility of beneficial congestion, which is related to the role of k_1 and k_2 on i's incentives to fully reveal his signals (at least for some realizations).

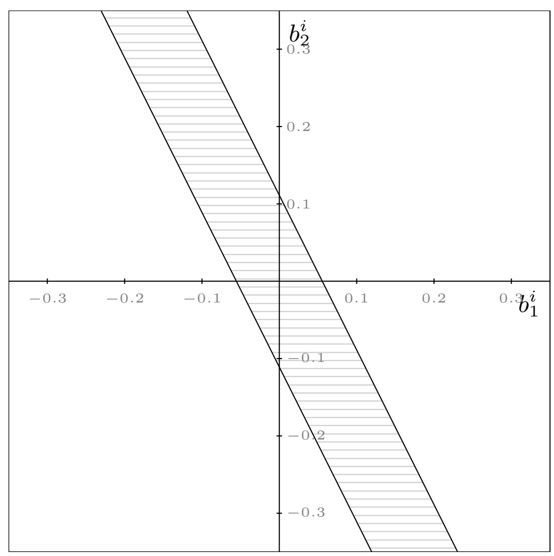
4.2 Beneficial Congestion Effects

When senders observe noisy signals about the same state their incentives for communication are interdependent: what each of them is willing to reveal depends on what the other is expected to reveal about the same state. This is the congestion effect in Galeotti et al. (2013), also captured in my framework captures by the fact that i's incentives to reveal S_1^i depends on k_1 –see equation (1). But in my framework there is the credibility loss due to the possibility of ambiguous information, for which i's incentives to reveal information about θ_1 also depend on the information the receiver gets about θ_2 on path.

Figure 5: Beneficial Congestion effect on incentives to reveal S_1



(a) $k_1 = 0,\ k_2 = 0$



(b) $k_1 = 0,\ k_2 = 1$

Note: $\mathrm{Corr}(\delta_1, \delta_2) = 0.8$.

When sender i expects the receiver to be well informed about θ_2, his information represents a relatively small proportion of what drives her decision. In the case of binary signals, in particular, i realizes that the receiver will be better informed than himself on path (about θ_2) and, thus, his incentives to lie on S_1^i under ambiguous information are smaller. This 'beneficial congestion' is reflected in the IC constraint for revealing S^i_r: it becomes looser as $k_{\tilde{r}}$ increases, as the figure below shows.

Beneficial congestion in the case of one signal is increasing in the interdependence, as shown in the following expression:

$$\frac{\partial IC_{(1)}}{\partial k_{\tilde{r}}} = \frac{C_r}{(k_{\tilde{r}}+3)^2}$$

Beneficial congestion also arises for DNS strategies, but the derivatives of the corresponding IC constraints are less intuitive. To show the result I then look at the set of preferences for which the beneficial congestion takes place. The proposition below describes the conditions under which increasing the number of senders revealing information about θ_1, k_1, induces i to reveal more information.

Proposition 4. *Let* $\mathbf{b} = \{\mathbf{b}^i, \mathbf{b}^j\}$ *and* $\tilde{\mathbf{b}} = \{\mathbf{b}^i, \tilde{\mathbf{b}}^j\}$ *be two collections of bias vectors, such that* $\mathbf{b}^j \neq \tilde{\mathbf{b}}^j$*. Let* $k_1, k_2 \in \{0,1\}$ *be the number other truthful signals the receiver has from other senders in the equilibrium under* $\mathbf{b}$*, and* $\tilde{k}_1 = k_1 + 1$ *and* $\tilde{k}_2 = k_2$ *be those for the equilibrium of the game under* $\tilde{\mathbf{b}}$*. For any* $1/2 < w < 1$*, if*

$$\frac{(k_2+3)}{(k_1+3)} < \min\left\{\frac{(k_1+4)}{(k_2+3)}\,;\,\frac{4\,w\,(1-w)}{[w^2+(1-w)^2]}\right\}$$

Then, there exist $\mathbf{b}^i \in \Re^2$ *such that i's equilibrium message strategies are:*

- *Revealing both signals when they coincide and nothing otherwise, when preferences are given by* $\mathbf{b}$*; and*
- *Revealing both signals, when preferences are given by* $\tilde{\mathbf{b}}$*.*

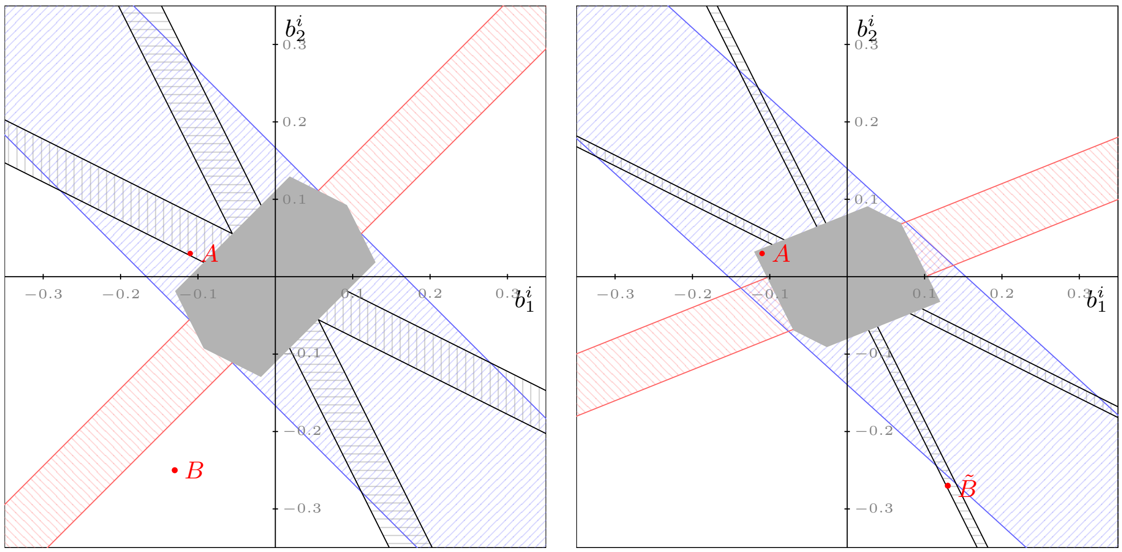
Proof. See Appendix D.2. □

Figure 6 presents a simple intuition of Proposition 4. There are two senders, A and B, each characterized by his bias vector; the figure represents A's IC constraints. I show how sender A's incentives change for different biases of sender B. The different biases will lead to different equilibrium message strategies for B, which change sender A incentives for communication. In the graph on the left, B's equilibrium message strategy is babbling, while A fully reveals when signals when they coincide and play babbling strategy otherwise.[23] Note that A's bias on the first dimension is relatively large, so if he happens to observe $(S^A_1, S^A_2) = (1,0)$ he prefers to announce $\mathbf{m}^A = \{(0,1)\}$ and that is why full revelation cannot be an equilibrium in this case.

Sender A's incentives change when B's preferences are as shown in $\tilde{B}$ in the right panel. Because $\tilde{B}$ is expected to reveal his information about θ_1 on path, A's influence from revealing both signals is smaller in y_1 relative to y_2. As a consequence, A is now willing to fully reveal if he observes $(S^A_1, S^A_2) =$

[23] Formally, $\mathbf{m}^A_{(a)} = \{\{(0,0)\};\{(1,1)\};\{(0,1);(1,0)\}\}$ and $\mathbf{m}^B = \{\{(0,0);(1,1);(0,1);(1,0)\}\}$.

Figure 6: Effect of increasing the receiver's equilibrium information under Proposition 4 on A's incentives



(a) $k_1 = 0,\ k_2 = 0$ (b) $k_1 = 1,\ k_2 = 0$

Note: $\mathrm{Corr}(\delta_1, \delta_2) = 0.8$. Both k_1 and k_2 are A's equilibrium conjectures.

$(1, 0)$ and his equilibrium message strategy is full revelation.[24] Thus, increasing the receiver's precision on y_2 is now more important for A than the cost associated to y_1.

In summary, increasing the amount of information about θ_1 on path (k_1) reduces the influence of revealing both signals on y_1 which, in this case, is the dimension of higher conflict of interest for A. On top of that, this effect is possible because neither b_1^i nor b_2^i are too large in magnitude and the latter is sufficiently close to zero. This is a novel result arising in the presence of informational interdependence, as modelled in my framework. One of its implications relates to the conformation executive cabinets or legislative committees, which have been among the most studied application of cheap talk models in communication (see Austen-Smith, 1990; Dewan and Hortala-Vallve, 2011; Dewan et al., 2015); but it could also be applied more broadly to organizational design (see Alonso et al., 2008, 2015; Rantakari, 2008).

5 Concluding Remarks

I studied cheap talk communication between two senders and a receiver who decides on two issues, where information relevant for one decision also affects the other. The paper analysed the effects of such informational interdependence on senders' incentives for communication. I characterized com-

[24]Numerically, preferences for E_1 are $b_1^1 = -0.11$ and $b_2^1 = -0.03$.

munication incentives when the informational interdependence is linear. I showed that such incentives depend on how the interdependence aggregates decision-specific biases, but also on the information the senders observe. In particular, when they observe noisy signals about both states there exist signals realizations that increase incentives to deviate from truth-telling, which leads to a credibility loss harming communication.

I analysed two different information structures. In the first senders are specialists; that is, each of them observes a perfectly informative signal about a different state. This information structure is widely used in applications of multidimensional cheap talk (see Alonso et al., 2008, 2015; Rantakari, 2008). Specialization and orthogonality of states make each sender's strategy independent of the other sender's; incentives for communication thus depend on the aggregate conflict of interest, and the characterization is similar to the uniform-quadratic case in Crawford and Sobel (1982). I also characterized informational spillovers by comparing the two-dimensional decision problem with the 'most relevant' unidimensional problem according to each sender's information. Negative informational spillovers arise when the introduction of a secondary decision reduces the amount of information transmitted in equilibrium, similar to Levy and Razin (2007). I showed that positive spillovers can also arise: the introduction of a secondary decision improves communication because it reduces the conflict of interest between the sender in question and the receiver.

The second information structure consists of each sender observing two noisy signals, one associated to each state. Each sender's incentives now depend on the other sender's equilibrium message strategy. On the one hand, each sender has less incentives to reveal information if the other is expected to reveal *the same information* (see Austen-Smith, 1990; Galeotti et al., 2013). On the other hand, given that states are orthogonal, a sender may be 'trapped' in an equilibrium in which he is expected to reveal an unfavourable signal but cannot credibly transmit information about a favourable one; the sender has incentive to lie on the former conveying information learned about the latter which leads to a credibility loss in equilibrium. As a consequence, incentives to reveal information about one state also depend on whether the other sender reveals information about the *the other state*. Incentives for communication depend overall on both the aggregate conflict of interest and the credibility loss due to the possibility of ambiguous information. Increasing interdependence exacerbates these two effects.

The equilibrium characterization in this framework is not only based on communication on separate dimensions. In particular, there are equilibrium message strategies in which a sender fully reveals some signals realizations and sends uninformative messages for other realizations. Whenever a sender is fully revealing his signals, the relative influence of each of them depends on whether the other sender reveals the same information. This can lead to congestion as in Galeotti et al. (2013), but can also reduce the harmful effect of ambiguous information on credibility. As a consequence of the latter, I have shown that congestion can be beneficial. A sender will fully reveal his signals when the receiver is expected to get more information about a controversial decision; in doing so the sender provides information useful for the low-conflict decision, given the reduced influence on the controversial decision.

Appendix A Senders as specialists

A.1 Proof of Lemma 1

Proof. The proof has two steps. First I show that all equilibrium message strategies involve finite partitions of the state space, which is related to Lemma 1 in Crawford and Sobel (1982). For this to be true, it has to be that $\frac{\partial^2 U^i}{\partial y_d^2} < 0$ and $\frac{\partial^2 U^i}{\partial y_d \partial \theta_r} > 0$, both of which are true given preferences are quadratic and additive separable. This implies that the equilibrium communication strategy consist in finite partitions of the state space, which induce a finite number of actions. Let denote by $a_0^i, a_1^i, ..., a_N^i$ the boundaries of this partitions.

For the second part, I work out the arbitrage condition for a generic boundary type. In order to keep notation simple, I assume that sender 1 observes the realization of θ_1 and sender 2 that of θ_2. Denote the posterior induced on θ_i after message m^i by $\nu^i = E(\theta_i|m^i)$. Then, sender i's incentives for communication will be given by:

$$\begin{aligned}
E\left[U^i|\theta_i,\nu_i^{i*}\right] &= -E\left[(y_1^* - w_{11}\theta_1 - w_{12}\theta_2 - b_1^i)^2 + (y_2^* - w_{21}\theta_1 - w_{22}\theta_2 - b_2^i)^2|\theta_i, m^{i*}\right] \\
&= -E\big[[w_{11}(\nu_1^{i*} - \theta_1) + w_{12}(\nu_2^{i*} - \theta_2)]^2 - 2b_1^i[w_{11}(\nu_1^{i*} - \theta_1) + w_{12}(\nu_2^{i*} - \theta_2)] + (b_1^i)^2 + \\
&\quad + [w_{21}(\nu_1^{i*} - \theta_1) + w_{22}(\nu_2^{i*} - \theta_2)]^2 - 2b_2^i[w_{21}(\nu_1^{i*} - \theta_1) + w_{12}(\nu_2^{i*} - \theta_2)] + (b_2^i)^2|\theta_i, m^{i*}\big]
\end{aligned}$$

From the last expression it is easily noted that:

$$[w_{d1}(\nu_1^{i*} - \theta_1) + w_{d2}(\nu_2^{i*} - \theta_2)]^2 = w_{d1}^2(\nu_1^{i*} - \theta_1)^2 + 2w_{d1}w_{d2}(\nu_1^{i*} - \theta_1)(\nu_2^{i*} - \theta_2) + w_{d2}^2(\nu_2^{i*} - \theta_2)^2$$

Leading to:

$$\begin{aligned}
E\left[U^i|\theta_i,\nu_i^{i*}\right] = -E\Big[&(\nu_1^{i*} - \theta_1)^2(w_{11}^2 + w_{21}^2) + 2(w_{11}w_{12} + w_{21}w_{22})(\nu_1^{i*} - \theta_1)(\nu_2^{i*} - \theta_2) + (\nu_2^{i*} - \theta_2)^2(w_{12}^2 + w_{22}^2) - \\
&- 2(\nu_1^{i*} - \theta_1)[b_1^i w_{11} + b_2^i w_{21}] - 2(\nu_2^{i*} - \theta_2)[b_1^i w_{12} + b_2^i w_{22}] + (b_1^i)^2 + (b_2^i)^2|\theta_i, m^{i*}\Big]
\end{aligned}$$

By independence of θ_1 and θ_2, $E\left[\nu_j^{i*}|\theta_i, m^{i*}\right] = E\left[\theta_j|\theta_i, m^{i*}\right] = E(\theta_j)$. Then, the IC constraint becomes:

$$\begin{aligned}
E\left[U^i|\theta_i,\nu_i^{i*}\right] = &- E\Big[(\nu_i^{i*} - \theta_i)^2(w_{1i}^2 + w_{2i}^2) - 2(\nu_i^{i*} - \theta_i)[b_1^i w_{1i} + b_2^i w_{2i}]|\theta_i, m^{i*}\Big] \\
&- (\nu_j^{i*} - \theta_j)^2(w_{1j}^2 + w_{2j}^2) + (b_1^i)^2 + (b_2^i)^2
\end{aligned} \tag{7}$$

Because of continuity of the state space, for any equilibrium partition there exist boundary types that would be indifferent between announcing two messages. More precisely, denote by a_n^i a generic boundary type for $n = \{0, 1, ..., N\}$ (equilibrium partitions of the state space). Then, by definition of boundary type:

$$E[U^i(m_n^i) - U^i(m_{n+1}^i)|a_n^i]$$

Now solving the above expression with (7) and noting that:

$$\nu_n^i = \frac{a_{n-1}^i + a_n^i}{2} \qquad \text{and} \qquad \nu_{n+1}^i = \frac{a_{n+1}^i + a_n^i}{2}$$

I get:

$$(a_{n-1}^i - a_{n+1}^i)\left[(w_{1i}^2 + w_{2i}^2)(a_{n+1}^i - 2a_n^i + a_{n-1}^i) - 4(b_1^i w_{1i} + b_2^i w_{2i})\right] = 0$$

Which holds if and only if:

$$a^i_{n+1} = 2a^i_n - a^i_{n-1} + 4\frac{(b^i_1 w_{1i} + b^i_2 w_{2i})}{(w^2_{1i} + w^2_{2i})}$$

The same Arbitrage condition as the uniform-quadratic case in Crawford and Sobel (1982), with the conflict of interest between sender i and the receiver defined by:

$$B^i \equiv \frac{(b^i_1 w_{1i} + b^i_2 w_{2i})}{(w^2_{1i} + w^2_{2i})}$$

□

Appendix B Incentives for Communication

B.1 Derivation of IC Constraints

Suppose that the decision-maker holds k^*_r signals about one of the state, for $r = \{1,2\}$. Let ℓ^*_r denote the number of such signals that equal 1; then the conditional pdf is:

$$f(\ell^*_r|\theta_r, k^*_r) = \frac{k^*_r!}{\ell^*_r!(k^*_r - \ell^*_r)!}\theta_r^{\ell^*_r}(1-\theta_r)^{k^*_r - \ell^*_r}$$

And her posterior is:

$$h(\theta_r|\ell^*_r, k^*_r) = \frac{(k^*_r + 1)!}{\ell^*_r!(k^*_r - \ell^*_r)!}\theta_r^{\ell^*_r}(1-\theta_r)^{k^*_r - \ell^*_r}$$

Consequently:

$$E(\theta_r|\ell^*_r, k^*_r) = \frac{(\ell^*_r + 1)}{(k^*_r + 2)}$$
$$\mathrm{Var}(\theta_r|\ell^*_r, k^*_r) = \frac{(\ell^*_r + 1)(k^*_r - \ell^*_r + 1)}{(k^*_r + 2)^2(k^*_r + 3)}$$

For $r = \{1,2\}$.

Proof. Recall that $\mathbf{S}^i$ is the vector of signals actually received by sender i and $\mathbf{m}^*$ is the one containing equilibrium message strategies for all senders. Now, let $\mathbf{m}^{i*}$ denote sender i's pure message strategy in equilibrium, and $\hat{\mathbf{m}}^i$ the deviation under consideration. In addition, denote by $y_d(\mathbf{m}^i, \mathbf{m}^{-i})$ the action the receiver would take in dimension $d = \{1,2\}$, when she receives $\mathbf{m}^{-i}$ from players other than i, and i is influential under both $\mathbf{m}^{i*}$ and $\hat{\mathbf{m}}^i$. Given that i takes other senders' equilibrium strategies as given, and in order to simplify notation let be $y_d(\mathbf{m}^{i*}(\mathbf{S}^i), \mathbf{m}^{-i}) = y_d(\mathbf{m}^{i*})$ and $y_d(\hat{\mathbf{m}}^i(\mathbf{S}^i), \mathbf{m}^{-i}) = y_d(\hat{\mathbf{m}}^i)$

Then, i reports $\mathbf{m}^{i*}$ instead of $\hat{\mathbf{m}}^i$ if and only if:

$$-\int_0^1\int_0^1\Bigg[\Big[\left(\delta_1 + b^i_1 - y_1(\mathbf{m}^{i*})\right)^2 + \left(\delta_2 + b^i_2 - y_2(\mathbf{m}^{i*})\right)^2\Big] - \\ - \Big[\left(\delta_1 + b^i_1 - y_1(\hat{\mathbf{m}}^i)\right)^2 + \left(\delta_2 + b^i_2 - y_2(\hat{\mathbf{m}}^i)\right)^2\Big]\Bigg] f(\theta_1, \mathbf{m}^{-i}|\mathbf{S}^i)\, f(\theta_2, \mathbf{m}^{-i}|\mathbf{S}^i)\, d\theta_1\, d\theta_2 \geq 0$$

Using the identity $a^2 - b^2 = (a+b)(a-b)$, and by noting that $f(\theta_1, \mathbf{m}^{-i}|\mathbf{S}^i) = f(\theta_1|\mathbf{m}^{-i}, S_1^i)\, P(\mathbf{m}^{-i}|S_1^i)$ and that $f(\theta_2, \mathbf{m}^{-i}|\mathbf{S}^i) = f(\theta_2|\mathbf{m}^{-i}, S_2^i)\, P(\mathbf{m}^{-i}|S_2^i)$ I get:

$$
\begin{aligned}
-\int_0^1 \int_0^1 \Bigg[& \left[\left(y_1(\mathbf{m}^{i*}) + y_1(\hat{\mathbf{m}}^i)\right) - 2(\delta_1 + b_1^i)\right] \left(y_1(\mathbf{m}^{i*}) - y_1(\hat{\mathbf{m}}^i)\right) + \\
& + \left[\left(y_2(\mathbf{m}^{i*}) + y_2(\hat{\mathbf{m}}^i)\right) - 2(\delta_2 + b_2^i)\right] \left(y_2(\mathbf{m}^{i*}) - y_2(\hat{\mathbf{m}}^i)\right) \Bigg] \\
& f(\theta_1|\mathbf{m}^{-i}, S_1)\, f(\theta_2|\mathbf{m}^{-i}, S_2)\, P(\mathbf{m}^{-i}|S_1^i)\, P(\mathbf{m}^{-i}|S_2^i)\, d\theta_1\, d\theta_2 \geq 0
\end{aligned}
$$

Next, observing that:

$$
\begin{aligned}
y_1(\mathbf{m}^i, \mathbf{m}^{-i}) = E(\delta_1|\mathbf{m}^i, \mathbf{m}^{-i}) \\
y_2(\mathbf{m}^i, \mathbf{m}^{-i}) = E(\delta_2|\mathbf{m}^i, \mathbf{m}^{-i})
\end{aligned}
$$

Then, rearranging and denoting by:

$$
\begin{aligned}
\Delta(\delta_1) = E(\delta_1|\mathbf{m}^{i*}, \mathbf{m}^{-i}) - E(\delta_1|\hat{\mathbf{m}}^i, \mathbf{m}^{-i}) \\
\Delta(\delta_2) = E(\delta_2|\mathbf{m}^{i*}, \mathbf{m}^{-i}) - E(\delta_2|\hat{\mathbf{m}}^i, \mathbf{m}^{-i})
\end{aligned}
$$

I get:

$$
\begin{aligned}
-\int_0^1 \int_0^1 \Bigg[& \Big[\frac{E(\delta_1|\mathbf{m}^{i*}, \mathbf{m}^{-i}) + E(\delta_1|\hat{\mathbf{m}}^i, \mathbf{m}^{-i})}{2} - \delta_1 - b_1^i\Big] \left[\Delta(\delta_1)\right] + \\
& + \Big[\frac{E(\delta_2|\mathbf{m}^{i*}, \mathbf{m}^{-i}) + E(\delta_2|\hat{\mathbf{m}}^i, \mathbf{m}^{-i})}{2} - \delta_2 - b_2^i\Big] \left[\Delta(\delta_2)\right] \Bigg] \\
& f(\theta_1|\mathbf{m}^{-i}, S_1^i)\, f(\theta_2|\mathbf{m}^{-i}, S_2^i)\, P(\mathbf{m}^{-i}|S_1^i)\, P(\mathbf{m}^{-i}|S_2^i)\, d\theta_1\, d\theta_2 \geq 0
\end{aligned} \tag{8}
$$

Given that the equilibrium message strategies for players other than i, $\mathbf{m}^{-i}$, are independent of i's actual signal realizations, the expressions $P(\mathbf{m}^{-i}|S_1^i)$ and $P(\mathbf{m}^{-i}|S_2^i)$ can be taken out the double-integral. Then, noting that:

$$
\begin{aligned}
\int_0^1 \int_0^1 \delta_1\, f(\theta_1|S_1, \mathbf{m}^{-i})\, f(\theta_2|S_2, \mathbf{m}^{-i})\, P(\mathbf{m}^{-i}|S_1)\, P(\mathbf{m}^{-i}|S_2)\, d\theta_1\, d\theta_2 &= E\left(\delta_1|S_1, S_2, \mathbf{m}^{-i}\right) \\
&= E\left(w_{11}\, \theta_1 + w_{12}\, \theta_2 \middle| S_1, S_2, \mathbf{m}^{-i}\right) \\
&= w_{11} E(\theta_1|S_1, \mathbf{m}^{-i}) + w_{12} E(\theta_2|S_2, \mathbf{m}^{-i})
\end{aligned}
$$

and equivalently for δ_2.

Note that $y_d(\mathbf{m}^i, \mathbf{m}^{-i}) = w_{d1} E(\theta_1|\mathbf{m}^i, \mathbf{m}^{-i}) + w_{d2} E(\theta_2|\mathbf{m}^i, \mathbf{m}^{-i})$ for any $\mathbf{m}^i = \{\mathbf{m}^{i*}, \hat{\mathbf{m}}^i\}$. Let define the receiver's updated beliefs with respect to δ_d from i's perspective as:

$$
\begin{aligned}
\nu_d^{i*} =& E(\delta_d|\mathbf{m}^{i*}) \\
\hat{\nu}_d^i =& E(\delta_d|\hat{\mathbf{m}}^i) \\
\nu_d^i =& E(\delta_d|\mathbf{S}^i)
\end{aligned}
$$

And let $\nu_{1r}^{i*} = \nu_{2r}^{i*} = E(\theta_r|\mathbf{m}^{i*})$, $\hat{\nu}_{1r}^i = \hat{\nu}_{2r}^i = E(\theta_r|\hat{\mathbf{m}}^i)$, and $\nu_{1r}^i = \nu_{2r}^i = E(\theta_r|S_r^i)$ the posteriors associated to θ_r, respectively. Then, equation (8) becomes:

$$-\left[\left[(\nu_1^{i*}+\hat{\nu}_1^i)-2(\nu_1^i-b_1^i)\right]\Delta(\delta_1)+\left[(\nu_2^{i*}+\hat{\nu}_2^i)-2(\nu_2^i-b_2^i)\right]\Delta(\delta_2)\right]P(\mathbf{m}^{\text{-}i}|S_1)\,P(\mathbf{m}^{\text{-}i}|S_2)\geq 0 \quad (9)$$

Note that when either ν_d^{i*} or $\hat{\nu}_d^i$ coincide with i's posterior about y_d (ν_d^i), then the expression reduces to:

$$-\left[\left[-\frac{\Delta(\delta_1)}{2}-b_1^i\right]\Delta(\delta_1)+\left[-\frac{\Delta(\delta_2)}{2}-b_2^i\right]\Delta(\delta_2)\right]P(\mathbf{m}^{-i}|S_1)\,P(\mathbf{m}^{-i}|S_2)\geq 0 \quad (10)$$

But the message strategy may consist of revealing S_r^i only, in which case $\nu_{\tilde{r}}^{i*}=\hat{\nu}_{\tilde{r}}^i\neq\nu_{\tilde{r}}^i$, being $\theta_{\tilde{r}}$ the state for which the sender *does not reveal* any information. ☐

In order to facilitate future derivations, I proceed to derive the expected influence of each sender type if believed to be telling the truth. Let $m^i\in\mathbf{m^i}$ denote a particular message from the message strategy i plays.

Lemma 7. *Suppose that m^i is taken at face value by the receiver, then the expected induced decision (from i's perspective) is given by.*

- *When revealing both signals:*

$$\begin{aligned}
E\left(\delta_d|m^i=\{(0,0)\},\mathbf{m}^{\text{-}i}\right)=&w_{d1}\frac{(k_1+2)}{2(k_1+3)}+w_{d2}\frac{(k_2+2)}{2(k_2+3)}\\
E\left(\delta_d|m^i=\{(1,1)\},\mathbf{m}^{\text{-}i}\right)=&w_{d1}\frac{(k_1+4)}{2(k_1+3)}+w_{d2}\frac{(k_2+4)}{2(k_2+3)}\\
E\left(\delta_d|m^i=\{(0,1)\},\mathbf{m}^{\text{-}i}\right)=&w_{d1}\frac{(k_1+2)}{2(k_1+3)}+w_{d2}\frac{(k_2+4)}{2(k_2+3)}\\
E\left(\delta_d|m^i=\{(1,0)\},\mathbf{m}^{\text{-}i}\right)=&w_{d1}\frac{(k_1+4)}{2(k_1+3)}+w_{d2}\frac{(k_2+2)}{2(k_2+3)}
\end{aligned}$$

- *When revealing one signal only:*

$$\begin{aligned}
E\left(\delta_d|m^i=\{(0,0);(0,1)\},\mathbf{m}^{\text{-}i}\right)=&w_{d1}\frac{(k_1+2)}{2(k_1+3)}+\frac{w_{d2}}{2}\\
E\left(\delta_d|m^i=\{(1,0);(1,1)\},\mathbf{m}^{\text{-}i}\right)=&w_{d1}\frac{(k_1+4)}{2(k_1+3)}+\frac{w_{d2}}{2}\\
E\left(\delta_d|m^i=\{(0,0);(1,0)\},\mathbf{m}^{\text{-}i}\right)=&\frac{w_{d1}}{2}+w_{d2}\frac{(k_2+2)}{2(k_2+3)}\\
E\left(\delta_d|m^i=\{(0,1);(1,1)\},\mathbf{m}^{\text{-}i}\right)=&\frac{w_{d1}}{2}+w_{d2}\frac{(k_2+4)}{2(k_2+3)}
\end{aligned}$$

Now suppose sender i is not believed to be telling the truth, then his expected induced decision is given by:

$$E\left(\delta_d|\mathbf{m}^{\text{-}i}\right)=\frac{w_{d1}}{2}+\frac{w_{d2}}{2}$$

For $d=\{1,2\}$ indicating the corresponding decision.

Proof of Lemma 7 .

From equation (8) it is true that:

$$E\left(\delta_d | m^i = \{S_1^i, S_2^i\}, \mathbf{m}^{\text{-}i}\right) = w_{d1} E\left(\theta_1 | S_1^i, \mathbf{m}^{\text{-}i}\right) + w_{d2} E\left(\theta_2 | S_2^i, \mathbf{m}^{\text{-}i}\right)$$

Working out the expectation for each state conditional on the signal realization gives the following (for $\theta_r = \{\theta_1, \theta_2\}$, and $S_r^i = 0$):

$$E\left(\theta_r | S_r^i = 0, \mathbf{m}^{\text{-}i}\right) = \frac{(\ell_r + 1)}{(k_r + 3)}$$

By the Law of Iterated Expectations, recalling that $f(\ell_r|\theta_r, k_r) = h(\theta_r|\ell_r, k_r)/(k_r + 1)$, I get:

$$\begin{aligned} E\left(\theta_r | S_r^i = 0, \mathbf{m}^{\text{-}i}\right) &= E\left[E\left(\theta_r | \ell_r, S_r^i = 0, \mathbf{m}^{\text{-}i}\right)\right] \\ &= \frac{1}{(k_r+3)} \sum_{\ell_r = 0}^{k_r} \frac{(\ell_r + 1)}{(k_r + 1)} \\ &= \frac{(k_r+2)}{2(k_r+3)} \end{aligned}$$

And now the expectation conditional on $S_r^i = 1$:

$$\begin{aligned} E\left[E\left(\ell_r, \theta_r | S_r^i = 1, \mathbf{m}^{\text{-}i}\right)\right] &= \frac{1}{(k_r+3)} \sum_{\ell_r = 0}^{k_r} \frac{(\ell_r + 2)}{(k_r + 1)} \\ &= \frac{(k_r+4)}{2(k_r+3)} \end{aligned}$$

Finally, working out the expectations for both states according to the signals realizations corresponding to each type I get the expressions in Lemma 7. □

B.2 Incentives to reveal one signal

Proof of Lemma 3 .

Consider the interim equilibrium in which i is believed to be telling the truth with respect to one signal. The equivalent of equation (9) in terms of the posteriors on states if given by:

$$\sum_{y_d = \{y_1, y_2\}} - \left[w_{d1}(\nu_{d1}^{i*} - \hat{\nu}_{d1}^i) + w_{d2}(\nu_{d2}^{i*} - \hat{\nu}_{d2}^i)\right] \left[w_{d1}(\nu_{d1}^{i*} + \hat{\nu}_{d1}^i - 2\nu_{d1}^i) + w_{d2}(\nu_{d2}^{i*} + \hat{\nu}_{d2}^i - 2\nu_{d2}^i) - 2b_d^i\right] \geq 0 \tag{11}$$

Consider the case in which i reveals S_1^i only. This implies that $\nu_{11}^{i*} = \nu_{21}^{i*} = \nu_1^i$, and $\nu_{12}^{i*} = \nu_{22}^{i*} = 1/2$ (see last equation in Lemma 7). The above IC constraint then becomes:

$$\sum_{y_d = \{y_1, y_2\}} - \left[w_{d1}(\nu_{d1}^{i*} - \hat{\nu}_{d1}^i)\right] \left[-w_{d1}(\nu_{d1}^{i*} - \hat{\nu}_{d1}^i) - 2w_{d2}\left(\nu_{d2}^i - \frac{1}{2}\right) - 2b_d^i\right] \geq 0$$

Now I analyse for each possible pair of signal realizations.

1. $\mathbf{S^i} = \{(0,0)\}$, then according to Lemma 9:

$$\begin{aligned}\nu_{d1}^i =& \nu_{d1}^{i*} = \frac{(k_1+2)}{2(k_1+3)} \\ \hat{\nu}_{d1}^i =& \frac{(k_1+4)}{2(k_1+3)} \\ \nu_{d2}^i =& \nu_{d1}^i\end{aligned}$$

And the IC constraint (11) becomes:

$$\sum_{y_d=\{y_1,y_2\}} -\left[w_{d1}\frac{(-1)}{(k_1+3)}\right]\left[-\frac{w_{d1}}{2}\frac{(-1)}{(k_1+3)} - \frac{w_{d2}}{2}\frac{(-1)}{(k_2+3)} - b_d^i\right] \geq 0$$

Operating the sum I get:

$$w_{11}b_1^i + w_{12}b_2^i \leq \frac{(w_{11}^2+w_{12}^2)}{2(k_1+3)} + \frac{(w_{11}w_{12}+w_{21}w_{22})}{2(k_2+3)}$$

Multiplying both sides by $(w_{11}^2+w_{12}^2)^{-1}$ and defining $C_1 = \frac{(w_{11}w_{12}+w_{21}w_{22})}{(w_{11}^2+w_{12}^2)}$ I get:

$$B_1^i \leq \frac{1}{2}\left[\frac{1}{(k_1+3)} + \frac{C_1}{(k_2+3)}\right]$$

2. $\mathbf{S^i} = \{(1,1)\}$, then according to Lemma 9:

$$\begin{aligned}\nu_{d1}^i =& \nu_{d1}^{i*} = \frac{(k_1+4)}{2(k_1+3)} \\ \hat{\nu}_{d1}^i =& \frac{(k_1+2)}{2(k_1+3)} \\ \nu_{d2}^i =& \nu_{d1}^i\end{aligned}$$

Following the same steps as above it can be shown to lead to:

$$-B_1^i \leq \frac{1}{2}\left[\frac{1}{(k_1+3)} + \frac{C_1}{(k_2+3)}\right]$$

3. $\mathbf{S^i} = \{(1,0)\}$, then according to Lemma 9:

$$\begin{aligned}\nu_{d1}^i =& \nu_{d1}^{i*} = \frac{(k_1+4)}{2(k_1+3)} \\ \hat{\nu}_{d1}^i =& \frac{(k_1+2)}{2(k_1+3)} \\ \nu_{d2}^i =& \frac{(k_2+2)}{2(k_2+3)}\end{aligned}$$

The IC constraint becomes:

$$\sum_{y_d=\{y_1,y_2\}} -\left[w_{d1}\frac{(1)}{(k_1+3)}\right]\left[\frac{w_{d1}}{2}\frac{(1)}{(k_1+3)}+\frac{w_{d2}}{2}\frac{(-1)}{(k_2+3)}-b_d^i\right]\geq 0$$

Which, following the same steps leads to

$$-B_1^i \leq \frac{1}{2}\left[\frac{1}{(k_1+3)}-\frac{C_1}{(k_2+3)}\right]$$

4. $\mathbf{S^i} = \{(0,1)\}$, then according to Lemma 9:

$$\begin{aligned}\nu_{d1}^i =& \nu_{d1}^{i*} = \frac{(k_1+2)}{2(k_1+3)}\\ \hat{\nu}_{d1}^i =& \frac{(k_1+4)}{2(k_1+3)}\\ \nu_{d2}^i =& \frac{(k_2+4)}{2(k_2+3)}\end{aligned}$$

Which, following the same steps leads to

$$B_1^i \leq \frac{1}{2}\left[\frac{1}{(k_1+3)}-\frac{C_1}{(k_2+3)}\right]$$

Because the four conditions apply to the same bias, the more restrictive of them (lower RHS) constitutes necessary and sufficient conditions. In addition, because the LHS must hold for both positive and negative B_1^i irrespective of the signs of b_1^i and b_2^i, then it is equivalent to take its absolute value.

Finally, the proof for incentives to reveal S_2^i is equivalent. □

B.3 Incentives to lie on both signals

Proof of Lemma 4.

In this case the receiver takes i's messages at face value, while the deviation consists in lying on both signals simultaneously. Recall that when i reveals all his information, then $\nu_{dr}^{i*} = \nu_{dr}^i$ for both decisions and states $-y_d = \{y_1.y_2\}$ and $\theta_r = \{\theta_1.\theta_2\}$. Then, this message strategy will be incentive compatible if equation (10) holds, for which the expressions for $\Delta(\delta_d)$ can be computed with results in Lemma 7. Let $\beta_r^i = w_{1r}b_1^i + w_{2r}b_2^i$ ($\beta_r^i = B_r^i\,(w_{1r}^2+w_{2r}^2)$), then:

(a). For type $(0,0)$ not announcing $(1,1)$:

$$\frac{\beta_1^i}{(k_1+3)}+\frac{\beta_2^i}{(k_2+3)}\leq\frac{1}{2}\left[\frac{(w_{11}^2+w_{21}^2)}{(k_1+3)^2}+\frac{(w_{12}^2+w_{22}^2)}{(k_2+3)^2}+\frac{2[w_{11}w_{12}+w_{21}w_{22}]}{(k_1+3)(k_2+3)}\right]$$

(b). For type $(0,1)$ not pooling to $(1,0)$:

$$\frac{\beta_1^i}{(k_1+3)}-\frac{\beta_2^i}{(k_2+3)}\leq\frac{1}{2}\left[\frac{(w_{11}^2+w_{21}^2)}{(k_1+3)^2}+\frac{(w_{12}^2+w_{22}^2)}{(k_2+3)^2}-\frac{2[w_{11}w_{12}+w_{21}w_{22}]}{(k_1+3)(k_2+3)}\right]$$

No deviation for type $(1,1)$ requires a condition similar to (a) with the signs on the LHS changed, while no deviation for $(1,0)$ requires the same as (b) with the corresponding change in signs in the LHS. Note that (a) and that for $(1,1)$ imply equation (3) and the same for (c) and that for $(1,0)$ with equation (4). □

B.4 Incentive to play a babbling strategy when revealing both signals

Proof of Lemma 5.

In a similar fashion than the previous Lemma, the expected marginal influence in each decision for a type that fully reveals when other types are playing babbling strategies in equilibrium is given by Lemma 7 in the following way:

$$\Delta(\delta_d)_{(S_1^i,S_2^i)} = E\left[E(\delta_d|\ell_1,\ell_2,m^i=\{S_1^i,S_2^i\},\mathbf{m}^{\text{-}i})\right] - E\left[E(\delta_d|\ell_1,\ell_2,\mathbf{m}^{\text{-}i})\right]$$

Then, for each type:

$$\Delta(\delta_d)_{(0,0)} = -\frac{w_{d1}}{2(k_1+3)} - \frac{w_{d2}}{2(k_2+3)} \qquad \Delta(\delta_d)_{(1,1)} = \frac{w_{d1}}{2(k_1+3)} + \frac{w_{d2}}{2(k_2+3)}$$
$$\Delta(\delta_d)_{(0,1)} = -\frac{w_{d1}}{2(k_1+3)} + \frac{w_{d2}}{2(k_2+3)} \qquad \Delta(\delta_d)_{(1,0)} = \frac{w_{d1}}{2(k_1+3)} - \frac{w_{d2}}{2(k_2+3)}$$

Plugging each of these expression into the IC constraint defined in (9) and working each of them out as in the previous section I get:

(a). For type $(0,0)$:

$$\frac{\beta_1^i}{(k_1+3)} + \frac{\beta_2^i}{(k_2+3)} \leq \frac{1}{4}\left[\frac{(w_{11}^2+w_{21}^2)}{(k_1+3)^2} + \frac{(w_{12}^2+w_{22}^2)}{(k_2+3)^2} + \frac{2[w_{11}w_{12}+w_{21}w_{22}]}{(k_1+3)(k_2+3)}\right]$$

(b). For type $(0,1)$:

$$\frac{\beta_1^i}{(k_1+3)} - \frac{\beta_2^i}{(k_2+3)} \leq \frac{1}{4}\left[\frac{(w_{11}^2+w_{21}^2)}{(k_1+3)^2} + \frac{(w_{12}^2+w_{22}^2)}{(k_2+3)^2} - \frac{2[w_{11}w_{12}+w_{21}w_{22}]}{(k_1+3)(k_2+3)}\right]$$

And similarly for types $(1,1)$ –part (a)– and $(1,0)$ –part (b). Each of the above corresponds to the incentives for the *influential type* to play his equilibrium strategy. Now, for the *babbling types* in each case, the expected influence of no announcing the influential type will be the opposite to that of the corresponding influential type; that is:

$$\Delta(\delta_d)_{\text{babbling}} = -\Delta(\delta_d)_{(S_1^i,S_2^i)}$$

Which can be proved to lead to the same IC above. Finally, pairing the corresponding IC constraints in each case leads to the corresponding conditions. □

Appendix C Characterization of the Most Informative Equilibrium

Proposition 5. *The strategy profile* $(\mathbf{y}^*,\mathbf{m}^*)$ *constitutes the receiver-optimal equilibrium when for every possible message strategy from sender i to the decision-maker, then i:*

1. ***Reveals both signals*** *if and only if condition* (2) *holds for both signals, and* (4) *hold. The message strategy in this case is given by:*
$\mathbf{m}^i = \Big\{\{(0,0)\};\{(0,1)\};\{(1,0)\};\{(1,1)\}\Big\}$.

2. ***Reveals*** S_r ***only:***

   (a) *if* (1) *holds for* S_r^i *but not for* $S_{\tilde{r}}^i$

   (b) *if* (1) *and* (5) *hold together, then:*

$$\frac{[(w_{11})^2 + (w_{21})^2]}{(k_1+2)(k_1+3)} > \frac{[(w_{12})^2 + (w_{22})^2]}{(k_2+2)(k_2+3)} \tag{12}$$

3. ***Reveals both signals when is of type*** $(0,0)$ ***or*** $(1,1)$ ***and no information otherwise:***
$\mathbf{m}^i = \Big\{\{(0,0)\};\{(1,1)\};\{(0,1);(1,0)\}\Big\}$

   (a) *if condition* (5) *holds but* (1) *does not; or*

   (b) *if* (5) *and* (1) *hold, and condition* (12) *does not.*

4. ***Reveals both signals when is of type*** $(0,1)$ ***and*** $(1,0)$ ***and no information otherwise*** *if and only if condition* (6) *holds and* (1) *does not. The associated message strategy is:*
$\mathbf{m}^i = \Big\{\{(0,0);(1,1)\};\{(0,1)\};\{(1,0)\}\Big\}$.

5. ***Reveals no information*** *(babbling strategy) if and only if none of the previous applies. That is:* $\mathbf{m}^i = \Big\{\{(0,0);(1,1);(0,1);(1,0)\}\Big\}$.

Proof of Proposition 5.

This proof consists on two steps: in the first I construct the equilibrium for each of the message strategies in Proposition 2, while the second shows that each message strategy constitutes the receiver-optimal equilibrium (from the receiver's perspective) for the set of preferences it applies.

C.1 Equilibrium Construction

Part 1. Sender i revealing both signals truthfully constitutes the *Fully Separating* equilibrium. Thus, for b_1^i and b_2^i satisfying (2) for both signals, (3), and (4), then the following system of beliefs:

$$\mu^*\big((0,0)|m^i = \{(0,0)\}\big) = 1 \qquad \mu^*\big((1,0)|m^i = \{(1,0)\}\big) = 1$$
$$\mu^*\big((0,1)|m^i = \{(0,1)\}\big) = 1 \qquad \mu^*\big((1,1)|m^i = \{(1,1)\}\big) = 1$$

And the following equilibrium actions:

$$y_d^*\left(m^i = \{(0,0)\}, \mathbf{m}^{-i}\right) = w_{d1}\frac{(\ell_1+1)}{(k_1+3)} + w_{d2}\frac{(\ell_2+1)}{(k_2+3)} \qquad y_d^*\left(m^i = \{(1,0)\}, \mathbf{m}^{-i}\right) = w_{d1}\frac{(\ell_1+2)}{(k_1+3)} + w_{d2}\frac{(\ell_2+1)}{(k_2+3)}$$
$$y_d^*\left(m^i = \{(0,1)\}, \mathbf{m}^{-i}\right) = w_{d1}\frac{(\ell_1+1)}{(k_1+3)} + w_{d2}\frac{(\ell_2+2)}{(k_2+3)} \qquad y_d^*\left(m^i = \{(1,1)\}, \mathbf{m}^{-i}\right) = w_{d1}\frac{(\ell_1+2)}{(k_1+3)} + w_{d2}\frac{(\ell_2+2)}{(k_2+3)}$$

For $d = \{1,2\}$.

Are consistent with sender i revealing both signals truthfully, that is $\mathbf{m}^{i*} = \{(0,0);(0,1);(1,0);(1,1)\}$

Part 2. For sender i's biases that satisfy (1) for S_1 only, consider the following equilibrium beliefs for the receiver:

$$\mu^*\left((0,0)|\mathbf{m}^i=(0,0)\right)=\mu^*\left((0,1)|\mathbf{m}^i=(0,0)\right)=\frac{1}{2} \qquad \mu^*\left((0,1)|\mathbf{m}^i=(0,1)\right)=\mu^*\left((0,0)|\mathbf{m}^i=(0,1)\right)=\frac{1}{2}$$
$$\mu^*\left((1,0)|\mathbf{m}^i=(1,0)\right)=\mu^*\left((1,1)|\mathbf{m}^i=(1,0)\right)=\frac{1}{2} \qquad \mu^*\left((1,1)|\mathbf{m}^i=(1,1)\right)=\mu^*\left((1,0)|\mathbf{m}^i=(1,1)\right)=\frac{1}{2}$$

The above beliefs mean that upon hearing any message the receiver is certain that i reveals S_1 truthfully but not S_2. The receiver's optimal actions being the following:

$$y_d^*\left(m^i=(0,0),\mathbf{m}^{\text{-}i}\right)=y^*\left(m^i=(0,1),\mathbf{m}^{\text{-}i}\right)=w_{d1}\frac{(\ell_1+1)}{(k_1+3)}+w_{d2}\frac{(\ell_2+1)}{(k_2+2)}$$
$$y_d^*\left(m^i=(1,0),\mathbf{m}^{\text{-}i}\right)=y^*\left(m^i=(1,1),\mathbf{m}^{\text{-}i}\right)=w_{d1}\frac{(\ell_1+2)}{(k_1+3)}+w_{d2}\frac{(\ell_2+1)}{(k_2+2)}$$

For $d=\{1,2\}$.

When $\mathbf{b}^i$ satisfies (1) o but not (3) the above system of beliefs is consistent with i optimally revealing S_1 only, that is $\mathbf{m}^{i*}=\big\{\{(0,0),(0,1)\};\{(1,0),(1,1)\}\big\}$, and i being influential trough S_1 only.

Part 3. The case of i revealing S_2 only is equvalent to the previous and, thus, omitted.

Part 4. Note that when (5) but not (1), the only incentives to lie for sender i happens between types $(0,1)$ and $(1,0)$. In this equilibrium types $(0,1)$ and $(1,0)$ play the corresponding babbling strategy, while the other types are revealing truthfully both signals.

Let consider the following equilibrium beliefs for the receiver upon having seen sender i's message $\mathbf{m}^i$, given that i's bias satisfy condition (5) but not (1).

$$\mu^*\left((0,0)|\mathbf{m}^i=(0,0)\right)=1 \qquad \mu^*\left((1,1)|\mathbf{m}^i=(1,1)\right)=1$$
$$\mu^*\left((0,1)|\mathbf{m}^i=(0,1)\right)=\mu^*\left((1,0)|\mathbf{m}^i=(0,1)\right)=\frac{1}{2} \qquad \mu^*\left((1,0)|\mathbf{m}^i=(1,0)\right)=\mu^*\left((0,1)|\mathbf{m}^i=(1,0)\right)=\frac{1}{2}$$

And the optimal actions by the receiver:

$$y_d^*\left(m^i=(0,0),\mathbf{m}^{\text{-}i}\right)=w_{d1}\frac{(\ell_1+1)}{(k_1+3)}+w_{d2}\frac{(\ell_2+1)}{(k_2+3)}$$
$$y_d^*\left(m^i=(1,1),\mathbf{m}^{\text{-}i}\right)=w_{d1}\frac{(\ell_1+2)}{(k_1+3)}+w_{d2}\frac{(\ell_2+2)}{(k_2+3)}$$
$$y_d^*\left(m^i=(1,0),\mathbf{m}^{\text{-}i}\right)=y_d^*\left(m^i=(0,1),\mathbf{m}^{\text{-}i}\right)=w_{d1}\frac{(\ell_1+1)}{(k_1+2)}+w_{d2}\frac{(\ell_2+1)}{(k_2+2)}$$

For $d=\{1,2\}$.

Meaning that upon hearing either $(0,0)$ or $(1,1)$ the receiver updates her estimation of both θ_1 and θ_2, but she does not update any of them otherwise. From the sender's perspective, types $(0,1)$ and $(1,0)$ should prefer an equilibrium babbling strategy (mixing between these two messages) rather than announcing any of the influential messages.

The lack of incentives for types $(0,0)$ and $(1,1)$ to pool to each other given both of them are influential is given by condition (3), which is implied by (5).

Now, incentives to reveal both signals given types $(0, 1)$ and $(1, 0)$ are babbling are given by Lemma 5. It is worth noting that condition (5) guarantees type $(0, 0)$ as well as $(1, 1)$ has no incentives to play the babbling strategy when he is believed about both signals if announces truthfully. At the same time, these conditions guarantee types $(0, 1)$ and $(1, 0)$ prefers to play the babbling strategy available rather than announcing any of the influential messages.

Part 5. In this equilibrium types $(0, 0)$ and $(1, 1)$ send the corresponding babbling messages, while the other types are revealing truthfully both signals. The proof is similar to that in Part 4, adapting the equilibrium beliefs to the messages and types that are playing each strategy.

Part 6. The babbling equilibrium is always part of the available equilibria in any cheap talk game. In Part 6 of the following section I show that for the set of preferences that do not satisfy any of the conditions above, the unique equilibrium available is the babbling.

C.2 Most Informative Equilibrium

Part 1. Trivially, there cannot exist a more informative equilibrium given this represent the fully separating case.

Part 2. Any strategy that is more informative than revealing only one signal has at least one type fully revealing in equilibrium (and the others revealing partially). Consider the following message strategies that would improve upon revealing S_1^i only:

- $\tilde{\mathbf{m}}^i = \{\{(0, 0)\}; \{(0, 1)\}; \{(1, 0); (1, 1)\}\}$
- $\hat{\mathbf{m}}^i = \{\{(0, 0); (0, 1)\}; \{(1, 0)\}; \{(1, 1)\}\}$

(a) Condition (1) holds for S_1 only. For $\tilde{\mathbf{m}}^i$ to be an equilibrium, types $(0, 0)$ and $(0, 1)$ must have incentives to separate from each other, which requires condition (1) to hold for S_2, leading to a contradiction. The same applies to $\hat{\mathbf{m}}^i$ for separation between types $(1, 0)$ and $(1, 1)$.

(b) Condition (1) hold for both signals. The set of preferences for which both $\tilde{\mathbf{m}}^i$ and $\hat{\mathbf{m}}^i$ can be sustained in equilibrium are the same, leading to multiple equilibria. Moreover, both $\tilde{\mathbf{m}}^i$ and $\hat{\mathbf{m}}^i$ improve the receiver's ex-ante payoff[25] in exactly the same way –i.e. by obtaining information about S_2^i when sender is one of the fully revealing types, which occurs with probability $1/2$; thus, receiver ex-ante utility fails to select among these equilibrium message strategies. Finally, the implementation of any $\tilde{\mathbf{m}}^i$ or $\hat{\mathbf{m}}^i$ depends upon the pooling types playing mixed strategies and, thus, is restricted to all possible message strategies being played with positive probability *on path*. In other words, there are no out-of-equilibrium strategies/beliefs that sustain any of these strategies –i.e. neologism-proofness (Farrell, 1993) fails.

Part 3. Any strategy that is more informative than revealing only S_2^i must be of the form:

- $\tilde{\mathbf{m}}^i = \{\{(0, 0)\}; \{(1, 0)\}; \{(0, 1); (1, 1)\}\}$
- $\hat{\mathbf{m}}^i = \{\{(0, 0); (1, 0)\}; \{(0, 1)\}; \{(1, 1)\}\}$

Then, the same arguments as in Part 2.(a) and 2.(b) apply.

[25] With respect to revealing S_1^i only.

Part 4. Given the equilibrium message strategy $\mathbf{m}^i = \{\{(0,0)\};\{(1,1)\};\{(0,1);(1,0)\}\}$ then a more informative message strategy would necessarily involve at least one more type revealing both signals, since revelation of one signal is not available due to pooling types not sharing any single realization –i.e. $(0,1)$ and $(1,0)$. As a consequence, the only message strategy that is more informative than $\mathbf{m}^i$ is the fully separating one, for which condition (4) must hold; a contradiction.

Part 5. Given the equilibrium message strategy $\mathbf{m}^i = \{\{(0,0);(1,1)\};\{(0,1)\};\{(1,0)\}\}$ then a more informative message strategy would involve at least one more type revealing both signals, since revelation of one signal is not available due to pooling types not sharing any single realization –i.e. $(0,0)$ and $(1,1)$. As a consequence, the only message strategy that is more informative than $\mathbf{m}^i$ is the fully separating one, for which condition (4) must hold; a contradiction.

Part 6. The equilibrium message strategy is given by $\mathbf{m}^i = \{(0,0);(0,1);(1,0);(1,1)\}$.

Let first consider the partition in which a single type fully separates. Lemma 5 implies that for types $(0,0)$ and $(1,1)$ (individually) the corresponding condition leads to the equilibrium in Part 4, which rules out this possibility. The same argument applies to the cases in which either types $(0,1)$ or $(1,0)$, the receiver-optimal equilibrium for the set of preferences satisfying the corresponding condition would involve the following message strategy $\mathbf{m}^i = \{\{(0,0);(0,1)\};\{(1,0)\};\{(1,1)\}\}$.

Secondly, consider the cases in which one type with coincident signals and another with non-coincident signals separate and the others play babbling. By Lemma 5 whenever type $(0,0)$ separates, type $(1,1)$ has incentives to do so and the deviation is profitable for the receiver; while type $(0,1)$ has incentives to separate if and only if $(1,0)$ has.

Finally, I must rule out the message strategy in which only one realization of a given signal is revealed –i.e. the equilibrium message strategies being either $\mathbf{m}^i = \{\{(0,0);(0,1)\};\{(1,0);(1,1)\}\}$ or $\mathbf{m}^i = \{\{(0,0);(1,0)\};\{(0,1);(1,1)\}\}$, with only one message being influential in each. The argument is simple: if, for instance, sender i were willing to reveal only $S_1^i = 1$; then the receiver would infer $S_1^i = 0$ when i plays the babbling strategy, which implies the equilibrium in Part 1. $\square$

C.3 Selection between two equilibrium strategies with the same number of influential messages.

Proof. The ex-ante expected utility for a generic player in equilibrium $(\mathbf{y},\mathbf{m})$ is given by:

$$\begin{aligned} E\left[U^i(\boldsymbol{\delta},\mathbf{b});\mathbf{m}\right] &= -E\left[(y_1-\delta_1-b_1^i)^2\right] - E\left[(y_2-\delta_2-b_2^i)^2\right] \\ &= -\sum_{d=\{1,2\}} E\left[(y_d-\delta_d)^2 - 2(y_d-\delta_d)b_d^i + \left(b_d^i\right)^2;\mathbf{m}\right] \end{aligned}$$

Which, by definitions of y_d andδ_d yield:

$$E\left[U^i(\boldsymbol{\delta},\mathbf{b});\mathbf{m}\right] = -\sum_{d=\{1,2\}} \left(b_d^i\right)^2 + E\left[\Big(w_{d1}\big(E(\theta_1|\mathbf{m})-\theta_1\big) + w_{d2}\big(E(\theta_2|\mathbf{m})-\theta_2\big)\Big)^2\right] + E\left[E(\delta_d|\mathbf{m})-\delta_d\right] b_d^i$$

With some rearranging and given the last term equals zero, I have:

$$\begin{aligned} E\left[U^i(\boldsymbol{\delta},\mathbf{b});\mathbf{m}\right] = &-\left[\left(b_1^i\right)^2 + \left(b_2^i\right)^2\right] - \left[(w_{11})^2 + (w_{21})^2\right] E\left[\left(E(\theta_1|\mathbf{m})-\theta_1\right)^2;\mathbf{m}\right] \\ &- \left[(w_{12})^2 + (w_{22})^2\right] E\left[\left(E(\theta_2|\mathbf{m})-\theta_2\right)^2;\mathbf{m}\right] \end{aligned} \tag{13}$$

Now, following Galeotti et al. (2013) and Dewan and Squintani (2015), the expectation of the squared deviation for each state is given by:

$$
\begin{aligned}
E\left[\left(E(\theta_r|\mathbf{m})-\theta_r\right)^2;\mathbf{m}\right] &= \int_0^1 \sum_{\ell_r=0}^{k_r+1}\left(E(\theta_r|\mathbf{m})-\theta_r\right)^2 f(\ell_r|k_r,\theta_r)\,d\theta_r \\
&= \int_0^1 \sum_{\ell_r=0}^{k_r+1}\left(E(\theta_r|\mathbf{m})-\theta_r\right)^2 \frac{h(\theta_r|\ell_r,k_r)}{(k_r+2)}\,d\theta_r \\
&= \frac{1}{(k_r+2)}\sum_{\ell_r=0}^{k_r+1}\int_0^1 \left(E(\theta_r|\mathbf{m})-\theta_r\right)^2 h(\theta_r|\ell_r,k_r)\,d\theta_r \\
&= \frac{1}{(k_r+2)}\sum_{\ell_r=0}^{k_r+1}\mathrm{Var}(\theta_r|\ell_r,k_r)
\end{aligned}
$$

Plugging the above into 13 gives the expression for player i's ex-ante expected utility:

$$
E\left[U^i(\boldsymbol{\delta},\mathbf{b});\mathbf{m}\right] = -\left[\left(b_1^i\right)^2+\left(b_2^i\right)^2\right] - \frac{[(w_{11})^2+(w_{21})^2]}{6\,(k_1+2)} - \frac{[(w_{12})^2+(w_{22})^2]}{6\,(k_2+2)}
$$

In order to analyse the ex-ante optimality of each equilibrium in Proposition 2.6, I "break" $\mathbf{m}$ into $\mathbf{m}^i$ and $\mathbf{m}^{-i}$. Then, denoting by $\tilde{k}_r$ the equilibrium number of truthful messages for senders other than i, the expected variance of each possible message strategy for i become:

$$
E\left[U_R\left(\mathbf{y},\mathbf{m}^i=\{\{(0,0);(0,1)\};\{(1,0);(1,1)\}\},\mathbf{m}^{-i}\right)\right] = -\frac{[(w_{11})^2+(w_{21})^2]}{6\,(\tilde{k}_1+3)} - \frac{[(w_{12})^2+(w_{22})^2]}{6\,(\tilde{k}_2+2)} \quad (14)
$$

$$
E\left[U_R\left(\mathbf{y},\mathbf{m}^i=\{\{(0,0);(1,0)\};\{(0,1);(1,1)\}\},\mathbf{m}^{-i}\right)\right] = -\frac{[(w_{11})^2+(w_{21})^2]}{6\,(\tilde{k}_1+2)} - \frac{[(w_{12})^2+(w_{22})^2]}{6\,(\tilde{k}_2+3)} \quad (15)
$$

$$
\begin{aligned}
E\left[U_R\left(\mathbf{y},\mathbf{m}^i=\{\{(0,0)\};\{(1,1)\};\{(0,1);(1,0)\}\},\mathbf{m}^{-i}\right)\right] = &-\frac{1}{2}\left[\frac{[(w_{11})^2+(w_{21})^2]}{6\,(\tilde{k}_1+3)} + \frac{[(w_{12})^2+(w_{22})^2]}{6\,(\tilde{k}_2+3)}\right] \\
&-\frac{1}{2}\left[\frac{[(w_{11})^2+(w_{21})^2]}{6\,(\tilde{k}_1+2)} + \frac{[(w_{12})^2+(w_{22})^2]}{6\,(\tilde{k}_2+2)}\right]
\end{aligned} \quad (16)
$$

Where the fact that in (14) sender i reveals S_1 only can be seen in the different numerators of its first and second term, and the same applies to (15). Now, when i reveals $(0,0)$ and $(1,1)$ only (Part 4.c) the receiver's ex-ante expected utility weights the probability of i being one of these types, and the complementary probability of being $(0,1)$ or $(1,0)$ and not receiving any information. The following step consist in finding the conditions under which each of the above expressions are ex-ante optimal for the receiver, given the equilibrium strategies of the other senders.

1. Sender i reveals S_1 only:

(a) (14) $\geq$ (15):

$$-\frac{[(w_{11})^2+(w_{21})^2]}{6\,(\tilde{k}_1+3)}-\frac{[(w_{12})^2+(w_{22})^2]}{6\,(\tilde{k}_2+2)}+\frac{[(w_{11})^2+(w_{21})^2]}{6\,(\tilde{k}_1+2)}+\frac{[(w_{12})^2+(w_{22})^2]}{6\,(\tilde{k}_2+3)}\geq 0$$

Which leads to:

$$\frac{[(w_{11})^2+(w_{21})^2]}{(\tilde{k}_1+2)(\tilde{k}_1+3)}\geq\frac{[(w_{12})^2+(w_{22})^2]}{(\tilde{k}_2+2)(\tilde{k}_2+3)} \tag{17}$$

(b) (14) $\geq$ (16):

$$\frac{[(w_{11})^2+(w_{21})^2]}{2}\left[\frac{1}{(\tilde{k}_1+2)}-\frac{1}{(\tilde{k}_1+3)}\right]-\frac{[(w_{12})^2+(w_{22})^2]}{2}\left[\frac{1}{(\tilde{k}_2+2)}-\frac{1}{(\tilde{k}_2+3)}\right]\geq 0$$

Which is straightforward to note that leads to (17).

2. Sender i reveals S_2 only: requires that (15) $\geq$ (14) and (15) $\geq$ (16); which following the above algebra happen if and only if

$$\frac{[(w_{11})^2+(w_{21})^2]}{(\tilde{k}_1+2)(\tilde{k}_1+3)}\leq\frac{[(w_{12})^2+(w_{22})^2]}{(\tilde{k}_2+2)(\tilde{k}_2+3)} \tag{18}$$

3. Sender i reveals $(0,0)$ and $(1,1)$ only: requires that (16) $\geq$ (14) and (16) $\geq$ (15); which happen if and only if,

$$\frac{[(w_{11})^2+(w_{21})^2]}{(\tilde{k}_1+2)(\tilde{k}_1+3)}\geq\frac{[(w_{12})^2+(w_{22})^2]}{(\tilde{k}_2+2)(\tilde{k}_2+3)}$$

and

$$\frac{[(w_{11})^2+(w_{21})^2]}{(\tilde{k}_1+2)(\tilde{k}_1+3)}\leq\frac{[(w_{12})^2+(w_{22})^2]}{(\tilde{k}_2+2)(\tilde{k}_2+3)}$$

Both equations above are compatible if and only if:

$$\frac{[(w_{11})^2+(w_{21})^2]}{(\tilde{k}_1+2)(\tilde{k}_1+3)}=\frac{[(w_{12})^2+(w_{22})^2]}{(\tilde{k}_2+2)(\tilde{k}_2+3)} \tag{19}$$

The above equation tells us that the receiver's ex-ante utility is maximal when i reveals $(0,0)$ and $(1,1)$ only, when she has similar amount of truthful messages for each state in the equilibrium being played. In addition, equation (19) implies that for i revealing S_1 only to be the optimal ex-ante equilibrium for the receiver (17) must hold with inequality, and the same applies for S_2 and (18). $\square$

Observation 1. *The RHS of* (4) *is positive if and only if:*

$$[w_{11}(k_2+3)-w_{12}(k_1+3)]^2+[w_{22}(k_1+3)-w_{21}(k_2+3)]^2>0$$

Since $\frac{w_{11}}{(k_1+3)}$ represents sender i's influence on y_1 through revealing S_1, then $w_{11}(k_2+3)-w_{12}(k_1+3)=0$ implies that the influence on y_1 through S_1 perfectly outweighs that of S_2. Consequently, RHS of (4) equal to zero implies sender i has no influence on any decision when he reveals both signals (and he is either type $(0,1)$ or $(1,0)$), which again depends on the number of other sender revealing truthfully their signals. In section 4.1 I show this plays an important role on the effect of changes in the number of senders other than i revealing truthfully in equilibrium.

Appendix D Comparative Statics

Proposition 6. *Let $1/2 \leq w \leq 1$. How informational interdependence affects incentives for communication depend on two effects: the aggregate conflict of interest and the credibility loss; as follows.*

Incentives to reveal one signal only:

$$\frac{\partial IC_{(1)}}{\partial w} = -\frac{1}{(k_{\tilde{r}}+3)}\frac{\partial C_r}{\partial w} - 2\frac{\partial |B_r^i|}{\partial w} \tag{20}$$

Incentives to fully reveal $(S_1^i, S_2^i) = \{(0,0);(1,1)\}$:

$$\frac{\partial IC_{(3)}}{\partial w} = \frac{2}{(k_1+3)(k_2+3)}\left[\frac{\partial C_r}{\partial w} \pm (k_2+3)\frac{\partial B_1^i}{\partial w} \pm (k_1+3)\frac{\partial B_2^i}{\partial w}\right] \tag{21}$$

Incentives to fully reveal $(S_1^i, S_2^i) = \{(0,1);(1,0)\}$:

$$\frac{\partial IC_{(4)}}{\partial w} = \frac{2}{(k_1+3)(k_2+3)}\left[-\frac{\partial C_r}{\partial w} \pm (k_2+3)\frac{\partial B_1^i}{\partial w} \mp (k_1+3)\frac{\partial B_2^i}{\partial w}\right] \tag{22}$$

for $\theta_{\tilde{r}} \neq \theta_r$

D.1 Proof of Proposition 6 and Lemma 6

Proof. Let $w_{11} = w_{22} = w$; then, note that:

$$\begin{aligned}\frac{\partial B_1^i}{\partial w} &= \frac{b_1^i - b_2^i}{w^2+(1-w)^2} - \frac{[2w-2(1-w)][wb_1^i+(1-w)b_2^i]}{[w^2+(1-w)^2]^2}\\ &= \frac{b_1^i - b_2^i}{w^2+(1-w)^2} - \frac{2(2w-1)}{w^2+(1-w)^2}B_1^i\end{aligned}$$

By the rule of derivative of absolute value functions, $\frac{\partial |y(x)|}{\partial x} = \frac{\partial y(x)}{\partial x}\frac{y(x)}{|y(x)|}$, and noting that $|B_1^i| = \frac{|wb_1^i+(1-w)b_2^i|}{w^2+(1-w)^2}$, the above expression yields:

$$\frac{\partial |B_1^i|}{\partial w} = \text{sign}(B_1^i) \times \frac{(b_1^i - b_2^i)}{[w^2+(1-w)^2]} + |B_1^i| \times \frac{2(1-2w)}{[w^2+(1-w)^2]}$$

For higher interdependence to reduce the aggregate conflict of interest, I need $\frac{\partial |B_1^i|}{\partial w} > 0$.[26] The second term of the above equation is clearly negative; so, for the derivate to be positive a necessary condition is that the first term is positive as well. I then need that $\text{sign}(B_1^i) = \text{sign}(b_1^i - b_2^i)$, which depends on the signs of the decision-specific biases as follows.

1. if $\text{sign}(b_1^i) = \text{sign}(b_2^i)$, then $\text{sign}(B_1^i) = \text{sign}(b_1^i)$ and the necessary condition above depends on $\text{sign}(b_1^i - b_2^i)$. It is straightforward to check that the necessary condition is satisfied when $|b_1^i| > |b_2^i|$.
2. if $\text{sign}(b_1^i) \neq \text{sign}(b_2^i)$, then $\text{sign}(b_1^i - b_2^i) = \text{sign}(b_1^i)$ and the necessary condition above depends on $\text{sign}(B_1^i)$. Again, it is straightforward to notice that $\text{sign}(B_1^i) = \text{sign}(b_1^i)$ requires that $w|b_1^i > (1-w)b_2^i$.

[26] Recall that highe w means less informational interdependence.

Now, let proceed to compute the credibility loss, given by $\frac{\partial C_r}{\partial w}$. Note that $w_{11} = w_{22}$ implies that $C_1 = C_2 = \frac{2w(1-w)}{1-2w(1-w)}$; for which, after some algebra I get:

$$\frac{\partial C_r}{\partial w} = \frac{2(1-2w)}{[1-2w(1-w)]^2} \leq 0$$

□

D.2 Increasing the decision-maker's equilibrium information

Proof. For $k_1' > k_1$, the statement in Proposition 4 is equivalent to preferences that satisfy the following conditions:

$$|\beta_1^i| \leq \frac{w^2 + (1-w)^2}{2\,(k_1'+3)} \tag{23}$$

$$|\beta_2^i| \leq \frac{w^2 + (1-w)^2}{2\,(k_2+3)} \tag{24}$$

$$\left|\frac{\beta_1^i}{(k_1'+3)} - \frac{\beta_2^i}{(k_2+3)}\right| \leq \frac{1}{2}\left[\frac{w^2+(1-w)^2}{(k_1'+3)^2} + \frac{w^2+(1-w)^2}{(k_2+3)^2} - \frac{4\,w\,(1-w)}{(k_1'+3)(k_2+3)}\right] \tag{25}$$

$$\left|\frac{\beta_1^i}{(k_1+3)} - \frac{\beta_2^i}{(k_2+3)}\right| > \frac{1}{2}\left[\frac{w^2+(1-w)^2}{(k_1+3)^2} + \frac{w^2+(1-w)^2}{(k_2+3)^2} - \frac{4\,w\,(1-w)}{(k_1+3)(k_2+3)}\right] \tag{26}$$

It is worth noting that condition (23) implies the equivalent for k_1.

The proof proceeds as follow: I first find β_1^i and β_2^i for which both (25) and (26) hold, and then derive the conditions under which they satisfy (23) and (24).

First note that LHS of both (25) and (26) depend on the signs of β_1^i and β_2^i:

1. If $\text{sign}[\beta_1^i] \neq \text{sign}[\beta_2^i]$, then

$$\left|\frac{\beta_1^i}{(k_1'+3)} - \frac{\beta_2^i}{(k_2+3)}\right| = \frac{|\beta_1^i|}{(k_1'+3)} + \frac{|\beta_2^i|}{(k_2+3)}$$

2. If $\text{sign}[\beta_1^i] = \text{sign}[\beta_2^i]$, then

$$\left|\frac{\beta_1^i}{(k_1'+3)} - \frac{\beta_2^i}{(k_2+3)}\right| = \left|\frac{|\beta_1^i|}{(k_1'+3)} - \frac{|\beta_2^i|}{(k_2+3)}\right|$$

The signs of β_1^i and β_2^i are determined by both the sign and the relative magnitude of b_1^i and b_2^i. In the first of the above cases, biases go in opposite directions and are relatively similar in magnitude according to the interdependence: $\frac{(1-w)}{w} < \frac{|b_1^i|}{|b_2^i|} < \frac{w}{(1-w)}$. For $\text{sign}[\beta_1^i] = \text{sign}[\beta_2^i]$ arises either when $\text{sign}[b_1^i] = \text{sign}[b_2^i]$, or $\text{sign}[b_1^i] \neq \text{sign}[b_2^i]$ but one of them being sufficiently larger in magnitude.

For the sake of simplicity, assume the increase in the receiver's information on θ_1 is minimal –i.e. $k_1' = k_1 + 1$.

Case 1: sign$[\beta_1^i] \neq$ sign$[\beta_2^i]$. Then, (25) becomes

$$\frac{|\beta_1^i|}{(k_1'+3)} + \frac{|\beta_2^i|}{(k_2+3)} \leq \frac{1}{2}\left[\frac{w^2+(1-w)^2}{(k_1'+3)^2} + \frac{w^2+(1-w)^2}{(k_2+3)^2} - \frac{4\,w\,(1-w)}{(k_1'+3)(k_2+3)}\right]$$

And (26):

$$\frac{|\beta_1^i|}{(k_1+3)} + \frac{|\beta_2^i|}{(k_2+3)} > \frac{1}{2}\left[\frac{w^2+(1-w)^2}{(k_1+3)^2} + \frac{w^2+(1-w)^2}{(k_2+3)^2} - \frac{4\,w\,(1-w)}{(k_1+3)(k_2+3)}\right]$$

Operating both conditions I get that the biases for which increasing information in θ_1 improves communication must satisfy:

$$\frac{(k_1+3)}{2}\left[\frac{w^2+(1-w)^2}{(k_1+3)^2} + \frac{w^2+(1-w)^2}{(k_2+3)^2} - \frac{4\,w\,(1-w)}{(k_1+3)(k_2+3)}\right] - \frac{|\beta_2^i|\,(k_1+3)}{(k_2+3)} < |\beta_1^i| \leq$$
$$\leq \frac{(k_1'+3)}{2}\left[\frac{w^2+(1-w)^2}{(k_1'+3)^2} + \frac{w^2+(1-w)^2}{(k_2+3)^2} - \frac{4\,w\,(1-w)}{(k_1'+3)(k_2+3)}\right] - \frac{|\beta_2^i|\,(k_1'+3)}{(k_2+3)}$$

and

$$\frac{(k_2+3)}{2}\left[\frac{w^2+(1-w)^2}{(k_1+3)^2} + \frac{w^2+(1-w)^2}{(k_2+3)^2} - \frac{4\,w\,(1-w)}{(k_1+3)(k_2+3)}\right] - \frac{|\beta_1^i|\,(k_2+3)}{(k_1+3)} < |\beta_2^i| \leq$$
$$\leq \frac{(k_2+3)}{2}\left[\frac{w^2+(1-w)^2}{(k_1'+3)^2} + \frac{w^2+(1-w)^2}{(k_2+3)^2} - \frac{4\,w\,(1-w)}{(k_1'+3)(k_2+3)}\right] - \frac{|\beta_1^i|\,(k_2+3)}{(k_1'+3)}$$

Then, the necessary conditions for each of the above to hold is, respectively:

$$|\beta_2^i| \leq \frac{[w^2+(1-w)^2]}{2\,(k_2+3)} - \frac{[w^2+(1-w)^2]\,(k_2+3)}{2\,(k_1+3)(k_1+4)} \tag{27}$$

and

$$|\beta_1^i| > \frac{[w^2+(1-w)^2]}{2}\left[\frac{1}{(k_1+3)} + \frac{1}{(k_1+4)}\right] - \frac{2\,w(1-w)}{(k_2+3)} \tag{28}$$

Note that condition (27) implies condition (24). Then, for it to hold it must be that the RHS is greater than zero, which occurs if:

$$(k_1+3)(k_1+4) > (k_2+3)^2$$

Now, for condition (28) to hold together with (23) it must be the case that the value of $|\beta_1^i|$ is in between two magnitudes: RHS(28)$< |\beta_1^i| \leq$ RHS(23); which holds if:

$$\frac{(k_2+3)}{(k_1+3)} < \frac{4\,w\,(1-w)}{[w^2+(1-w)^2]}$$

It is worth noting that both conditions above require k_1 to be sufficiently large with respect to k_2, which has the interpretation of *saturation*. After the increase in k_1 takes place, sender i is willing to reveal all his information because he will move y_1 very little and y_2 (where preferences are aligned) very much.

Case 2: sign$[\beta_1^i]$ = sign$[\beta_2^i]$. I consider two cases: in the first the conflict of interest in the first dimension is so large in magnitude that it outweighs that of the second in terms of incentives (even in the case in which they go in opposite directions across dimensions); secondly, I analyse the case in which the bias in the second dimension outweighs that of the first.

a. $\frac{|\beta_1^i|}{(k_1'+3)} > \frac{|\beta_2^i|}{(k_2+3)}$

Then, (25) becomes

$$\frac{|\beta_1^i|}{(k_1'+3)} - \frac{|\beta_2^i|}{(k_2+3)} \leq \frac{1}{2}\left[\frac{w^2+(1-w)^2}{(k_1'+3)^2} + \frac{w^2+(1-w)^2}{(k_2+3)^2} - \frac{4\,w\,(1-w)}{(k_1'+3)(k_2+3)}\right]$$

And (26) becomes

$$\frac{|\beta_1^i|}{(k_1+3)} - \frac{|\beta_2^i|}{(k_2+3)} > \frac{1}{2}\left[\frac{w^2+(1-w)^2}{(k_1+3)^2} + \frac{w^2+(1-w)^2}{(k_2+3)^2} - \frac{4\,w\,(1-w)}{(k_1+3)(k_2+3)}\right]$$

Operating both conditions as before and getting restrictions over $|\beta_1^i|$ and $|\beta_2^i|$ the necessary conditions are:

$$|\beta_2^i| > \frac{[w^2+(1-w)^2]}{2\,(k_2+3)} - \frac{[w^2+(1-w)^2]\,(k_2+3)}{2\,(k_1+3)(k_1+4)} \tag{29}$$

and

$$|\beta_1^i| > \frac{[w^2+(1-w)^2]}{2}\left[\frac{1}{(k_1+3)} + \frac{1}{(k_1'+3)}\right] - \frac{4\,w(1-w)}{(k_2+3)} \tag{30}$$

In this case both (23) and (24) will be upper bounds for $|\beta_1^i|$ and $|\beta_2^i|$, such that (29) holds if:

$$(k_1+3)(k_1+4) > (k_2+3)^2$$

While (30) holds if:

$$\frac{(k_2+3)}{(k_1+3)} < \frac{4\,w\,(1-w)}{[w^2+(1-w)^2]}$$

Note that the conditions are the same as the previous case. The differences are on the restrictions over b_1^i and b_2^i: $\text{sign}[\beta_1^i] = \text{sign}[\beta_2^i]$ implies that when $\text{sign}[b_1^i] \neq \text{sign}[b_2^i]$ then $(1-w)|b_1^i| > g|b_2^i|$; while in the previous case both conflict of interest had to be of similar magnitude.[27]

All the above lead to the conclusion that when conflict of interests go in opposite directions across dimensions, "beneficial congestion effects" on θ occur when the magnitude of the conflict in the first dimension is sufficiently large with respect to that on the second dimension.

b. $\frac{|\beta_1^i|}{(k_1+3)} < \frac{|\beta_2^i|}{(k_2+3)}$

Then, (25) becomes

$$\frac{|\beta_2^i|}{(k_2+3)} - \frac{|\beta_1^i|}{(k_1'+3)} \leq \frac{1}{2}\left[\frac{w^2+(1-w)^2}{(k_1'+3)^2} + \frac{w^2+(1-w)^2}{(k_2+3)^2} - \frac{4\,w\,(1-w)}{(k_1'+3)(k_2+3)}\right]$$

And (26) becomes

$$\frac{|\beta_2^i|}{(k_2+3)} - \frac{|\beta_1^i|}{(k_1+3)} > \frac{1}{2}\left[\frac{w^2+(1-w)^2}{(k_1+3)^2} + \frac{w^2+(1-w)^2}{(k_2+3)^2} - \frac{4\,w\,(1-w)}{(k_1+3)(k_2+3)}\right]$$

Operating both conditions as before and getting restrictions over $|\beta_1^i|$ and $|\beta_2^i|$ the necessary conditions are:

[27] More precisely, when $\text{sign}[\beta_1^i] \neq \text{sign}[\beta_2^i]$, $(1-w)|b_2^i| < |b_1^i| < w|b_2^i|$.

$$|\beta_2^i| < \frac{[w^2 + (1-w)^2]}{2\,(k_2+3)} - \frac{[w^2 + (1-w)^2]\,(k_2+3)}{2\,(k_1+3)(k_1+4)} \tag{31}$$

and

$$|\beta_1^i| < \frac{2\,w(1-w)}{(k_2+3)} - \frac{[w^2 + (1-w)^2]}{2}\left[\frac{1}{(k_1+3)} + \frac{1}{(k_1+4)}\right] \tag{32}$$

Given that (31) implies (24), it is only necessary to check when its RHS is greater than zero. As before, this happens for:

$$(k_1+3)(k_1+4) > (k_2+3)^2$$

Similarly, (32) is greater than zero when:

$$(k_2+3)\left[\frac{1}{(k_1+3)} + \frac{1}{(k_1+4)}\right] < \frac{4\,w\,(1-w)}{[w^2 + (1-w)^2]}$$

It is straightforward to note that the last condition is more restrictive than the previous cases' equivalents, thus it can be taken as a sufficient condition for the existence of positive congestion effects for a large range of biases.

Moreover, it can be noted that when $w = 1$ the above condition cannot be satisfied.

Note: I am leaving aside the case of $\frac{|\beta_1^i|}{(k_1'+3)} < \frac{|\beta_2^i|}{(k_2+3)} < \frac{|\beta_1^i|}{(k_1+3)}$, since the conditions above guarantee the beneficial effect of increasing the receiver's information for some range of biases. $\square$

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