Designing Complex Organizations*

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Abstract

I study organizational design under informational interdependence—the fact that information most relevant to a set of actions also affect others. In my model, a principal faces biased agents who have noisy information about multiple states. Because of the interplay between interdependence and (decision-specific) biases, communication depends on the allocation of decision rights. I find the principal prefers to delegate a high-conflict decision if that improves information transmission in other dimensions. When agents have extreme preferences, centralization may discipline decision-specific conflict of interests and improve communication. I also analyse how complexity affects information acquisition. Under delegation, investment in information is expected to be lower overall and concentrated on states that are more important for the decision at hand. Finally, I show that specialization signals an agent's commitment not to manipulate information and thus enhances his credibility.

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1 Introduction

Organizations operate in complex environments. The problems they face have multiple causes and the solutions typically involve many courses of action. In such environments information is a crucial input. Organizations spend vast amounts of resources in collecting data and producing information, generally dispersed among its members. Making this information available for decision-making requires an appropriate organizational structure. This paper studies organizational design to produce and aggregate information effectively in complex environments.

The features just described are common in many types of organizations. Research-based multinational corporations (MNC), for instance, usually engage in parallel product-development projects. Some of these projects involve updating products the firm already commercializes, its profitability mainly depends on cost-reducing innovations. Some projects involve creating new products, and profitability rely on finding commercially viable innovations from new technologies. Information about old and new technologies is dispersed among the firm's subsidiaries. Typically, cost-reducing innovations also apply to new products, increasing their commercial viability. In a similar way, attractive new technologies can be used on established products to increase their market appeal. These decisions are *informationally interdependent*—information mostly relevant for one set of decisions also affects other decisions.

Additional examples of such interdependence can be found in policy-making. Problems requiring public intervention are generally defined in terms of multiple goals. Policy instruments are limited by technology and political considerations, such that they address the policy goals with different degree of success. Any policy then consists of many interrelated decision,s and the relevant information is dispersed among several political actors. Because of the limits of policy instruments, environmental changes affecting one of multiple goals trigger re-calibration of many decisions. Informational interdependence constitutes a prominent feature of many complex environments. In this paper I show it plays a crucial role in designing organizations to produce and aggregate information.

To see how organizations respond to informational interdependence, consider agricultural policy in the United States after World War Two.[1] Pesticides were seen as the solution to eradicate world hunger in a context of American renewed enthusiasm about the progress of science. Important legislation was passed with the support of both agricultural and chemical industry, while consumers and environmental groups were excluded from participation. The Congress and the executive branch facilitated the industry's growth by setting an institutional structure based in the Agriculture Department (USDA), promoting the use of pesticides for years to come. But the optimism about the glory of pesticides started a dramatic decay in the

[1] Based on (Baumgartner and Jones, 2009), Ch. 5, pp. 93-102.

late 1950s after two major policy disasters. The USDA launched two pest eradication campaigns, both involving massive aerial spraying that resulted in devastation of local flora and fauna. A number of congress members started paying attention to pesticide issues they had ignored in the past. After a first round of hearing in 1958, the Food and Drug Administration (FDA) was granted expanded authority to restrict toxic residues on food. In the late 1960s, in response to pressure from a broad environmental movement, the Congress passed major revisions of pesticides regulation that resulted in less policy influence for the USDA and a prominent role for the Environmental Protection Agency.

The example illustrates interdependence: reporting pesticides' health and environmental effects influenced the extent to which they had to be promoted by USDA. It also illustrates that relevant information may be in hands of interested players: the department's close relationships with agricultural lobbies and the chemical industry.[2] In such cases, incentives to transmit information depend on how it is expected to affect decisions. The interplay between informational interdependence and decision-specific biases can lead to *informational externalities*—when informed agents' incentives for communication change depending on how many decisions the policy-maker is in charge of.[3] In the example, the USDA could not be relied on information about environmental and health matters because of its links with agricultural lobbies and the chemical industry. After the Congress realized this, the department's influence was restricted to decisions in which efficiency played a central role. Partial delegation of high-conflict decisions allowed more effective aggregation of policy-relevant information.[4]

In this paper I construct a model of organizational design under informational interdependence. An organization faces a problem defined in terms of two state variables and the solution comprises two decisions. The organization consists of a principal and n biased agents, each of whom observes noisy signals about each state. Before any communication takes place, the principal allocates decision-rights among all members of the organization (including herself). Incentives for communication are shaped by the conflict of interest between each agent and each decision-maker, depending on the decision(s) he/she is in charge of. When authority is centralized, information about each state affects both decisions and, thus, the measure of conflict of interest aggregates decision-specific biases. The model features linear informational interdependence, which allows me to study its consequences in a parsimonious way.

The optimal organizational structure resolves a trade-off between informational gains and loss of control. Informational gains can be of two types. Direct gains refer to the additional

[2]Baumgartner and Jones refer to these type of relationship between policy implementation bodies and special interest groups as 'policy monopolies,' but political scientist have also used the names 'iron triangles' and 'subsystem politics' (see chapter 1 in Baumgartner and Jones (2009).

[3]This definition is consistent with the notion of "informational spillovers" in Levy and Razin (2007).

[4]Large (1973) discusses the implications of the Federal Environmental Pesticide Control Act of 1972.

information an agent is expected to receive in equilibrium when the principal delegates a given decision to him. These gains reflect the traditional argument under which the principal delegates to an agent with sufficiently large 'informational advantage' (see Dessein, 2002; Dewan et al., 2015). [5] Indirect gains, on the contrary, refer to the additional information the principal expects to receive when she delegates one decision and retain authority on the other (partial delegation). These gains arise when there exists at least one agent who reveals more information under partial delegation than under centralization. This means high conflict of interest (with the principal) in one decision, and low on the other. Delegation of the high-conflict decision allows the principal to recover communication: any information transmitted only affects the low-conflict decision.

I find that organizational design plays a crucial role in cases of informational externalities. Proposition 1 shows negative externalities require delegation of high-conflict decisions and retaining authority over low-conflict ones. The optimal organizational structure is thus partial delegation. But informational externalities can also be positive in this context. When, for instance, an agent's preferences are extreme in each dimension but the aggregate conflict of interest with the principal is low (because of how interdependence aggregates decision-specific biases).[6] Agents with such preferences reveal more information to the principal under centralization. Hence, if the number of agents with such preferences is sufficiently large, the principal prefers to centralize decision-making. The mechanism behind this result is similar to 'mutual discipline' in public communication with multiple audiences (Farrell and Gibbons, 1989; Goltsman and Pavlov, 2011), and 'persuasive cheap talk' in multidimensional communication (Chakraborty and Harbaugh, 2010): information affects decisions in a way that decision-specific conflict of interest compensate each other.

Informational interdependence not only affects communication, it also affects information acquisition. In the second part of the paper I introduce endogenous information acquisition. After the allocation of decision rights, each agent decides on the information he wants to observe (if any) at a given cost. Similar to Di Pei (2015), no agent acquires information he is not willing to reveal on the equilibrium path. But this investment is worth making only if it is cost-effective, i.e. the expected utility gains from revealing the acquired information must compensate its cost. Because agents can only acquire signals with a fixed amount of noise,[7] expected utility gains are decreasing in the amount of information the decision-maker receives on path. This imposes an upper bound on the number of agents who become informed in any equilibrium.

[5] Informational advantage conducive to delegation arises ex-post and on path, as in Dewan et al. (2015).

[6] The intuition of positive externalities exists in international relations since "an agreement leading to the peaceful resolution of an international crisis often becomes possible when an issue, not originally in contention, is brought into the bargaining for linkage purposes" (Morgan, 1990). The literature on international relations has recently started to use cheap talk to model credibility (see Ramsay, 2011; Trager, 2011, 2017; Lindsey, 2017).

[7] I assume signals have a fixed amount of noise and agents can acquire at most one of such signals per state.

I find that the *overall investment* in information is expected to be lower under delegation. The intuition is that, under delegation, an agent typically influences one decision; under centralization, on the contrary, influential agents affect both decisions. Investing in any piece of information thus yields higher expected utility gains under centralization and, hence, more agents are willing to acquire information as compared to delegation. This result reflects empirical findings showing that MNCs tend to centralize advanced R&D activities, independent of their nature (Ecker et al., 2013),[8] sometimes the whole business is centred around few key research activities (Gassmann and von Zedtwitz, 1999; Boutellier et al., 2008).

I also find that the organizational structure affects the *relative investment* in information. Information about one state has a higher influence on one of the decisions, but this difference in salience is compensated in the aggregate. This means that, under delegation, an agent expects to have higher influence on a specific decision when he acquires information about the salient state. Hence, each decision-maker expects to receive more information about the salient state.

Finally, the possibility to acquire information about multiple states can lead to specialization, and this can benefit communication. When and agent's information about one state is favourable to his interest and that about the other is not, he has incentives to follow the former. These incentives impose a penalty on credibility, incentive compatibility constraints are tighter than if he observes only one signal. But then specialization—observing information about one state only—kills the loss of credibility and, thus, works as a commitment device for the agent.

Related Literature. The paper builds on contributions of the literature on multidimensional cheap talk. In a seminal paper, Battaglini (2002) shows that this multidimensionality allows a receiver to extract all the information from perfectly-informed senders, by restricting the influence of each of them on the dimension of common interest. When there are as many senders as decisions, the receiver can commit to ignore part of the information provided by each sender because it is provided (in equilibrium) by the others. Note that, as a consequence, the receiver does not need delegation. Dewan and Hortala-Vallve (2011) apply this result to explain the design of jurisdictions in parliamentary cabinets. They argue a Prime Minister uses her prerogatives on ministerial appointments and allocation of portfolios to limit each minister's influence to the dimension of common interest, this way there is no need for effective delegation of authority. But the argument above relies on the assumption that ministers are perfectly informed. When this is not the case and decisions are interdependent, the receiver loses her (equilibrium) commitment power. Levy and Razin (2007) show this leads to communication

[8] Decisions on advanced R&D activities (which are significantly more costly) tend to be more centralized even for basic research, which typically tends to be decentralized.

breakdown if the conflict of interest in one dimension is sufficiently large.[9] In such scenario, senders' incentives depend on how information affects decisions and, thus, their interests. My paper shows delegation substitutes the principal's ability to ignore information in Battaglini (2002) and, more generally, presents how the allocation of decision rights helps her to cope with informational externalities (as defined in Levy and Razin, 2007).

The main contribution of the paper is on organizational design, in which strategic communication has important consequences on the allocation of decision rights. In unidimensional decision problems with a perfectly informed sender, delegation involves a trade-off between informational gains and loss of control (biased decision). Dessein (2002) shows that delegation dominates cheap talk communication for all conflict of interest for which the latter is influential. The principal's willingness to delegate, however, is decreasing in the sender's informational advantage (measured by the ex-ante variance of the state variable). The same intuitions underlie the organization of legislative institutions (Austen-Smith and Riker, 1987; Austen-Smith, 1990), policy-making cabinets (Dewan and Hortala-Vallve, 2011; Dewan et al., 2015), political parties (Dewan and Squintani, 2015), and multi-divisional firms (Alonso et al., 2008; Rantakari, 2008). All these scenarios, but not the papers, feature complex environments. Introducing informational interdependence thus represents a step towards a more realistic understanding of the underlying forces. Only Alonso et al. (2008) and Rantakari (2008) allow for multidimensional decisions, but they focus on the trade-off between coordination and adaptation.

Multi-divisional firms trade off the need for adaptation to local shocks with the need for coordination between divisions. Communication frictions may lead to inefficiencies in terms of giving-up benefits from specialization of production (Dessein and Santos, 2006) or the need for coordination through scheduled task instead of using communication (Dessein et al., 2016). When divisional managers' information is not verifiable, the allocation of decision rights —along with non-separability of preferences and divisional conflict of interest— shape incentives for communication (Alonso et al., 2008, 2015; Rantakari, 2008). But the trade-off between adaptation and coordination may not be the relevant problem in many applications. In multinational corporations, for instance "R&D is torn between the pressures for scientific and commercial results. Control and coordination needed for the sake of internal consistency seems to apply little to research-based organization. Bureaucratic and hierarchical control as well as social control does not work as much as scientists feel more affiliated with their profession than to their employers." (Boutellier et al., 2008). My framework isolates the effects of informational interdependence on communication, and shows how organization can be designed to produce and

[9] In a complementary paper, Chakraborty and Harbaugh (2007) show that, in two-player games, the sender can credibly communicate any ranking of the decision dimensions that reflects (at least partially) the order of the realization of the states across dimensions.

aggregate information effectively.

The allocation of decision rights also affects incentives to acquire information. Aghion and Tirole (1997) show that delegation of formal authority can motivate an agent to acquire information; again, delegation is optimal when the expected utility gains from a better-informed decision outweigh the loss of control. But delegation may discourage information acquisition in this context, either because he prefers to put more effort in decisions that are good for him personally (Rantakari, 2012), or because he no longer has to convince a principal with divergent opinion about the best course of action (Che and Kartik, 2009). When communication is strategic and the sender acquires noisy signals in bunches, the principal can induce him to over-invest in information if she can credibly punish the agent upon deviations at the information acquisition stage (Argenziano et al., 2016). In such cases, delegation is of little use for the principal and centralization dominates. My paper shows that informational interdependence has meaningful consequences on incentives to acquire information. By allowing information acquisition about multiple states, my framework also captures the different drivers of specialization.

More recently, Deimen and Szalay (2019) have also integrated the ideas of information acquisition and strategic communication. In their framework, a decision-maker and an expert face an uni-dimensional problem that depends on information about two states variables. Players disagree on the state upon which the decision has to be calibrated, such that ex-ante correlation between the states implies lower conflict of interest between them. If the decision-maker transfers formal authority to the expert, he acquires perfect information about his preferred state and calibrate the decision accordingly. The decision-maker's benefits from such a policy are increasing in the correlation between states. Under centralization, however, the expert's credibility at the communication stage is constrained by his information, such that in equilibrium he acquires perfect information about the principal's preferred state and a noisy signal about his own preferred state. Decisions in my paper also require information about two states, but interdependence affects communication non-monotonically because decision-specific conflict of interest are state independent (see Crawford and Sobel, 1982). Besides, I focus on a multidimensional decision problem and show that it has consequences on incentives to invest in information under different organizational structures.

The next section presents the baseline model with no information acquisition and the results on optimal allocation of decision-rights. In section 3 I integrate the allocation of decision-rights with endogenous information acquisition. Section 4 discusses some extensions and conclude.

2 Organizational Design to Aggregate Information

An organization is characterized by a principal, n agents, and two decisions, $\mathbf{y} \in \Re^2$.[10] The outcome of each decision depends on two states, θ_1 and θ_2. Agents—but not the principal—have access to information about the states, such that information about any of them *affects both decisions*. In the application to product development, the states can be interpreted as the relevant technological attributes, while decisions represent the different product lines using these technologies (arguably, salience of each technology differs among products lines). In the policy implementation example, states can be interpreted as the different policy goals of a policy intervention, while decisions represent the different policy instruments that address those goals (arguably, different instruments address goals with different degree of success). In both cases, information about any of the states affects both decisions, but individual decisions are more sensitive to information about a specific state. I represent the (state-dependent) 'optimal calibration' of decisions as a composite state $\boldsymbol{\delta} = (\delta_1, \delta_2)$, such that:

$$\begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} \equiv \begin{bmatrix} w_{11}\,\theta_1 + w_{12}\,\theta_2 \\ w_{21}\,\theta_1 + w_{22}\,\theta_2 \end{bmatrix}$$

I assume θ_1 and θ_2 are uniformly distributed with support in the interval $[0,1]$, and that $\theta_1 \perp \theta_2$. These assumptions are not without loss of generality. The uniform distribution has been extensively used in the cheap talk literature (e.g. Crawford and Sobel, 1982); while independence of states allows me to isolate the effect of informational interdependence through $\boldsymbol{\delta}$. The elements of the weighting matrix W are indexed by w_{dr}, where d represents decisions, $y_d = \{y_1, y_2\}$, and r represent states, $\theta_r = \{\theta_1, \theta_2\}$. All the weights are weakly positive and I normalize them to satisfy $w_{d1} + w_{d2} = 1$. Without loss, I also take $w_{11}, w_{22} > \frac{1}{2}$, so that the index corresponding to the state also reflects which decision that state is more important for. As a consequence, the informational interdependence between decisions is linear, and captured by the ex-ante correlation between the composite states.

$$\mathrm{Corr}(\delta_1, \delta_2) = \frac{(w_{11}w_{12} + w_{21}w_{22})}{[(w_{11}^2 + w_{21}^2)(w_{12}^2 + w_{22}^2)]}$$

At the beginning of the game, the principal decides on the allocation of decision rights. Informed agents can then send private cheap talk messages to each decision maker who, after receiving all the messages, decides. Preferences for player $i = \{P, 1, \ldots, n\}$ are given by:

$$U^i(\boldsymbol{\theta}, \mathbf{y}, \mathbf{b}^i) = -\left(y_1 - \delta_1(\theta_1, \theta_2) - b_1^i\right)^2 - \left(y_1 - \delta_2(\theta_1, \theta_2) - b_2^i\right)^2$$

[10] As usual in this literature I use female pronouns to refer to the principal, and male pronouns for each agent.

Where $\mathbf{b}^i \in \Re$ represents i's bias vector, which is normalized to $\mathbf{b}^P = (0,0)$ for the principal.

Agents have access to information about both states. Each agent observes one signal associated to each state (two in total). Let $\mathbf{S}^i = (S_1^i, S_2^i) \in \mathcal{S} \equiv \{0,1\}^2$ be i's signals, and $\tilde{\mathbf{S}} \in \mathcal{S}$ be the vector of realizations. Signals are independent across players, conditionally on $\boldsymbol{\theta}$. The prior probability distribution for each signal is characterized by $\Pr(\tilde{S}_1^i = 1) = \theta_1$ and $\Pr(\tilde{S}_2^i = 1) = \theta_2$.[11]

Each agent sends private, cheap talk messages to decision-maker $j = \{P, 1, .., n\}$. Let $\mathbf{m}_j^i\left(\mathbf{S}^i\right) \in \{0,1\}^2$ denote i's message to decision-maker j, in charge of y_d.[12] Note that i's message strategies associated to each signal can take one of two forms (up to relabelling messages): the truthful one, $m_j^i(S_r^i) = \tilde{S}_r^i$ for all S_r^i, and the babbling one, $m_j^i(\tilde{S}_r^i = 0) = m_j^i(\tilde{S}_r^i = 1)$. Besides, the set of message strategies is not just based on babbling or revealing information on separate dimensions independently. An agent could, for instance, truthfully reveal both signals for some realizations and send the babbling message for others. Such message strategies are possible here because states are orthogonal and any information about one state does not reveal information about the other. I call these strategies 'dimensional non-separable' (DNS).

Let $\mathbf{m}_j = \{..., \mathbf{m}_j^i, ...\}$ denote the matrix containing all the messages decision-maker j receives from agents (including himself if applicable). The updated expectation and variance for each state depend on the number of agents revealing the corresponding signal truthfully, $k_r(j) \leq n$, and the the number of those agents who report a 1, $\ell_r(j)$, for $\theta_r = \{\theta_1, \theta_2\}$, as follows.

$$E\left(\theta_r|\mathbf{m}_j\right) = \frac{(\ell_r(j)+1)}{(k_r(j)+2)} \qquad \mathrm{Var}\left(\theta_r|\mathbf{m}_j\right) = \frac{(\ell_r(j)+1)(k_r(j)-\ell_r(j)+1)}{(k_r(j)+2)^2(k_r(j)+3)}$$

Decision-rights are allocated before each agent learns his information. Formally, the principal decides on a set of assignments that grants decision-making authority over the set of decisions. The assignment grants complete jurisdiction over the delegated decision, such that authority over each decision is granted to a unique individual. Decision-makers are also granted the possibility of private communication with each agent. Different allocations of decision-rights lead to different organizational structures. I group these structures into three categories: under *centralization* the principal decides on both issues; under *full delegation* the principal grants authority to two different agents, each of them assigned to a different decision; under *partial delegation* the principal retains authority over one issue and delegates the other to an agent.

At this point two clarifications are necessary. First, I am not considering the case of delegation of both decisions to a single agent. This is without loss since agents' incentives for

[11]A similar information structure, for unidimensional problems with one state variable, has been used in Austen-Smith and Riker (1987); Austen-Smith (1993); Morgan and Stocken (2008) among others.

[12]I adopt the convention of indexing typical agents with i and a typical decision-maker with j.

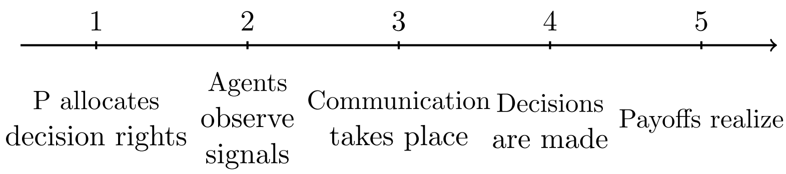


Figure 1: Timing of the Organizational Structure Game

communication will use the same measure of conflict of interest as in centralization, and optimality of such allocation would depend on the degree of centrality of preferences. As I shall show later, the two forms of delegation considered in the paper involve different measures of conflict of interest. The second clarification relates to the distinction between delegation of decision authority and decentralization on the access to information. In my framework, authority can be centralized or decentralized, the latter is called 'delegation' throughout the paper. Information, on the contrary, is always decentralized because it is dispersed among agents.

The equilibrium concept is pure-strategies Perfect Bayesian Equilibria (equilibrium, henceforth). A full characterization including mixed strategies is cumbersome and does not provide much intuitions beyond the pure-strategies case.[13]

I introduce some notation before defining equilibrium strategies. Let $k_r^*(j) \equiv k_r(\mathbf{m}_j^*)$ and $\ell_r^*(j)$ denote the number of truthful messages and 'ones' decision-maker j receives in equilibrium. Also, let k_r^j be agent i's conjecture about the number of agents other than him who reveal information about θ_r to j (on path).[14] In order to keep track of who decides what, let $j', j'' \in \{P, 1, ..., n\}$ be the decision-makers for y_1 and y_2, respectively (note that the number of apostrophes coincide with the index of the decision allocated to each player).

A Perfect Bayesian Equilibrium of this game is then characterized by the triple $\left(\mathbf{y}^*, \mathbf{m}_{j'}^*, \mathbf{m}_{j''}^*\right)$ which represents the vector of decisions, and the vectors of message strategies to j' and j'', respectively. Optimal actions satisfy:

$$y_1^* = w_{11}E(\theta_1|\mathbf{m}_{j'}^*) + w_{12}E(\theta_2|\mathbf{m}_{j'}^*) + b_1' \qquad y_2^* = w_{21}E(\theta_1|\mathbf{m}_{j''}^*) + w_{22}E(\theta_2|\mathbf{m}_{j''}^*) + b_2''$$

Where b_1' (b_2'') represents the bias of decision-maker j' (j'') with respect to y_1 (y_2). Note that centralization means $j' = j'' = P$ and the associated biases are equal to zero. From the principal's perspective, delegation of decision-rights has two payoff-relevant consequences. On the one hand, it implies a biased agent decides on behalf of the principal, resulting in a biased decision. On the other hand, individual incentives for communication depend on the conflict

[13]For an analysis of mixed-strategies in a similar framework see Habermacher (2018).

[14]Note that i's conjecture will be correct in equilibrium, and whenever his message strategy involves revealing the corresponding signal then $k_r^*(j) = k_r(j) + 1$, otherwise $k_r^*(j) = k_r(j)$.

of interest between each decision-maker and the agent in question. Different organizational structures, and decision-makers, then result in different communication incentives. Agent i's optimal message strategy to decision-maker j solves:

$$\mathbf{m}_j^{i*}(\mathbf{S}^i, \mathbf{b}^i, b_1', b_2'') = \arg\max_{\mathbf{m}_j^i} \left\{ E\Big[-\left(y_1\left(m_{j'}^i, \mathbf{m}_{j'}^{-i}\right) - \delta_1 - b_1^i\right)^2 - \left(y_2\left(m_{j''}^i, \mathbf{m}_{j''}^{-i}\right) - \delta_2 - b_2^i\right)^2 \Big] \Big| \mathbf{S}^i \right\}$$

Before the analysis of communication incentives, I clarify the equilibrium-selection criterion and introduce some notational conventions. Communication equilibria is selected on the basis of the decision-maker's ex-ante expected utility, a natural extension of the notion of 'most informative equilibrium' used in the literature.In order to simplify notation, let $k_r^{\mathrm{C}} \equiv \{k_r(j) | \, j' = j'' = P\}$ denote the number of truthful messages about $\theta_r = \{\theta_1, \theta_2\}$ the principal receives under centralization; let $k_r^{\mathrm{P1}} \equiv \{k_r(P) | \, j' = P\}$ the number of messages she receives when decides on y_1 only, and $k_r^{\mathrm{P2}} \equiv \{k_r(P) | \, j'' = P\}$ when she decides on y_2 only. For when P does not decide at all, let $k_r' \equiv k_r^*(j')$ and $k_r'' \equiv k_r^*(j'')$ refer to the number of truthful messages for decision makers of y_1 and y_2, respectively, while keep $k_r^j \equiv k_r^*(j)$ for a generic decision-maker.

Note that the principal's expected utility from different allocations of decision rights depends on the amount of information the different decision-makers are expected to receive on the equilibrium path (and their biases). I first analyse incentives for communication for a typical agent under different organizational structures; then, I proceed to study the role of informational interdependence on the principal-optimal structure.

Incentives for communication. I first describe agent i's communication incentives under delegation; that is, how much of his information he reveals to decision-maker j in charge of y_d. Because communication between i and j is private, incentives depend on decision-specific conflict of interest represented by b_d^i and b_d^j. But i's incentives to reveal information also depend on how many other agents are expected to be truthful to j on the equilibrium path. Since message strategies associated to each signal are degenerated, the precision of beliefs induced by truth-telling cannot be controlled by any individual.[15] As a consequence, communication of any signal depends on how many other agents are expected to reveal the same information in equilibrium. The two determinants just described constitute the traditional argument for communication of imperfect information via cheap talk (see Austen-Smith and Riker, 1987; Krishna and Morgan, 2001; Galeotti et al., 2013). In my framework, agents observe signals about two independent states, which introduces a third determinant of communication incentives.

Information about any state affects both decisions, but states' independence can produce signals that, if fully revealed, move decisions against the informational interdependence. To see

[15]This is true because signals are binary and I focus on pure strategies.

this, consider the case in which agent i's information consists of $(\tilde{S}^i_1, \tilde{S}^i_2) = \{(0,1)\}$. The overall influence of revealing both signals is smaller than, say, revealing $\tilde{S}^i_1 = 0$; in the former case, signals' influence compensate each other. Agent i will then be tempted to follow the signal that is more favourable to his interests.[16] As I show below, such ambiguous information creates a credibility loss that harms communication incentives. The Lemma below states the result.

Lemma 1 (Incentive Compatibility of Communication on y_d.)**.** *Consider an equilibrium* $(\mathbf{y}^*, \mathbf{m}^*)$ *in which the principal delegates y_d to agent j (and he does not decide on the other decision).*[17] *Then, revealing any information (either S^i_r or both signals) is incentive compatible for i if:*

$$|b^i_d - b^j_d| \leq \frac{1}{2}\left|\frac{w_{d1}}{(k^j_1+3)} - \frac{w_{d2}}{(k^j_2+3)}\right| \tag{1}$$

And revealing both signals when $\tilde{\mathbf{S}}^i\{(0,0);(1,1)\}$ and announcing the babbling message for other realizations if:

$$|b^i_d - b^j_d| \leq \frac{1}{4}\left[\frac{w_{d1}}{(k^j_1+3)} + \frac{w_{d2}}{(k^j_2+3)}\right] \tag{2}$$

Where $y_d = \{y_1, y_2\}$.

Proof. See Appendix B.3 □

The left-hand side of equation (1) represents the conflict of interest between agent i and decision-maker j; the right-hand side is a measure of i's expected utility gains from a inducing more precision on decision y_d, by means of revealing information to j. To see the role of each term of the right-hand side, suppose j decides over y_1 such that $w_{11} > 1/2$. If agent i expects many other agents to reveal information about θ_1 (higher k_1), the marginal influence of revealing S^i_1 is small because j is *ex-post* well-informed. In other words, i expects j's expectation of θ_1 to be very precise and, thus, has little incentives to reveal information about that state. These are the congestion effects studied in Galeotti et al., 2013. But i has information about θ_2 as well, and this introduces an additional effect.

Consider an equilibrium in which i is expected to reveal S^i_1 only. In such a case, whatever he says about θ_2 is not influential on path. As a consequence, when i's information about θ_1 is unfavourable and that about θ_2 is favourable, his incentives to deviate from revealing S^i_1 are larger because *he believes* the overall influence on y_1 should be smaller than what his information about θ_1 conveys. But in equilibrium j acknowledges this higher incentives to deviate, leading

[16]Here the notion of favourable (unfavourable) information is that of signals that would move decisions towards (against) his bias.

[17]By assumption, communication between decision-makers only involves own signals (not information transmitted by other agents). For an analysis on hierarchies as information intermediation see Migrow (2017).

to a loss of credibility in the form of a tighter IC constraint. This is the reason why incentives to reveal $S^i - 1$ also depend on information the decision-maker has about θ_2—k_2^j in equation (1). Note that i's incentives to deviate from revealing S_1^i are decreasing k_2^j, because they depend on i's expectation about the influence S_2^i would have on y_1 (which is decreasing in k_2). In a companion paper I discuss how such incentives can lead to congestion effects that benefit the decision-maker (see Habermacher, 2018). In Appendix B.3 I show that this credibility loss rules out message straegies in which i reveals only one signal under delegation.[18]

Incentives to reveal both signals depend on how balanced is j's equilibrium information. Here the notion of 'balance' relates to both the number of truthful messages received on path (k_r^j) and the relative importance of the corresponding state (w_{dr}). For instance, if $\frac{w_{11}}{(k_1^j+3)} > \frac{w_{12}}{(k_2^j+3)}$, decision-maker j is not receiving enough information about θ_1 relative to how important it is for y_1 (recall that $w_{11} > w_{12}$).

Equation (2) shows when dimensional non-separable messages arise under delegation. In the appendix I show the only of such strategies arising in equilibrium has i truthfully revealing both signals when $(\tilde{S}_1^i, \tilde{S}_2^i) = \{(0,0),(1,1)\}$ and sending the babbling message otherwise. Since this strategy involves babbling messages, i expects (ex-ante) to be influential for half of the possible pairs of signals realizations. As a consequence, the IC constraint in (2) is typically more restrictive than (1) and holds when the right-hand side in (1) is close to zero.[19]

Finally, note that what i says to j affects y_d but not the other decision, meaning that delegation kills the informational interdependence between decisions. The principal can then use the allocation of decision rights to *govern* the interdependence. Proposition 8 in Appendix A shows the equilibrium communication under delegation.

Under centralization, any information transmitted to the principal affects both decisions. Besides the informational interdependence (governed by $\mathbf{W}$), the influence of information on decisions depend on whether i reveals one or both signals, or play dimensional non-separable strategies. To see this, note that revealing only S_1^i has a larger influence on y_1 than on y_2, such that the bias on the first dimension weighs more heavily in determining i's incentives. When he reveals both signals truthfully, the overall influence is balanced (due to the normalization on the weighting matrix) and so the biases weights in determining communication incentives. The lemma presents the IC constraints and shows the different measures of conflict of interest depending on the information revealed.

Lemma 2 (Incentive Compatibility of Communication under Centralization)**.** *Consider an equilibrium* $(\mathbf{y}^*, \mathbf{m}^*)$*, truthful communication is be incentive compatible for agent i in the following*

[18]Formally, the IC constraints for any message strategy in which i reveals one signals coincides with that of full revelation such that the latter will always be selected by the decision maker.

[19]In Appendix B.3 I show that the RHS of (2) is larger than that of (1) if $\frac{3w_{d\tilde{r}}}{w_{dr}} \geq \frac{(k_{\tilde{r}}+3)}{(k_r+3)}$.

cases:

- *Revealing S_r^i, if:*

$$|\beta_r^i| \leq \frac{(w_{1r})^2 + (w_{2r})^2}{2} \left[\frac{1}{(k_r^{\mathrm{C}}+3)} - \frac{C_r}{(k_{\tilde{r}}^{\mathrm{C}}+3)} \right] \tag{3}$$

- *Revealing both when $\tilde{\mathbf{S}}^i = \{(0,0);(1,1)\}$, if:*

$$\frac{(w_{11})^2 + (w_{21})^2}{(k_1^{\mathrm{C}}+3)} \left[\frac{1}{(k_1^{\mathrm{C}}+3)} + \frac{C_1}{(k_2^{\mathrm{C}}+3)} \pm 2\beta_1 \right] + \frac{(w_{12})^2 + (w_{22})^2}{(k_2^{\mathrm{C}}+3)} \left[\frac{1}{(k_2^{\mathrm{C}}+3)} + \frac{C_2}{(k_1^{\mathrm{C}}+3)} \pm 2\beta_2 \right] \geq 0 \tag{4}$$

- *Revealing both when $\tilde{\mathbf{S}}^i = \{(0,1);(1,0)\}$, if:*

$$\frac{(w_{11})^2 + (w_{21})^2}{(k_1^{\mathrm{C}}+3)} \left[\frac{1}{(k_1^{\mathrm{C}}+3)} - \frac{C_1}{(k_2^{\mathrm{C}}+3)} \pm 2\beta_1 \right] + \frac{(w_{12})^2 + (w_{22})^2}{(k_2^{\mathrm{C}}+3)} \left[\frac{1}{(k_2^{\mathrm{C}}+3)} - \frac{C_2}{(k_1^{\mathrm{C}}+3)} \mp 2\beta_2 \right] \geq 0 \tag{5}$$

Where $\beta_r = b_1^i w_{1r} + b_2^i w_{2r}$, $C_r = \frac{w_{11}w_{12}+w_{21}w_{22}}{w_{1r}^2+w_{2r}^2} \in [0,1]$, and $\pm$ means that the condition must hold for the most restrictive of these operations, given the sign of the corresponding β.

Proof. See Appendix in Habermacher (2018). □

Lemma 2 shows how communication incentives depend on different measures of conflict of interest and how informational interdependence determines the way decision-specific biases aggregate. Communication is incentive compatible if the (expected) utility gains from increasing the decision-maker's precision compensate the relevant aggregate conflict of interest, measured by β_1^i and β_2^i. In this case, dimensional non-separable strategies arise in equilibrium for non-degenerate parameter values. The associated IC constraints can be found in Appendix A.

The fact that different message strategies affect decisions differently under centralization leads to two main differences with respect to delegation. First, revealing only one signal is incentive-compatible for non-empty set of $\mathbf{b}^i$, as shown in equation (3)—it constitutes the most-informative message strategy for some $\mathbf{b}^i$, as shown in Proposition 9 in the appendix. The overall influence of revealing information about a single state depends on how it affects decisions, which is governed by $\mathbf{W}$. Because i is imperfectly informed about both states, the influence also depends on the information the principal receives from other agents on the equilibrium path, which follows the same intuition of the delegation case.[20] The relevant measure of conflict

[20] When more other agents are expected to reveal information about the same state on path, i's incentives decrease; while if more agents are expected to revel information about *the other state* i's incentives for communication increases due to lower credibility loss.

of interest for this strategy aggregates the decision-specific biases according to their weight on each decision, as shown on the left-hand side of equation (3).

The second difference with delegation relates to the effects of ambiguous information. Under centralization, revealing both signals has a balanced overall influence on decisions. If those signals consist of ambiguous information, $\tilde{\mathbf{S}}^i = \{(0,1),(1,0)\}$), decisions move in opposite directions; for instance, if i credibly announces $\mathbf{m}_P^i = (0,1)$, then updated y_1 would be closer to 0 and y_2 closer to 1. Because the influence determines how decision-specific biases aggregate in the IC constraint, such influence on decisions can lead to dimensional non-separable strategies in which i credibly reveals ambiguous signals and sends babbling messages otherwise. The complete characterization of the most-informative equilibrium under centralization can be found in Appendix A. I now focus on the analysis of the optimal organizational structure.[21]

Optimal Organizational Structure. In this subsection I characterize the optimal organizational structure, and present the main result of this section: the optimal allocation of decision-rights depends on the existence and the extent of informational externalities.

Individual incentives for communication depend on the relevant measure of conflict of interest with the decision-maker, and the information transmitted by other agents. As a consequence, the profile of agents' preferences, $\mathbf{b}$, constitutes one of the main determinants of the equilibrium at the communication stage (the other being the informational interdependence). With the allocation of decision rights, the principal affects agents' incentives for communication and the resulting equilibrium; effectively, she chooses among the different equilibria induced by $\mathbf{b}$. Her actual choice takes into account two main elements: the amount of information revealed to each decision maker and their biases. The lemma below formalizes these two elements on the principal's ex-ante expected utility, for two generic decision-makers j' and j''.

Lemma 3 (principal's ex-ante expected utility)**.** *Consider an equilibrium* $(\mathbf{y},\mathbf{m})$ *in which* $j', j'' = \{P, 1, ..., n\}$ *decide on* y_1 *and* y_2*, respectively. The equilibrium is characterized by the number of agents reporting truthfully to each decision-maker,* k'_1*,* k'_2*,* k''_1*, and* k''_2*. Then, the principal ex-ante expected utility is given by:*

$$\hat{U}^P(\mathbf{b}, j', j'') = -\left[(b'_1)^2 + \frac{(w_{11})^2}{6\big(k'_1+2\big)} + \frac{(w_{12})^2}{6\big(k'_2+2\big)}\right] - \left[(b''_2)^2 + \frac{(w_{21})^2}{6\big(k''_1+2\big)} + \frac{(w_{22})^2}{6\big(k''_2+2\big)}\right] \quad (6)$$

Proof. See Appendix B.2 □

The first term in square brackets represents the principal's ex-ante expected utility associated to delegation of y_1 to decision-maker j'. Because his decision will be biased, the principal's

[21] For a more thorough discussion of communication incentives under centralization see Habermacher (2018).

utility is decreasing in b'_1. But her utility also depends on his equilibrium beliefs about θ_1 and θ_2 after communication with other agents. The precision of these expectations is determined by k'_1 and k'_2, meaning that she would like to delegate authority to a player whose preferences are central on the associated dimension. This is the trade-off the traditional argument focuses on: informational gains $k'1$ and k'_2 must compensate the loss of control (b'_1); and the same argument applies to delegation of y_2 to j''.

But informational interdependence can lead to a different source of utility gains for the principal. There may be agents willing to transmit more information when she decides over one decision as compared to centralization. This happens when, for instance, agents' biases are very large on the first dimension but small on the second. In such a case the principal may find optimal to delegate y_1 and, thus, agents' incentives for communication with her will only depend on $\mathbf{b}_2$, the low-conflict dimension. In other words, the principal benefits from delegation of a controversial decision if agents' preferences on the other dimension are aligned. The principal gets agents to transmit more information to her because delegation breaks the *negative informational externalities*—when a high-conflict decision impedes communication mainly affecting a low-conflict decision. Informational interdependence is a necessary condition for such externalities. The following lemma summarizes the two sources of informational gains from delegation.

Lemma 4. *Consider the equilibrium in which the principal delegates y_1 to agent j' and retain authority on y_2; then, utility gains derived from better information (if any) consist of:*

- ***Direct Informational Gains****: increased precision in the delegated decision. Relates to j''s preferences being more central than the principal's, whose utility gains materialize if:*

$$DIG_{j'}(y_1) \equiv \frac{(w_{11})^2}{6}\left[\frac{1}{\left(k_1^{\mathrm{C}}+2\right)} - \frac{1}{\left(k'_1+2\right)}\right] + \frac{(w_{12})^2}{6}\left[\frac{1}{\left(k_2^{\mathrm{C}}+2\right)} - \frac{1}{\left(k'_2+2\right)}\right] \geq (b'_1)^2$$

- ***Indirect Informational Gains****: increased precision on the retained decision. Relate to partial delegation breaking informational interdependencies. Utility gains for the principal take place if:*

$$IIG(y_2) \equiv \frac{(w_{21})^2}{6}\left[\frac{1}{\left(k_1^{\mathrm{C}}+2\right)} - \frac{1}{\left(k_1^{\mathrm{P}2}+2\right)}\right] + \frac{(w_{22})^2}{6}\left[\frac{1}{\left(k_2^{\mathrm{C}}+2\right)} - \frac{1}{\left(k_2^{\mathrm{P}2}+2\right)}\right] \geq 0$$

Moreover, whenever the optimal allocation of decision rights involves delegation of y_1 to j', then either $DIG_{j'}(y_1) > b'_1$, or $IIG(y_2) > 0$, or both.

Proof. The expressions for the informational gains arises from comparing equation (6) for different allocations of decision rights. The last part of the lemma results from the complete characterization of the optimal organizational structure of Proposition 11 in Appendix A. □

Direct informational gains (DIG, onwards) arise when more agents transmit information to the new decision-maker under delegation than to the principal under centralization (for at least one state). Similarly, indirect informational gains (IIG onwards) arise when more agents transmit information to the principal when she delegates one decision, as compared to centralization. In both cases, informational gains are more 'valuable' if associated to the state that is more relevant for the decision under consideration (θ_1 for y_1 and θ_2 for y_2). The last part of the lemma points out the intuitive results that informational gains (of any type) are necessary conditions for delegation to be optimal. Below I summarize conditions for existence of informational gains.

Lemma 5. *Consider an equilibrium in which the principal delegates y_1 to agent j' and retain authority on y_2. Let $h, i \in N$, be two agents such that $h, i \neq j$.*

Direct Informational Gains arise if and only if for every h who is willing to reveal a given signal to the principal under centralization, then there exist an agent i who is willing to reveal the same information to j' under delegation. In addition, among the second group of agents, there must be an agent whose preferences satisfy:

$$|b_1^i - b_1'| < \left| b_1^i + b_2^i \frac{w_{21}}{w_{11}} \right| \tag{7}$$

Indirect Informational Gains arise if and only if for every h who is willing to reveal information to the principal under centralization, then there exists an agent i who is willing to reveal the same information to the principal when she decides on y_2 only. In addition, among the second group there must be an agent whose preferences satisfy:

$$|b_2^i| < \left| b_1^i \frac{w_{12}}{w_{22}} + b_2^i \right| \tag{8}$$

Proof. See Appendix B.5 □

Lemma 5 presents necessary conditions for each type of informational gains. Direct informational gains arise if and only if j' has more central preferences on y_1; that is, there is at least one agent who is willing to reveal information to j' but not to the principal under centralization. Equation (7) reflects this condition, it shows that the relevant conflict of interest between i and j' (left-hand side) is lower than that between i and the principal under centralization (right-hand side). As mentioned earlier, delegation to agents with more central preferences constitutes the 'traditional' argument in the literature of organizational design with imperfectly informed senders (see Dewan et al., 2015; Dewan and Squintani, 2015). The presence of informational interdependencies, however, can lead to other mechanisms.

Indirect gains arise only if there is at least an agent who reveals more information to the principal when she decides on y_2 only than when she decides on both issues. Equation (8) shows that the conflict of interest with the principal under partial delegation (left-hand side) is lower than the aggregate conflict of interest. In particular, since b_2^i is in both sides of the equation, the inequality requires that the bias on the first dimension is so large that communication under centralization is less informative than when the principal only decides on y_2. This is the notion of *negative informational externalities* I first described by Levy and Razin (2007).[22] The presence of such externalities is a necessary condition for indirect informational gains.

Full Delegation is optimal when there are two different agents with central preferences and no informational externalities associated to retaining authority. The following observation summarizes the intuition behind Full Delegation.

Observation 1. *When the principal delegates both decisions, it is due to Direct Informational Gains in both —that is, $k_1^{'} > k_1^{\mathrm{C}}$ and $k_2^{''} > k_2^{\mathrm{C}}$.*

The allocation of decision rights depends then on the different communication equilibria induced by the profile of preferences. The previous discussion made clear that the argument reduces to whether the direct and/or indirect informational gains are sufficiently large, where sufficiency involves the loss of control from delegation. Proposition 11 in Appendix A provides a precise formulation of this argument, serving as a formal counterpart of the discussion below.

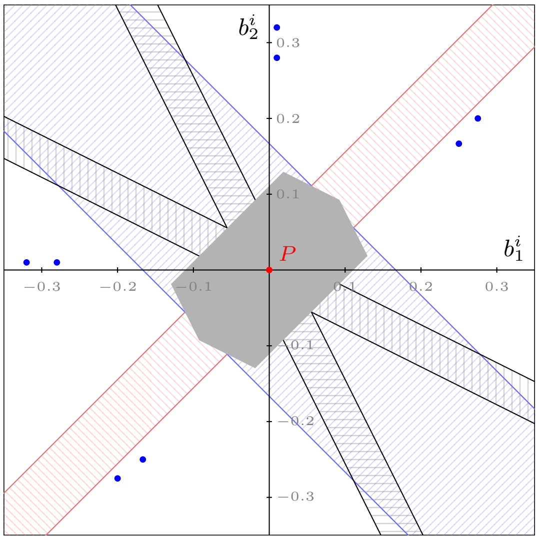
The principal finds optimal to delegate both decisions when there are two agents with central preferences on the corresponding dimension. Again, because delegation breaks interdependence, the conflict of interest is measured by the decision-specific bias. But informational gains must at least compensate the principal's utility losses from biased decisions, as well as the potential indirect informational gains if she retained any single decision. Full delegation is then optimal if either there are no indirect informational gains or they are small relative to the direct gains on the corresponding decision.

The direct informational gains associated to one decision can be so large that the principal is willing to tolerate some information loss (with respect to centralization). In such a case, full delegation dominates partial delegation only if minimizing the informational loss requires delegating the other decision; but the informational gains must compensate for these losses.

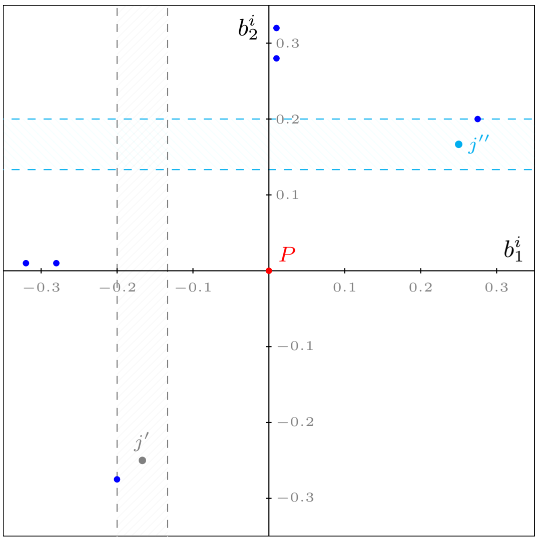
In order to see this, Figure 2 depicts incentive compatibility constraints under different organizational structures, for a given distribution of preferences —agents' ideal decisions in blue

[22] Levy and Razin describe negative informational spillovers as the case when i's bias with respect to y_1 is so large that he would not reveal any information under centralization, even though $|b_2^i|$ is very small.

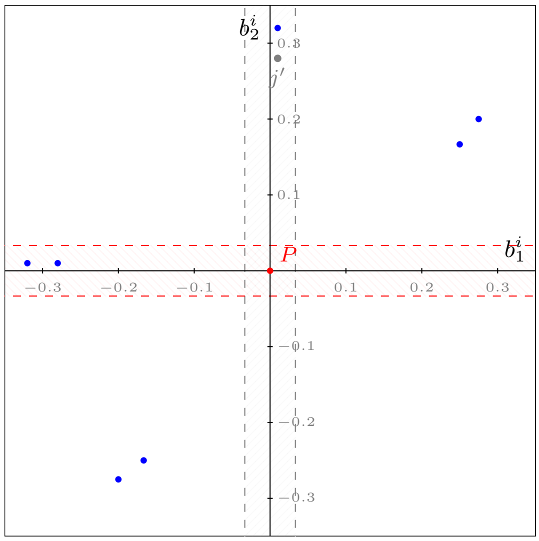
Figure 2: Different Organizational Structures



(a) Centralization: $k_1^{\mathrm{C}} = k_2^{\mathrm{C}} = 0$



(b) Full Delegation: $k'_1 = k'_2 = 2$ and $k''_1 = k''_2 = 2$



(c) Partial Delegation: $k'_1 = k'_2 = 2$ and $k_1^{\mathrm{P}2} = k_2^{\mathrm{P}2} = 2$

Full Revelation | S_1 only | S_2 only | Babbling

FR to j' | FR to j'' | FR to P | Babbling

Note: $w_{11} = w_{22} = \frac{2}{3}$

dots, principal's as a red dot. The top panel shows the incentive compatibility of communication under centralization; no agent's ideal decisions lie on the IC constrains, meaning that no agent is revealing any information in the centralization equilibrium. In panel (b) the principal delegates y_1 to agent j' (grey dot) and y_2 to j'' (magenta dot); leading to an equilibrium in which each decision-maker receives two signals per state –one pair is observed by himself, the other truthfully reveal by another agent. This shows the informational gains from delegation, since decision-makers have more central preference than the principal under centralization.

Panel (c) shows the case of Partial Delegation. By delegating y_1 to agent j' (in grey), the principal's conflict of interest with the two agents in the II quadrant depends on the bias in y_2, which is close to zero. This is a clear example of negative informational externalities: the presence of a conflictual decision (y_1) under centralization breaks communication between these agents and the principal. Delegation of y_1 to j' breaks this interdependence and the agents are willing to truthfully reveal information associated to y_2.[23]

Finally, centralization is ex-ante optimal when the informational gains from delegation are insufficient (if any) relative to the loss of control. In other words, the principal's preferences are sufficiently central in both decisions given the correlation between them. This has to be true even in the presence of negative externalities, meaning that the utility loss from delegation is larger than the informational gains derived from breaking those externalities.

Negative informational externalities are necessary conditions for indirect gains. Since large informational gains lead to delegation, one can think that delegation will be optimal when informational externalities are large. But these externalities can also be beneficial for the principal, which happens when decision-specific biases are large but the aggregate conflict of interest under centralization is small. The proposition below presents the optimal organizational structure under informational externalities.

Proposition 1. *Let* $\mathbf{b} = (\mathbf{b}^1, ..., \mathbf{b}^n)$ *denote a given profile of biases for the* n *informed agents. The optimal organizational structure is characterized by the number of truthful messages decision-makers receive in equilibrium,* $\mathbf{k}'$, $\mathbf{k}''$, *and the number of truthful messages the principal receives under centralization,* $\mathbf{k}^{\mathrm{C}}$. *Suppose that* $k_1' = k_2'$, $k_1'' = k_2''$, *and* $k_1^{\mathrm{C}} = k_2^{\mathrm{C}}$. *Now consider the associated game consisting of* $n + m$ *agents, with the profile of biases for the first* n *agents remaining the same as in the original profile, that is* $\tilde{\mathbf{b}} = (\mathbf{b}^1, ..., \mathbf{b}^n, \tilde{\mathbf{b}}^{n+1}, ..., \tilde{\mathbf{b}}^{n+m})$.

1. *For* $\tilde{\mathbf{b}}^{n+1} = \tilde{\mathbf{b}}^{n+m} = (\infty, 0)$*; then, there exists a sufficiently large* m *such that the optimal organizational structure in the game with* $n + m$ *agents is partial delegation of* y_1 *only.*

2. *For* $\tilde{\mathbf{b}}^{n+1} = \tilde{\mathbf{b}}^{\frac{2n+m+1}{2}} = (-\infty, \infty)$ *and* $\tilde{\mathbf{b}}^{\frac{2n+m+1}{2}} = \tilde{\mathbf{b}}^{n+m} = (\infty, \infty)$*; then, there exists a sufficiently large* m *such that the optimal organizational structure in the game with* $n + m$ *agents is centralization.*

Proof. See Appendix B.6 □

[23] It is straightforward to note that the principal could have obtained the same ex-ante welfare by delegating y_2 to the leftmost agent with preferences in the bottom right corner. Note that the optimality of partial delegation does not depends on asymmetries on agents' preferences as in Rantakari (2008).

Informational externalities in the above result are captured by the preferences of the 'additional' agents, such that m reflects the intensity of the externalities. The proposition shows that sufficiently large negative externalities lead to partial delegation, because the principal finds optimal to delegate the controversial decision in order to induce the additional agents to reveal the information they have. Sufficiently large externalities means the additional information the principal expects to receive brings her a higher expected utility than the allocation of decision rights with the original profile of preferences.

Similarly, the second case in Proposition 1 reflects the case of positive informational externalities. The additional agents in both groups of preferences are willing to play dimensional non-separable strategies under centralization. Senders in the first group fully reveal their signals when $\mathbf{S}^i = \{(0,0);(1,1)\}$ and announce the babbling message for the other possible realizations; while senders in the second group fully reveal their signals when $\mathbf{S}^i = \{(0,1);(1,0)\}$ and announce the babbling message otherwise. As in the previous case, the additional information the principal expects to receive from these senders bring her a higher expected utility than the optimal allocation under the original profile of preferences.

Because the choice among organizational structure depends on the trade-off between informational gains and loss of control, in the following subsection I study it in more depth.

2.1 Relationship between informational gains and loss of control

Since agents are imperfectly informed about the two states, informational gains from delegation are always imperfect—unlike models assuming perfectly-informed senders, e.g. Gilligan and Krehbiel (1987, 1989) and Dessein (2002). Indeed, the principal will consider delegation if informational gains compensate the loss of control due to the agent's bias. The larger his bias, the larger the informational gains necessary to compensate the principal for the loss of control. This defines a relationship between minimum informational gains and loss of control, which I present in Proposition 2 after the following notational convention.

Let player j decide on y_d, then the ex-ante expected variance associated to state θ_r on the equilibrium path is given by.

$$\mathrm{Var}\left(\theta_r|\mathbf{m}_j\right) \equiv \frac{1}{6(k_r^j+2)}$$

Proposition 2 (Maximum Admissible Loss of Control)**.** *For any profile of biases, denote by* $\mathbf{m}_{\mathrm{C}}$ *the vector of messages the principal receives in the equilibrium under centralization, and by* $\mathbf{m}_j = \{\mathbf{m}_{j'}, \mathbf{m}_{j''}\}$ *the equilibrium messages for* j' *and* j'' *in the equilibrium under delegation, for* $j', j'' = \{P, 1, 2, ...\}$*. In addition, let* $\mathbf{b}^{\mathrm{D}} = (b_1', b_2'')$ *be the decision-makers' biases on the corresponding decision. Then, the maximum* $\mathbf{b}^{\mathrm{D}}$ *for which the principal is willing to delegate at* *least one decision is given by:*

$$||\mathbf{b}^{\mathrm{D}}|| \equiv \left[\sum_{y_d}\left[w_{d1}^2\big(\mathit{Var}(\theta_1|\mathbf{m}_{\mathrm{C}}) - \mathit{Var}(\theta_1|\mathbf{m}_j)\big) + w_{d2}^2\big(\mathit{Var}(\theta_2|\mathbf{m}_{\mathrm{C}}) - \mathit{Var}(\theta_2|\mathbf{m}_j)\big)\right]\right]^{\frac{1}{2}}$$

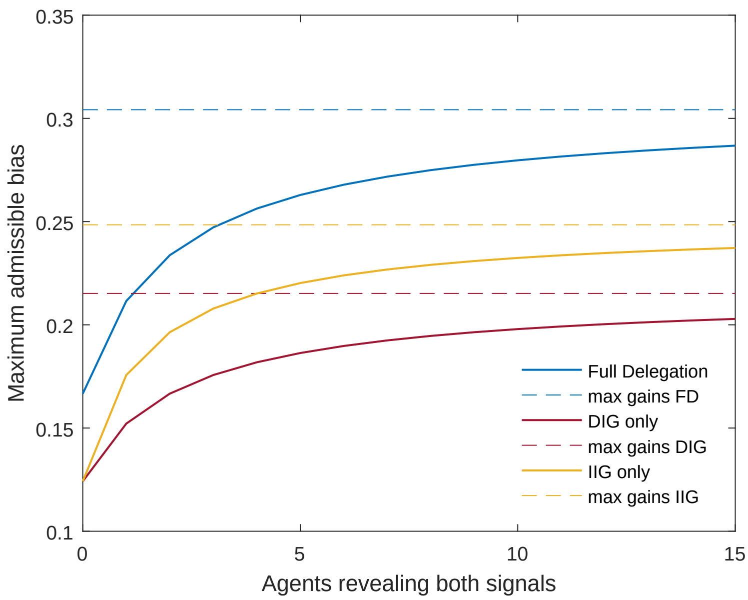
Moreover, the 'marginal value' of an additional signal is decreasing in the amount on information the principal receives under centralization. Let $\Delta\,\mathit{Var}(\theta_r|k_r^{\mathrm{C}}) = \mathit{Var}(\theta_r|\mathbf{m}_{\mathrm{C}}) - \mathit{Var}(\theta_r|\mathbf{m}_j, k_r^j = k_r^{\mathrm{C}}+1)$*, then:*

$$\frac{\partial \Delta\,\mathit{Var}(\theta_r|k_r^{\mathrm{C}})}{\partial k_r^{\mathrm{C}}} = \frac{1}{6}\left[\frac{1}{(k_r^{\mathrm{C}}+3)^2} - \frac{1}{(k_r^{\mathrm{C}}+2)^2}\right] < 0 \qquad (9)$$

Proof. See Appendix B.7 □

The maximum bias the principal is willing to tolerate is positively associated to informational gains from delegation. The expression $||\mathbf{b}^{\mathrm{D}}||$ represents the relevant measure for the loss of control: how far are both decisions to the principal's (state-dependent) preferred ones. Under partial delegation one of the elements in $\mathbf{b}^{\mathrm{D}}$ is zero, and the possibility of breaking negative externalities could lead to delegation even if the new decision-maker is uninformed in equilibrium. The right-hand side represents the reduction of the ex-post variance for each state that results from delegation of at least one decision.

Figure 3: Admissible loss of control as a function of informational gains



Note: $w_{11} = w_{22} = \frac{2}{3}$, $k_1^{\mathrm{C}} = k_2^{\mathrm{C}} = 0$

To see this, Figure 3 shows $||\mathbf{b}^{\mathrm{D}}||$ when $k_1^{\mathrm{C}} = k_2^{\mathrm{C}} = 0$ and $k_1^j = k_2^j = 0$ for $j = \{j', j''\}$. I am thus assuming the principal does not receive any truthful message under centralization;

ultimately, a restriction on the profile of biases. The blue lines represent the relationship between informational gains and bias under Full Delegation. Minimal informational gains in this case means decision-makers only have their own signals in equilibrium, and the principal will tolerate up to $||\mathbf{b}^{\text{D}}|| = 0.176$. But this figure increases dramatically as long as both decision-makers start receiving information from other agents. In particular, the first 5 agents revealing both signals explain 85% of the maximum admissible bias—the hypothetical case in which both decision-makers are perfectly informed ex-post (dashed line).[24] Note that the horizontal axis in this case represents the number of agents revealing both signals to each decision-maker.

The red lines represent the case of direct informational gains only: the loss of control the principal tolerates as a function of $k_1^j = k_2^j$ when there are no indirect informational gains. A pattern similar to the full delegation case appears.[25] Note that 70% of the maximum admissible loss under Full Delegation is explained by informational gains in one decision only. Large informational gains in one dimension can thus justify delegation to biased agent; in such a case the principal will not tolerate much additional loss of control from the other decision-maker. In other words, the presence of a biased agent with sufficiently central preferences in one dimension can lead to partial delegation, mainly because it blocks delegation of the other decision to more moderate agents with less central preferences.

The yellow lines represent the case of partial delegation when informational gains are only indirect, i.e. the principal receives more information on the retained decision. This is a measure of how important negative externalities can be. It measures the loss of control she is willing to tolerate in order to recover communication on the decision affected by externalities. The maximum bias when indirect informational gains are maximal ($k^{\text{P}d} = \infty$) is $||\mathbf{b}^{\text{D}}|| = 0.2484$. She would tolerate some loss even when not receiving any information in equilibrium, because j observes himself two signals. This relationship is increasing in the information held by j.

Finally, expression (9) shows that the reduction in expected residual variance from an additional signal (in a delegation equilibrium) is decreasing in the number of truthful messages the principal receives under centralization. In other words, an additional signal from delegation represent less of an 'informational gain' when principal receives more information under centralization. The maximum bias she is willing to tolerate thus decreases as the profile of biases allows more agents to truthfully reveal information under centralization. This can be seen as an extension of Corollary 1 in Dessein (2002) to the case of imperfectly informed senders: incentives to delegate depend on the magnitude of the residual variance. The residual variance here depends on the amount of information received by the principal under centralization.[26]

[24] The maximum admissible bias with $k_1' = k_2' = k_1'' = k_2'' = \infty$ is $||\mathbf{b}^{\text{D}}|| = 0.3042$.

[25] When j' is fully informed in equilibrium the principal will tolerate a bias of $|0.2152|$.

[26] In Dessein's paper there is a single, perfectly informed sender who advices the principal on a one-dimensional

3 Endogenous Information Acquisition

In this section I analyse agents' incentives to acquire information before the communication stage, but after decision-rights have been allocated. I first present the extended model. Secondly, I derive the two incentive-compatibility constraints involved in information acquisition decisions—communication IC and cost-effectiveness— and show how information costs impose restrictions on informational gains from delegation. I then characterize the equilibrium strategies for a generic agent, showing the cases in which he decides to specialize. Finally, I characterize how the optimal organizational structure of Proposition 11 changes with the cost of information, and show that delegation typically leads to *ex-post specialization*.

In principle, agents could have the choice on how much information about each state to observe, involving information acquisition at the intensive and the extensive margins. Here, however, I focus on the extensive margin, meaning that each agent decides whether to observe at most one binary signal per state. This approach allows me to capture in a parsimonious way the effects of informational interdependence on information acquisition. In section 4, I discuss some implications of allowing agents to acquire information on the intensive margin.

Each agent has thus access to one binary trial per state and decides which realizations to observe (if any). Formally, let $\mathfrak{s}^i \in \left\{\{\emptyset\},\{\tilde{S}_1\},\{\tilde{S}_2\},\{\tilde{S}_1,\tilde{S}_2\}\right\}$ be agent i's information-acquisition decision. With this formulation i's type is given by the realizations of both signals, but he decides the extent to which he observes his type.

Definition 1. *The information structure for agent i in the Information Acquisition game consists of the following elements: $\mathbf{S}^i = (S_1^i, S_2^i)$ are the signals available to him, $\tilde{\mathbf{S}}^i = (\tilde{S}_1^i, \tilde{S}_2^i)$ the realization of the corresponding signals (his type), and $\mathfrak{s}^i \in \left\{\{\emptyset\},\{\tilde{S}_1\},\{\tilde{S}_2\},\{\tilde{S}_1,\tilde{S}_2\}\right\}$ the information he actually decides to observe.*

The costs of different information structures are captured by the function $C(\circ)$, with $C\left(\{\tilde{S}_1,\tilde{S}_2\}\right) > C\left(\{\tilde{S}_1\}\right) = C\left(\{\tilde{S}_2\}\right) > C\left(\emptyset\right) = 0$. The principal has no direct access to information. The preferences of agent $i = \{1,\dots,n\}$ are given by:[27]

$$U^i(\boldsymbol{\theta},\mathbf{x},\mathbf{b}^i,\mathfrak{s}^i) = -\sum_{y_d=\{y_1,y_2\}} \left(y_d - \delta_d(\theta_1,\theta_2) - b_d^i\right)^2 - C(\mathfrak{s}^i)$$

Beside the costs of information, agents' incentives to acquire any piece of information depend on its expected utility gains. The information in hands of an agent affects his payoffs only

decision problem. He analyses incentives for delegation as a function of the ex-ante variance associated to the state, against the babbling equilibrium. Delegation dominates communication for every bias in which the sender would reveal any information to the principal (Proposition 2).

[27]The principal's preferences are captured by $U^P(\boldsymbol{\theta},\mathbf{x}) = -\left(y_1 - \delta_1(\theta_1,\theta_2)\right)^2 - \left(y_2 - \delta_2(\theta_1,\theta_2)\right)^2$.

if it induces updates in beliefs and thus affects decisions. This implies the agent expects to reveal that information truthfully to at least one decision-maker. As a consequence, agents only acquire information they are willing to reveal at the communication stage on the equilibrium path. But because information is costly and there may be other agents willing to reveal the same information, the expected utility gains from acquiring and revealing any signal must be sufficiently large. Note also that acquiring information about a single state gives an agent relatively higher influence over one decision, which leads to the notion of *specialization.* I denote a *specialist* an agent who has information about one state only.

Figure 4 shows the timing of the game. The allocation of decision rights is observed by all agents. Knowing who decides what, each agent chooses the information he will observe (if any). I assume overt information acquisition—individual decisions (but not information) is common knowledge. In Section 4 I discuss the implications of relaxing this assumption.

The communication stage is similar to the previous section, so I keep the notation. Let i be a generic agent and j a generic decision-maker, let $j', j'' \in \{P, 1, ..., n\}$ be the decision-makers for y_1 and y_2, respectively, such that $i = \{1, ...n\}$ and $j = \{j', j''\}$. Let $k_r^j \equiv k_r^*(\mathbf{m}_j^*(\mathfrak{s}^*))$ be the number of truthful messages decision-maker j receives in equilibrium, and $k_r' \equiv k_r^*(j')$ and $k_r'' \equiv k_r^*(j'')$ refer to the number of truthful messages to decision makers j' and j'' (respectively).

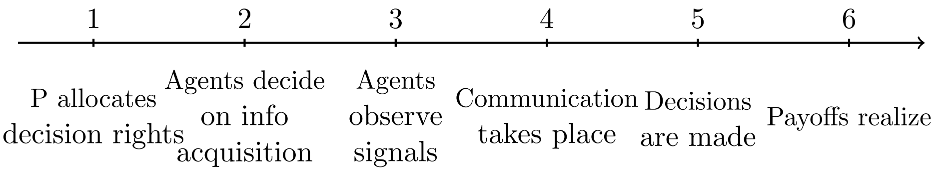


Figure 4: Timing of the Org. Structure / Info Acquisition game.

A PBE in this game is then characterized by the decision vector, $\mathbf{y}_d^*$, and collections of message and information acquisition strategies for each agent and decision-maker j, $\mathbf{m}_j^* = \{\dots, \mathbf{m}_j^{i*}, \dots\}$ and $\mathfrak{s}^* = \{\dots, \mathfrak{s}^{i*}, \dots\}$. The expressions for optimal actions and messages are similar to those of the previous section, noting that $k_r^*(\mathbf{m}^*(\mathfrak{s}^*))$, $y_d^*(\mathbf{m}_j^*(\mathfrak{s}^*))$, and $\mathbf{m}_j^{*i}\left(\mathfrak{s}^i, \mathbf{m}^{\text{-}i}(\mathfrak{s}^{\text{-}i})\right)$. Agent i's information acquisition strategy is given by:

$$\mathfrak{s}^{i*} = \arg\max_{\mathfrak{s}^i} \left\{ E\Big[-\left(y_1\left(\mathbf{m}_{j'}^i(\mathfrak{s}^i), \mathbf{m}_{j'}^{\text{-}i}\right) - \delta_1 - b_1^i\right)^2 - \left(y_2\left(\mathbf{m}_{j''}^i(\mathfrak{s}^i), \mathbf{m}_{j''}^{\text{-}i}\right) - \delta_2 - b_2^i\right)^2 \Big] - C(\mathfrak{s}^i) \right\}$$

The expectation is based on equilibrium beliefs. Agent i's equilibrium message strategy depends on the information he acquired in an earlier stage of the game. Both his message and information acquisition decisions depend on beliefs about other agents' strategies. The fact that information acquisition is common knowledge at the communication stage (overt game) simplifies the beliefs space. In other words, each agent forms a conjecture about other agents' acquisition strategies when deciding about his own, and also a conjecture about other agents' message strategies, the latter depending on i's knowledge of the information they have.

At the communication stage, incentive compatibility depends on the signals agent i has previously acquired. When he acquires information about both states, the IC constraints for communication are the same as in Lemmas 1 and 2. When he acquires information about one state, however, the IC constraints change dramatically because his incentives to reveal information are not affected by beliefs about the other state. This kills the credibility loss and truthful communication is incentive compatible for a broader set of bias vectors. Now, let me focus on the details of these arguments.

First, costly information acquisition means that each agent will invest in a signal only if he expects to benefit from it. Having more signals *per se* do not affect other agents' communication incentives. Agent i only affects k_r^j by truthfully revealing the acquired signal.[28] In equilibrium, then, he only acquires signals he is willing to reveal; if he fails to reveal any piece of information (off path), no other agent will change his on-path message strategy. The number of truthful messages does not change when i acquires a signal he does not reveal, but this information acquisition strategy is suboptimal because he bears the costs. The lemma below formalizes the fact that incentive compatibility at the information acquisition stage requires incentive compatibility at the communication stage.

Lemma 6. *Let $(\mathbf{y}^*, \mathbf{m}^*, \mathfrak{s}^*)$ be equilibrium strategy profiles for the principal and all agents. The equilibrium is characterized by the number of truthful messages decision-makers receive, $k_1^j\left(\mathbf{m}_j^*(\mathfrak{s}^*)\right)$ and $k_2^j\left(\mathbf{m}_j^*(\mathfrak{s}^{i*})\right)$, for $j = \{j', j''\}$. Then, agent i equilibrium information acquisition strategy, $\mathfrak{s}^{i*}$, satisfies:*

- *$S_r \in \mathfrak{s}^{i*}$ only if truthful revelation to j is incentive compatible, given $k_r^j\left(\mathbf{m}_j^*(\mathfrak{s}^*)\right)$;*
- *$\{S_1, S_2\} \in \mathfrak{s}^{i*}$ only if full revelation to j is incentive compatible, given $k_r^j\left(\mathbf{m}_j^*(\mathfrak{s}^*)\right)$.*

Proof. See Appendix B.8. □

The main implication of Lemma 6 is that the choice of organizational structure will affect agents' incentives for information acquisition, because it determines the relevant IC constraints at the communication stage. Incentives to acquire information depends on the possibility of being influential. But credibility hinges on both the conflict of interest and the number of other agents expected to reveal similar information on path. It is not surprising that agents

[28]Formally, i's incentives for communication depend on having acquired the signal, $\mathbf{b}^i$, and on his conjecture about k_1^j and k_2^j. Then, for i acquiring S_r^i *off-path* to change another agent h's conjecture about k_r^j, b^i should be such he is willing to reveal that signal. In such a case, h (off-path) conjecture for k_r^j should be larger than the equilibrium value, but then i would be willing to reveal S_r^i in equilibrium and would have acquired it.

only acquire information their expect to truthfully reveal on the equilibrium path, given the allocation of decision rights. The conclusion is similar to Di Pei (2015): information structures available to agents and the cost function satisfy assumptions 1 and 2 of his paper ("Richness" and "Monotonicity"). Both seem rather natural assumptions in my framework; for a given information structure a 'coarser' alternative means investing in less signals, which will also be cheaper than the original choice.

Another implication of Lemma 6 is that i's information acquisition decision is equivalent to choosing between message strategies given other agents' behaviour in the equilibrium under consideration. In particular, conditional on having acquired information about one state only, i's incentives for communication do not depend on his information about the other state. This eliminates the credibility loss due to ambiguous information, thus enlarging the set of biases for which revealing that signal is incentive compatible. The proposition below shows the result.

Proposition 3. *Let $S_r \in \mathfrak{s}^{i*}$ and $S_{\tilde{r}} \notin \mathfrak{s}^{i*}$ for $\theta_r \neq \theta_{\tilde{r}}$, and $k_r^j\left(\mathbf{m}_j^*(\mathfrak{s}^*)\right)$ be i's conjecture about other agents revealing their information about θ_r to decision-maker j. Then, agent i's IC constraint for revealing S_r^i is:*

- *When $j = P$ decides on both issues (centralization),*

$$|\beta_r^i| \leq \frac{(w_{1r})^2 + (w_{2r})^2}{2\left(k_r^{\mathrm{C}} + 3\right)} \tag{10}$$

- *When j decides on y_d only,*

$$|b_d^i - b_d^j| \leq \frac{w_{dr}}{2\left(k_r^j + 3\right)} \tag{11}$$

Proof. See Appendix B.9 □

When an agent decides to acquire information about one state only, he does not face the possibility of ambiguous information. This eliminates the temptation to lie when information about the other state is more favourable, thus killing the credibility loss. The formal implication of this is that truthful revelation of information about that state will be incentive compatible for a larger set of biases. In other words, *specialization* acts as a commitment device because the agent does not know when an unfavourable signal produces an excessive update against his preferences. Another implication of this results relates to incentives to acquire information when costs are zero, as shown in the following corollary.

Corollary 1. *Let $C(S_1^i) = C(S_2^i) = 0$. If agent i's preferences satisfy:*

$$|\beta_1^i| \in \left(\frac{(w_{1r})^2 + (w_{2r})^2}{2}\left[\frac{1}{(k_r^{\mathrm{C}} + 3)} - \frac{C_r}{(k_{\tilde{r}}^{\mathrm{C}} + 3)}\right] \; ; \; \frac{(w_{1r})^2 + (w_{2r})^2}{2\left(k_r^{\mathrm{C}} + 3\right)}\right]$$

Then, in the most informative equilibrium under centralization he acquires and truthfully reveals information about θ_1 only.

The result above says that *even i has costless access to information* both he and the principal prefer that he specializes in information about θ_1. In other words, the principal prefers a less informed agent because it ensures her that he will not be tempted to manipulate information. The same result can be derived for the case of delegation.

Note that the two results above hinge on the assumption that the information acquisition decisions are observable. In Section 4, I characterize incentives to acquire and reveal information under covert information acquisition. Both results may have interesting implications about how firms organize subunits' access to information, because raising the costs of acquiring some type of information may raise the credibility on information they already have low costs.

Another relevant implication of Proposition 3 is that it rules out dimensional non-separable message strategies under delegation. To see this, compare equation (11) with (2). First, note that the measures of conflict of interest (left-hand sides) are the same, while the expected utility gains (right-hand sides) are larger when i acquires one signal only—equation (11). Second, the principal ex-ante prefers to receive information about one state for sure to receive information about both states half of the time in expectation (see Appendix B.9).

Incentive compatibility at the communication stage is only a necessary condition for information acquisition. Whether the utility gains from revealing the information acquired are enough also depend on its costs. Utility gains from revealing any piece of information are decreasing in the number of other agents revealing the same information (k_r^j). Given costs are strictly positive, there exists a k_r^j for which acquiring S_r^i will not be incentive compatible (despite revealing it could still be). The following lemma presents the *cost-effectiveness* condition, which captures this idea.

Lemma 7. *Let k_r^j denote i's conjecture about other agents revealing S_r truthfully to j.*

Centralization: *acquiring signal S_r^i is cost-effective for i under centralization if:*

$$C(S_r^i) \leq \frac{(w_{1r})^2 + (w_{2r})^2}{6(k_r^{\mathrm{C}} + 2)(k_r^{\mathrm{C}} + 3)} \tag{12}$$

When willing to play DNS message strategies, cost-effectiveness requires:

$$2\,C(S_1^i, S_2^i) \leq \frac{(w_{11})^2 + (w_{21})^2}{6(k_1^* + 2)(k_1^* + 3)} + \frac{(w_{12})^2 + (w_{22})^2}{6(k_2^* + 2)(k_2^* + 3)} \tag{13}$$

Delegation: *acquiring signal S_r^i is incentive compatible for agent i if for at least one decision, y_d, with the corresponding decision maker j is true that:*

$$C(S_r^i) \leq \frac{(w_{dr})^2}{6(k_r^j+2)(k_r^j+3)} \tag{14}$$

Proof. See Appendix B.10. □

An agent acquires a signal if its expected influence on decision(s) is sufficiently large. The influence of revealing information about a state depends on how many other agents are expected to reveal the same information, which in turn depends on the organizational structure. Under centralization, revealing a given signal influences both decisions. Under delegation, the influence depends on whether i reveals the signal to one or both decision-makers—affects one or both decisions. The ex-ante expected utility gains of acquiring (and revealing) a given signal is thus weakly lower under delegation.

Dimensional non-separable message strategies face a more restrictive cost-effectiveness condition, because i expects to reveal information for half of the possible signal realizations. The costs of acquiring both signals must then be sufficiently low for such a strategy to be cost-effective. The latter does not hold if revealing one signal is also incentive compatible for i, in which case the most-informative equilibrium consists of acquiring and revealing information about one state.

Because the expected influence of truthful revelation is decreasing in the number of other agents revealing the same information, there exists a maximum number of agents for whom cost-effectiveness hold with respect to any signal.

Corollary 2. *The maximum number of agents acquiring S_r in any equilibrium under centralization is given by:*

$$K_r^{\mathrm{C}} = \left\lfloor \left[\frac{1}{4} + \frac{[(w_{1r})^2 + (w_{2r})^2]}{6\, C(S_r^i)}\right]^{1/2} - \frac{5}{2} \right\rfloor + 1 \tag{15}$$

And under delegation, the maximum number of agents acquiring S_r in any equilibrium is:

$$K_r^{\mathrm{D}} \in \left[\left\lfloor \left[\frac{1}{4} + \frac{(\hat{w}_{dr})^2}{6\, C(S_r)}\right]^{1/2} - \frac{5}{2} \right\rfloor + 1\, ;\, K_r^{\mathrm{C}} \right] \tag{16}$$

Where $\hat{w}_{dr} \equiv \min\{w_{1r}, w_{2r}\}$

Expression (15) represents the maximum number of agents, other than i, for whom investing on S_r is cost-effective under centralization (denoted by K_r^{C}). This number depends on the overall influence of θ_r and the cost of acquiring the associated signal. Under delegation, the same measure (K_r^{D}) depends on how many agents are willing to reveal that information to both decision-makers. If the profile of biases features many of such agents, then $K_r^{\text{D}} = K_r^{\text{C}}$ because each agent affects both decisions. But the influence for agents revealing their signals to one decision-maker only is lower, and so the maximum number of those willing to acquire that signal.

I now discuss agent i's equilibrium strategies under centralization and delegation, both of which are formally characterized in Appendix A. Agent i's incentives to acquire and reveal any piece of information combines the results of Lemmas 6 and 7. Under any allocation of decision-rights, the number of agents revealing the same piece of information is an important determinant of cost-effectiveness. It could lead some agents to acquire less information than what is IC at the communication stahe, resulting in either specialization or non-investment.

Dimensional non-separable message strategies can arise when the principal centralizes decision-making. As in Section 2, these strategies take the form of full revelation for some signals realizations and announcing the babbling message for the other realizations. Such strategies only arise when the costs of information are sufficiently low and revealing information about one state in not IC. Then, agent i typically prefers to reveal a single signal than play a DNS strategy, provided both are incentive compatible and cost-effective.[29]

Similar intuitions apply to the case of delegation. In presence of two decision-makers, agents acquire a signal if it is cost-effective and incentive compatible to reveal it to at least one of them. As discussed in Proposition 3, the possibility of not acquiring one of the signals makes credible the strategy of revealing only one signal. When such a strategy is incentive compatible it always refers to the state the decision-maker is less informed in equilibrium. In other words, the agent will acquire the signal for which the influence is larger, which in turn implies that dimensional non-separable strategies are dominated by this strategy. As a consequence, no dimensional non-separable strategy can emerge in equilibrium under delegation.

Optimal Organizational Structure. As discussed in Section 2, the principal delegates a decision if the informational gains compensate the loss of control. When information is costly for agents, there is a limit on the informational gains the principal can obtain from delegation (Corollary 2). Since the limit on informational gains is decreasing on the cost of each signal, the allocation of decision rights that was optimal with no information acquisition is, in principle, still optimal when costs are sufficiently low. In many cases, however, these costs affect the ex-ante expected benefits of the different organizational structures, and this is particularly true for those involving delegation. The following result shows that information costs impose stronger restrictions to delegation than centralization.

[29] For a formal discussion see Appendix B.10.

Proposition 4. *Let κ be the maximum number of agents willing to reveal information about $\theta_r = \{\theta_1, \theta_2\}$ to both decision-makers under delegation. Then, for every $\kappa < n$, there exist costs values for which the maximum number of agents willing to acquire (and reveal) information about θ_r is strictly lower under delegation than under centralization. Formally,*

$$C(S_r) \in \left(\frac{(\hat{w}_{dr})^2}{6(\kappa+2)(\kappa+3)}, \frac{(w_{1r})^2 + (w_{2r})^2}{6(\kappa+3)(\kappa+4)} \right] \Rightarrow K_r^{\mathrm{C}} > K_r^{\mathrm{D}}$$

Where $\hat{w}_{dr} = \max\{w_{1r}, w_{2r}\}$.

Proof. See Appendix B.13. □

When information is costly, the maximum informational gains under delegation are weakly lower than under centralization. Typically, agents reveal their information to one decision-maker under delegation (probably the reason why authority was given to that agent). Under centralization, any information transmitted affects both decisions and agents willing to reveal any signal have larger overall influence; the expected utility gains for such agents are thus larger under centralization and there will typically be more of them willing to invest in information.

Proposition 4 does not mean that centralization is always optimal. For a non-empty set of cost values, there exist profiles of biases, **b**, for which delegation of some sort is preferred to centralization ($K_r^{\mathrm{C}} > K_r^{\mathrm{D}}$ is not binding). But for sufficiently large costs for which centralization always dominates (see Corollary 4 in Appendix A). This is the case when cost of each signal is so high that no agent acquires information under delegation.

One last implication of Proposition 4 relates to the relationship between loss of control and informational gains in Proposition 2. In the previous section we learned that $||\mathbf{b}^{\mathrm{D}}||$ is increasing in the informational gains from delegation. In the present section I showed that information costs impose limits on informational gains; now I will show how $||\mathbf{b}^{\mathrm{D}}||$ is affected by costs. The maximum bias as a function of K^{D} (maximum informational gains) is given by:

$$||\mathbf{b}^{\mathrm{D}}|| = \left[\frac{[w^2 + (1-w)^2]}{6} \left[1 - \frac{2}{(K^{\mathrm{D}} + 2)} \right] \right]^{\frac{1}{2}}$$

Which together with equation (16) in Corollary 2 leads to the following result.

Corollary 3. *The effect of information costs on $||\mathbf{b}^{\mathrm{D}}||$ is given by:*

$$\frac{\partial ||\mathbf{b}^{\mathrm{D}}||}{\partial C(S_r)} < 0$$

The maximum admissible bias the principal tolerates decreases as the cost of information increases. Increasing costs reduces the number of agents willing to acquire any signal, decreasing the informational gains that can be achieved through delegation. This is relevant when information costs are large and the principal is not sure about the exact profile of biases at the moment of 'designing' the organization. In such a case, she can expect the benefits from delegation to be low relative to centralization.

There is a different way to see this. When information costs are high, the expected maximal informational gains from delegation are low and the principal thus tolerates less bias on decisions. So, her willingness to delegate depends on the amount of information a given agent is expected to have in equilibrium which, in turn, depends on information costs. This relationship between the information costs and distributional loss could be related to information acquisition on the intensive-margin—the higher the costs, the lower the amount of information a single agent is expected to acquire and, thus, the lower his 'informational advantage' with respect to the principal. I explore this intuition in Section 4.

I now analyse different consequences of delegation with costly information acquisition: decision-maker *ex-post specialization.*

3.1 Information flows under different organizational structures

In this subsection I analyse the information the principal can expect to receive from agents randomly allocated in the bias space. This way I try to capture a notion of 'long-run informational effects' of different organizational structures, which relates to the idea that institutions have persistent effects on policy outcomes. According to Baumgartner and Jones (2009), by allocating authority over a set of issues to a governmental agency, the policy maker is effectively granting policy influence to interest groups linked to the agency. Part of that influence is about 'feeding' the agency with information on states likely to be more favourable to the groups' preferences. In such cases, the policy becomes too responsive to these issues but insensitive to other issues it should also take into account.

In order to isolate the mechanism I assume signals have zero cost, $C(S_1) = C(S_2) = 0$, which does not necessary mean agents will acquire all the information available (see Corollary 1). The analysis focuses on the relative amount of information each decision-maker expects to receive in equilibrium under different organizational structures. I say that a given decision-maker receives 'balanced' information when $k_1^j \simeq k_2^j$. To do that I consider an arbitrarily large number of agents whose preferences represent the same conflict of interest with the principal. I then analyse the expected pattern of information transmission resulting from those agents' incentives.

Note that agents with small conflict of interest reveal both signals. From such agents, thus, the principal expects to receive balanced information in equilibrium, i.e. information about both states arrive in similar proportion ($k_1^{\mathrm{C}} \simeq k_2^{\mathrm{C}}$). The same conclusion applies to dimensional non-separable message strategies; agents with the corresponding preferences reveal both signals for some realizations and reveal nothing otherwise. As a consequence, the only way decision-makers can receive more information about a single state is when many agents reveal the associated signal and few agents reveal the other. I then focus on the proportion of agents revealing information about one state under different organizational structures.

First consider the case of centralization. Let $\varepsilon \in \Re_+$ and $N_\varepsilon = \{1, 2, ..., n_\varepsilon\}$ be a group of agents that satisfy the following property: for all $i \in N_\varepsilon$ then $||\mathbf{b}^i|| = \varepsilon$. Now, let $\lambda_r \equiv \left\{\mathbf{z} \in \Re^2 | \; \mathbf{z}' \, \mathbf{W}_r = 0\right\}$ be the locus with slope $-\frac{w_{1r}}{w_{2r}}$ related to θ_r. This locus represents the bias vectors for which incentives to reveal S_r to the principal are maximal.The IC constraint for revealing one signal under centralization –equation (10)– can be expressed as:

$$||\mathbf{b}^i - \text{Proj}_{\lambda_r}(\mathbf{b}^i)|| \leq \frac{\left[(w_{1r})^2 + (w_{2r})^2\right]^{\frac{1}{2}}}{2\,(k_r^{\text{C}} + 3)}$$

Where $\text{Proj}_{\lambda_r}(\mathbf{b}^i)$ is the projection of i's bias vector onto the locus λ_r. The result below follows:

Lemma 8. *Let $w \equiv w_{11} = w_{22}$. Given $\varepsilon \in \Re_+$ and arbitrarily large n_ε, then for every integer κ and $i \in N_\varepsilon$ with $||\mathbf{b}^i - Proj_{\lambda_1}(\mathbf{b}^i)|| \leq \frac{\left[(w)^2+(1-w)^2\right]^{\frac{1}{2}}}{2\,(\kappa+3)}$, there exists a $j \in N_\varepsilon$ with $||\mathbf{b}^j - Proj_{\lambda_2}(\mathbf{b}^j)|| \leq \frac{\left[(1-w)^2+(w)^2\right]^{\frac{1}{2}}}{2\,(\kappa+3)}$.*

Proof. See appendix B.14 □

Under centralization, for any agent who is willing to reveal information about one state, the principal expects to find another agent with the same conflict of interest who is willing to reveal information about the other state.[30] The expectation relates to the uncertainty about the profile of preferences, given the magnitude of the conflict of interest ($||\mathbf{b}^i|| = \varepsilon$ for all $i \in N_\varepsilon$).

The formal argument goes as follows. If the conflict of interest is sufficiently large ($\gamma\varepsilon$, with $\gamma > 1$) agents will either play dimensional non-separable strategies or reveal information about one state.[31] Incentives to reveal one signal for any agent depends on the distance between his bliss point and the loci λ_1 and λ_2. Lemma 8 says that the IC constraints for revealing one signal have the same 'width' with respect to the corresponding locus, when number of agents revealing each signal is fixed. In other words, for any given k_1^{C} the number of agents still willing to reveal information about θ_1 is always the same as those willing to reveal information about θ_2, for $k_1^{\text{C}} = k_2^{\text{C}}$. Panel (a) in Figure 5 illustrates this.

[30]Throughout this analysis I assume states are equally important across decisions –i.e. $w_{11} + w_{21} = w_{12} + w_{22}$. The analysis still goes through without it.

[31]If the fixed conflict of interest (ε) is sufficiently small there is a finite number of agents for which revealing both signals is incentive compatible, as shown in the inner circle in Figure 5.a. The principal receives similar amount of information from these agents.

Now consider the case of delegation. Let $\lambda_r^d \equiv \{\mathbf{z} \in \Re^2 | \ \mathbf{z}' \mathbf{I}_d = 0\}$ be the locus of maximal incentives to reveal S_r when deciding on y_d.[32] The locus λ_r^d captures the fact that communication depends only on the conflict of interest associated to y_d, so coincides with either the vertical or the horizontal axis —for λ_r^1. and λ_r^2, respectively. Note that $\lambda_1^d = \lambda_2^d$ for any $y_d = \{y_1, y_2\}$. Condition (11) for communication with the principal can be expressed as:

$$||\mathbf{b}^i - \text{Proj}_{\lambda_r^d}(\mathbf{b}^i)|| \leq \frac{w_{dr}}{2\big(k_r^{\text{P}d} + 3\big)}$$

Then, the equivalent of Lemma 8 in this case is:

Lemma 9. *Let $w \equiv w_{11} = w_{22}$. Given $\varepsilon \in \Re_+$ and an arbitrarily large n_ε, there exists an integer κ and $i \in N_\varepsilon$ with $||\mathbf{b}^i - Proj_{\lambda_r^d}(\mathbf{b}^i)|| \leq \frac{w_{dr}}{2(\kappa+3)}$ such that $||\mathbf{b}^i - Proj_{\lambda_{\tilde{r}}^d}(\mathbf{b}^i)|| > \frac{w_{d\tilde{r}}}{2(\kappa+3)}$.*

Moreover, this is true for the state associated to w_{dr} because $w_{dr} > w_{d\tilde{r}}$

Proof. Given that $\lambda_r^d = \lambda_{\tilde{r}}^d$ and $w_{dr} > w_{d\tilde{r}}$, the result holds for any $b_d^i \in \left(\frac{w_{d\tilde{r}}}{2(\kappa+3)}; \frac{w_{dr}}{2(\kappa+3)}\right]$ □

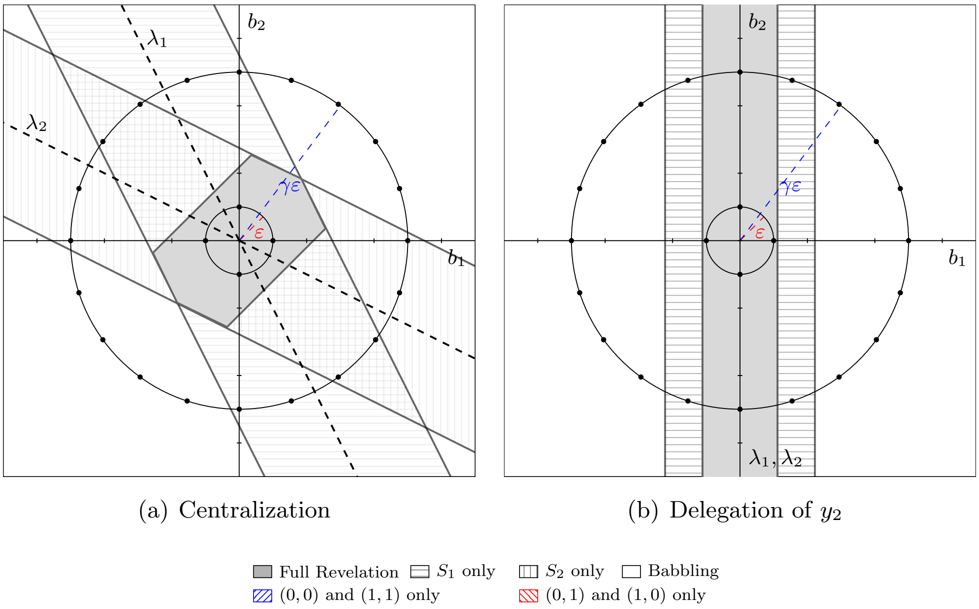
When the principal decides over y_1 only, agents with sufficiently large $|b_1|$ will only reveal information about θ_1. In other words, she can expect to have more information about the state that is more relevant for the decision she is in charge of. The intuition relates to the fact that each state is more relevant for a different decision. Under delegation, agents have a relatively larger influence by revealing information about the more relevant state, such that there will be more agents who find revealing the associated signal incentive compatible. The principal can expect to have a larger proportion of advisers *specialized* in the more relevant state.

Panel (b) in Figure 5 illustrates the information flows when the principal decides on y_1 only. Agents' incentives for communication then depend on b_1 and, given $w_{11} > 1/2$, information about θ_1 has a higher influence (for given $k_1^{\text{P}d} = k_2^{\text{P}d}$). As a consequence, the IC constraints for revealing S_1 only will hold for a larger sets of biases than that of revealing both signals; which in turn implies there will be more agents willing to reveal S_1 than those willing to reveal both signals. The principal then expects to have more information about θ_1 when she delegates y_2.

Lemmas 8 and 9 have important implications for the benefits of delegation when the principal does not know the exact profile of biases. In Section 2 I showed that negative informational externalities can provide a just foundation for partial delegation, but this argument requires the principal to know the profile of biases of informed agents, $\mathbf{b}$. When she does not observe $\mathbf{b}$, however, delegation may lead to losing control over payoff-relevant information (beyond the distributional loss). In particular, each decision-maker under delegation is expected to receive more information on the salient state, becoming an *ex-post specialist*. In the proposition below

[32]Where $\mathbf{I}$ is the 2-by-2 identity matrix, and $\mathbf{I}_d$ is its dth column, which matches the index of the decision under consideration.

Figure 5: Information transmission under different organizational structures



Note: $w_{11} = w_{22} = \frac{2}{3}$

I formalize this argument; to abstract from distributional losses, I allow partial delegation to an agent, $i = 0$, whose preferences are perfectly aligned with the principal.

Proposition 5. *Let, $w_{11} = w_{22} = w$. For any number of agents, $n+1$; for any conflict of interest $\varepsilon \in \Re_+$, such that the profile of biases, $\mathbf{B} = \{\mathbf{b}^0, \mathbf{b}^1, ..., \mathbf{b}^n\}$, is uniformly distributed between $[0, \varepsilon]$ for agents $i = \{1, .., n\}$ and $\mathbf{b}^0 = (0, 0)$; for any integer κ. Then, the principal expects to receive more balanced information under centralization than when she delegates any decision to agent $i = 0$; that is,*

$$|E\left[Var(\theta_1|\mathbf{m}_C^*)\right] - E\left[Var(\theta_2|\mathbf{m}_C^*)\right]| < |E\left[Var(\theta_1|\mathbf{m}_0^*)\right] - E\left[Var(\theta_2|\mathbf{m}_0^*)\right]|$$

Proof. See Appendix B.15 □

There are more potentially negative consequences of delegation besides distributional losses. If the principal does not know the exact distribution of biases of informed agents, she can expect delegation to lead to specialization on the issues that are more salient to each decision. Proposition 5 shows this specialization in the form of higher expected precision of beliefs about the most important state for each decision. This specialization could, in principle, be compensated by a smaller absolute variance on the salient state under delegation, i.e. $E\left[\text{Var}(y_d|\mathbf{m}_C^*)\right] > E\left[\text{Var}(y_d|\mathbf{m}_j^*)\right]$. But this seems not to be the case and Figure 5 can help to estimate the expected number of agents willing to reveal each signal under different allocation of decision rights. Within a given conflict of interest (circle) the mass of points that lies in any IC constraint represents the ex-ante expected 'mass' of agents willing to reveal the corresponding information (for a given κ). It can be seen that the areas around loci λ_1 and λ_2 in panel (a) are larger than the area around the vertical axis in panel (b),[33] suggesting that the amount of information about each state the principal expects to receive under centralization is larger than that for the most salient state under delegation. The intuition behind that relates to the smaller overall influence of delegation (which determines the 'width' of the IC constraints).

4 Discussion and Conclusion

Incentives for specialization

Here, I analyse the conditions under which i acquires information about one state in the most informative equilibrium. Incentives for communication in such cases are given by the IC constraints in Proposition 3. I present the intuitions in an example with 2 agents.

Let assume $w_{11} = w_{22} = w > \frac{1}{2}$ and $C(\mathfrak{s}^i) = c \times (\# \mathfrak{s}^i)$, and let denote the agents by A_1 and A_2. I focus on the centralization equilibrium in which A_1 acquires information about θ_1 and A_2 acquires information about θ_2, $\mathfrak{s}^{1*} = \{\tilde{S}_1^1\}$ and $\mathfrak{s}^{2*} = \{\tilde{S}_2^2\}$. In the equilibrium under consideration the principal is (ex-post) more informed than each of the agents since $k_1^* = 1$ and $k_2^* = 1$.[34] The Corollary below formalizes the result and panel (a) in Figure 6 illustrates the set of biases for which $\mathfrak{s}^{1*} = \{\tilde{S}_1^1\}$ under centralization.

Proposition 6 (Specialization under centralization)**.** *Suppose that there are only two agents, A_1 and A_2, and the marginal cost of each signal is linear and equal to c. The most-informative equilibrium under centralization, $(\mathbf{y}^*, \mathbf{m}^*, \mathfrak{s}^*)$, consist in A_1 acquiring and revealing information on θ_1 only, and A_2 acquiring and revealing information about θ_2 only, in the following cases:*[35]

1. *For $c \leq \frac{w^2+(1-w)^2}{72}$, if and only if A_1's preferences satisfy* (10) *for S_1^1, A_2's satisfy that for S_2^1, and neither satisfies* (5)*;*
2. *For $\frac{w^2+(1-w)^2}{72} < c \leq \frac{w^2+(1-w)^2}{36}$, if and only if condition* (10) *holds for the corresponding signal for A_1 and A_2.*

Proof. See Appendix B.12 □

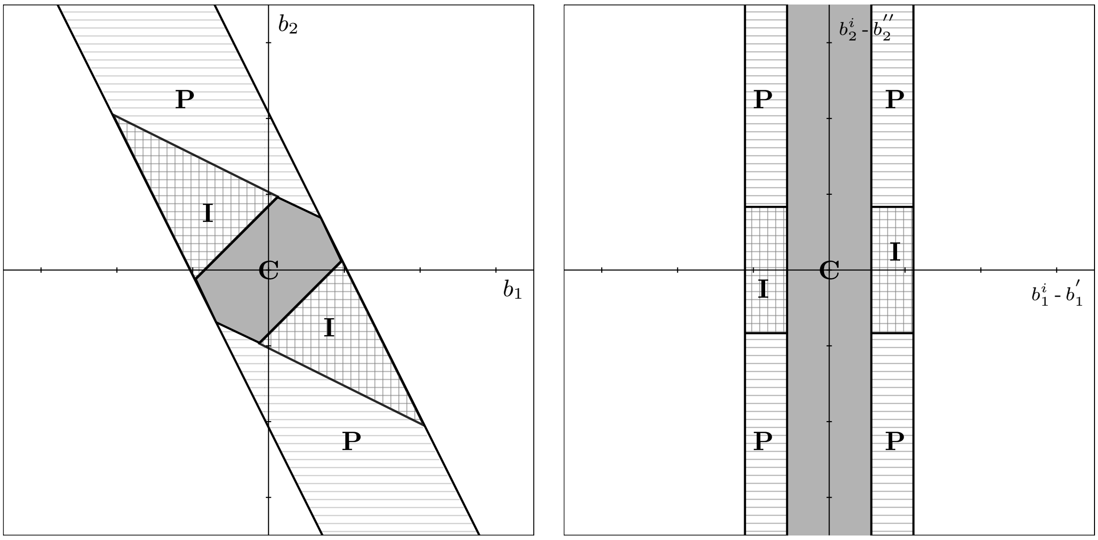
[33] Check the right-hand sides of the IC constraints in Lemmas 2 and 1

[34] The paper by Alonso et al. (2015) analyses a similar situation in the form of generalist-specialist information structure, where each agent specializes in a different piece of information and fully transmit it to the principal.

[35] Formally, the equilibrium consists in $\mathfrak{s}^{1*} = \{\tilde{S}_1^1\}$ and $m^{1*} = \big\{\{(0,0),(0,1)\},\{(1,0),(1,1)\}\big\}$ for A_1, and $\mathfrak{s}^{2*} = \{\tilde{S}_2^2\}$ and $m^{2*} = \big\{\{(0,0),(1,0)\},\{(0,1),(1,1)\}\big\}$ for A_2.

When the cost of a signal is close to zero, information acquisition does not impose restrictions on communication. Agents will then acquire any information they are willing to reveal and, thus, they specialize on different signals only if these are the only IC message strategies. The stripped region in panel (a) of Figure 6 shows specialization *driven by preferences.*

Figure 6: Specialization in the 2-agents model — Driven by preferences (**P**), influence (**I**), and costs (**C**).



(a) A_1 specializes on θ_1 – Centralization (b) A_1 specializes on θ_1 – Delegation

Notes: $w = \frac{2}{3}$ and $k_1^* = k_2^* = 1$

The cross-hatched region in panel (a) represents the set of biases for which A_1 find IC to reveal information about any state individually, but not if he acquired both signals. On the equilibrium path he expects A_2 to acquire and reveal information about θ_2, such that he expects larger utility gains from specialization on θ_1. This specialization decision is *driven by expected larger influence* on the principal's beliefs, given A_2's equilibrium strategy.[36]

When A_1 is willing to reveal information about both states, specialization only emerges if acquiring such information is too costly. Whether he acquires information about θ_1 or θ_2 depends on what A_2 is expected to do: A_1 acquires information about the state the principal is expected to be less informed on path. The solid gray region in panel (a) illustrates the case of specialization *driven by costs.* Now I focus attention on specialization under delegation.

Panel (b) in Figure 6 shows A_1's incentives for specialization under delegation.== Given $w_{11} = w_{22} > 1/2$ and A_1 in equilibrium acquires information about θ_1, his incentives for

[36]An alternative equilibrium exists when both agents' bias vectors lie on cross-hatched regions. The strategies $\mathfrak{s}^{1*} = \{\tilde{S}_2^1\}$ and $\mathfrak{s}^{2*} = \{\tilde{S}_1^2\}$ can also be sustained; agents thus face a coordination problem for which there is no clear selection criterion—the principal is ex-ante (and ex-post) indifferent between any of these. Both equilibria involve specialization mainly because no agent is willing to reveal both signals.

communication are stronger on the first dimension. Indeed, whenever his preferences are close to the decision-maker of the second dimension,j'', A_1 prefers to acquire information about θ_2 (so he becomes specialized on this information).The intuitions for the different drivers of specialization—preferences, influence, and costs— are the same as for centralization, and are illustrated in the different regions of panel (b).

Covert Information Acquisition

In the covert game, the decision-maker(s) does not observe agents' information acquisition decisions, for which any PBE must also specify his/her beliefs about those decisions. Throughout this analysis I focus on the case of centralization. Also, following the main model on information acquisition in this paper, I focus on pure strategy equilibria at the information acquisition stage. Based on Lemma 3 in Argenziano et al. (2016), it is without loss to focus on equilibria in which messages do not convey information about the acquisition of information (Lemma 12 in Appendix C). As a consequence, agents cannot change the decision-maker's beliefs about individual investment on information at the communication stage; with the additional implication that any deviation at the information acquisition stage results in a deviation (from truthtelling) at the communication stage (Lemma 13 in Appendix C). To see this, note that deviations at the information acquisition stage can be of two forms: acquiring fewer or more signals than on-path. In the former, the agent in question will induce beliefs about the signal(s) not acquired that are not consistent with their realizations. In the latter, he cannot convey the fact that he acquired more information, the utility gains from the deviation must then come in the form of lying for some signals realizations.

In the appendix I derive the IC constraints for a typical agent i in the covert information acquisition game. The intuitions relate to the different deviations available to i. When he deviates by *acquiring less signals* than expected on the equilibrium path, he saves on information costs on the one hands, but he induces lower-than-optimal precision on beliefs associated to state(s) for which information has not been acquired. Suppose i is expected to acquire information about both states on path; if, instead, he decides to acquire information about θ_1 only, his message corresponding to θ_2 will induce wrong beliefs for half of its possible realizations. Because his off-path message strategy will be to announce the most favourable realization of S_2^i, the deviation is equivalent to lying on this signal. As a consequence, incentive compatibility requires these utility gains—lying towards his bias *plus* saving on information costs— are lower than expected utility losses from inducing 'more variance' on decisions. It then easily noted that IC constraints of the covert game are more restrictive than those of the overt game—equations (25), (26), and (27) in Appendix C, and Lemma 6 (respectively).

The other type of deviations in the covert game consists of *acquiring more signals* than those specified on the equilibrium path. When i is expected to acquire and reveal information about θ_1, he can profitably deviate by also acquiring information about θ_2. Since, on path, he will only be influential on the message associated to first state, the expected utility gains from such deviation have to do with ambiguous information, $\tilde{\mathbf{S}}^i = \{(0,1),(1,0)\}$; that is, conveying favourable information about θ_2 when that for θ_1 is unfavourable. Despite such combination of signals realization happens with ex-ante probability of 1/4, for very low costs of information agent i will not be able to credibly acquire information about θ_1 only. The result below shows the sufficient condition on i's preferences that allows him to credibly commit to acquire information about θ_1 only.

Proposition 7. *Let $(\mathfrak{s}^*, \mathbf{m}^*, \mathbf{y}^*)$ characterize an equilibrium in the covert game under centralization, and let k_r^{C} be agent i's equilibrium conjecture about other agents truthfully revealing information about state $\theta_r = \{\theta_1, \theta_2\}$. Agent i's equilibrium strategies consist of acquiring and revealing information about θ_1 only, $\mathfrak{s}^{i*} = \{\tilde{S}_1^i\}$ and $m_1^{i*} = \{\tilde{S}_1^i\}$ if and only if:*

$$\frac{|\beta_1^i|}{(k^{\mathrm{C}}+3)} \leq \frac{(w_{11}^2 + w_{21}^2)}{2(k_1^{\mathrm{C}}+3)^2} - \max\left\{C(S_1^i)\,;\, \frac{(w_{11}w_{12} + w_{21}w_{22})}{2(k_1^{\mathrm{C}}+3)(k_2^{\mathrm{C}}+3)} - 2C(S_2^i)\right\} \tag{17}$$

Proof. See Appendix C. □

There are two relevant deviations for agent i when he is expected to specialize on θ_1 in equilibrium. The first consists of not acquiring the signal he is expected on path, for which he would lie at the communication stage towards his bias but also save on information costs. This deviation binds when the first term in bracket on the right-hand side is larger. The second deviation consists in acquiring more information and lying when the signal i is supposed to reveal represents unfavourable information (according to his biases), which is captured by the second term in bracket. As mentioned earlier, the second term is tied to the credibility loss due to ambiguous information, which only makes sense in the presence of informational interdependence. In other words, the second term in brackets in equation (17) represents an additional constraint to communication in the covert game due to informational interdependence.

More than one binary signal per state

Throughout the paper I assumed each agent's information consists of one binary signal associated to each state, two in total. Here I analyse what happens if I relax this assumption, allowing each agent to observe many binary trials associated to each state. I proceed in two steps. First, I show what happens with communication incentives when *a single agent* observes κ signals about *a single* state in an *unidimensional* cheap talk problem, based on Förster (2019). Secondly, I argue that, under the notion of informational interdependence used in this paper, incentives for communication of *perfectly informed* specialists are characterized by the same measures of conflict of interests.

Förster (2019) studies a sender's incentives for communication to a receiver in charge of one decision. The sender observes $\kappa \geq 1$ binary signals that are independent conditional on the state $\theta \in \Theta = [0, 1]$. Players' preferences consist of $u_i(y, \theta, b)$, which is twice continuously differentiable and strictly concave in y, with unique maximum for fixed θ and b, and satisfying single-crossing with respect to y, θ and y, b. In the most informative equilibrium, a sender plays an influential message strategy if his bias is below a threshold $\bar{b}(\kappa)$, which involves full revelation of his information for sufficiently low bias, $b \leq \underline{b}(\kappa) < \bar{b}(\kappa)$.[37] Interestingly, the threshold for influential equilibria is increasing in the number of binary signals the sender observes; in particular, when the state is uniformly distributed, $\bar{b}(\kappa) = \frac{(\kappa+1)}{4(\kappa+2)}$ (see Example 2 in that paper). This suggests that a better informed sender has more flexibility in deciding his message strategy and, thus, finds optimal to transmit some information for larger biases (note that $\lim_{\kappa \to \infty} \bar{b}(\kappa) = 1/4$). Back to the present paper, these results imply that restricting attention to one binary signal represent the 'most conservative' estimation of each agent's incentives to transmit any information.[38]

Now, what happens when agents observe more than one binary signal in the presence of informational interdependence? Due to the complexity posed by the possibility of ambiguous information, in a companion paper, I analysed the case of 2 specialist—each of whom is perfectly informed about a single (different) state and observe no information on the other. In that paper I show that each sender's message strategy consists of increasing partitions of the state space, and individual IC constraints are isomorphic to those in Crawford and Sobel (1982) where sender i's bias is represented by $\frac{\beta_r^i}{w_{1r}^2 + w_{2r}^2}$. Translated into the present framework, these IC constraints would represent the maximal incentives an agent would have to reveal information a single state, and could be compared to expression (10) on incentives of a specialist under centralization.

A last question that may arise relates to asymmetries on agents' information. In the paper I showed that individual incentives to reveal information depends on how much information the decision-maker is expected to have in equilibrium.[39] If one agent,A_1, observes more than one binary signals in my framework, his incentives should follow a more complex form of Förster's analysis, i.e. partitional communication equilibria. If agents other than A_1 still observe one binary signal, their incentives depend on the conjectures about the information the decision-maker

[37] See Propositions 2 and 3 in his paper.

[38] In appendix A I show that the principal strictly prefer to have a single, perfectly informed sender than infinitely many senders each observing single binary signals.

[39] In Krishna and Morgan (2001), for example, two senders are perfectly informed and incentives for communication of each of them depends on the other sender's bias, since it allows to predict how well-informed will the principal be in equilibrium.

receives in equilibrium, including the ex-ante expected residual variance from A_1's equilibrium message strategy. Indeed, the fact that $\underline{b}(\kappa)$ is decreasing in κ (see Remark 3 in Förster, 2019) implies that the principal (weakly) prefers to delegate any decision to A_1 rather than another agent (A_2) with the same conflict of interest, even if he willing to fully reveal his signal to him.[40] This suggests that, *ceteris paribus*, the principal prefers to delegate decisions to more informed agents.

Concluding remarks

Most organizations operate in complex environments, they face multi-causal problems and solutions involve many interrelated courses of action. Decision-relevant information is dispersed among its members, who communicate it strategically. This paper has studied how information is acquired and aggregated under such complexity. I showed that the allocation of decision rights constitutes a key tool to govern the conflict of interests in an organization. In particular, I found a principal may want to delegate controversial decisions if that improves transmission of information on other, less controversial ones. When preferences over all decisions are extreme, centralization can 'discipline' these conflict of interests such that more information is transmitted. I have shown that complexity affects incentives to acquire information under different organizational structures. Under delegation, expected investment in information is not only lower overall but also more concentrated on issues that are salient for the corresponding decision. The analysis presented here has broad applications to the organization of policy-making bodies, advisory committees, knowledge creation in multinational corporations, and other settings where information needs to be obtained and communicated in complex environments.

[40]The argument is that, if A_1's conflict of interest with A_2 is greater than $|\underline{b}(\kappa_1)|$, his message strategy is less than fully revealing; while $\underline{b}(\kappa_1) < \underline{b}(\kappa_2 = 1)$.

Appendix A Complementary results

The proposition below summarizes the equilibrium communication in the case of delegation.

Proposition 8 (Equilibrium Communication for y_d)**.** *Let agent j be the decision-maker of y_d. In the most-informative equilibrium $(\mathbf{y}^*, \mathbf{m}^*_{j'}, \mathbf{m}^*_{j''})$ between agents i and j, i fully reveals his information if and only if condition (1) hold. If the right-hand side of (2) is larger than that of (1), then agents with $|b^i_d - b^j_d|$ within these two values reveal both signals when $(\tilde{S}^i_1, \tilde{S}^i_2) = \{(0,0),(1,1)\}$ and send the corresponding babbling message otherwise. For other values of $\mathbf{b}^i$, i always send the babbling message consistent with his bias.*

Proof. See appendix B.4. □

Equilibrium communication in the case of one decision is characterized by the IC constraint in Lemma 1. As already discussed, full revelation dominates message strategies in which i reveals one signal because the IC constraints are the same and the decision-maker always prefer the former. The same happens with dimensional non-separable message strategies for most parameter values, so they can be overlooked in this case with little loss. Now is time to analyse communication between i and the principal when she retains authority over both decisions.

Lemma 10 (IC constraints for dimensional non-separable strategies under centralization)**.** *In any equilibrium $(\mathbf{y}^*, \mathbf{m}^*)$ in which $\mathbf{m}^{i*}$ includes a babbling strategy; then, i's incentives to reveal both signals against this deviation are characterized by:*

- *For $\mathbf{S}^i = \{0,0\}$ and $\mathbf{S}^i = \{1,1\}$, if:*

$$\left|\frac{\beta^i_1}{(k_1+3)} + \frac{\beta^i_2}{(k_2+3)}\right| \le \frac{1}{4}\left[\frac{(w_{11})^2+(w_{21})^2}{(k_1+3)^2} + \frac{(w_{12})^2+(w_{22})^2}{(k_2+3)^2} + \frac{2\left[w_{11}\,w_{12}+w_{21}\,w_{22}\right]}{(k_1+3)(k_2+3)}\right] \tag{18}$$

- *For $\mathbf{S}^i = \{0,1\}$ and $\mathbf{S}^i = \{1,0\}$, if:*

$$\left|\frac{\beta^i_1}{(k_1+3)} - \frac{\beta^i_2}{(k_2+3)}\right| \le \frac{1}{4}\left[\frac{(w_{11})^2+(w_{21})^2}{(k_1+3)^2} + \frac{(w_{12})^2+(w_{22})^2}{(k_2+3)^2} - \frac{2\left[w_{11}\,w_{12}+w_{21}\,w_{22}\right]}{(k_1+3)(k_2+3)}\right] \tag{19}$$

Proof. See Habermacher (2018) □

Proposition 9 (Characterization of P-Optimal equilibrium under centralization —Proposition 2 in Habermacher, 2018)**.** *The P-optimal Perfect Bayesian Equilibrium for sender i consists of the following strategies:*

1. ***Revealing both signals**, if $\mathbf{b}^i$ satisfies conditions (4), (5) and (10) with respect to both states.*

2. ***Revealing one signal only**, if $\mathbf{b}^i$ satisfies condition (3) for S^i_r only.*

3. ***Dimensional non-separable message strategies*** *in the following cases:*
   (a) *Fully revealing* $\mathbf{S}^i = \{(0,0);(1,1)\}$ *if* $\mathbf{b}^i$ *satisfies condition* (18) *only;*
   (b) *Fully revealing* $\mathbf{S}^i = \{(0,1);(1,0)\}$ *if* $\mathbf{b}^i$ *satisfies condition* (19) *only.*
4. ***No communication*** *(babbling strategy), if none of the above holds.*[41]

Proof. See Habermacher (2018). □

Lemma 11 (Optimal Organizational Structure)**.** *Given the vector of preferences,* $\mathbf{b} = \{\mathbf{b}_1, \ldots, \mathbf{b}_n\}$, *and generic agents* i, j', *and* j''; *the organizational structure that maximizes the principal's ex-ante welfare is:*

Full Delegation. *That is, agents* j' *and* j'' *decide on* y_1 *and* y_2 *(resp.) if and only if:*

1. $DIG_{j'}(y_1) - (b_1')^2 > \max\Big\{DIG_i(y_1) - (b_1^i)^2\,,\ IIG(y_1)\,,\ -\{DIG_{j''}(y_2) - (b_2'')^2\}\Big\}$ *for any* $i \neq j'$; *and*
2. $DIG_{j''}(y_2) - (b_2'')^2 > \max\Big\{DIG_i(y_2) - (b_2^i)^2\,,\ IIG(y_2)\,,\ -\{DIG_{j'}(y_1) - (b_1')^2\}\Big\}$ *for any* $i \neq j''$.

Partial Delegation. *That is, agent* j *decides on* y_d *and the principal retains decision authority over* $y_{\tilde{d}}$; *if and only if there exist both Direct and Indirect informational gains such that:*

1. $DIG_j(y_d) - (b_d^j)^2 > \max\Big\{DIG_i(y_d) - (b_d^i)^2\,,\ IIG(y_d)\,,\ -IIG(y_{\tilde{d}})\Big\}$ *for any* $i \neq j$; *and*
2. $IIG(y_{\tilde{d}}) > \max\Big\{DIG_i(y_{\tilde{d}}) - (b_{\tilde{d}}^i)^2\,,\ -\{DIG_j(y_d) - (b_d^j)^2\}\Big\}$ *for any* $i \neq j$.

Centralization. *That is, the principal decides on both issues, if and only if there are no agent* i *and* j *such that:*

1. $DIG_j(y_d) - (b_d^j)^2 + IIG(y_{\tilde{d}}) > 0$; *nor*
2. $DIG_{j'}(y_1) - (b_1')^2 + DIG_{j''}(y_2) - (b_2'')^2 > 0$

Proof. The proof is constructive. The optimal organizational structure maximizes the principal's ex-ante expected utility. Optimality of delegation implies some informational gains, otherwise she can retain authority over both issues and decide with the information transmitted under centralization. Full Delegation is then optimal if there are two agents j' and j'' who decide on y_1 and y_2, respectively; such that the corresponding informational gains more than compensate

[41] Where Full Revelation corresponds to the equilibrium message strategy $\mathbf{m}^i = \Big\{\{(0,0)\};\{(0,1)\};\{(1,0)\};\{(1,1)\}\Big\}$; revealing S_1^i or S_2^i only correspond to $\mathbf{m}^i = \Big\{\{(0,0);(0,1)\};\{(1,0);(1,1)\}\Big\}$ and $\mathbf{m}^i = \Big\{\{(0,0);(1,0)\};\{(0,1);(1,1)\}\Big\}$; DNS message strategies correspond to $\mathbf{m}^i = \Big\{\{(0,0)\};\{(1,1)\};\{(0,1);(1,0)\}\Big\}$ when $(0,0)$ and $(1,1)$ fully reveal their types, and $\mathbf{m}^i = \Big\{\{(0,0);(1,1)\};\{(0,1)\};\{(1,0)\}\Big\}$ the case in which $(0,0)$ and $(1,1)$ do so; and finally the babbling strategy $\mathbf{m}^i = \Big\{\{(0,0);(1,1);(0,1);(1,0)\}\Big\}$.

each decision-maker's bias. These gains must be maximal among all agents, and strictly larger than if the principal retained any single decision (IIG).

Partial Delegation is optimal in either of two cases (non-exclusive). First, when direct informational gains from delegation are possible only on one decision, the principal prefers to retain authority on the other. If the DIG are sufficiently large, she may be willing to tolerate some informational losses on the retained decision; that is, receiving less information than under centralization.

The second and most interesting case is when indirect informational gains are large. From Lemma 5 we know that the presence of negative informational externalities under centralization is a necessary condition. Delegating y_d thus breaks the interdependence between decisions and allows communication on the low-conflict dimension. This may hold even if there are no informational gains in the delegated decision, as long as the indirect ones are sufficiently large.

Finally, Centralization is optimal when any potential informational gain is small, such that it does not compensate the loss of control on the delegated decision(s). □

Corollary 4. *Let $w \equiv w_{11} = w_{22}$, $\kappa = 1$, and $\frac{\hat{w}_{dr}^2}{72} < C(S_r) \leq \frac{[(w_{1r})^2+(w_{2r})^2]}{72}$ for both θ_r. If there are no agents j' and j'' whose preferences represent a conflict of interest within $||\mathbf{b}^{\mathrm{D}}|| \leq 1 - 2w(1-w)$, then the principal strictly prefers centralization over any form of delegation. If there where such agents, the principal still prefers centralization (strictly) as long as there is at least one agent i who fully reveals his information —i.e. $\mathbf{b}^i$ satisfy conditions* (3) *and* (5) *with respect to both signals.*

Proof. When $\kappa = 1$, decision-makers have the stronger incentives to acquire both signals under delegation. Noting that $6(\kappa+2)(\kappa+3) = 72$, from Proposition 4 I get that $\frac{\hat{w}_{dr}^2}{72} < C(S_r) \leq \frac{[(w_{1r})^2+(w_{2r})^2]}{72}$ implies $K_r^{\mathrm{C}} > K_r^{\mathrm{D}} = 1$. In addition, from Proposition 2 we know that, for $w_{11} = w_{22} = w$ and $k_r^j = 1$:

$$||\hat{\mathbf{b}}^{\mathrm{D}}|| = \left[\frac{w^2}{6}\left(1 - \frac{2}{3}\right) + \frac{(1-w)^2}{6}\left(1 - \frac{2}{3}\right)\right] = 1 - 2w(1-w)$$

If there are no agents j' and j'' such that $||\mathbf{b}^{\mathrm{D}}|| \leq ||\hat{\mathbf{b}}^{\mathrm{D}}||$, then the distributional loss from delegation is never compensated by the maximal informational gain; as a consequence, centralization yields higher ex-ante expected utility to the principal. On the other hand, if there were agents j' and j'' such that $(b_1', b_2'') \in ||\hat{\mathbf{b}}^{\mathrm{D}}||$ the principal prefers delegation when there is no agent i whose preferences satisfy conditions (3) and (5). For if there were such an agent, he reveals both signals under centralization and, thus, delegation yields no informational gains.

Under the parameters of Corollary 4, costs are so high that the maximum number of truthful messages under delegation is zero. Having the chance to influence both decisions, i's acquisition of one signal is cost-effective (provided communication is IC). The result also illustrates the restrictions imposed by information costs on the optimality of different organizational structures (Proposition 11). For sufficiently high costs the principal always prefer to retain authority over both issues, but restrictions weaken as the costs of acquiring a signal decreases. □

A.1 Agents' equilibrium strategies.

In this subsection I combine the results of Lemma 6 and Lemma 7 to characterize agent i's equilibrium information acquisition and message strategies. I start with the case of centralization ($j' = j'' = P$) and then proceed to the case of delegated decisions. The following result summarizes the intuitions developed in the previous discussion, presenting the equilibrium acquisition and message strategies for a typical agent under centralization.

Proposition 10 (Equilibrium under Centralization)**.** *In the most informative equilibrium under centralization* $(\mathbf{y}^*, \mathbf{m}^*, \mathfrak{s}^*)$*, agent i only acquires signals that are cost-effective and incentive compatible. In particular, i's equilibrium strategies are given by:*

Acquiring and revealing both signals: *if and only if conditions* (12) *and* (10) *hold for both signals, and* (5) *hold.*

Acquiring both signals and playing a dimensional non-separable strategy: *if condition* (13) *hold for both signals and* (10) *does not at all, in the following cases:*

- *Fully revealing both signals when they coincide and babbling otherwise, if condition* (18) *holds;*

- *Fully revealing both signals when they do not coincide and babbling otherwise, if condition* (19) *holds.*

Acquiring and revealing one signal only. *Agent i acquires and reveals* S_1^i *if* (10) *and* (12) *hold with respect to* θ_1 *and one of the following is true:*

- *Revealing* S_2^i *is not IC —i.e.* (10) *does not hold for* θ_2*; or*

- *Acquiring* S_2^i *is not CE —i.e.* (12) *does not hold for* θ_2*; or*

- *Acquiring* S_2^i *is CE and revealing it is IC, but revealing both signals is not IC —i.e.*(3) *and* (12) *hold for both signals, but* (5) *does not and* $\frac{(w_{1r})^2+(w_{2r})^2}{(k_r^*+2)(k_r^*+3)} \geq \frac{(w_{1\tilde{r}})^2+(w_{2\tilde{r}})^2}{(k_{\tilde{r}}^*+2)(k_{\tilde{r}}^*+3)}$

For $r \neq \tilde{r}$*.*

Acquiring no signal, *if only if any of the statements below is true:*

- *No signal is CE to acquire —i.e. condition* (12) *does not holds for any signal; and/or*

- *No signals is IC to reveal —i.e. condition* (10) *does not hold for any signal, nor* (5) *holds.*

Proof. See Appendix B.11 □

As discussed earlier, the number of agents revealing the same piece of information is an important determinant of cost-effectiveness. It could lead some agents to acquire less information than what each is willing to reveal, resulting in either specialization or non-investment. The possibility of abstaining to acquire a given signal can enhance incentives for communication (for the other signal) because it kills the effects of ambiguous information.

Dimensional non-separable message strategies can arise under centralization. As in the pure communication game, these strategies take the form of full revelation for some realizations and babbling for the rest. Because any of these involves acquiring both signals and revealing them half of the time, they arise when costs are sufficiently low and only if revealing one signal is not IC. Then, agent i typically prefers to reveal a single signal than play a DNS strategy, provided both are incentive compatible and cost-effective.[42]

Similar intuitions apply to the case of delegation. The result below characterizes equilibrium strategies for agent i and decisions y_d and $y_{\tilde{d}}$.

Proposition 11 (Equilibrium under Delegation)**.** *When the organizational structure involves more than one decision-maker, agent i only acquires signals that are cost-effective and for which communication is incentive compatible. In the most informative equilibrium* ,$(\mathbf{y}^*, \mathbf{m}^*, \mathfrak{s}^*)$*, i's equilibrium strategies are:*

[42] For a formal discussion see Appendix B.10.

Acquiring and revealing both signals: *if and only if conditions* (1) *and* (14) *hold for both signals and at least one decision-maker –and the associated decision.*

Acquiring and revealing S_1 ***only****, if acquiring this signal is both cost-effective and incentive compatible for agent i in the following cases:*

1. *Revealing* S_2 *is not IC for any decision —i.e. condition* (11) *does not hold for* S_2^i*; or*

2. *Acquiring* S_2 *is not CE for any decision —i.e. condition* (14) *does not hold for* S_2^i *for any decision; or*

3. *Both* S_1 *and* S_2 *are CE and IC, but revealing both is not IC with respect to any decision-maker —i.e. conditions* (11) *and* (14) *hold for both signals and at least one decision-maker, but* (1) *does not hold for any of them and* $\frac{(w_{dr})^2}{(k_r^*+2)(k_r^*+3)} \geq \frac{(w_{d\tilde{r}})^2}{(k_{\tilde{r}}^*+2)(k_{\tilde{r}}^*+3)}$

For $r \neq \tilde{r}$*.*

Acquiring no signal *if only if any of the statements below are true:*

1. *Condition* (11) *does not hold for any signal and any decision, nor* (5) *hold; and/or*

2. *Condition* (14) *does not holds for any signal, any decision.*

Proof. See Appendix B.11. □

In presence of two decision-makers agents acquire a signal if it is cost-effective and incentive compatible to reveal it to at least one of them. As discussed in Proposition 3, the possibility of not acquiring one of the signals makes credible the strategy of revealing only one signal. When such a strategy is incentive compatible it always refers to the state the decision-maker is less informed in equilibrium. In other words, the agent will acquire the signal for which the influence is larger, which in turn implies that dimensional non-separable strategies are dominated by this strategy. As a consequence, no dimensional non-separable strategy can emerge in equilibrium under delegation.

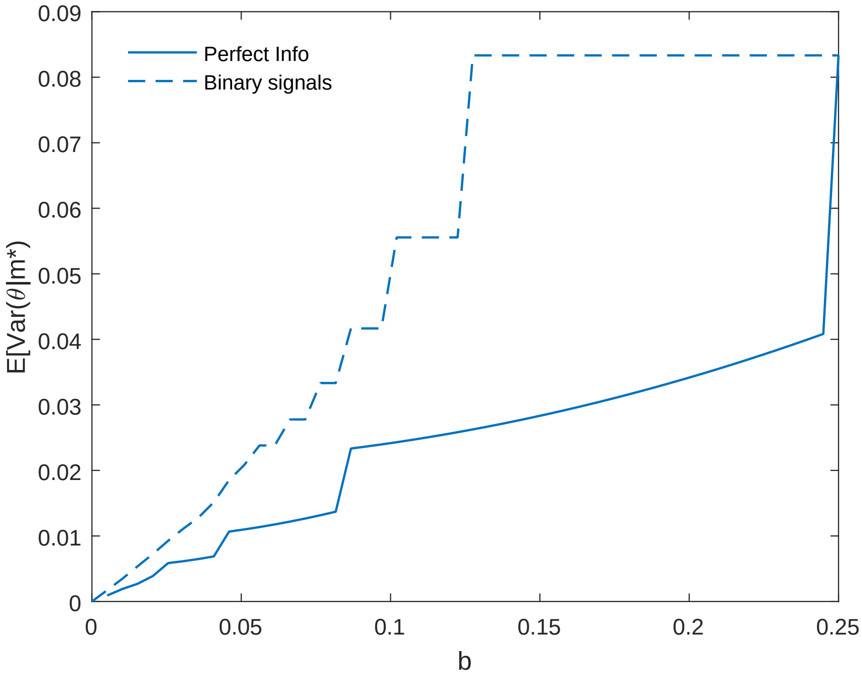
A.2

Here, I compare the uniform-quadratic case in Crawford and Sobel (1982) with a beta-binomial model in which infinitely many senders with the same bias receive individual binary signals. The difference between the two formulations is that agents' message strategies are degenerated in the latter; either they reveal information truthfully or announce the corresponding babbling message. Figure 7 shows the residual variance for the two models for the relevant range of biases,[43] the fact that the beta-binomial model leads to higher variance for each bias means that *the receiver strictly prefers to have a single, perfectly informed agent* (expert) rather than infinitely many of them each of which is imperfectly informed. This conclusion holds despite the fact that each of the many imperfectly informed agents who reveals information, reveals all of it (truthful adviser). The intuition then relates to the expert's ability in equilibrium to control the precision on the receiver's decision; that is, to send equilibrium messages that induce beliefs which variance is increasing on the realization of the state. In other words, a truthful adviser's incentives to reveal his information depends on how much information other senders are expected to reveal in equilibrium.[44]

[43] A necessary condition for influential equilibria in the uniform-quadratic is $b \leq 1/4$.

[44] Note that if there were more than one expert with the same biases, the amount of information transmitted in equilibrium remains the same.

Figure 7: Residual variance from optimal decision with a perfectly informed senders versus infinitely many senders with binary signals — $b \in [0, 1/4]$



Appendix B Proofs

B.1 Generic IC constraints for communication

Proof. Let $j', j'' \in \{P, 1, ..., n\}$ be the decision-makers for y_1 and y_2, respectively; and $i \in \{1, ..., n\}$ be a generic sender. Let $\mathbf{m}_j^{i*}$ denote i's equilibrium message strategy with respect to j, and $\hat{\mathbf{m}}_j^i$ an alternative message strategy (deviations to be considered in each case). Then, $y_d(\mathbf{m}_j^i, \mathbf{m}_j^{-i})$ represents the action j takes when i is expected to play $\mathbf{m}_j^{i*}$ and other senders are playing $\mathbf{m}_j^{-i}$. Given i's conjectures about others' strategies are correct in equilibrium, I can simplify notation in the following way: $y_d(\mathbf{m}_j^{i*}(\mathbf{S}^i), \mathbf{m}_j^{-i}) = y_d(\mathbf{m}_j^{i*})$ and $y_d(\hat{\mathbf{m}}_j^i(\mathbf{S}^i), \mathbf{m}_j^{-i}) = y_d(\hat{\mathbf{m}}_j^i)$.

Message strategy $\mathbf{m}^{i*} = \{m_{j'}^{i*}, m_{j''}^{i*}\}$ is then incentive compatible for sender i if and only if for any alternative $\hat{\mathbf{m}}^i$:

$$\begin{aligned}
-\int_0^1 \int_0^1 \Bigg[\Big[\left(\delta_1 + b_1^i - y_1(\mathbf{m}_{j'}^{i*})\right)^2 + \left(\delta_2 + b_2^i - y_2(\mathbf{m}_{j''}^{i*})\right)^2 \Big] - \\
- \Big[\left(\delta_1 + b_1^i - y_1(\hat{\mathbf{m}}_{j'}^{i})\right)^2 + \left(\delta_2 + b_2^i - y_2(\hat{\mathbf{m}}_{j''}^{i})\right)^2 \Big] \Bigg] f(\theta_1, \mathbf{m}^{-i}|\mathbf{S}^i)\, f(\theta_2, \mathbf{m}^{-i}|\mathbf{S}^i)\, d\theta_1\, d\theta_2 \geq 0
\end{aligned}$$

By operating inside the square brackets with the identity $a^2 - b^2 = (a+b)(a-b)$, by definition of optimal decisions, $y_d^* = E(\delta_d|\mathbf{m}_j) + b_d^j$, and by denoting:

$$\Delta(\delta_1) = E(\delta_1|\mathbf{m}_{j'}^{i*}, \mathbf{m}_{j'}^{-i}) - E(\delta_1|\hat{\mathbf{m}}_{j'}^{i}, \mathbf{m}_{j'}^{-i}) \qquad \Delta(\delta_2) = E(\delta_2|\mathbf{m}_{j''}^{i*}, \mathbf{m}_{j''}^{-i}) - E(\delta_2|\hat{\mathbf{m}}_{j''}^{i}, \mathbf{m}_{j''}^{-i})$$

I get:[45]

$$
\begin{aligned}
-\int_0^1\int_0^1 \Bigg[& \Big[\frac{E(\delta_1|\mathbf{m}^{i*}_{j'},\mathbf{m}^{\text{-}i}_{j'})+E(\delta_1|\hat{\mathbf{m}}^{i}_{j'},\mathbf{m}^{\text{-}i}_{j'})}{2}-\delta_1-(b^i_1-b'_1)\Big]\Delta(\delta_1)+\\
& +\Big[\frac{E(\delta_2|\mathbf{m}^{i*}_{j''},\mathbf{m}^{\text{-}i}_{j''})+E(\delta_2|\hat{\mathbf{m}}^{i}_{j''},\mathbf{m}^{\text{-}i}_{j''})}{2}-\delta_2-(b^i_2-b''_2)\Big]\Delta(\delta_2)\Bigg]\\
& f(\theta_1|\mathbf{m}^{\text{-}i},S^i_1)\,f(\theta_2|\mathbf{m}^{\text{-}i},S^i_2)\,P(\mathbf{m}^{\text{-}i}_{j'}|S^i_1)\,P(\mathbf{m}^{\text{-}i}_{j'}|S^i_2)\,P(\mathbf{m}^{\text{-}i}_{j''}|S^i_1)\,P(\mathbf{m}^{\text{-}i}_{j''}|S^i_2)\,d\theta_1\,d\theta_2\geq 0
\end{aligned}
$$

Given that the equilibrium message strategies for players other than i, $\mathbf{m}^{\text{-}i}$, are independent of i's actual signal realizations, the expressions $P(\mathbf{m}^{\text{-}i}_j|S^i_1)$ and $P(\mathbf{m}^{\text{-}i}_j|S^i_2)$ can be taken out the double-integral.

I denote the receiver's updated beliefs with respect to δ_d from i's perspective as:

$$
\nu^{i*}_d = E(\delta_d|\mathbf{m}^{i*}_j,\mathbf{m}^{\text{-}i}_j) \qquad \hat{\nu}^i_d = E(\delta_d|\hat{\mathbf{m}}^i_j,\mathbf{m}^{\text{-}i}_j) \qquad \nu^i_d = E(\delta_d|\mathbf{S}^i_j,\mathbf{m}^{\text{-}i}_j)
$$

Such that $\Delta(\delta_d)=\nu^{i*}_d-\hat{\nu}^i_d$. The generic IC constraint is then given by:

$$
-\Bigg[\Big[(\nu^{i*}_1+\hat{\nu}^i_1)-2(\nu^i_1+b'_1-b^i_1)\Big]\,\Delta(\delta_1)+\Big[(\nu^{i*}_2+\hat{\nu}^i_2)-2(\nu^i_2+b''_2-b^i_2)\Big]\,\Delta(\delta_2)\Bigg]\,P(\mathbf{m}^{\text{-}i}|S_1)\,P(\mathbf{m}^{\text{-}i}|S_2)\geq 0 \tag{20}
$$

Note that under Centralization $b'_1=b''_2=0$ and we are back to the IC constraints in Habermacher (2018). More importantly, when i reveals one signal only ν^{i*}_d and $\hat{\nu}^i_d$ are different from ν^i. Sender i's strategies in equilibrium and in the deviation under analysis do not transmit all the information he has, and the beliefs he induces on j are different from what he believes are the optimal decisions (in the equilibrium under consideration). As I show later, this generates credibility losses for i because of the possibility of ambiguous information —i.e. signals that move decisions in opposite directions if fully revealed. □

B.2 Proof of equation (6)

Proof. The principal's ex-ante expected utility in the equilibrium characterized by by $\{\mathbf{y},\mathbf{m}_{j'},\mathbf{m}_{j''}\}$ is given by:

$$
E\left[U^{\text{P}}(\boldsymbol{\delta},\mathbf{b});\mathbf{m}\right]=-E\left[(y_1-\delta_1)^2;\mathbf{m}_{j'}\right]-E\left[(y_2-\delta_2)^2;\mathbf{m}_{j''}\right]
$$

Which, by definitions of equilibrium y_d and δ_d yield:[46]

$$
E\left[U^{\text{P}}(\boldsymbol{\delta},\mathbf{b});\mathbf{m}\right]=-(b'_1)^2-(b''_2)^2-\sum_{y_d=\{y_1,y_2\}}E\left[\Big(w_{d1}\big(E(\theta_1|\mathbf{m}_j)-\theta_1\big)+w_{d2}\big(E(\theta_2|\mathbf{m}_j)-\theta_2\big)\Big)^2\right]
$$

With some rearrangement and given $\theta_1\perp\theta_2$, I have:

$$
E\left[U^{\text{P}}(\boldsymbol{\delta},\mathbf{b});\mathbf{m}\right]=-\left[(b'_1)^2+(b''_2)^2\right]-\sum_{y_d=\{y_1,y_2\}}\sum_{\theta_r=\{\theta_1,\theta_2\}}\left[(w_{dr})^2\,E\left[\big(E(\theta_r|\mathbf{m}_j)-\theta_r\big)^2;\mathbf{m}_j\right]\right] \tag{21}
$$

[45] Note that $f(\theta_1,\mathbf{m}^{\text{-}i}|\mathbf{S}^i)=f(\theta_1|\mathbf{m}^{\text{-}i},S^i_1)\,P(\mathbf{m}^{\text{-}i}|S^i_1)$ and that $f(\theta_2,\mathbf{m}^{\text{-}i}|\mathbf{S}^i)=f(\theta_2|\mathbf{m}^{\text{-}i},S^i_2)\,P(\mathbf{m}^{\text{-}i}|S^i_2)$

[46] Note that the terms $E\left(E(\delta_d|\mathbf{m}_j)-\delta_d\right)b^j_d=0$.

Now, the expectation of the squared deviation for each state is given by:

$$
\begin{aligned}
E\left[\left(E(\theta_r|\mathbf{m}_j)-\theta_r\right)^2;\mathbf{m}_j\right] &= \int_0^1 \sum_{\ell_r=0}^{k_r^j}\Big(E(\theta_r|\mathbf{m}_j)-\theta_r\Big)^2 f(\ell_r^j|k_r^j,\theta_r)\,d\theta_r \\
&= \int_0^1 \sum_{\ell_r=0}^{k_r^j}\Big(E(\theta_r|\mathbf{m}_j)-\theta_r\Big)^2 \frac{h(\theta_r|\ell_r^j,k_r^j)}{(k_r^j+1)}\,d\theta_r \\
&= \frac{1}{(k_r^j+1)}\sum_{\ell_r^j=0}^{k_r^j}\int_0^1 \Big(E(\theta_r|\mathbf{m}_j)-\theta_r\Big)^2 h(\theta_r|\ell_r^j,k_r^j)\,d\theta_r \\
&= \frac{1}{(k_r^j+1)}\sum_{\ell_r^j=0}^{k_r^j}\mathrm{Var}(\theta_r|\ell_r^j,k_r^j) \\
&= \frac{1}{(k_r^j+1)}\sum_{\ell_r^j=0}^{k_r^j}\frac{(\ell_r^j+1)(k_r^j-\ell_r^j+1)}{(k_r^j+2)^2(k_r^j+3)}
\end{aligned}
$$

Solving the sum and plugging the above into (21) yields:

$$
\hat{U}^{\mathrm{P}}(\mathbf{b},\mathbf{m}_{j'},\mathbf{m}_{j''}) = -\left[(b_1')^2+\frac{(w_{11})^2}{6\big(k_1'+2\big)}+\frac{(w_{12})^2}{6\big(k_2'+2\big)}\right]-\left[(b_2'')^2+\frac{(w_{21})^2}{6\big(k_1''+2\big)}+\frac{(w_{22})^2}{6\big(k_2''+2\big)}\right]
$$

Let denote by $\mathrm{Var}(\theta_r|\mathbf{m}_j)\equiv E\left[\left(E(\theta_r|\mathbf{m}_j)-\theta_r\right)^2;\mathbf{m}_j\right]$. Now, the expected decision-specific uncertainty in the communication equilibrium in which j decides over y_d is given by $\mathrm{Var}(\delta_d|\mathbf{m}_j)=(w_{d1})^2\,\mathrm{Var}(\theta_1|\mathbf{m}_j)+(w_{d2})^2\,\mathrm{Var}(\theta_2|\mathbf{m}_j)$, and the above expression can thus be written as:

$$
\hat{U}^{\mathrm{P}}(\mathbf{b},\mathbf{m}_{j'},\mathbf{m}_{j''}) = -\left[(b_1')^2+\mathrm{Var}(\delta_1|\mathbf{m}_{j'})\right]-\left[(b_2'')^2+\mathrm{Var}(\delta_2|\mathbf{m}_{j''})\right] \tag{22}
$$

□

B.3 Proof of Lemma 1

Proof. Consider the equilibrium $(\mathbf{y}^*,\mathbf{m}^*_{\mathbf{j}'},\mathbf{m}^*_{\mathbf{j}''})$ in which the principal delegates y_d to agent j, who does not decide on the other decision. By the assumption on private information with each decision-maker, i's messages to j only affect y_d. The IC constraint in (20) then becomes:

$$
-\Big[(\nu_d^{i*}+\hat{\nu}_d^i)-2(\nu_d^i-b_d^i)\Big]\,\Delta(\delta_d)\geq 0
$$

I denote by $\nu_{dr}^{i*}=E(\theta_r|\mathbf{m}_j)$ and $\hat{\nu}_{dr}^i=E(\theta_r|\hat{\mathbf{m}}_j)$ sender i's expectations of j's posterior beliefs about θ_r in equilibrium when he plays strategies m_j^i and $\hat{m}_j$ (eqm and deviation), respectively. In addition, denote by $\nu_{dr}^i=E(\theta_r|S_r^i,\mathbf{m}_j^{-i})$ sender i's expectation of j's posterior beliefs about θ_r if j knew i's information about that state.

The IC constraint then becomes:

$$
-\left[w_{d1}(\nu_{d1}^{i*}-\hat{\nu}_{d1}^i)+w_{d2}(\nu_{d2}^{i*}-\hat{\nu}_{d2}^i)\right]\Big[w_{d1}(\nu_{d1}^{i*}+\hat{\nu}_{d1}^i)+w_{d2}(\nu_{d2}^{i*}+\hat{\nu}_{d2}^i)-2[w_{d1}\nu_{d1}^i+w_{d2}\nu_{d2}^i-(b_d^i-b_d^j)]\Big]\geq 0
$$

When he reveals only one signal, however, the previous is true for the state associated to the signal he reveals truthfully but not for the other state. To see this recall that he is not being influential with respect to the latter, so his expectations about j's beliefs in equilibrium are different from his conjectures including his own information.[47] It is easy to check that:

$$\nu^i_{dr} = E\left(\theta_r|\tilde{S}^i_r, \mathbf{m}^{\text{-}i}_j\right) = \frac{(\ell^j_r + 1 + \tilde{S}_r)}{(k^j_r + 3)}$$
$$= \frac{\left(k^j_r + 3 - (-1)^{\tilde{S}^i_r}\right)}{2(k^j_r + 3)}$$

The second equality follows from taking expectation over the realization of others' signals (by the Law of Iterated Expectations); whereas $E(\theta_r|m^{\text{-}i}_j) = \nu^{\text{-}i}_{dr} = 1/2$ —i.e. what i expects j's beliefs on θ_r are if only considers other senders' truthful messages.

Consider the equilibrium in which i reveals S^i_1 only, the IC constraint becomes:

$$-\left[w_{d1}(\nu^{i*}_{d1} - \hat{\nu}^i_{d1})\right]\left[w_{d1}(\hat{\nu}^i_{d1} - \nu^{i*}_{d1}) + 2w_{d2}(\nu^{i*}_{d2} - \nu^i_{d2}) + 2[(b^i_d - b^j_d)]\right] \geq 0$$

When i's type is given by $\tilde{\mathbf{S}}^i = (0,0)$, then $\nu^{i*}_{d1} = \nu^i_{d1} = \frac{(k^j_1+2)}{2(k^j_1+3)}$, $\hat{\nu}^i_{d1} = \frac{(k^j_1+4)}{2(k^j_1+3)}$; and $\nu^{i*}_{d2} = \hat{\nu}^i_{d2} = 1/2$, while $\nu^i_{d2} = \frac{(k^j_1+2)}{2(k^j_1+3)}$. Replacing these values on the above IC constraint I get:

$$2(b^i_1 - b^j_1) \leq \frac{w_{d1}}{(k^j_1 + 3)} + \frac{w_{d2}}{(k^j_2 + 3)}$$

The case of $\tilde{\mathbf{S}}^i = (1,1)$ is analogous, with the LHS having negative signs. For the case of $\tilde{\mathbf{S}}^i = (0,1)$, all i's conjectures about j's posteriors are the same as the previous case (i reveals the same realization of the same signal), except for $\nu^i_{d2} = \frac{(k^j_1+4)}{2(k^j_1+3)}$. It can be easily checked that this leads to:

$$2(b^i_1 - b^j_1) \leq \frac{w_{d1}}{(k^j_1 + 3)} - \frac{w_{d2}}{(k^j_2 + 3)}$$

Whereas for $\tilde{\mathbf{S}}^i = (1,0)$ the LHS above has a negative sign.

Because all these IC constraints have to be satisfied in order for i to be credible in the equilibrium under consideration, and given that all of them hold for the same measure of conflict of interest between i and j, the following is a necessary and sufficient conditions for i not having incentives to lie on S^i_1:

$$|b^i_1 - b^j_1| \leq \frac{1}{2}\left[\frac{w_{d1}}{(k^j_1 + 3)} - \frac{w_{d2}}{(k^j_2 + 3)}\right]$$

Now, in equilibria in which i reveal both signals truthfully $\nu^{i*}_{dr} = \nu^i_{dr}$; that is, what he expects j's beliefs to be in equilibrium is the same as what he conjectures the optimal decisions will be according to his information. The IC constraint then becomes:

$$-\left[w_{d1}(\nu^{i*}_{d1} - \hat{\nu}^i_{d1}) + w_{d2}(\nu^{i*}_{d2} - \hat{\nu}^i_{d2})\right]\left[w_{d1}(\nu^{i*}_{d1} - \hat{\nu}^i_{d1}) + w_{d2}(\nu^{i*}_{d2} - \hat{\nu}^i_{d2}) - 2(b^i_d - b^j_d)\right] \geq 0$$

Incentive compatibility in this case means i prefers truthful revelation of both signals to

[47] In other words, if i is expected to reveal S^i_1, then $\nu^{i*}_{d2} = \hat{\nu}^i_{d2} = E(\theta_s|\mathbf{m}^{\text{-}i}_j)$ because i is not influential with respect to θ_2, and $\nu^{i*}_{d2} \neq \nu^i_{d2}$.

any deviation, taking into account that any message he sends is believed to be truthful. So, for each type $\tilde{\mathbf{S}}^i = \{(0,0);(0,1);(1,0);(1,1)\}$ I consider deviations to announce a message different from his own. This leads to three generic deviations: lying in both signals,[48] lying on S^i_1,[49] and lying on S^i_2.[50] A substantial amount of algebra shows that the IC constraints for not lying in one signal are similar to those for revealing one signal, but without the negative term on the RHS (see Habermacher, 2018 for a detailed derivation of these IC). Incentives not to lie on both signals depend on whether the signals coincide or not, enthusiast readers can check that replacing the values for ν^{i*}_{dr} and $\hat{\nu}^i_{dr}$ for each type and deviation leads to the following IC constraints.

$$\text{For } \tilde{S}^i = \{(0,0);(1,1)\}: \qquad |b^i_1 - b^j_1| \le \frac{1}{2}\left[\frac{w_{d1}}{(k^j_1+3)} + \frac{w_{d2}}{(k^j_2+3)}\right]$$

$$\text{For } \tilde{S}^i = \{(0,1);(1,0)\}: \qquad |b^i_1 - b^j_1| \le \frac{1}{2}\left|\frac{w_{d1}}{(k^j_1+3)} - \frac{w_{d2}}{(k^j_2+3)}\right|$$

The last of the above is the necessary condition for full revelation given it is more restrictive (RHS is smaller). Note moreover that the expression is similar to that for revealing one signal only, which mean that whenever 'both' hold the decision-maker will prefer full revelation and, thus, will be the strategy arising in equilibrium.

Finally, I analyse the existence of Dimensional Non-separable (DNS) message strategies. I consider two of them:[51] $m^{i*}_j = \{\{(0,0)\},\{(1,1)\},\{(0,1),(1,0)\}\}$ and $m^{i*}_j = \{\{(0,1)\},\{(1,0)\},\{(0,0),(1,1)\}\}$. Deviation incentives between the two influential messages translate into the same IC constrains above. Note that this rules out the DNS in which the agent fully reveals ambiguous information because the IC constraint will be similar to that for Full Revelation.

Now I show that the strategy $m^{i*}_j = \{\{(0,0)\},\{(1,1)\},\{(0,1),(1,0)\}\}$ cannot arise in equilibrium. The argument relies on the incentives of the 'non-influential types' to announce influential messages according to their bias. For type $\mathbf{S}^i = (0,1)$ not having incentives to announce $m^i_j = \{(1,1)\}$ we have that $\nu^{i*}_{1r} = 1/2$ and $\hat{\nu}^i_{1r} = \frac{(k^j_r+4)}{2(k^j_r+3)}$ for both signals; whereas $\nu^i_{11} = \frac{(k^j_1+4)}{2(k^j_1+3)}$ and $\nu^i_{12} = \frac{(k^j_2+2)}{2(k^j_2+3)}$. Then, solving the IC constraint I get the following:

$$(b^i_1 - b^j_1) \le \frac{1}{4}\left[\frac{w_{11}}{(k^j_1+3)} - \frac{w_{12}}{(k^j_2+3)}\right]$$

Which, solving for all the relevant cases, leads to:

$$|b^i_1 - b^j_1| \le \frac{1}{4}\left|\frac{w_{11}}{(k^j_1+3)} - \frac{w_{12}}{(k^j_2+3)}\right|$$

Note that this IC constraint is more restrictive than that for Full Revelation, and the decision maker will prefer the latter. As a consequence, no DNS strategy arises under delegation. □

[48] Meaning that type $(0,0)$ announces $(1,1)$ or vice-versa, and type $(0,1)$ announces $(1,0)$ or vice-versa.
[49] Meaning that type $(0,0)$ announces $(1,0)$ or vice-versa, and type $(0,1)$ announces $(1,1)$ or vice-versa.
[50] Meaning that type $(0,0)$ announces $(0,1)$ or vice-versa, and type $(1,0)$ announces $(1,1)$ or vice-versa.
[51] In the companion paper I show that full revelation of two types and babbling for the other two are the only two DNS message strategies arising in any equilibrium. The argument is based on the equilibrium selection criterion given the similarity of IC constraints, and applies to the case of one decision as well.

B.4 Proof of Proposition 8

Proof. Having derived the necessary conditions for communication (Lemma 1), getting the receiver-optimal (R-optimal) equilibrium consist of finding the system of beliefs consistent with each message strategy (if those exist). Let denote by $\mu_j^*\big(\mathbf{S}^i|m_j^{i*}\big)$ the beliefs of decision-maker j about i's information (her type) upon receiving message m_j^{i*}.

A Fully Revealing message strategy means any of i's messages is taken at face value, that is:

$$\mu^*\left((0,0)|m_j^i = \{(0,0)\}\right) = 1 \qquad \mu^*\left((1,0)|m_j^i = \{(1,0)\}\right) = 1$$
$$\mu^*\left((0,1)|m_j^i = \{(0,1)\}\right) = 1 \qquad \mu^*\left((1,1)|m_j^i = \{(1,1)\}\right) = 1$$

From Lemma (1) we know that if i's preferences satisfy condition (1), then he truthfully announces his type and the beliefs above are consistent with that strategy in equilibrium. Because j also knows that, the system of beliefs defined above are implemented only if i's preferences satisfy condition (1); otherwise, there is always a deviation for which the beliefs are not consistent. Now, because the IC constraints for revealing one signal are the same and given I focus on R-optimal equilibria, fully revealing dominates.

For DNS, the Proof of Lemma 1 showed the only of such strategies emerging in equilibrium is $m_j^{i*} = \{\{(0,0)\},\{(1,1)\},\{(0,1),(1,0)\}\}$. Beliefs consistent with such a strategy are given by:

$$\mu^*\left((0,0)|m_j^i = \{(0,0)\}\right) = 1 \qquad \mu^*\left((1,1)|m_j^i = \{(1,1)\}\right) = 1$$
$$\mu^*\left((0,1)|m_j^i = \{(0,1)\}\right) = \mu^*\left((1,0)|m_j^i = \{(0,1)\}\right) = 1/2$$
$$\mu^*\left((0,1)|m_j^i = \{(1,0)\}\right) = \mu^*\left((1,0)|m_j^i = \{(1,0)\}\right) = 1/2$$

By the same argument as above, these are equilibrium beliefs if and only if i's preferences satisfy (2). □

B.5 Proof of Lemma 5

Proof. Informational gains arise when more agents reveal information to al least one decision-maker, as compared to those revealing to the principal under centralization.

Consider direct informational gains first. For every agent who would reveal information under centralization, there must exist an agent revealing at least the same amount of information to the new decision-maker under delegation. Strict gains require that there also exist at least one agent revealing strictly more information to the new decision-maker. Let j' decides on y_1 and the principal on y_2. For every other agent $h \in N$ such that β_1^h satisfies (3) there must exist a $i \in N$ such that b_1^i satisfies (1); otherwise, k_1' will be lower than k_1^{C}.[52] At this point, there must also exist an agent such that:

$$|b_1^i w_{11} + b_2^i w_{21}| > \frac{1}{2}\left[\frac{(w_{11})^2 + (w_{21})^2}{(k_1^{\mathrm{C}}+3)} - \frac{w_{11}w_{12} + w_{21}w_{22}}{(k_2^{\mathrm{C}}+3)}\right]$$
$$|b_1^i - b_1^j| \leq \frac{1}{2}\left[\frac{w_{11}}{(k_1^j+3)} - \frac{w_{12}}{(k_2^j+3)}\right]$$

Multiplying the last inequality by w_{1r}, its right-hand side is strictly lower than that of the expression above. I thus get the condition that $|b_1^i - b_1^j| < |b_1^i + b_2^i \frac{w_{21}}{w_{11}}|$ for direct info gains from delegation.

[52] Indeed, any of the agents revealing under delegation are revealing both signals.

Indirect informational gains mean that in equilibrium $k_2^{\mathrm{P}2} > k_2^{\mathrm{C}}$; the above equations must hold for y_2. An argument similar to the previous leads to: for every $h \in N$ such that β_2^h satisfies (3), there must exist a $i \in N$ such that b_2^i satisfies (1) and, in addition, there exist exists an agent i such that:

$$|b_1^i w_{12} + b_2^i w_{22}| > \frac{1}{2}\left[\frac{(w_{12})^2 + (w_{22})^2}{(k_2^{\mathrm{C}} + 3)} - \frac{w_{11}w_{12} + w_{21}w_{22}}{(k_1^{\mathrm{C}} + 3)}\right]$$

$$|b_2^i| \leq \frac{1}{2}\left[\frac{w_{22}}{(k_2^j + 3)} - \frac{w_{21}}{(k_1^j + 3)}\right]$$

Again, multiplying the last inequality be w_{22} evidences that its RHS is larger than that of the first one. This reduces the necessary condition for IIG to $w_{22}|b_2^i| < |b_1^i w_{12} + b_2^i w_{22}|$. It follows that the previous is only possible when $|b_1^i|$ is sufficiently large. □

B.6 Proof of Proposition 1

Proof. Let $\mathbf{b} = (\mathbf{b}^1, ..., \mathbf{b}^n)$ denote a given profile of biases for n informed agents. The optimal organizational structure is characterized by the number of truthful messages decision-makers receive in equilibrium, $\mathbf{k}'$, $\mathbf{k}''$, and the number of truthful messages the principal receives under centralization, $\mathbf{k}^{\mathrm{C}}$, as per Proposition 11. Suppose that $k_1' = k_2'$, $k_1'' = k_2''$, and $k_1^{\mathrm{C}} = k_2^{\mathrm{C}}$. Now consider the associated game consisting of $n + r$ agents, with the profile of biases for the first n agents remaining the same as in the original profile, that is $\tilde{\mathbf{b}} = (\mathbf{b}^1, ..., \mathbf{b}^n, \tilde{\mathbf{b}}^{n+1}, ..., \tilde{\mathbf{b}}^{n+r})$.

Now, consider the following cases for the preferences of agents $n + 1$ to $n + r$:

1. $\tilde{\mathbf{b}}^{n+1} = ... = \tilde{\mathbf{b}}^{n+r} = (\infty, 0)$.

   If the optimal organizational structure with the original profile $\mathbf{b}$ was full delegation, the principal prefers to retain authority over y_2 if $IIG(y_2) \leq DIG_{j''}(y_2) - (b_2'')^2$; which by Lemma 4 and Proposition 11 translates into:

$$\frac{w_{21}^2}{6}\left[\frac{1}{(k_1''+2)} - \frac{1}{(k_1^{\mathrm{P}2} + r + 2)}\right] + \frac{w_{22}^2}{6}\left[\frac{1}{(k_2''+2)} - \frac{1}{(k_2^{\mathrm{P}2} + r + 2)}\right] \geq -(b_2'')^2$$

   It can be easily noted that for any $k_1'', k_2'', k_1^{\mathrm{P}2}, k_2^{\mathrm{P}2} \leq n$, there exists a r for which the above holds.

   If the optimal organizational structure under $\mathbf{b}$ involved centralization, then the additional information she receives from partial delegation must also compensate the fact that she is now delegating y_1 to an agents. By Lemma 4 and Proposition 11 this means:

$$\frac{w_{21}^2}{6}\left[\frac{2}{(k_1^{\mathrm{C}}+2)} - \frac{1}{(k_1^{\mathrm{P}2} + r + 2)}\right] + \frac{w_{22}^2}{6}\left[\frac{1}{(k_2^{\mathrm{C}}+2)} - \frac{1}{(k_2^{\mathrm{P}2} + r + 2)}\right] \geq$$
$$\geq \frac{w_{11}^2}{6}\left[\frac{1}{(k_1'+2)} - \frac{1}{(k_1^{\mathrm{C}}+2)}\right] + \frac{w_{12}^2}{6}\left[\frac{1}{(k_2'+2)} - \frac{1}{(k_2^{\mathrm{C}}+2)}\right] + b_1'$$

   Note that the RHS of the expression above is non-negative because of the optimality of centralization under $\mathbf{b}$. Then, if there is at least one agent j' whose bias satisfies:[53]

$$(b_1')^2 \leq \frac{w_{11}^2 + w_{21}^2}{6(n+2)} + \frac{w_{12}^2 + w_{22}^2}{6(n+2)} - \frac{w_{11}^2}{18} - \frac{w_{12}^2}{18}$$

Then, there exists a finite r such that Partial Delegation (of y_1) is preferred by the principal over centralization.

2. $\tilde{\mathbf{b}}^{n+1} = ... = \tilde{\mathbf{b}}^{\frac{2n+m+1}{2}} = (-\infty, \infty)$ and $\tilde{\mathbf{b}}^{\frac{2n+m+1}{2}} = ... = \tilde{\mathbf{b}}^{n+m} = (\infty, \infty)$.

   Lemma 10 implies that, for $k_1^{\mathrm{C}} = k_2^{\mathrm{C}}$, any sender with any of these preferences will have maximal incentives to play DNS strategies. In particular, those in the first group satisfy:

$$\left|\frac{\beta_1^i}{(k_1^{\mathrm{C}}+3)} + \frac{\beta_2^i}{(k_2^{\mathrm{C}}+3)}\right| = \left|\frac{w_{11}b_1^i + w_{12}b_2^i + w_{21}b_1^i + w_{22}b_2^i}{(k^{\mathrm{C}}+3)}\right| = \left|\frac{b_1^i + b_2^i}{(k^{\mathrm{C}}+3)}\right| = 0$$

   And those in the second group: those in the first group satisfy:

$$\left|\frac{\beta_1^i}{(k_1^{\mathrm{C}}+3)} - \frac{\beta_2^i}{(k_2^{\mathrm{C}}+3)}\right| = \left|\frac{w_{11}b_1^i + w_{12}b_2^i - w_{21}b_1^i - w_{22}b_2^i}{(k^{\mathrm{C}}+3)}\right| = \left|\frac{b_1^i - b_2^i}{(k^{\mathrm{C}}+3)}\right| = 0$$

   Then, $r = 2n$ is a sufficient condition for the principal to prefer centralization over any other organizational structure that was optimal with the original profile of biases.

□

B.7 Proof of Proposition 2

Proof. Suppose the equilibrium organizational structure involve some form of delegation, and let $j', j'' \in \{P, 1, 2, ...\}$ be decision makers for y_1 and y_2, respectively. The principal's ex-ante utility gain with respect to centralization is given by:

$$\begin{aligned}(b_1')^2 + (b_2'')^2 \leq & \frac{(w_{11})^2}{6}\left[\frac{1}{(k_1^{\mathrm{C}}+2)} - \frac{1}{(k_1'+2)}\right] + \frac{(w_{12})^2}{6}\left[\frac{1}{(k_2^{\mathrm{C}}+2)} - \frac{1}{(k_2'+2)}\right] + \\ & \frac{(w_{21})^2}{6}\left[\frac{1}{(k_1^{\mathrm{C}}+2)} - \frac{1}{(k_1''+2)}\right] + \frac{(w_{22})^2}{6}\left[\frac{1}{(k_2^{\mathrm{C}}+2)} - \frac{1}{(k_2''+2)}\right]\end{aligned}$$

According to expression (22), and denoting by $\mathbf{m}_{\mathrm{C}}$ the messages sent to the principal under centralization, the above can be expressed as:

$$\left[(b_1')^2 + (b_2'')^2\right]^{\frac{1}{2}} \leq \left[\sum_{y_d}\left[w_{d1}^2\big(\mathrm{Var}(\theta_1|\mathbf{m}_{\mathrm{C}}) - \mathrm{Var}(\theta_1|\mathbf{m}_j)\big) + w_{d2}^2\big(\mathrm{Var}(\theta_2|\mathbf{m}_{\mathrm{C}}) - \mathrm{Var}(\theta_2|\mathbf{m}_j)\big)\right]\right]^{\frac{1}{2}}$$

Denote by $\hat{\mathbf{b}}^{\mathrm{D}}$ the vector whose component satisfy the above expression with equality. Because the LHS above represents the euclidean distance of a vector with components b_1' and b_2'' to the origin, then $\hat{\mathbf{b}}^{\mathrm{D}}$ represents the maximum conflict of interest the principal will tolerate as a function of the informational gains from delegation of the corresponding decision(s).

[53] The expression reflects the case in which all n agents fully reveal their signals under centralization ($k_r^{\mathrm{C}} = n$), j' does not receive any signals from other agents in equilibrium ($k_r' = 1$), and the indirect informational gains are maximal ($r = \infty$).

For the second part of the proof let assume that $k_r^j = k^{\mathrm{C}} + 1$, and denote the reduction in variance under delegation by $\Delta\mathrm{Var}(\theta_r|\mathbf{m}_j) = \mathrm{Var}(\theta_r|\mathbf{m}_{\mathrm{C}}) - \mathrm{Var}(\theta_r|\mathbf{m}_j, k_r^j = k_r^{\mathrm{C}} + 1)$, which then is given by:

$$\Delta\mathrm{Var}(\theta_r|\mathbf{m}_j) = \frac{1}{6}\left[\frac{1}{(k_r^{\mathrm{C}}+2)} - \frac{1}{(k_r^{\mathrm{C}}+3)}\right]$$

Then, taking the derivative with respect to k_r^{C} gives expression (9). □

B.8 Proof of Lemma 6

Proof. The proof proceed by contradiction. I focus on the centralization case, since decentralization follows the same logic. Let $(\{\mathbf{y}^*\}, \{\mathbf{m}^*, \mathfrak{s}^*\})$ be the equilibrium strategy profiles for the receiver and all agents, respectively. Recall the equilibrium is characterized by k_1^* and k_2^*.

Acquisition of S_1. Suppose that i's equilibrium info acquisition strategy has $S_1 \in \mathfrak{s}^{i*}$ but condition (3) does not hold for S_1. In such a case revealing information about θ_1 is not incentive compatible for i despite he acquired information about it. Other agents base their message strategies on conjectures about k_1^*, but i is not included among agents revealing S_1 truthfully. At the information acquisition stage, i's expected payoff of $\mathfrak{s}^{i*}$ is thus given by:

$$E\left[U^i\left(\mathbf{y}^*(\mathbf{m}^*(\mathfrak{s}^*)), \delta, b^i\right)\right] = -E\left[\left(\mathbf{y}_1\left(m^{i*}(\mathfrak{s}^{i*}), \mathbf{m}^{\text{-}i*}\right) - \delta_1 - b_1^i\right)^2 + \left(\mathbf{y}_2\left(m^{i*}(\mathfrak{s}^{i*}), \mathbf{m}^{\text{-}i*}\right) - \delta_2 - b_2^i\right)^2\right] - C(\mathfrak{s}^{i*})$$

Now, consider the following deviation: $\hat{\mathfrak{s}}^i = \mathfrak{s}^{i*}\backslash\{S_1\}$. Note that this deviation does not affect k_1^* or k_2^*, and i's overall influence on j's decision(s) is thus unaltered—i.e. $y_d\left(m^i(\hat{\mathfrak{s}}^i), \mathbf{m}_j^{\text{-}i}\right) = y_d\left(m^i(\mathfrak{s}^{i*}), \mathbf{m}_j^{\text{-}i}\right)$. Note also that $C(\mathfrak{s}^{i*}) > C(\hat{\mathfrak{s}}^i)$, given $\#\mathfrak{s}^{i*} > \#\hat{\mathfrak{s}}^i$. Consequently,

$$E\left[U^i\left(\mathbf{y}^*(\mathbf{m}^*(\mathfrak{s}^*)), \delta, b^i\right)\right] - E\left[U^i\left(\mathbf{y}\left(m^i(\hat{\mathfrak{s}}^i), \mathbf{m}^{\text{-}i*}\right), \delta, b^i\right)\right] = -C(\mathfrak{s}^{i*}) + C(\hat{\mathfrak{s}}^i) < 0$$

So, $\hat{\mathfrak{s}}^i$ is a profitable deviation from $\mathfrak{s}^{i*}$. $\Rightarrow\Leftarrow$

Acquisition of both signals. The proof is similar to the previous, with i's proposed equilibrium strategy being $\mathfrak{s}^{i*} = \{S_1, S_2\}$ but conditions for Full Revelation do not hold. Then, a profitable deviation for i will be to acquire the signal he is willing to reveal on the equilibrium path (if any). □

B.9 Proof of Proposition 3

Proof. I analyse the case of delegation, as centralization follows the same argument but requires more algebra (see Habermacher, 2018 for a reference). Suppose agent i's acquires only S_1^i; then strategy $\mathbf{m}^{i*} = \{m_{j'}^{i*}, m_{j''}^{i*}\}$ is preferred to any alternative $\hat{\mathbf{m}}$ iff (IC constraint in section B.3):

$$-\left[w_{d1}(\nu_{d1}^{i*} - \hat{\nu}_{d1}^i) + w_{d2}(\nu_{d2}^{i*} - \hat{\nu}_{d2}^i)\right]\left[w_{d1}(\nu_{d1}^{i*} + \hat{\nu}_{d1}^i) + w_{d2}(\nu_{d2}^{i*} + \hat{\nu}_{d2}^i) - 2[w_{d1}\nu_{d1}^i + w_{d2}\nu_{d2}^i - (b_d^i - b_d^j)]\right] \geq 0$$

But since i has information about θ_1 only, then $E(\theta_2|S_1^i, \mathbf{m}_j^{\text{-}i}) = \nu_{d2}^i = \nu_{d2}^{i*} = \hat{\nu}_{d2}^i$. Moreover, the strategy space when i has only one signal is degenerated, such that he can only reveal it or lie. Revealing S_1^i is thus IC iff:

$$-\left[w_{d1}(\nu_{d1}^{i*} - \hat{\nu}_{d1}^i)\right]\left[w_{d1}(\nu_{d1}^{i*} - \hat{\nu}_{d1}^i) - 2(b_d^j - b_d^i)\right] \geq 0$$

Following the same steps as in section B.3 it is easy to note that the above expression becomes:

$$\text{For } \tilde{S}_1^i = 0: \qquad 2(b_d^i - b_d^j) \leq \frac{w_{d1}}{(k_1^j + 3)}$$
$$\text{For } \tilde{S}_1^i = 1: \quad -2(b_d^i - b_d^j) \leq \frac{w_{d1}}{(k_1^j + 3)}$$

Which together imply equation (11). □

B.10 Equilibrium Information Acquisition and Cost-effectiveness condition

Before the proof of Lemma 7 I derive the information-acquisition IC constraint.

Observation 2. *Let* $k_r^{j*} \equiv k_r^j \left(\mathbf{m}_j^i(\mathfrak{s}^{i*}, \mathfrak{s}^{-i}), \mathbf{m}_j^{-i}(\mathfrak{s}^{i*}, \mathfrak{s}^{-i})\right)$ *and* $\hat{k}_r^j \equiv k_r^j \left(\mathbf{m}_j^i(\hat{\mathfrak{s}}^i, \mathfrak{s}^{-i}), \mathbf{m}_j^{-i}(\mathfrak{s}^{i*}, \mathfrak{s}^{-i})\right)$ *for* $\theta_r = \{\theta_1, \theta_2\}$. *Let* $\mathfrak{s}^{i*}$ *denote i's information acquisition strategy in an equilibrium characterized by* $(\mathbf{y}^*, \mathbf{m}^*, \mathfrak{s}^*)$. *Then, i's ex-ante expected utility from* $\mathfrak{s}^{i*}$ *is given by:*

$$E\left[U^i\left(\mathbf{m}^*, \mathfrak{s}^{i*}, \mathfrak{s}^{-i}, \boldsymbol{\delta}, \mathbf{b}^i\right)\right] = -\left[(b_1^i)^2 + (b_2^i)^2\right] - \sum_{y_d = \{y_1, y_2\}} \left[\frac{(w_{d1})^2}{6\left(k_1^{j*} + 2\right)} + \frac{(w_{d2})^2}{6\left(k_2^{j*} + 2\right)}\right]$$

Now, let $(\mathbf{y}^*, \mathbf{m}^*, \mathfrak{s}^*)$ *be equilibrium strategy profiles. Then,* $\mathfrak{s}^{i*}$ *is incentive compatible for agent i if and only if, for every alternative* $\hat{\mathfrak{s}}^i$*:*

$$\sum_{y_d = \{y_1, y_2\}} \sum_{\theta_r = \{\theta_1, \theta_2\}} \frac{(w_{dr})^2}{6} \left[\frac{1}{\left(\hat{k}_r^j + 2\right)} - \frac{1}{\left(k_r^{j*} + 2\right)}\right] \geq \left[C(\mathfrak{s}^{i*}) - C(\hat{\mathfrak{s}}^i)\right] \tag{23}$$

Proof of Lemma 7

Proof. I first derive the cost-effectiveness condition (12) and then the maximum number of agents for which acquiring a given piece of information is cost-effective –condition (15). In order to derive cost-effectiveness (CE), I consider each possible info acquisition strategy in equilibrium.

The number of agents revealing truthfully their signals in equilibrium, k_r^*, includes i's message strategy when he acquires (and reveals) it in equilibrium.[54] Here I need to make two clarifications. Firstly, if there were agents who acquired information on θ_1 but were not willing to reveal it when $\hat{k}_1 = k_1^* + 1$, then only one of them changes his message strategy because when one of these agents stop revealing, then $\hat{k}_1 = k_1^*$ again. As a consequence, $\hat{k}_1 = \{k_1^*, k_1^* + 1\}$. I take the most conservative of these approaches by making $\hat{k}_1 = k_1^* + 1$ whenever i acquires S_1 off-path.

The second clarification relates to what happens when i acquires a signal and does not reveal it. Since other agents' message strategies will depend on conjectures about k_r, i not revealing the signal acquired off-path does not affect their equilibrium behaviour at the communication stage. In other words, Lemma 6 holds: i gains nothing from acquiring a signal he will not reveal.

Centralization. Let first consider the acquisition of both signals in equilibrium; that is $\mathfrak{s}^{i*} = \{S_1^i, S_2^i\}$. Expression (23) for each possible alternative strategy becomes:

[54] In equilibria in which i does not acquire S_1^i, k_1^* does not count him; but in any deviation in which he does acquire it, then $\hat{k}_1 = k_1^* + 1$.

1) $\tilde{\mathfrak{s}}^i = \{\emptyset\}$

$$\sum_{\theta_r} \frac{(w_{1r})^2 + (w_{2r})^2}{6(k_r^* + 2)(k_r^* + 3)} \geq C(S_1^i, S_2^i)$$

2) $\tilde{\mathfrak{s}}^i = \{S_r^i\}$

$$\frac{(w_{1\tilde{r}})^2 + (w_{2\tilde{r}})^2}{6(k_{\tilde{r}}^* + 2)(k_{\tilde{r}}^* + 3)} \geq C(S_{\tilde{r}}^i)$$

Now, when the equilibrium strategy consists of one signal only, $\mathfrak{s}^{i*} = \{\tilde{S}_r^i\}$, the IC constraints become:

3) $\tilde{\mathfrak{s}}^i = \{\emptyset\}$

$$\frac{(w_{1r})^2 + (w_{2r})^2}{6(k_r^* + 2)(k_r^* + 3)} \geq C(S_r^i)$$

4) $\tilde{\mathfrak{s}}^i = \{S_{\tilde{r}}^i\}$

$$\frac{(w_{1r})^2 + (w_{2r})^2}{6(k_r^* + 2)(k_r^* + 3)} \geq \frac{(w_{1\tilde{r}})^2 + (w_{2\tilde{r}})^2}{6(k_{\tilde{r}}^* + 2)(k_{\tilde{r}}^* + 3)}$$

5) $\tilde{\mathfrak{s}}^i = \{S_1^i, S_2^i\}$

$$\frac{(w_{1\tilde{r}})^2 + (w_{2\tilde{r}})^2}{6(k_{\tilde{r}}^* + 2)(k_{\tilde{r}}^* + 3)} < C(S_{\tilde{r}}^i)$$

Case 3) represents the necessary condition to acquire any individual signal S_r^i, since it implies case 1) (in which it holds for both signals) and case 4) (in which the agent acquires the signal that would have the highest influence). This case corresponds to equation (12).

Now I work out the expression for the maximum number of agents to acquire a given signal under centralization. According to the equation (12) as k_r increases the ex-ante expected utility of acquiring S_r decreases. So, the maximum number of agents who will acquire that signal is given by the largest k^* for which the cost-effectiveness condition hold. Re-arranging this condition I get the following polynomial:

$$-(k_r^*)^2 - 5k_r^* - \left[6 - \frac{(w_{1r})^2 + (w_{2r})^2}{6\,C(S_r^i)}\right] \geq 0$$

Then, solving for the highest positive root I get K_r^{C} in (15).

Delegation. As before, i is not willing to acquire signals he is not willing to reveal on-path (Lemma 6). But in this case there are two decision-makers and IC can refer to any of them (or both). From (23) we know that acquiring S_r^i requires that i is willing to reveal it to at least one decision-maker, say:

$$C(S_r^i) \leq \frac{(w_{dr})^2}{6(k_r^j + 2)(k_r^j + 3)}$$

For at least one y_d. Consider the case of acquiring both signals, which is cost-effective in two generic cases. First, when i is willing to reveal at least one signal to a different decision-maker, the CE condition above should hold for each decision-maker. Second, when i is willing to reveal both signals to a single j the RHS of the above expression becomes larger. As a consequence, equation (14) is a necessary condition for investing in any individual signal.

To get the expression for the maximum number of agents to invest in S_r I need to analyse also two cases. The minimal incentives to reveal are given by the case in which all agents are willing to reveal S_r to decide on the dimension it is less important. This will define the minimum upper-bound, since the CE condition becomes

$$C(S_r^i) \leq \min_{w_{dr}} \left\{ \frac{(w_{dr})^2}{6(k_r^j+2)(k_r^j+3)} \right\}$$

Now consider the case in which all agents are willing to reveal *both signals* to *both decision-makers*. This is the maximum upper-bound. In such a case, the CE condition will be just like the centralization case; that is:

$$C(S_r^i) \leq \frac{(w_{1r})^2 + (w_{2r})^2}{6(k_r^j+2)(k_r^j+3)}$$

Then, following the same steps as the proof of Lemma 7 (Appendix B.10) I have the first and the second expressions in square bracket in equation (16), respectively. □

Proof of equation (13) *(DNS under Centralization).* For any dimensional non-separable strategy in which i fully reveals some realizations and play babbling on the others, the expected payoff in equation (23) becomes.

$$\frac{[(w_{11})^2 + (w_{21})^2]}{6}\left[\frac{1}{(\hat{k}_1+2)} - \frac{1}{2(k_1^*+2)} - \frac{1}{2(k_1^*+3)}\right] + \\ + \frac{[(w_{12})^2 + (w_{22})^2]}{6}\left[\frac{1}{(\hat{k}_2+2)} - \frac{1}{2(k_2^*+2)} - \frac{1}{2(k_2^*+3)}\right] \geq [C(\mathfrak{s}^{i*}) - C(\hat{\mathfrak{s}}^i)]$$

Agent i fully reveals his signals half of the time (in expectation), where k_r^* indicates the equilibrium number of agents revealing information about θ_r apart from i. Then, solving for deviations as in the previous result, I get that acquiring both signals to play a DNS message strategy is cost effective if:

$$\frac{(w_{11})^2 + (w_{21})^2}{6(k_1^*+2)(k_1^*+3)} + \frac{(w_{12})^2 + (w_{22})^2}{6(k_2^*+2)(k_2^*+3)} \geq 2C(S_1^i, S_2^i)$$

Now, i would prefer to acquire both signals and play DNS strategy to acquire only S_r and reveal it for sure if:

$$\frac{(w_{1\tilde{r}})^2 + (w_{2\tilde{r}})^2}{6(k_{\tilde{r}}^*+2)(k_{\tilde{r}}^*+3)} - \frac{(w_{1r})^2 + (w_{2r})^2}{6(k_r^*+2)(k_r^*+3)} \geq 2C(S_{\tilde{r}}^i)$$

Which is easily shown that never holds when $w_{11} = w_{22}$ and $w_{d1} + w_{d2} = 1$. □

B.11 Proof of Proposition 10

Proof. We know from Lemmas 6 and 7 that any equilibrium information acquisition strategy must be CE and IC at the communication stage. Recall that the principal observes agents' choices of information, for which she knows the relevant message space for each agent. Equilibrium communication is then characterized as in Proposition 8 and its equivalent under centralization (Proposition 2 in Habermacher, 2018).

But cost-effectiveness can impose restrictions on equilibrium communication; for instance, when i cannot afford to acquire all the information he is willing to reveal on-path. Consider the case in which i revealing both signals is IC but acquiring only one of them is CE. Revelation of each signal individually is a necessary condition for Full Revelation, so i acquires one signal if CE and reveal it in equilibrium. Now, which of those signals he actually acquires depend on the ex-ante expected utility –see case 4) in the previous proof.

Similar argument applies to any equilibria in which i is willing to reveal S_r^i only. If acquiring is CE, then he reveals that information in equilibrium; if not, he does not reveal any information –indeed, he has no message to send and the receiver observes that.

Finally, no information is transmitted when i's preferences are such that he is not willing to reveal any information, or when acquiring any signal is not cost-effective.

For the case of delegation the same argument applies, with Lemmas 1 and 6, and Corollary 2. $\square$

B.12 Proof of Corollary 6

Proof. When costs do not impose restrictions on information acquisition, incentive compatibility at the communication stage dictates agents' equilibrium strategies (expressions in Lemma 2). In the case of two agents and linear costs, CE does not restrict agents' communication strategies if the cost of any signal is lower than the ex-ante expected utility in the equilibrium when $k_r^* = 1$; that is:

$$C(S_r^i) \leq \frac{(w_{1r})^2 + (w_{2r})^2}{6(k_r^*+2)(k_r^*+3)}$$
$$c \leq \frac{(w)^2 + (1-w)^2}{72}$$

Agent A_1 acquires and reveals S_1^1 in equilibrium if his preferences satisfy expression (10) associated to θ_1 and with $k_1^* = 1$ (see Section B.4 for the supporting system of beliefs on communication). But his preferences should not be such that full revelation is IC, otherwise the principal would be better-off under this equilibrium.[55] A similar argument applies for A_2 with respect to S_2^2.

When $\frac{(w)^2+(1-w)^2}{72} < c \leq \frac{(w)^2+(1-w)^2}{36}$ it is not CE to acquire both signals, so agents acquire the signal each of them is willing to reveal. In other words, the necessary and sufficient condition for specialization in this case is that (10) holds for different signals for each agent. $\square$

B.13 Proof of Proposition 4

Proof. Let $(\mathfrak{s}, \mathbf{m}, \mathbf{y})$ denote a generic equilibrium in which $\kappa < n$ is the maximum number of agents willing to reveal S_r to both decision-makers (suppose $\kappa > 0$). For any of such agents cost-effectiveness under delegation –condition (14)– is given by:

$$C(S_r^i) \leq \frac{(w_{1r})^2 + (w_{2r})^2}{6(k_r^j+2)(k_r^j+3)}$$

But for any other agent, the CE condition is at most:

$$C(S_r^i) \leq \frac{(\hat{w}_{dr})^2}{6(k_r^j+2)(k_r^j+3)}$$

Then, $C(S_r^i) \leq \frac{(w_{1r})^2+(w_{2r})^2}{6(\kappa+2)(\kappa+3)}$ implies acquiring information about θ_r is CE for agents willing to reveal it to both decision-makers. But if at the same time $C(S_r^i) > \frac{(\hat{w}_{dr})^2}{6(\kappa+2)(\kappa+3)}$, agents willing

[55] Note that if condition (10) holds for S_2^1 (but still not those for full revelation) he will still find optimal to acquire S_1^1 only, *given A_2 is acquiring the other signal in equilibrium.*

to reveal S_r to at most one decision-maker do not acquire this signal because it is not CE.

On the other hand, the first of the above equations (15) determines the maximum number of agents for which acquiring S_r is CE under centralization. But since $C(S_r^i) \leq \frac{(w_{1r})^2+(w_{2r})^2}{6(\kappa+3)(\kappa+4)}$, it should be greater or equal to $\kappa+1$. Then, $K_r^{\text{D}} = \kappa < K_r^{\text{C}}$ □

B.14 Proof of Lemma 8

Proof. Let $w \equiv w_{11} = w_{22}$, let κ be an non-negative integer, and let $\varepsilon \in \Re_+$ with associated integer n_ε. Also, let $i \in N_\varepsilon$ be an agent whose preferences satisfy equation (10) with respect to θ_r for $k_r = \kappa$. Note that $\boldsymbol{\lambda}_r = \left(1, -\frac{w_{1r}}{w_{2r}}\right)$, the associated unit vector is $\hat{\boldsymbol{\lambda}}_r = \frac{\boldsymbol{\lambda}_r}{||\boldsymbol{\lambda}_r||} = \left(\frac{w_{1r}}{(w_{1r}^2+w_{2r}^2)^{1/2}}, \frac{w_{2r}}{(w_{1r}^2+w_{2r}^2)^{1/2}}\right)$, and that $\text{Proj}_{\boldsymbol{\lambda}_r}(\mathbf{b}^i) = (\mathbf{b}^i \cdot \hat{\boldsymbol{\lambda}}_r)\boldsymbol{\lambda}_r$. Then, careful algebra leads to condition (10) expressed as:

$$||\mathbf{b}^i - \text{Proj}_{\lambda_1}(\mathbf{b}^i)|| \leq \frac{\left[(w)^2 + (1-w)^2\right]^{\frac{1}{2}}}{2\,(\kappa+3)}$$

Now consider an agent j with the following preferences: $b_1^j = b_2^i$ and $b_2^j = b_1^i$. I need to show 1) $j \in N_\varepsilon$, and 2) $\mathbf{b}^j$ satisfies equation (10) for θ_2. Proving the first claim is straightforward, since j's preference vector is just i's with its components swapped. This says that both i and j agents have exactly the same conflict of interest with the principal.

The second part of the proof requires work out $||\mathbf{b}^j - \text{Proj}_{\lambda_2}(\mathbf{b}^j)||$, which yield:

$$\begin{aligned}
||\mathbf{b}^j - \text{Proj}_{\lambda_2}(\mathbf{b}^j)|| &= |b_1^j\,(1-w) + b_2^j\,w|\ \left[(w)^2 + (1-w)^2\right]^{-\frac{1}{2}} \\
&= |b_2^i\,(1-w) + b_1^i\,w|\ \left[(w)^2 + (1-w)^2\right]^{-\frac{1}{2}} \\
&= ||\mathbf{b}^i - \text{Proj}_{\lambda_1}(\mathbf{b}^i)||
\end{aligned}$$

□

B.15 Proof of Proposition 5

Proof. Note first that $\text{Var}(y_d|\mathbf{m}^*) = w_{d1}^2\text{Var}(\theta_1|\mathbf{m}^*) + w_{d2}^2\text{Var}(\theta_2|\mathbf{m})$ which, given $w_{11} = w_{22} = w$, implies that $\text{Var}(y_1|\mathbf{m}^*) - \text{Var}(y_2|\mathbf{m}^*) = w^2(1-w)^2\left[\text{Var}(\theta_1|\mathbf{m}^*) - \text{Var}(\theta_2|\mathbf{m}^*)\right] = w^2(1-w)^2\left[\frac{1}{6(\tilde{k}_1^*+2)} - \frac{1}{6(\tilde{k}_2^*+2)}\right]$.

Under centralization, the principal's expectation about her update beliefs on each state—and, thus, the residual variance— depends on how much information she expects to receive from agents on the equilibrium path. Since biases are uniformly distributed between $[0, \varepsilon]$, Lemma 8 implies that for any agent she expects to be revealing information about θ_1, she expects another agent with the same conflict of interest who will be willing to reveal information about θ_2. Then, in expectation, the number of agents revealing each signal coincide, that is $\tilde{k}_1^{\text{c}} = \tilde{k}^{\text{c}} = 2$, which implies that $E\left[\text{Var}(\theta_1|\mathbf{m}^*) - \text{Var}(\theta_2|\mathbf{m}^*)\right] = 0$.

Now suppose the principal delegates y_2 to agent $i = 0$. Again, given the uniform distribution of biases, by Lemma 9 she expects more agents willing to reveal information about θ_1 than θ_2—and the opposite pattern for $i = 0$. Then, her ex-ante expectations about equilibrium information on each state satisfy: $\tilde{k}_1^{\text{P1}} > \tilde{k}_2^{\text{P1}}$ and $\tilde{k}_1^0 < \tilde{k}_2^0$; as a consequence, $E\left[\text{Var}(\theta_1|\mathbf{m}^{\text{P1}})\right] < E\left[\text{Var}(\theta_2|\mathbf{m}^{\text{P1}})\right]$ and $E\left[\text{Var}(\theta_1|\mathbf{m}^0)\right] > E\left[\text{Var}(\theta_2|\mathbf{m}^0)\right]$. □

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Appendix C (Online Appendix)

C.1 Covert Information Acquisition Game

In the covert game the decision-maker does not observe agents' information acquisition decisions. This implies that a Perfect Bayesian Equilibrium must also specify the decision-maker's beliefs about agents' investments in information. I will focus on pure strategy equilibria at the information acquisition stage. In principle, the agent in question may try to convey information about which signals he acquired to the decision-maker by means of his cheap talk message. However, a result from Argenziano et al. (2016) allows me to restrict attention to equilibria in which agents do not signal how much information each has acquired, and this is without loss of generality. Below I present the result.

Lemma 12 (Argenziano et al., 2016)**.** *Any outcome supported in a Perfect Bayesian Equilibrium of the covert game in which an agent follows a pure strategy in the choice of information can be supported in a Perfect Bayesian Equilibrium in which the decision makers beliefs about his information acquisition decision do not vary with the agents message.*

There would be two classes of deviations available to agents if the decision-maker's beliefs about information acquisition decisions could be affected by the choice of messages. First, an agent could acquire an off-path amount of information but still send the message corresponding to the equilibrium amount of information. Secondly, the agent could acquire an off-path amount information and send a message corresponding to an off-path information acquisition choice, which in turn may no be true. The lemma says that any equilibrium outcome under the second class of deviations can be supported as an equilibrium in which the agent cannot change the decision-maker's beliefs about his information acquisition decision.

When an agent acquires an off-path amount of information he can choose among the equilibrium messages according to his preferences. As a consequence, any deviation at the information acquisition stage implies a deviation at the communication stage. The result below summarizes this.

Lemma 13. *When agent i acquires fewer signals than what is expected on the equilibrium path, the messages used under the deviation are a strict subset of the equilibrium messages available. When i acquires more signals than expected on path, he uses the additional information to deviate from truth-telling for some signal realizations.*

The argument of the above lemma is straightforward. When i acquires fewer signals, he will not be able to condition his message on the information that has not been observed. As a consequence, the messages effectively used under the deviation will be fewer than those available on the equilibrium path, which implies that i is inducing beliefs that do not reflect his signal realizations. When i acquires more signals he cannot transmit that additional information on the equilibrium path (there is no way of signalling he acquired more information). Because additional information implies additional costs, i must be obtaining some utility gains with respect to equilibrium communication—by inducing beliefs according to his preferences for some signals realizations. This clearly implies that he will deviate from truth-telling when he observes the corresponding realizations.

Let $(\mathfrak{s}^*, \mathbf{m}^*(\mathfrak{s}^*), \mathbf{y}^*(\mathbf{m}^*(\mathfrak{s}^*)))$ be the equilibrium information acquisition decisions, message strategies, and decisions (respectively). Then, agent i's IC constraint at the information acquisition stage must consider any possible deviation $\hat{\mathfrak{s}}^i$ and the corresponding message strategy

$\hat{m}^i(\hat{\mathfrak{s}}^i)$; that is,

$$E\left[\int_0^1\int_0^1 -\sum_{y_d}^{\{y_1,y_2\}}\left(y_d\left(m^{i*},\mathbf{m}^{\text{-}i*}\right)-\delta_d-b_d^i\right)^2 f(\theta_1|\mathfrak{s}^{i*},\mathbf{m}^{\text{-}i*})f(\theta_2|\mathfrak{s}^{i*},\mathbf{m}^{\text{-}i*})+\right.$$
$$\left.+\int_0^1\int_0^1\sum_{y_d}^{\{y_1,y_2\}}\left(y_d\left(\hat{m}^{i},\mathbf{m}^{\text{-}i*}\right)-\delta_d-b_d^i\right)^2 f(\theta_1|\hat{\mathfrak{s}}^{i},\mathbf{m}^{\text{-}i*})f(\theta_2|\hat{\mathfrak{s}}^{i},\mathbf{m}^{\text{-}i*})\right]\geq C(\mathfrak{s}^{i*})-C(\hat{\mathfrak{s}}^i) \tag{24}$$

Then, given that deviations at the info acquisition stage do not affect the set of influential messages (Lemma 12) and that this deviations necessarily imply deviations at the communication stage (Lemma 13), the above expression can be solved by computing the expectation over all possible signals realizations and the corresponding messages on- and off-path. In particular, the utility gains from deviations will be given by the realizations in which the messages on- and off- path are different. Formally, let $\tilde{\mathbf{S}}^i$ represent the realization of the signals corresponding to agent i, independent of which of these he observes (determined by $\mathfrak{s}^i$). I can then express and compare message strategies on- and off-path as functions of i's type and the information he observes. In other words, *before deciding on information acquisition* and given the equilibrium under play, he can assess the utility gains from any info acquisition strategy and the corresponding messages he expects to send conditional on each possible pair of signal realizations. Equation 24 then becomes:

$$\sum_{\tilde{\mathbf{S}}\in\mathcal{S}}\Pr(\tilde{\mathbf{S}}^i)\times\int_0^1\int_0^1-\sum_{y_d}^{\{y_1,y_2\}}\left[\left(y_d\left(m^i(\mathfrak{s}^{i*},\tilde{\mathbf{S}}^i)\right)-\delta_d-b_d^i\right)^2-\left(y_d\left(m^i(\hat{\mathfrak{s}}^i,\tilde{\mathbf{S}}^i)\right)-\delta_d-b_d^i\right)^2\right]\times$$
$$\times f(\theta_1|\tilde{S}_1^i,\mathbf{m}^{\text{-}i*})f(\theta_2|\tilde{S}_2^i,\mathbf{m}^{\text{-}i*})\geq C(\mathfrak{s}^{i*})-C(\hat{\mathfrak{s}}^i)$$

Now, I proceed to analyse deviations from different equilibrium info acquisition strategies.

Agent i acquires both signals in equilibrium ($\mathfrak{s}^{i*}=\mathbf{S}^i$)

Let denote by $\nu_r^{i*}(\tilde{\mathbf{S}}^i)=E\left(\theta_r|m^i(\mathfrak{s}^{i*},\tilde{\mathbf{S}}^i),\mathbf{m}^{\text{-}i*}\right)$ the beliefs about $\theta_r=\{\theta_1,\theta_2\}$ induced by i under the equilibrium information acquisition strategies and the message corresponding to the realizations given by $\tilde{\mathbf{S}}\in\mathcal{S}$. Equivalently, denote by $\hat{\nu}_r^i(\tilde{\mathbf{S}}^i)=E\left(\theta_r|m^i(\hat{\mathfrak{s}}^i,\tilde{\mathbf{S}}^i),\mathbf{m}^{\text{-}i*}\right)$ be the beliefs induced under the deviation at the information acquisition stage (for the same signals realizations). Then, the IC constraint at the information acquisition stage for agent i becomes:

$$\sum_{\tilde{\mathbf{S}}^i\in\mathcal{S}}\Pr(\tilde{\mathbf{S}}^i)\left[-\sum_{y_d}\left[w_{d1}\left(\nu_1^{i*}(\tilde{\mathbf{S}}^i)-\hat{\nu}_1^i(\tilde{\mathbf{S}}^i)\right)+w_{d2}\left(\nu_2^{i*}(\tilde{\mathbf{S}}^i)-\hat{\nu}_2^i(\tilde{\mathbf{S}}^i)\right)\right]\times\right.$$
$$\left.\left[-w_{d1}\left(\nu_1^{i*}(\tilde{\mathbf{S}}^i)-\hat{\nu}_1^i(\tilde{\mathbf{S}}^i)\right)-w_{d2}\left(\nu_2^{i*}(\tilde{\mathbf{S}}^i)-\hat{\nu}_2^i(\tilde{\mathbf{S}}^i)\right)-2b_d^i\right]\right]\geq C(\mathfrak{s}^{i*})-C(\hat{\mathfrak{s}}^i)$$

First consider the deviation in which i only acquires information about θ_1; that is, $\hat{\mathfrak{s}}^i=\{S_1^i\}$. It is straightforward to note that this deviation *per se* does not imply any difference in induced beliefs with respect to θ_1, formally $\nu_1^{i*}(\tilde{\mathbf{S}}^i)=\hat{\nu}_1^i(\tilde{\mathbf{S}}^i)$ for all $\tilde{\mathbf{S}}^i\in\mathcal{S}$. Now, i's message associated with S_2^i does not depend on the signal's realization, but depends on $\mathbf{b}^i$ and may also depend on S_1^i.

Let consider the case in which $\hat{m}^i=\{\tilde{S}_1^i,1\}$, i.e. i truthfully reveals his information about

θ_1 and announces always a 1 for θ_2. Then, $\hat{\nu}_1^i(\tilde{\mathbf{S}}^i) = \frac{(k_2+4)}{2(k_2+3)}$ and it is different from $\nu_1^{i*}(\tilde{\mathbf{S}}^i)$ only when $\tilde{S}_2^i = \{0\}$ which, in turn, happens for $\tilde{\mathbf{S}}^i = \{(0,0);(1,0)\}$. The IC constraint in such a case is:

$$\Pr\left(\tilde{\mathbf{S}} = \{(0,0)\}\right)\left[\sum_{y_d}\left[w_{d2}\left[\frac{(k_2+2)}{2(k_2+3)} - \frac{(k_2+4)}{2(k_2+3)}\right]\right]\left[w_{d2}\left[\frac{(k_2+2)}{2(k_2+3)} - \frac{(k_2+4)}{2(k_2+3)} - 2b_d^i\right]\right]\right] +$$
$$\Pr\left(\tilde{\mathbf{S}} = \{(1,0)\}\right)\left[\sum_{y_d}\left[w_{d2}\left[\frac{(k_2+2)}{2(k_2+3)} - \frac{(k_2+4)}{2(k_2+3)}\right]\right]\left[w_{d2}\left[\frac{(k_2+2)}{2(k_2+3)} - \frac{(k_2+4)}{2(k_2+3)} - 2b_d^i\right]\right]\right] \geq C(S_2^i)$$

Given that $\Pr\left(\tilde{\mathbf{S}} = \{(0,0)\}\right) = \Pr\left(\tilde{\mathbf{S}} = \{(1,0)\}\right) = 1/4$, the IC constraint becomes.

$$\frac{1}{(k_2+3)}\left[\frac{(w_{12}^2 + w_{22}^2)}{2(k_2+3)} - \beta_2^i\right] \geq C(S_2^i)$$

That is, the expected utility gains of inducing the correct beliefs about θ_2 should be greater than the extra utility from saving in the costs of becoming informed about that state. It is easy to show that the case of $\hat{m}^i = \{\tilde{S}_1^i, 0\}$ has the sign of β_2^i reversed, for which the generic IC constraint for not acquiring signal S_r^i becomes:

$$\frac{1}{(k_r+3)}\left[\frac{(w_{1r}^2 + w_{2r}^2)}{2(k_r+3)} - |\beta_r^i|\right] \geq C(S_r^i) \tag{25}$$

For the deviation involving no information acquisition, $\hat{\mathfrak{s}}^i = \{\emptyset\}$, the expression of the IC constraint will depend on the message i decides to announce at the communication stage. On the one hand, when the message is $\hat{m)}^i = \{(0,0),(1,1)\}$, the induced beliefs will coincide with the equilibrium strategy when $\tilde{\mathbf{S}}^i = (1,1)$, but also partially for other realizations. Formally $\nu_r^{i*}(1,1) = \hat{\nu}_r^i(1,1)$ for $\theta_r = \{\theta_1, \theta_2\}$, $\nu_1^{i*}(1,0) = \hat{\nu}_1^i(1,1)$, and $\nu_2^{i*}(0,1) = \hat{\nu}_r^i(1,1)$. Following the characterization of equilibrium communication under centralization, the IC constraint becomes:

$$\left[\frac{1}{(k_1+3)}\left[\frac{(w_{11}^2 + w_{21}^2)}{2(k_1+3)} + \frac{(w_{11}w_{12} + w_{21}w_{22})}{(k_2+3)} - |\beta_1^i|\right] + \right.$$
$$\left. + \frac{1}{(k_2+3)}\left[\frac{(w_{12}^2 + w_{22}^2)}{2(k_2+3)} + \frac{(w_{11}w_{12} + w_{21}w_{22})}{(k_1+3)} - |\beta_2^i|\right]\right] \geq 2C(S_1^i, S_2^i) + \frac{(w_{11}w_{12} + w_{21}w_{22})}{(k_1+3)(k_2+3)} \tag{26}$$

Which basically is a more strict version of the IC constraint for full revelation when signals coincide (under centralization).

Similarly, when the deviation involves announcing $\hat{m)}^i = \{(0,1),(1,0)\}$ the IC constraint becomes:

$$\left[\frac{1}{(k_1+3)}\left[\frac{(w_{11}^2 + w_{21}^2)}{2(k_1+3)} - \frac{(w_{11}w_{12} + w_{21}w_{22})}{(k_2+3)} - |\beta_1^i|\right] + \right.$$
$$\left. + \frac{1}{(k_2+3)}\left[\frac{(w_{12}^2 + w_{22}^2)}{2(k_2+3)} - \frac{(w_{11}w_{12} + w_{21}w_{22})}{(k_1+3)} - |\beta_2^i|\right]\right] \geq 2C(S_1^i, S_2^i) + \frac{(w_{11}w_{12} + w_{21}w_{22})}{(k_1+3)(k_2+3)} \tag{27}$$

Agent i acquires one signal on path ($\mathfrak{s}^{i*} = \{S_1^i\}$).

When i acquires only one signal on path, his assessment of the consequences of any deviation still conditions on each possible pair of signal realizations. As I show in this section, this becomes particularly important for deviation involving acquisition of more signals. Not is necessary to distinguish between the induced beliefs on- and off-path, and the actual information i has access to. Thus, in addition to the previously defined $\nu_r^{i*}(\tilde{\mathbf{S}}^i)$ and $\hat{\nu}_r^i(\tilde{\mathbf{S}}^i)$, I now denote by $\nu_r^i(\tilde{\mathbf{S}}^i) = E\left(\theta_r|\tilde{\mathbf{S}}^i, \mathbf{m}^{\text{-}i*}\right)$ the beliefs about θ_r that would result from the decision-maker observing the signals available to agent i (independent of his information acquisition strategy). Then, i's IC constraint at the information acquisition stage becomes:

$$\sum_{\tilde{\mathbf{S}}^i \in \mathcal{S}} \Pr(\tilde{\mathbf{S}}^i)\Bigg[-\sum_{y_d}\left[w_{d1}\left(\nu_1^{i*}(\tilde{\mathbf{S}}^i) - \hat{\nu}_1^i(\tilde{\mathbf{S}}^i)\right) + w_{d2}\left(\nu_2^{i*}(\tilde{\mathbf{S}}^i) - \hat{\nu}_2^i(\tilde{\mathbf{S}}^i)\right)\right] \times$$
$$\left[w_{d1}\left(\nu_1^{i*}(\tilde{\mathbf{S}}^i) + \hat{\nu}_1^i(\tilde{\mathbf{S}}^i) - 2\nu_1^i(\tilde{\mathbf{S}}^i)\right) + w_{d2}\left(\nu_2^{i*}(\tilde{\mathbf{S}}^i) + \hat{\nu}_2^i(\tilde{\mathbf{S}}^i) - 2\nu_2^i(\tilde{\mathbf{S}}^i)\right) - 2b_d^i\right]\Bigg] \geq C(S_1^i) - C(\hat{\mathfrak{s}}^i)$$

When i considers the deviation of not acquiring any signals and decides to announce $\hat{m}_1^i = \{1\}$, he induces incorrect beliefs as compared to the equilibrium in two cases, namely $\tilde{\mathbf{S}} = \{(0,0);(0,1)\}$. The ex-ante expected utility losses of such strategy depends on the signal realizations, as can be noted in the expression for the IC constraint below:

$$\Pr\left(\tilde{\mathbf{S}}^i = (0,0)\right)\Bigg[-\sum_{y_d}\left[w_{d1}\left(\frac{(k_1+2)}{2(k_1+3)} - \frac{(k_1+4)}{2(k_1+3)}\right)\right] \times$$
$$\left[w_{d1}\left(\frac{(k_1+2)}{2(k_1+3)} + \frac{(k_1+4)}{2(k_1+3)} - \frac{2(k_1+2)}{2(k_1+3)}\right) + w_{d2}\left(\frac{1}{2} + \frac{1}{2} - \frac{2(k_2+2)}{2(k_2+3)}\right) - 2b_d^i\right]\Bigg] +$$
$$\Pr\left(\tilde{\mathbf{S}}^i = (0,1)\right)\Bigg[-\sum_{y_d}\left[w_{d1}\left(\frac{(k_1+2)}{2(k_1+3)} - \frac{(k_1+4)}{2(k_1+3)}\right)\right] \times$$
$$\left[w_{d1}\left(\frac{(k_1+2)}{2(k_1+3)} + \frac{(k_1+4)}{2(k_1+3)} - \frac{2(k_1+2)}{2(k_1+3)}\right) + w_{d2}\left(\frac{1}{2} + \frac{1}{2} - \frac{2(k_2+4)}{2(k_2+3)}\right) - 2b_d^i\right]\Bigg] \geq C(S_1^i)$$

Which, after some algebra gives:

$$\frac{1}{(k_1+3)}\left[\frac{(w_{11}^2 + w_{21}^2)}{2(k_1+3)} - \beta_1^i\right] \geq C(S_1^i) \tag{28}$$

It is straightforward to note that the generic IC constraint involves the absolute value of β_r^i.

Deviations involving the acquisition of more information have the issue that i cannot signal this to the decision-maker. This additional information will thus be used to identify situations (i.e. signal realizations) under which i will deliberately lie to the decision-maker. Such deviations are thus related to the credibility loss, because i would like to induce beliefs about the signal he is not expected to acquire on path by means of messages on the signal he is believed on path.

As analysed in the communication game, the credibility loss takes place when signals do not coincide, $\tilde{\mathbf{S}}^i = \{(0,1);(1,0)\}$, so any deviation at the communication stage will take place in one of these cases. Moreover, given that β_1^i is typically not zero, i's incentives to lie will always be in a single direction, that is either when $\tilde{\mathbf{S}}^i = (0,1)$ or when $\tilde{\mathbf{S}}^i = (1,0)$ but not in both. The IC constraint for the deviation of acquiring both signals and announcing $\hat{m}_1^i = 0$ when $\tilde{\mathbf{S}}^i = (1,0)$

will be given by:

$$-\Pr\left(\tilde{\mathbf{S}}^i=(1,0)\right)\sum_{y_d}\left[\left[w_{d1}\left(\frac{(k_1+4)}{2(k_1+3)}-\frac{(k_1+2)}{2(k_1+3)}\right)\right]\times\right.$$

$$\left.\left[w_{d1}\left(\frac{(k_1+4)}{2(k_1+3)}+\frac{(k_1+2)}{2(k_1+3)}-\frac{(k_1+4)}{(k_1+3)}\right)+w_{d2}\left(1-\frac{(k_2+2)}{(k_2+3)}\right)-2b_d^i\right]\right]\geq C(S_2^i)$$

Which yields:

$$\left[\frac{(w_{11}^2+w_{21}^2)}{2(k_1+3)^2}-\frac{(w_{11}w_{12}+w_{21}w_{22})}{2(k_1+3)(k_2+3)}-\beta_1^i\right]\geq -2C(S_2^i) \tag{29}$$

Which is equivalent to say that the cost of acquiring the second signal is too large with respect to the utility gain from deviating under ambiguous information.

Incentive compatibility then depends on $|\beta_1^i|$ being within the limits imposed by equations (10), (28), and (29). Note that equation (28) implies (10), meaning that if i is willing to acquire $\tilde{S}_1^i$ instead of acquiring no signal, then he will certainly reveal it. Incentive compatibility thus is captured by equations (28), and (29), which lead to:

$$\frac{|\beta_1^i|}{(k^{\mathrm{C}}+3)}\leq\min\left\{\frac{(w_{11}^2+w_{21}^2)}{2(k_1^{\mathrm{C}}+3)^2}-C(S_1^i)\,;\,\frac{(w_{11}^2+w_{21}^2)}{2(k_1^{\mathrm{C}}+3)^2}-\frac{(w_{11}w_{12}+w_{21}w_{22})}{2(k_1+3)(k_2+3)}+2C(S_2^i)\right\}$$

C.2 Information acquisition on the intensive margin

This subsection tries to convey a sense of how restrictive is the assumption that sender can acquire at most one signal associated to each state. Despite the question relates to the model with information acquisition (Section 3), throughout this analysis I will 'endow' agents with different amount of information and study their communication incentives. The analysis with information acquisition on the intensive margin, and its consequences for optimal organizational design will be subject of future work.

I focus in a one-decision, one-state cheap talk model between a sender and a receiver, much in the spirit of the uniform-quadratic example in Crawford and Sobel (1982). I perform two exercises. First, I compare the uniform-quadratic case with the beta-binomial model in which infinitely many senders with the same bias receive a binary signal. Since each of these virtual senders will truthfully reveal his signal when the expected precision of the decision is sufficiently low, the exercise allows me to understand how much the *receiver* gains from allowing a single sender to partition the message space. I show that the receiver strictly prefers to have a single, perfectly-informed sender than infinitely many of them having noisy information, for each possible bias.

In order to gain further intuition, in the second analysis I parametrize the sender's information as a finite number of iid binary trials correlated to the state. I borrow this information structure from Argenziano et al. (2016), but I extend their work by characterizing the maximum number of partitions as a function of the number of signals available to the sender (and his bias). I show the maximum number of partitions in any influential equilibria converges to the perfectly informed case in Crawford and Sobel (1982) when the number of binary experiments goes to infinity. So far I cannot compare the receiver's ex-ante expected utility under centralization and delegation as a function of the sender's information, due to not being able to find a closed form solution for the residual variance under communication.

Binary signals vs perfect information. Let $i = \{1, 2, ..., N\}$ be senders, each of which observes an iid signal $S^i \in \{0, 1\}$ correlated with the state $\theta \sim U[0, 1]$ such that $\Pr(S^i = 1) = \tilde{\theta}$. The receiver has to decide on $y \in \Re$ and her preferences are given by $U_R = -(y - \theta)^2$; while that for a typical sender is given by $U_i = -(y - \theta - b^i)^2$. Let assume that $b^i = b \in \Re_+$ for all $i = \{1, 2, ..., N\}$. As has been the case so far, the senders first observe their signals, send cheap talk messages to the receiver, and then she decides on y.

I denote by k the number of sender revealing truthfully their signals to the receivers, and ℓ those that report a 1. Following Galeotti et al. (2013), a sender reveals his signal truthfully to the receiver if and only if:

$$b \leq \frac{1}{2(k+3)}$$

And the maximum number of senders willing to reveal their signals truthfully will be given by

$$k = \left\lfloor \frac{1}{2b} - 3 \right\rfloor$$

The receiver's ex-ante expected variance as a function of the number of agents truthfully revealing their binary signals is, then:

$$E[\text{Var}(\theta|\ell, k)] = \frac{1}{6(k+2)} \simeq \frac{b}{3(1-2b)}$$

Now, the ex-ante expected variance when there is only one sender who perfectly observes θ and has the same bias b as before—the uniform-quadratic case in Crawford and Sobel (1982)—is given by:

$$E[\text{Var}(\theta|N(b))] = \frac{1}{12N^2} + \frac{b^2(N-1)}{3}$$

Where the maximum number of partitions of the state space, $N(b)$, is given by:

$$N(b) = \left\lceil -\frac{1}{2} + \frac{1}{2}\left(1 + \frac{2}{b}\right)^{1/2} \right\rceil$$

Figure 7 below shows the comparison between the residual variance in both cases. The receiver derives higher ex-ante expected utility—expected variance is lower—when the sender is perfectly informed rather than when many senders fully reveal their information. The intuition behind this observation relates to the possibility of partitioning the state space when the sender is perfectly informed. In order to see that I now analyse a sender-receiver game in which the former observes k binary signals.