Overview

$$For \begin{cases} B_n \text{ : remaining balance at time n}, \forall n \in \mathbb{N} \\ D_n \text{ : \% defaults at time n}, \forall n \in \mathbb{N} \end{cases} \text{ we have the following}$$

$$B_n = B_{n-1} \cdot (1 - D_n) \Rightarrow B_n = B_0 \cdot \prod_{i=1}^{n} (1 - D_i)$$

Analysis

Our aim is to find an additional constant default rate α so that the remaining balance after n periods i lower by an arbitrary amount A. This can be summarized by the following proposition:

$$\{\, \alpha \,|\, B_0 \cdot \textstyle\prod_{i=1}^{n} \,[\,(1 - D_i) \cdot (1 - \alpha)\,] = B_n - A \,\}$$

Let's solve for α:

$$B_0 \cdot \prod_{i=1}^{n} [\,(1 - D_i) \cdot (1 - \alpha)\,] = B_n - A$$

Since $B_n = B_0 \cdot \prod_{i=1}^{n} (1 - D_i)$, we have

$$B_0 \cdot \prod_{i=1}^{n} [\,(1 - D_i) \cdot (1 - \alpha)\,] = B_0 \cdot \prod_{i=1}^{n} (1 - D_i) - A$$

$$\Leftrightarrow A = B_0 \cdot [\prod i = 1^n \,[\,(1 - D_i)\,] - \prod i = 1^n \,[\,(1 - D_i) \cdot (1 - \alpha)\,]\,]$$

$$\Leftrightarrow A = B_0 \cdot [\textstyle\prod_{i=1}^{n} \,[\,(1 - D_i)\,] - (1 - \alpha)^n \cdot \textstyle\prod_{i=1}^{n} \,[\,(1 - D_i)\,]\,]$$

$$\Leftrightarrow A = B_0 \cdot \prod_{i=1}^{n} (1 - D_i) \cdot [1 - (1 - \alpha)^n]$$

$$\Leftrightarrow (1 - \alpha)^n = 1 - \frac{A}{B_0 \cdot \prod_{i=1}^{n} (1 - D_i)}$$

$$\Leftrightarrow \alpha = 1 - \sqrt[n]{1 - \frac{A}{B_0 \cdot \prod_{i=1}^{n} (1 - D_i)}}$$

Application

For any period n in the forecast, the new/adjusted PD rates will be:

$$
\begin{gathered}
(1 - PD_{new}) = (1 - PD_{old}) \cdot (1 - \alpha) \\
\Leftrightarrow PD_{new} = PD_{old} + \alpha - PD_{old} \cdot \alpha
\end{gathered}
$$