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Mathematical Approach on Exploring Relativity - I

Special Relativity

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Disclaimer

These notes are made for the crash course I am currently teaching (as of October 2023) to the students who have registered through the collaboration of Brahmannd and Society of Mathematics at PDEU.

I am a postgraduate, teaching the topic for an audience for the first time, so there may be certain errors and informalities in the content, however, notes will certainly be updated and formalised more with the feedbacks from students and readers.

I am grateful to my Special and General Relativity professors at Imperial College London for teaching the subject with depth and rigour. I am also grateful to the professors at University of Cambridge and Dr. Dexter Chua who have shared their precious notes on the web from which I have studied these subjects in ever more depth (I may not be allowed to share them here, however you can also find them for your personal usage and preparation using a simple Web Search).

I have attempted to avoid any bias in my writing in regards to copying from what I have learnt at my MSc tenure as well as from the resources I have prepared from, yet the notes may reflect parts of language and pace of these lectures according to my learnings. Even then, all errors and mistakes are almost surely mine.

Recommended Resources and Books

The first set of resources are from YouTube for absolute beginners.

- Intro to Special Relativity Course by minutephysics
- Special relativity by Khan Academy Physics
- MIT 8.20 Introduction to Special Relativity, January IAP 2021 by MIT OpenCourseWare

The following two are lecture series by Prof Leonard Susskind on stanford YouTube channel,they are primarily recommended in accordance with my frequency of understanding the subject and my preparation. In my understanding, these two are better for students who have a decent grasp of undergraduate level mathematics. If you can smoothly sail through these, the notes mostly complement these lectures for beginners.

- Classical Mechanics
- Special Relativity

Finally, the formal set of recommendations - the Books. As this subject is a century old and extremely famous, there is a plethora of good and amazing books on it. The following is the combination of books suggested by my professors at ICL and Professor David Tong of University of Cambridge in his notes.

- Douglas Gregory, Classical Mechanics

This is a good book is for the preparation or revision on the fundamentals of Newtonian mechanics.

- L. Landau and E. Lifshitz, Mechanics
- H. Goldstein, C. Poole and J. Safko, Classical Mechanics

These two books are for the students who want to go in further depth with the Lagrangian and Hamiltonian formalisms of the first chapter. Book by Landau and Lifshitz is really concise and discuses the overview, whereas, Goldstein is rather for the absolute pros.

- L. Landau, The Classical Theory of Fields
- A. French, Special Relativity

The classical theory of field, suggested by my Special Relativity professor, is the second part of the series by Landau, and builds on the topics discussed in the first part - Mechanics towards Special Relativity (Electromagnetism and General Relativity if you wish to go further). The book by French is suggested by Prof Tong and I personally find it really beautifully written with all the diagrams, the figures and images of actual researchers and experiments.

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1.1 Newtonian Mechanics

Laws of Motion and Law of Gravity

1.2 Historical Development of Classical Dynamics

1.3 Why Do we need Relativity?

1.4 Galilean Relativity

1.5 Lagrangian and Hamiltonian (Brief Overview)

2. Special Relativity – I

2.1 Postulates of Relativity

2.2 Derivations of Lorentz Transformation

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2.5 Addition of Velocities in SR

2.6 Concept of Spacetime

2.7 Metric and Its Invariance

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2.9 Relativistic Doppler Effect

3. Special Relativity – II

3.1 Index Notations

3.2 4-Vectors and Examples

3.3 Momentum in Special Relativity

3.4 $E = mc^2$ Mass-Energy Equivalence

3.5 Conservation of Energy and Momentum

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