1. Introduction

Having studied probability in standard XII, now we shall proceed to study some probability distributions viz. Binomial Distribution, Poisson's Distribution and Normal Distribution.

2. Random Variable

In probability problems it is convenient to think of a variable and consider the outcomes as the values, the variable takes. In a toss of a die we think of a variable x which takes values 1, 2, 3, 4, 5, 6. In a toss of a coin we think of a variable x which takes values 0 and 1 (the number of heads obtained). Such a variable is called **a random variable.** In simple words a variable used to denote the numerical value of the outcome of an experiment is called **a random variable.**

If the random variable x takes only a finite or countably infinite values it is called **a discrete random variable** e.g. if x denotes the result of a toss of a die or if x denotes the result of shots fired till the target is hit, x is a discrete random variable. On the other hand if x takes uncountably infinite values then it is called **a continuous random variable.** For example, if x is the height of a person selected at random or x is the age of the person then x is a continuous random variable because x can take any value between a specified range.

3. Probability distributions

Suppose x denotes the outcome of the toss of a die. Then x takes the values 1, 2, 3, 4, 5, 6 each with probability 1/6. We can put this information in a table. Such a table giving the probabilities of x along with its values is called probability distribution table. The way in which the probabilities are distributed over the values of the variable or the totality of the probabilities together with the corresponding values of x is called the **probability distribution.**

x	$p(x)$
1	1 / 6
2	1 / 6
3	1 / 6
4	1 / 6
5	1 / 6
6	1 / 6
Total	1

In general if x is a discrete random variable and we know the probability p_i that x will take the value x_i for all values of x_i then the probability distribution is called a **discrete probability distribution.** The probabilities p_i may be given in a tabular form or by a formula satisfying the conditions that **(i)** $0 \le p_i \le 1$ for all i and **(ii)** $\Sigma p_i = 1$. Then p is called **probability density function.** Generally, the capital letter 'P' stands for the term 'probability' and small letter p or p_i or $p(x_i)$ for its value.

If x is a continuous random variable and we know the probability that x will be in a given interval (a, b) then the probability distribution is called ***a continuous probability distribution.*** Usually a continuous probability distribution is given in the form of a function $f(x)$ and $f(x)$ is called **probability density function. The area under the curve from a to b is equal to the probability that x will be in interval (a, b). The total area under the curve is equal to 1.**

$$\text{i.e.} \quad \int_{-\infty}^{\infty} f(x)\, dx = 1 \quad \text{and} \quad p(a < x < b) = \int_a^b f(x)\, dx \quad \text{and} \quad f(x)\ ^3\ 0 \text{ for all } x.$$

4. Discrete Probability Distributions

Ex. 1 : The probability density function of a random variable x is zero except at $x = 0, 1, 2$. At these points $P(0) = 3c^3$, $P(1) = 4c - 10c^2$, $P(2) = 5c - 1$. (i) Find c, (ii) Find $P(0 < x \le 2)$

Sol. : (i) Since $\Sigma P_i = 1$, we have

$$3c^3 + 4c - 10c^2 + 5c - 1 = 1 \qquad \therefore\ 3c^3 - 10c^2 + 9c - 2 = 0$$

$$\therefore\ (3c - 1)(c - 2)(c - 1) = 0$$

$$\therefore\ c = \frac{1}{3} \quad \text{The other values are inadmissible as } 0 \le p_i \le 1$$

(ii) The probability distribution now is

x	0	1	2	Total
$p(x)$	1/9	2/9	2/3	1

$$P(0 < x \le 2) = P(x = 1) + P(x = 2) = \frac{2}{9} + \frac{2}{3} = \frac{8}{9}.$$

Exercise

1. A random variable x has the following probability distribution

x	1	2	3	4	5	6	7
$p(x)$	k	$2k$	$3k$	k^2	$k^2 + k$	$2k^2$	$4k^2$