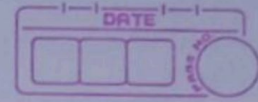


Tutorial No. 8



PUSH DOWN AUTOMATA

Q.1) Define.

a) PDA \Rightarrow

- A pushdown automata can be viewed as a finite automata with stack.
- An added stack provides memory and increases capability of the machine.
- PDA can read input from input string as like FA.
- PDA performs operation on stack as follows:
 - 1) push 3) Check empty
 - 2) pop 4) NOP
- PDA takes transitions from one state to another state.
- PDA is more powerful than FA.

b) Acceptance of a string by PDA \Rightarrow

- Let the PDA, $M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$
 then language accepted th^r an empty stack is given by,

$$L(M) = \{ w \mid (q_0, w, Z_0) \xrightarrow[M]{*} (q_1, \underset{\substack{\uparrow \\ \text{Input} \\ \text{empty}}}{\epsilon}, \underset{\substack{\uparrow \\ \text{stack empty}}}{\epsilon}) \}$$

where,

 q_1 = any state belonging to Q . ϵ = Input is empty. ϵ = Stack is empty.

- It means when PDA starts processing the input 'w' from start state q_0 and current topmost

Symbol on stack z_0 , after taking so many transitions input gets completely scanned i.e. 'w' contains 'ε', stack becomes empty. machine stops at any state q , then that string is said to be accepted by PDA through stack.

c) DPDA \Rightarrow

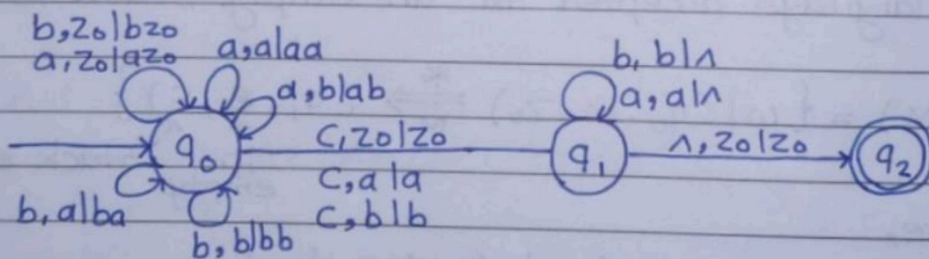
- A deterministic pushdown automata is a variation of pushdown automation. It accepts the deterministic context-free languages, a proper subset of context-free language.

- A deterministic pushdown automata has at most one legal transition for combination of input symbol, state and top stack symbol. This differs from non-deterministic PDA.

Q.2) Design PDA for:

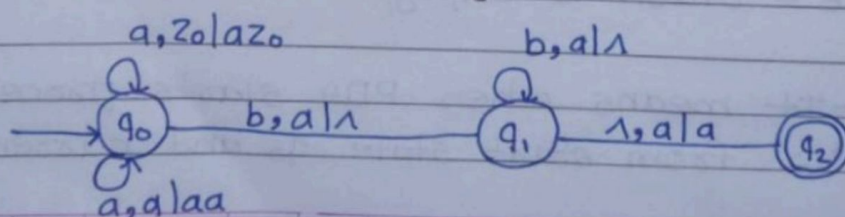
a) $L = \{ xcx^R \mid x \in \{a, b\}^* \}$

\rightarrow



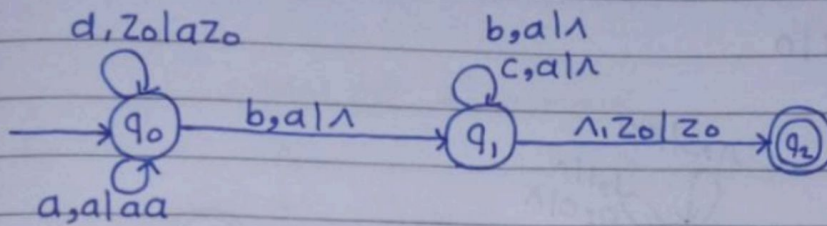
b) $L = \{ a^m b^n \mid m > n \text{ \& } n > 0 \}$

\rightarrow



c) $L = \{a^i b^j c^k \mid i = j + k\}$

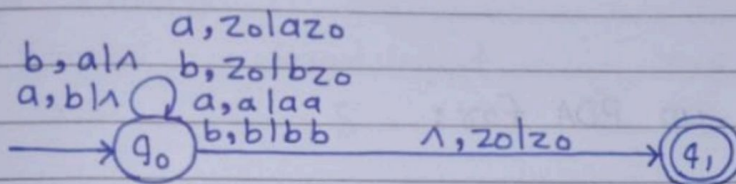
→



d) $L = \{x \mid n_a(x) = n_b(x), x \in \Sigma^*, \Sigma = \{a, b\}\}$

→

$L = \{ab, abab, abba, baba, baab, \dots\}$



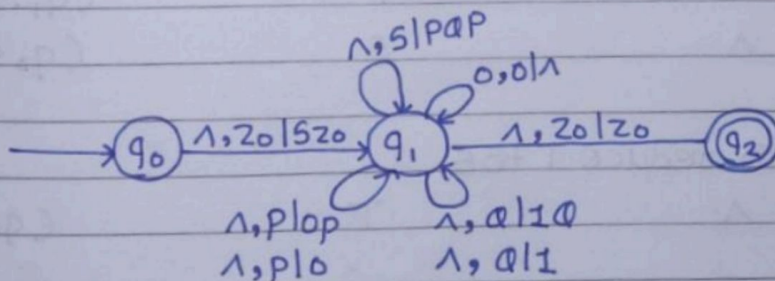
Q.3) Design Top-down PDA for:

a) $S \rightarrow P \mid Q \mid O$

$P \rightarrow OP \mid O$

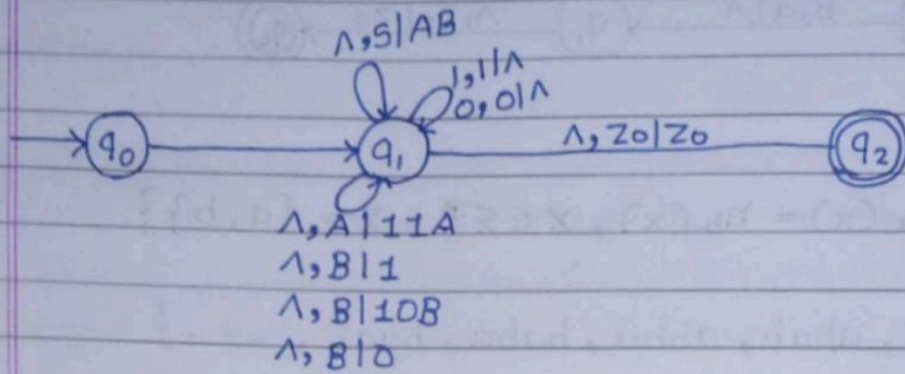
$Q \rightarrow 1Q \mid 1$

→



- b) $S \rightarrow AB$
 $A \rightarrow 11A \mid 1$
 $B \rightarrow 10B \mid 0$

→



Q.4) Design Bottom-up PDA for:

- a) $S \rightarrow S+T \mid T$
 $T \rightarrow T*a \mid a$

→

State	input	stack symbol	Moves(s)
q	S	x	(q, S x)
To reduce S+T to S			
q	Λ	T	(q _{1.1} , Λ)
q _{1.1}	Λ	+	(q _{1.2} , Λ)
	Λ	S	(q, S)
To reduce T to S			
q	Λ	T	(q, S)
To reduce T*a to T			
q	Λ	a	(q _{3.1} , Λ)
q _{3.1}	Λ	*	(q _{3.2} , Λ)
q ₃	Λ	T	(q, T)

To reduce a to T

q	Δ	a	(q, T)
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Moves to accept

q	Δ	S	(q, Δ)
---	----------	---	---------------

q ₁	Δ	Z ₀	(q_2, Δ)
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other combinations			None
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b) $S \rightarrow AB$

$A \rightarrow aA|a$

$B \rightarrow bB|b$

→ State	Read input	TOS	Move(S)
q	S	X	(q, SX)

Reduce AB to S

q	Δ	B	$(q, 1, \Delta)$
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q _{1.1}	Δ	A	(q, S)
------------------	----------	---	----------

Reduce aA to A

q.	Δ	A	$(q, 2A.1, \Delta)$
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q _{2.1}	Δ	a	(q, A)
------------------	----------	---	----------

Reduce a to A

q	Δ	a	(q, A)
---	----------	---	----------

bB to B

q	Δ	B	$(q, 3.1, \Delta)$
---	----------	---	--------------------

q _{3.1}	Δ	b	(q, B)
------------------	----------	---	----------

b to B

q	Δ	b	(q, B)
---	----------	---	----------

Moves to accept.

q	Δ	S	(q, Δ)
---	----------	---	---------------

q ₁	Δ	Z ₀	(q_2, Δ)
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