M T W T F S S

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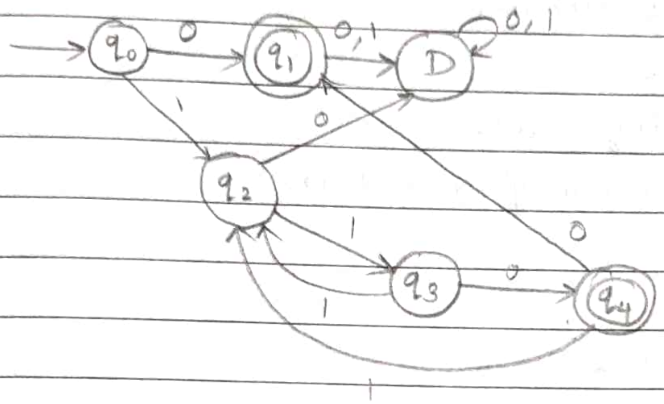
Date: 22/03/2021

YOUVA

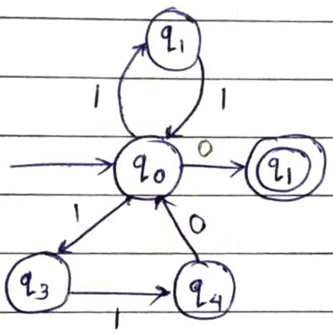
Q Design DFA for following regular expression.

$(11 + 110)^* \, 0$



NFA for this expression

i)

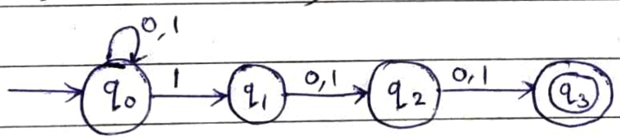


110 $\{q_0, q_1\}$

For q_1 it is accepting

ii) NFA

$(0+1)^* 1 (0+1)(0+1)$



111 $\{q_0, q_1, q_2, q_3\}$ (Results are in term of multiple states thus written as set.)

For q_3 (111) is accepting state

Results are in terms of set.

★ Non- Deterministic finite Automata :

NFA - ~~No any~~

NFA is a 5 tuple $(Q, \Sigma, q_0, A, \delta)$

Q - is set of states

Σ - set of input alphabet

q_0 → initial state

A - set of accepting states

δ - transition function given as follows

$$\delta : Q \times \Sigma \rightarrow 2^Q$$

where, 2^Q is power set of Q.

i) If $Q = \{q_0, q_1\}$

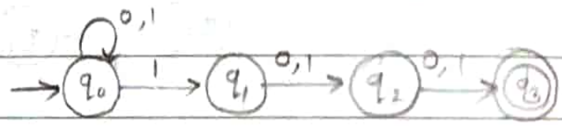
$\therefore \delta = 2^Q = \{ \phi, \{q_0\}, \{q_1\}, \{q_0, q_1\} \}$

ii) For $Q = \{q_0, q_1, q_2, q_3\}$

$\delta = 2^4 =$ Total 16 subsets

Result must be from that 16 subsets.

	0	1
q_0	$\{q_0\}$	$\{q_0, q_1\}$
q_1	$\{q_1\}$	$\{q_2\}$
q_2	$\{q_2\}$	$\{q_3\}$
q_3	ϕ	ϕ



* δ^* (Extended transition function for an NFA)

Let $M = (Q, \Sigma, q_0, A, \delta)$ is an NFA extended transition function

$$\delta^* : Q \times \Sigma^* \rightarrow 2^Q$$

Define as follows

1. for any $q \in Q$

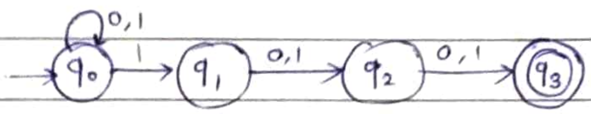
$$\delta^*(q, \Lambda) = \{q\}$$

2. For any $q \in Q$, for any $y \in \Sigma^*$

For any $a \in \Sigma$

$$\boxed{\delta^*(q, ya) = \cup \delta(r, a)} \quad (r \in \delta^*(q, y))$$

Q



Consider above NFA & calculate $\delta^*(q_0, 111)$

$\delta^*(q_0, 011)$

$\delta^*(q_0, 110)$

i) $\delta^*(q_0, \underbrace{11}_{y}\underbrace{1}_{a}) = \cup \delta(r, 1)$

$r \in \delta^*(q_0, 11)$

$r \in \{q_0, q_1, q_2\}$

$= \delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1)$

$\boxed{\delta^*(q_0, 111) = \{q_0, q_1, q_2, q_3\}}$ string is accepted $\because$ q_3 is accepting state which is part of result.

$\delta^*(q_0, \underbrace{1}_{y}\underbrace{1}_{a}) = \cup \delta(r, 1)$

$r \in \delta^*(q_0, 1)$

$r \in \{q_0, q_1\}$

$= \delta(q_0, 1) \cup \delta(\delta_1, 1)$

$\delta^*(q_0, 11) = \{q_0, q_1, q_2\}$

$\delta^*(q_0, \underbrace{}_{y}\underbrace{1}_{a}) = \cup \delta(r, 1)$

$r \in \delta^*(q_0, \Lambda)$

$r \in \{q_0\}$

$\delta^*(q_0,1) = \delta(q_0,1) = \{q_0, q_1\}$

$\delta^*(q_0, \Lambda) = \{q_0\}$

iii) $\delta^*(q_0, \underbrace{111}_{y}\underbrace{0}_{a}) = \bigcup \delta(r,0)$

$r \in \delta^*(q_0, 111)$

$r \in \{q_0, q_1, q_2, q_3\}$

$= \delta(q_0,0) \cup \delta(q_1,0) \cup \delta(q_2,0) \cup \delta(q_3,0)$

$= \{q_0, q_2, q_3, \phi\}$

∴ string is accepted.

HW ii) $\delta^*(q_0, \underbrace{01}_{y}\underbrace{1}_{a})$

For any NFA, $M = (Q, \Sigma, q_0, A, \delta)$, accepting language L then there exist the (DFA) recognizing same languange L, $M_1 = (Q_1, \Sigma, q_1, A_1, \delta_1)$

M_1 define as follows:

$Q_1 = 2^Q$

$q_1 = \{q_0\}$ (Initial state of DFA = Initial set of NFA)

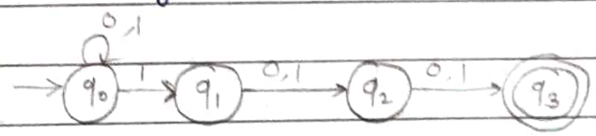
for any $q \in Q_1$ & any $a \in \Sigma$

$$\delta_1(q, a) = \bigcup_{r \in q} \delta(r, a)$$

A_1 = set of accepting states of DFA $= \{q \in Q_1 \mid q \cap A \neq \phi\}$

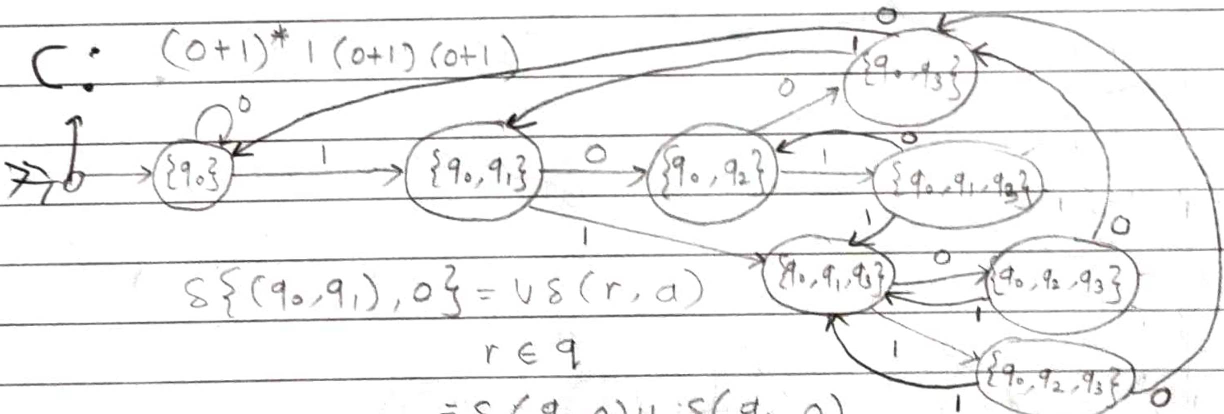
$$A_1 = \{q \in Q \mid q \cap A \neq \phi\}$$

Q Convert following NFA to DFA



$q_0 \cup \phi = q_0$

C: $(0+1)^* 1 (0+1)(0+1)$

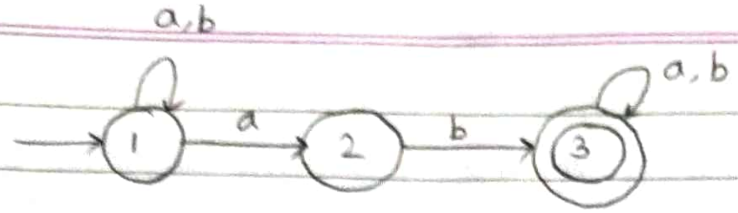


$$\delta\{(q_0, q_1), 0\} = \bigcup_{r \in q} \delta(r, a)$$

$$= \delta(q_0, 0) \cup \delta(q_1, 0)$$

$A_1 = \{q \in Q \mid q \cap A \neq \phi\}$

2)



$(a+b)^* ab (a+b)^*$

$Q = \{1, 2, 3\}$

$q_0 = 1$

$A = \{3\}$

$\Sigma = \{a, b\}$

	a	b
1	$\{1,2\}$	$\{1\}$
2	ϕ	$\{3\}$
3	$\{3\}$	$\{3\}$

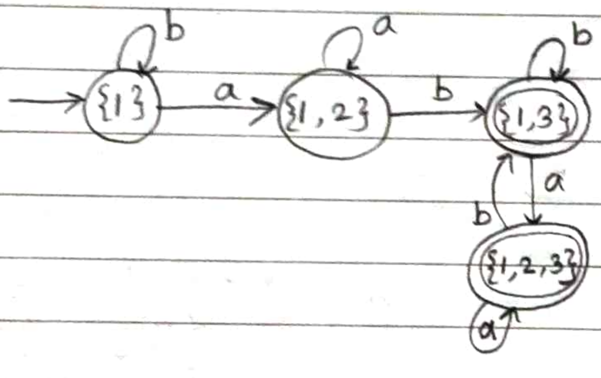
$Q_1 = 2^Q \; \{\{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}, \phi\}$

$q_1 = \{1\}$

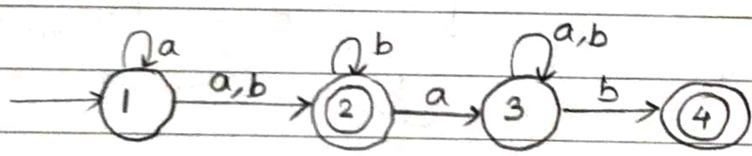
$\Sigma = \{a, b\}$

$A_1 \quad (A \in A_1)$

$A_1 = Q_1$ consisting A



3)



$a^* (a+b) b^* a (a+b)^* b.$

$Q = \{1, 2, 3, 4\}$

$q_0 = 1$

$A = \{2, 4\}$

$\Sigma = \{a, b\}$

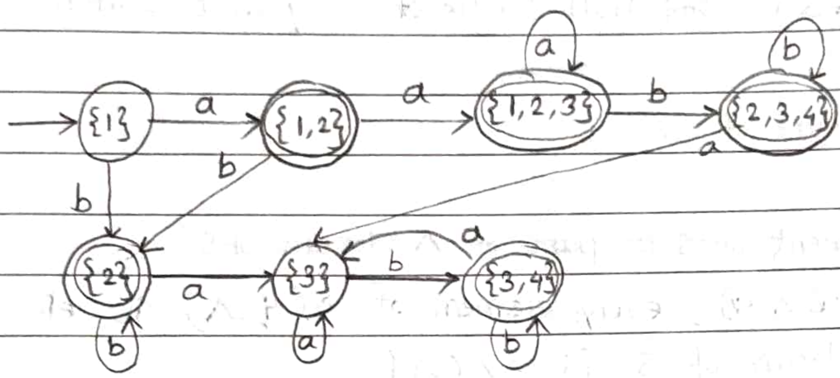
	a	b
1	$\{1,2\}$	$\{2\}$
2	$\{3\}$	$\{2\}$
3	$\{3\}$	$\{3,4\}$
4	ϕ	ϕ

$Q = 2^4 = 32$

$q_1 = \{1\}$

$\Sigma = \{a, b\}$

A_1 = Q_1 consisting A ({2, 4})



* Non-deterministic Finite Automata with null transition (Λ)

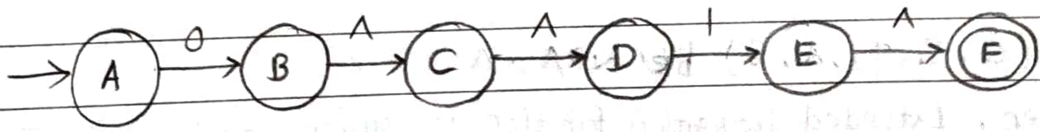
NFA-Λ is 5 tuple $(Q, \Sigma, q_0, A, \delta)$ where,

- Q - set of states
- Σ - set of all input symbols
- q_0 - Initial state
- A - set of accepting state
- δ - transition function

Given as follows :

$$\delta : Q \times (\Sigma \cup \{\Lambda\}) \rightarrow 2^Q$$

Q1)



(01)

* Null Closure

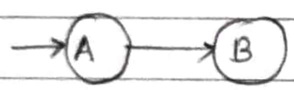
Let, $M = (Q, \Sigma, q_0, A, \delta)$ be NFA - Λ and S is subset of Q ($S \subseteq Q$), the null closure of any set which is subset

Λ closure of s $\Lambda(s)$

1) Every element of s is part of Λ closure of S
2) For any $q \in \Lambda(S)$, every element of $\delta(q, \Lambda)$ is element (or part) of Λ closure of S [i.e. $\Lambda(S)$]
3) No any other element of Q is in Λ closure of S.

Find out $\Lambda(\{B\})$ i.e. $S = \{B\}$ for given e.g.

$\Lambda(\{B\}) = \{B, C, D\}$
$\Lambda(\{C, D\}) = \{C, D\}$
$\Lambda(\{C, D, E\}) = \{C, D, E, F\}$



* Extended Transition function for NFA - Λ

δ^* NFA - Λ

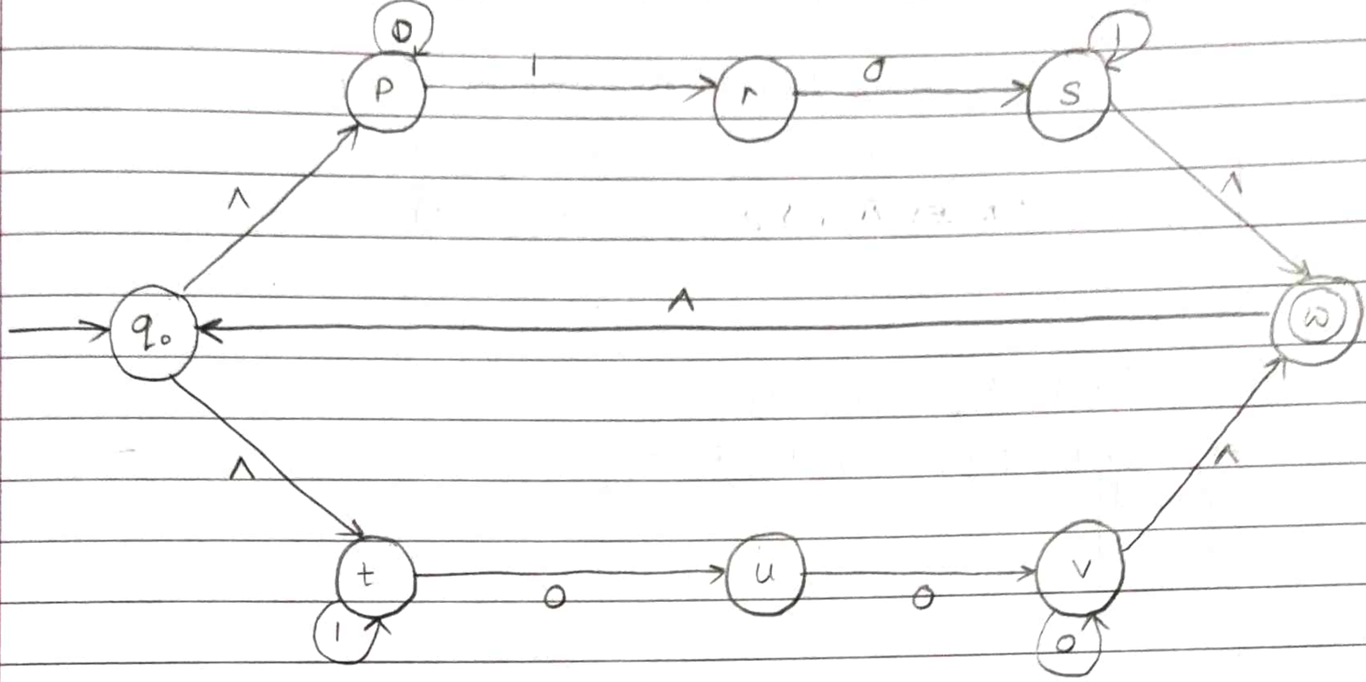
Let, $M = (Q, \Sigma, q_0, A, \delta)$ be NFA - Λ
then, Extended Transition function is given by :

$$\delta^* : Q \times \Sigma^* \to 2^Q$$

1) For any $q \in Q$, $\delta^*(q, \Lambda) = \Lambda(\{q\})$
2) For any $q \in Q$, any $y \in \Sigma^*$ & any $a \in \Sigma$

$$\delta^*(q, ya) = \Lambda\left(\bigcup_{r \in \delta^*(q,y)} \delta(r, a)\right)$$

Q) Consider following NFA-Λ & calculate $\delta^*(q_0, 010)$ & $\delta^*(q_0, 110)$



$\Lambda 010 \Lambda = \{s\}$

$\Lambda 010 \Lambda\Lambda = \{s, \omega, q_0, p, t\}$ $\therefore$ string (010) is accepted.

1) i) $\delta^*(q_0, \underbrace{01}_{y}\underbrace{0}_{a}) = \Lambda\left(\bigcup_{r \in \underbrace{\delta^*(q_0, 01)}_{\{r\}}} \delta(r, 0)\right)$

$= \Lambda(\delta(r, 0))$

$= \Lambda(\{s\})$

$= \{s, \underset{(s,\Lambda)}{\omega}, \underset{(\omega,\Lambda)}{q_0}, \underset{(q_0,\Lambda)}{p}, \underset{(p,\Lambda)}{t}\}$

$\therefore$ String is Accepted

ii) $\delta^*(q_0, \underbrace{0}_{y}\underbrace{1}_{a}) = \Lambda\left(\bigcup_{r \in \underbrace{\delta^*(q_0, 0)}_{\{p, u\}}} \delta(r, 1)\right)$

$= \Lambda(\delta(p, 1) \cup \delta(u, 1))$

$= \Lambda(\{r\} \cup \phi)$

$= \Lambda(\{r\})$

$\delta^*(q_0, 01) = \{r\}$

iii) $\delta^*(q_0, \underset{y}{\wedge}\underset{a}{0}) = \wedge\left(\bigcup_{r \in \underbrace{\delta^*(q_0,\wedge)}_{\{q_0, p, t\}}} \delta(r,0)\right)$

$= \wedge\left(\delta(q_0,0) \cup \delta(p,0) \cup \delta(t,0)\right)$

$= \wedge\left(\phi \cup \{p\} \cup \{u\}\right)$

$\delta^*(q_0,0) = \wedge(\{p,u\}) = \{p,u\}$

iv) $\delta^*(q_0,\wedge) = \wedge(\{q_0\})$

every element of set aretransferred to null closure

$\wedge(\{q_0\}) = \{q_0, p, t\}$

2) i) $\delta^*(q_0, \underset{y}{11}\,\underset{a}{0}) = \wedge\left(\bigcup_{r \in \underbrace{\delta^*(q_0,11)}_{\{t\}}} \delta(r, 0)\right)$

$= \wedge(\delta(t,0))$

$= \wedge(\{u\})$

$= \{u\}$

$\therefore$ String is rejected.

ii) $\delta^*(q_0, \underset{y}{1}\underset{a}{1}) = \wedge\left(\bigcup_{r \in \underbrace{\delta^*(q_0,1)}_{\{r,t\}}} \delta(r,1)\right)$

$= \wedge(\delta(r,1) \cup \delta(t,1))$

$= \wedge(\phi \cup t)$

$= \wedge(\{t\})$

$= \{t\}$

iii) $\delta^*(q_0, \underset{y}{\Lambda}\underset{a}{1}) = \Lambda\left(\bigcup_{r \in \underbrace{\delta^*(q_0,\Lambda)}_{\{q_0, p, t\}}} \delta(r, 1)\right)$

$= \Lambda(\delta(q_0,1) \cup \delta(p,1) \cup \delta(t,1))$

$= \Lambda(\phi \cup r \cup t)$

$= \Lambda(\{r, t\})$

$= \{r, t\}$

iv) $\delta^*(q_0, \Lambda) = \Lambda(\{q_0\})$

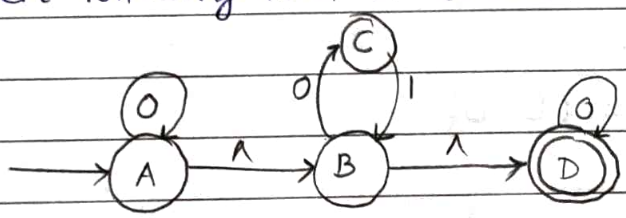
$\Lambda(\{q_0\}) = \{q_0, p, t\}$

* **Theorem**

If any language L is accepted by NFA-Λ given by -
$M = (Q, \Sigma, q_0, A, \delta)$ then there is NFA
$M_1 = (Q_1, \Sigma, q_1, A_1, \delta_1)$ that also recognize L.

Q Convert following NFA-Λ to NFA

8 Marks



(RE = $0^*(01)^*0^*$)

Transition table

(To avoid null)

	Λ	0	1	$\delta^*(q,0)$	$\delta^*(q,1)$
A	{B}	{A}	ϕ	{A, B, C, D}	ϕ
B	{D}	{C}	ϕ	{C, D}	ϕ
C	ϕ	ϕ	{B}	ϕ	{B, D}
D	ϕ	{D}	ϕ	{D}	ϕ

→

★ $$\delta^*(A,0) = \Lambda\left(\bigcup_{r\in\delta^*(A,\Lambda)} \delta(r,0)\right)$$

$$= \Lambda(\delta(A,0) \cup \delta(B,0) \cup \delta(D,0))$$

$$= \Lambda\{A,C,D\}$$

$$= \{A,B,C,D\}$$

$$\delta^*(A,\Lambda) = \Lambda(A) = \{A,B,D\}$$

★ $$\delta^*(A,1) = \Lambda\left(\bigcup_{r\in\delta^*(A,\Lambda)} \delta(r,1)\right)$$

$$= \Lambda(\delta(A,1) \cup \delta(B,1) \cup \delta(D,1))$$

$$= \Lambda(\phi)$$

$$= \phi$$

$$\delta^*(A,\Lambda) = \Lambda(A) = \{A,B,D\}$$

★ $$\delta^*(B,0) = \Lambda\left(\bigcup_{r\in\delta^*(B,\Lambda)} \delta(r,0)\right)$$

$$= \Lambda(\delta(B,0) \cup \delta(D,0))$$

$$= \Lambda\{C,D\} = \{C,D\}$$

$$\delta^*(B,\Lambda) = \Lambda(B) = \{B,D\}$$

★ $$\delta^*(C,1) = \Lambda\left(\bigcup_{r\in\delta^*(C,\Lambda)} \delta(r,1)\right)$$

$$= \Lambda(\delta(C,1))$$

$$= \Lambda\{B\}$$

$$= \{B,D\}$$

$$\delta^*(C,\Lambda) = \Lambda(C) = \{C\}$$

★ $$\delta^*(D,0) = \Lambda\left(\bigcup_{r \in \delta^*(D,\Lambda)} \delta(r,0)\right)$$

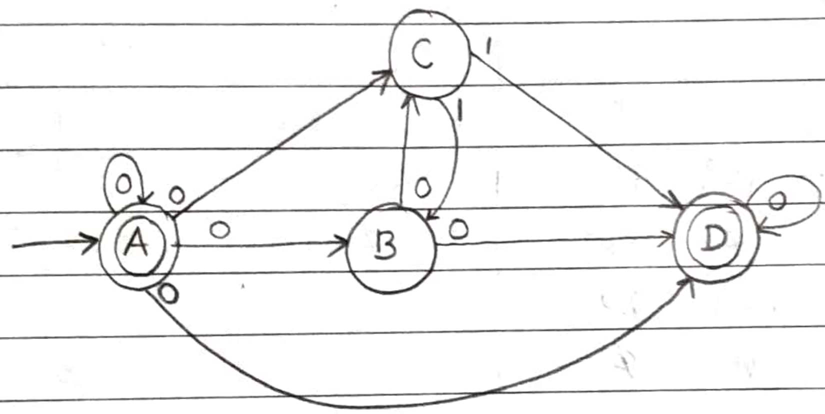
$$= \Lambda(\delta(D,0))$$

$$= \Lambda\{D\}$$

$$= \{D\}$$

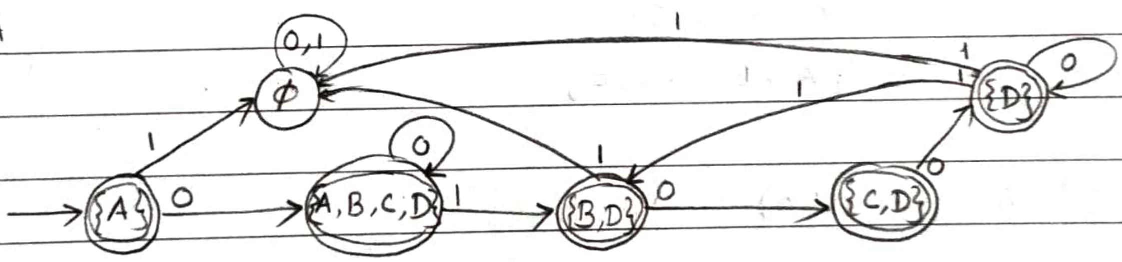
$$\delta^*(D,\Lambda) = \Lambda(D) = \{D\}$$

NFA :

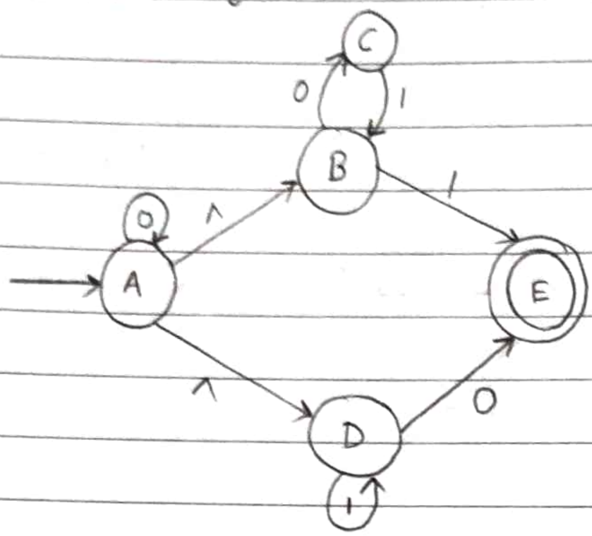


[A also accepted ∵ in NFA-Λ , Λ is accepted (i.e - ΛΛ = Λ)
∴ for NFA (ans) A is accepted]

DFA



Q) Conver following NFA-Λ to NFA & DFA



$\left(0^*((01)^*1 + 1^*0)\right)$

$\left(0^*(01)^*1 + 0^*1^*0\right)$

Transition Table

	Λ	0	1	$\delta^*(q,0)$	$\delta^*(q,1)$
A	{B,D}	{A}	ϕ	{A,B,C,D,E}	{D,E}
B	ϕ	{C}	{E}	{C}	{E}
C	ϕ	ϕ	{B}	ϕ	{B}
D	ϕ	{E}	{D}	{E}	{D}
E	ϕ	ϕ	ϕ	ϕ	ϕ

★ $\delta^*(A,0) = \Lambda\left(\bigcup_{r \in \delta^*(A,\Lambda)} \delta(r,0)\right)$

$= \Lambda(\delta(A,0) \cup \delta(B,0) \cup \delta(D,0))$

$= \Lambda\{A,C,E\}$

$= \{A,B,C,D,E\}$

$\delta^*(A,\Lambda) = \Lambda(A) = \{A,B,D\}$

★ $\delta^*(B,0) = \Lambda\left(\bigcup_{r \in \delta^*(B,\Lambda)} \delta(r,0)\right)$

$= \Lambda(\delta(B,0))$

$= \Lambda\{C\}$

$= \{C\}$

$\delta^*(B,\Lambda) = \Lambda(B) = \{B\}$

$$\star \quad \delta^*(B,1) = \Lambda\left(\bigcup_{r \in \delta^*(B,\Lambda)} \delta(r,1)\right)$$

$$= \Lambda(\delta(B,1))$$

$$= \Lambda(E)$$

$$= \{E\}$$

$$\delta^*(B,\Lambda) = \Lambda(B) = \{B\}$$

$$\star \quad \delta^*(D,0) = \Lambda\left(\bigcup_{r \in \delta^*(D,\Lambda)} \delta(r,0)\right)$$

$$= \Lambda(\delta(D,0))$$

$$= \Lambda(E)$$

$$= \{E\}$$

$$\delta^*(D,\Lambda) = \Lambda(D) = \{D\}$$

$$\star \quad \delta^*(D,1) = \Lambda\left(\bigcup_{r \in \delta^*(D,\Lambda)} \delta(r,1)\right)$$

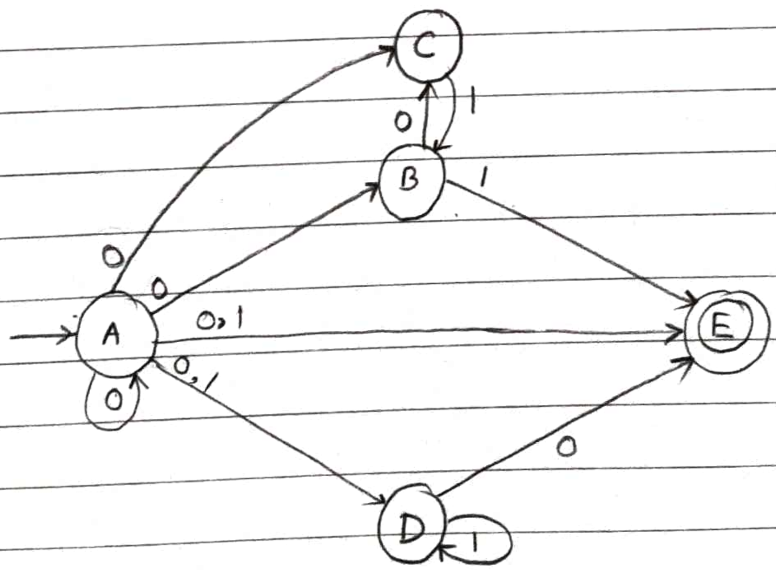
$$= \Lambda(\delta(D,1))$$

$$= \Lambda(D)$$

$$= \{D\}$$

$$\delta^*(D,\Lambda) = \Lambda(D) = \{D\}$$

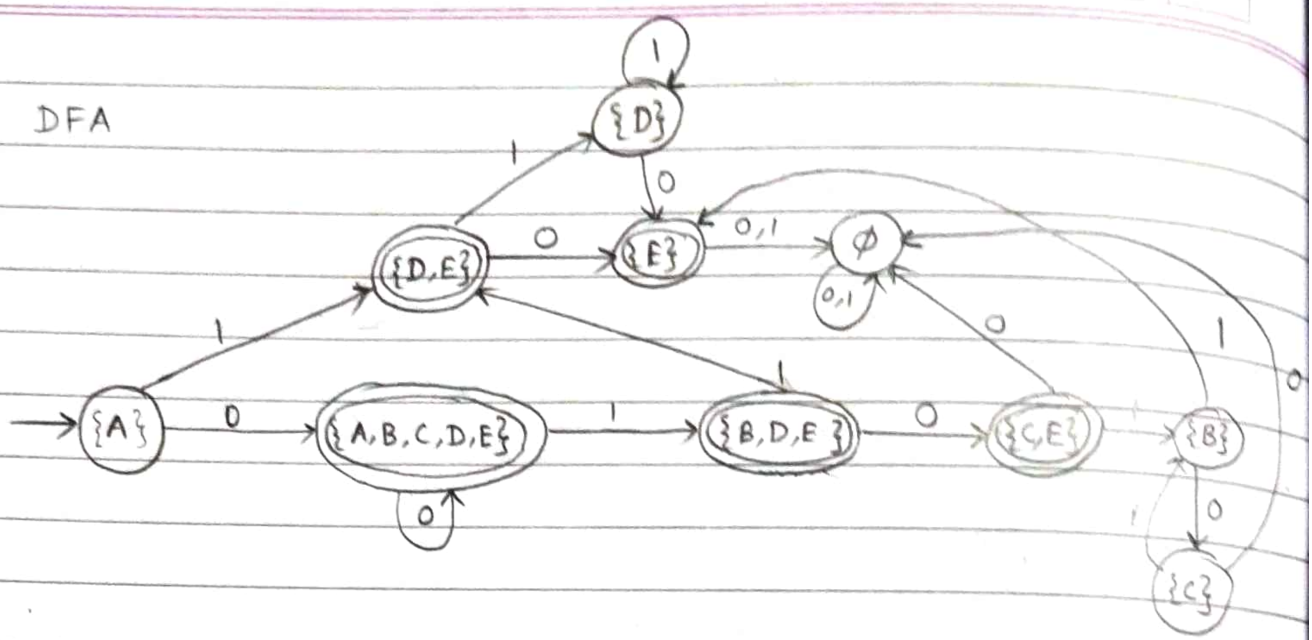
NFA:



E is final state

∴ Every combination of E is final state

DFA



∴ 5 final states.

Kleene's Theorem

Any language can be recognized by finite automata.

To prove above theorem, our aim is to show that for