6. Binomial Distribution

Definition : If (i) an experiment results only in two ways, success or failure, (ii) the probability of success is p and the probability of failure is q such that $p + q = 1$ and (iii) the experiment is repeated n times then probability of r successes is given by

$$\boxed{\begin{aligned} & p(x=r) = {}^nC_r\, p^r q^{n-r} \\ i.e.\quad & p(x=r) = \frac{n(n-1)(n-2).....(n-r+1)}{r!} p^r q^{n-r} \end{aligned}}$$

and the resulting probability distribution is called **Binomial.**

The probabilities that x will take the values 0, 1, 2, 3 ..... r ..... n can also be expressed as follows.

Thus, $p(x=0) = q^n$, $p(x=1) = npq^{n-1}$,

$$p(x=2) = \frac{n(n-1)}{2!} p^2 q^{n-2} \text{ etc.}$$

A variate having binomial distribution is called **binomial variate.** Further, the characteristics n, p, q **are called the parameters or the constants** of the distribution.

If the values of the parameters n, p, q are known the probability distribution is completely known, because we can then calculate the probabilities of 0, 1, 2, ... n successes by using the above formula.

Uses : Thus, in problems involving **(1)** the tossing of a coin-heads or tails, **(2)** the result of an examination-success or failure, **(3)** the result of a game-win or loss, **(4)** the result of inspection-acceptance or rejection where there are two mutually exclusive and exhaustive outcomes and where trials are finite, the binomial distribution can be used.

Properties of the binomial distribution

We give below some important properties of the binomial distribution without derivation.

(i) If x denotes binomial variate, the mean of the distribution is given by,

$$\boxed{\bar{x} = np}$$

(ii) The variance of the distributions is npq *i.e.*

$$\boxed{V(x) = npq}$$

Ex. 1 : If 10% bolts produced by a machine are defective, calculate the probability that out of a sample selected at random of 10 bolts, not more than one bolt will be defective. **(S.U. 1986, 90, 91, 92)**

Sol. : We have P (defective bolt) $= p = 0{\cdot}1,\ q = 0{\cdot}9$ and $n = 10$

P (not more than one bolt defective)

$= P$ (Zero defective) $+ P$ (one defective)

By Binomial Distribution

$$P(x) = {}^{n}C_{x}\, p^{x} q^{n-x} = {}^{10}C_{x}(0{\cdot}1)^{x}\,(0{\cdot}9)^{10-x}$$

$$\therefore \quad P(x=0) = {}^{10}C_{0}(0{\cdot}1)^{0}\,(0{\cdot}9)^{10} = 0{\cdot}3487$$

$$P(x=1) = {}^{10}C_{1}\,(0{\cdot}1)^{1}\,(0{\cdot}9)^{9} = 0{\cdot}3874$$

$\therefore$ Required probability $= P(x=0) + P(x=1)$

$= 0{\cdot}7361$

Ex. 2 : From a box containing 100 transistors 20 of which are defective, 10 are selected at random. Find the probability that (i) all will be defective (ii) all are non-defective (iii) at least one is defective. **(S.U. 1985, 88, 93, 98)**

Sol. : $$P(\text{defective transistor}) = p = \frac{20}{100} = 0{\cdot}2\,,\ q = 0{\cdot}8 \text{ and } n = 10$$

$$P(x\,\text{def.}) = {}^{n}C_{x}\, p^{x} q^{n-x} = {}^{10}C_{x}(0{\cdot}2)^{x}\,(0{\cdot}8)^{10-x}$$

(i) $P(\text{all def.}) = P(x=10) = {}^{10}C_{10}(0{\cdot}2)^{10}\,(0{\cdot}8)^{0} = 0{\cdot}0000001$

(ii) $P(\text{zero def.}) = P(x=0) = {}^{10}C_{0}(0{\cdot}2)^{0}\,(0{\cdot}8)^{10} = 0{\cdot}1074$

(iii) P (at least one defective) $= 1 - P$ (zero defective)

$= 1 - 0{\cdot}1074 = 0{\cdot}8926.$