CG.

Transformations.

Rotation operation perform on two dimension object.

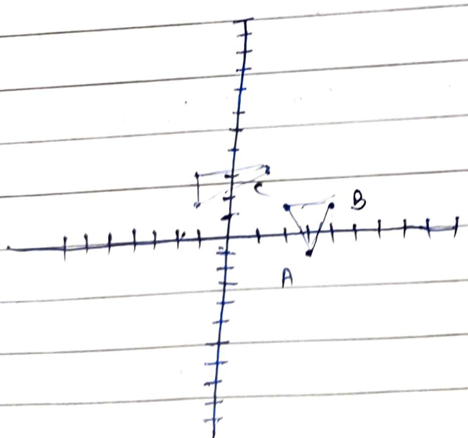
* To rotate the object about origin in +ve counter clockwise sense by angle 90°, the rotation matrix R is used, where $R = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

* for 180° rotation, the rotation matrix $R = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

* for 270° —"— $R = \begin{bmatrix} 0 & -1 \\ +1 & 0 \end{bmatrix}$

* for 360° —"— $R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

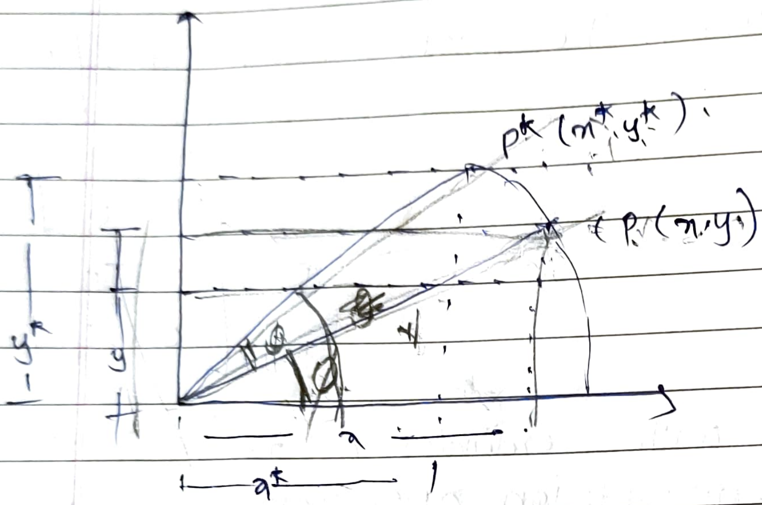
Q. The object ABC with co-ordinates A(3,-1), B(4,1), C(2,1). Perform rotation operation on given object in +ve counter clockwise sense by angle $\theta = 90°$. Find out the resultant matrix and show the resultant object.



$$\begin{array}{c} A \\ B \\ C \end{array} \begin{bmatrix} 3 & -1 \\ 4 & 1 \\ 2 & 1 \end{bmatrix}_{3\times 2} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}_{2\times 2} = \begin{bmatrix} 1 & 3 \\ -1 & 4 \\ -1 & 2 \end{bmatrix}_{3\times 2}$$

For 270°,

$$\begin{array}{c} A \\ B \\ C \end{array}\begin{bmatrix} 3 & -1 \\ 4 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & -3 \\ 1 & -4 \\ 1 & -2 \end{bmatrix}$$



$\sin\phi = \frac{y}{r}$. $\cos\phi = \frac{x}{r}$

$y = r\sin\phi$ $\quad x = r\cos\phi$.

$P^* = (x^*, y^*) = [\, r\cos(\phi+\theta) \;\; r\sin(\phi+\theta) \,]$

$= [\, r(\cos\phi\cos\theta - \sin\phi \cdot \sin\theta), \; r(\cos\phi\sin\theta + \sin\phi\cos\theta) \,]$

$(x^*, y^*) = [\, x\cos\theta - y\sin\theta \;\; x\sin\theta + y\cos\theta \,]$

Thus, the transformed pts has components

$x^* = x\cos\theta - y\sin\theta$

$y^* = x\sin\theta + y\cos\theta$.

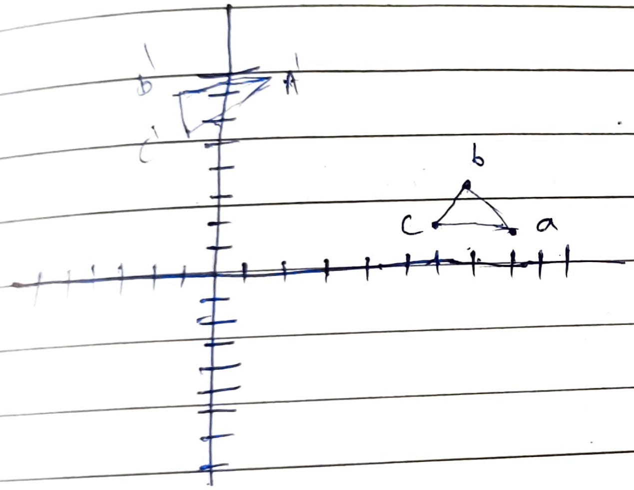
In matrix form it is represented as

$$[X]^* = [X][T] = (x^* \; y^*)$$

$$= [x \; y] \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

* $R = \begin{bmatrix} \cos\theta & -\sin\theta \\ +\sin\theta & \cos\theta \end{bmatrix}$ * $\boxed{A}_{c} : \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Q. Consider $\triangle ABC$; $A(8,1)$, $B(7,3)$, $C(6,2)$. Perform the rotation operation on given $\triangle ABC$ by angle $\theta = 75°$ about origin. Perform the matrix multi. operation and show the resultant matrix.



$$\begin{bmatrix} 8 & 1 \\ 7 & 3 \\ 6 & 2 \end{bmatrix} \begin{bmatrix} \cos 75 & \sin 75 \\ -\sin 75 & \cos 75 \end{bmatrix} = \begin{bmatrix} 8 & 1 \\ 7 & 3 \\ 6 & 2 \end{bmatrix} \begin{bmatrix} 0.2 & 0.9 \\ -0.9 & 0.2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.7 & 7.4 \\ -1.3 & 6.9 \\ -0.6 & 5.8 \end{bmatrix}$$

★ 2-D Reflection :-

To reflect 2 dimensional object about $y = 0$, i.e through x-axis, we have to use reflection matrix

$$[T] = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

To reflect the object above $x = 0$, i.e through y axis, we use matrix.

$$[T] = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}.$$

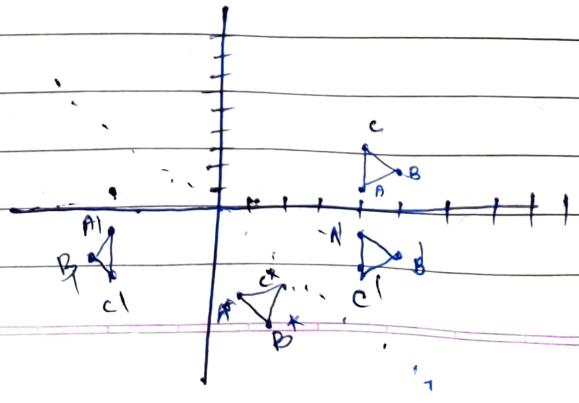
To reflect the object about the ~~the~~ line $y = x$, we use the matrix :-

$$[T] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

To reflect the object about the line $y = -x$, we use the matrix :-

$$[T] = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

Q. Consider the ΔABC, with co-ordinates $A(4,1)$, $B(5,2)$ $C(4,3)$ is 1st reflected about x axis and then about $y = -x$ axis and then finally rotated about origin by angle $\theta = 270°$. Find out the resultant matrix and show the resultant object.



① Reflc. about x.

$$\therefore \begin{bmatrix} 4 & 1 \\ 5 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 5 & -2 \\ 4 & -3 \end{bmatrix}$$

② Reflc. about y = -x

$$\therefore \begin{bmatrix} 4 & -1 \\ 5 & -2 \\ 4 & -3 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} +1 & -4 \\ +2 & -5 \\ +3 & -4 \end{bmatrix}$$

③ Rotation by 270°

$$\begin{bmatrix} 1 & -4 \\ 2 & -5 \\ 3 & -4 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -4 & -1 \\ -5 & -2 \\ -\cancel{3}\,4 & -3 \end{bmatrix}$$

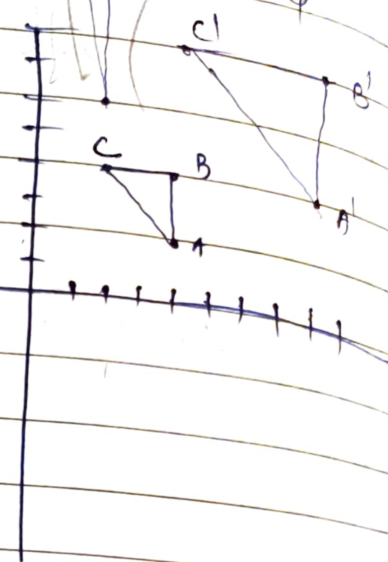
$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

* 2-D Scaling.

Their is generalize matrix form for scaling operation which is given by $[T] = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where a, b c and d are the matrix elements and if a = d, b = c = 0, this gives a uniform scaling. But if a ≠ d; ~~b≠~~ b = c = 0, this gives a non uniform scaling, and if a = d, b = c = 0 and if we want to scale the object about the centre of the object then whatever resultant object we are getting, that will purely scaled object.

Q. Consider the △ABC, with co-ordinates A(4, 2), B(4, 4), C(2, 4). Perform uniform scaling operation by considering the scaling factor 2 as well as non uniform scaling operation on same △le by

Considering scaling factors $\frac{1}{2}$, 3 for a & d resp.



①
$$\begin{array}{c} A \\ B \\ C \end{array}\begin{bmatrix} 4 & 2 \\ 4 & 4 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 4 \\ 8 & 8 \\ 4 & 8 \end{bmatrix}$$

②
$$\begin{array}{c} A \\ B \\ C \end{array}\begin{bmatrix} 4 & 2 \\ 4 & 4 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 0.5 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 6 \\ 2 & 12 \\ 1 & 12 \end{bmatrix}$$

* Homogeneous Co-ordinates:

The matrix for translation in the homogenous co-ordinates is given by $[T] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}_{3\times 3}$

Rotation:

$$[R] = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

For scaling, homogeneous Co-ordinates

$$[S] = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

For reflection, about y axis is given by

$$r_1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

reflection about x axis is given by

$$r_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

reflection about $x=y$ line

$$r_3 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

reflection about $y=-x$ line.

$$r_4 = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

reflection about origin.

$$r = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Q. Suppose the centre of the object at (4,3). and is decide to rotate the object 90° in Counter Clockwise sense about its Center. Perform the operation and show the resultant matrix.

$$[T] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & -3 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 7 & -1 & 1 \end{bmatrix}$$

Now,

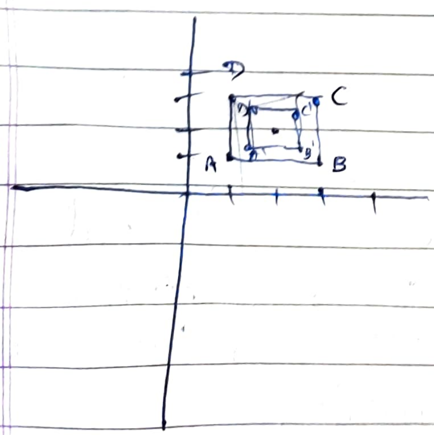
$$\begin{bmatrix} 4 & 3 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 7 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 3 & 1 \end{bmatrix}$$

Q. Find the transformation matrix that transforms the given square ABCD to half its size with centre still remaining at the same position. The co-ordinates of the square are A(1,1), B(3,1), C(3,3), D(1,3). Centre is at (2,2). Also perform rotation operation on given object about its centre by angle $\theta = -90^\circ$. Find resultant matrix & show resultant obj.

2 2

$-(0,0)$



$$[S] = [T][S_c][T]^{-1}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & -2 & 1 \end{bmatrix} \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$$

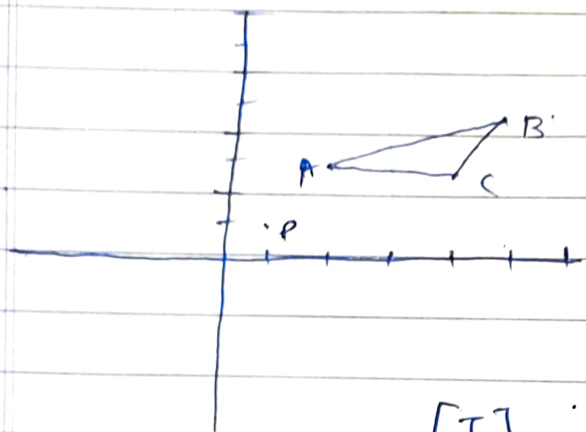
$$= \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Now,

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & 1 \\ 3 & 3 & 1 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1.5 & 1.5 & 1 \\ 2.5 & 1.5 & 1 \\ 2.5 & 2.5 & 1 \\ 1.5 & 2.5 & 1 \end{bmatrix}$$

Q. Perform a clockwise 45° rotation operation on a triangle ABC, A(2,3), B(5,5), C(4,3), about the pt. P (1,1). Find out resultant matrix and show resultant object.



$$[T] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0.7 & -0.7 & 0 \\ 0.7 & 0.7 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$[T] = \begin{bmatrix} 0.7 & -0.7 & 0 \\ 0.7 & 0.7 & 0 \\ -1.4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.7 & -0.7 & 0 \\ 0.7 & 0.7 & 0 \\ -0.4 & 1 & 1 \end{bmatrix}$$

Now $\begin{bmatrix} 2 & 3 & 1 \\ 5 & 5 & 1 \\ 4 & 3 & 1 \end{bmatrix} \begin{bmatrix} 0.7 & -0.7 & 0 \\ 0.7 & 0.7 & 0 \\ -0.4 & 1 & 1 \end{bmatrix}$

1.4
2.1
-0.4

$$= \begin{bmatrix} 3.1 & 1.7 & 1 \\ 6.6 & 1 & 1 \\ 4.5 & 0.3 & 1 \end{bmatrix}$$

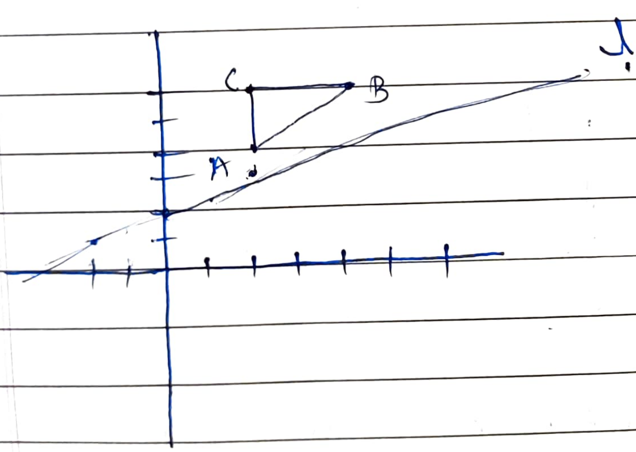
Reflection of two dimensional object through arbitrary line.

To perform such operation we have to go through 5 diff. steps.

1. Translate the line & the object so that the line passes through origin.
2. Rotate the line & object about the origin until the lines co-insides with one of the co-ordinate axis.
3. reflect the object through co-ordinate axis.
4. Apply the inverse rotation operation.
5. Apply the inverse translation operation.

$[T] = [O][T'][R][R'][R]^{-1}[T']^{-1}$.

Q. Consider the ΔABC with co-ordinates A(2,4), B(4,6), C(2,6). Reflect the given Δ^{le} through line L with eqn $y = \frac{1}{2}(x+4)$. Find out the resultant matrix & show reflected object.



$$O = \begin{bmatrix} 2 & 4 & 1 \\ 4 & 6 & 1 \\ 2 & 6 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 & 1 \\ 4 & 6 & 1 \\ 2 & 6 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} [R]^{-1}[T']^{-1}$$

$$= \begin{bmatrix} 2 & \cancel{4}\ 2 & 1 \\ 4 & 4 & 1 \\ 2 & 4 & 1 \end{bmatrix} \begin{bmatrix} 0.9 & -0.45 & 0 \\ 0.45 & 0.9 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0.9 & 2.7 & 1 \\ 1.8 & 5.4 & 1 \\ 0.9 & 5.4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.9 & -2.7 & 1 \\ 1.8 & -5.4 & 1 \\ 0.9 & -5.4 & 1 \end{bmatrix} \begin{bmatrix} 0.9 & 0.45 & 0 \\ -0.45 & 0.9 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.405 & -2.025 & 1 \\ -3.24 & & \\ & & \end{bmatrix}$$

Ans :

$$\begin{bmatrix} 2.7334 & 2.3694 & 1 \\ 5.5268 & 2.7388 & 1 \\ 4.3298 & 1.1724 & 1 \end{bmatrix}$$

3D Translations :

Q.

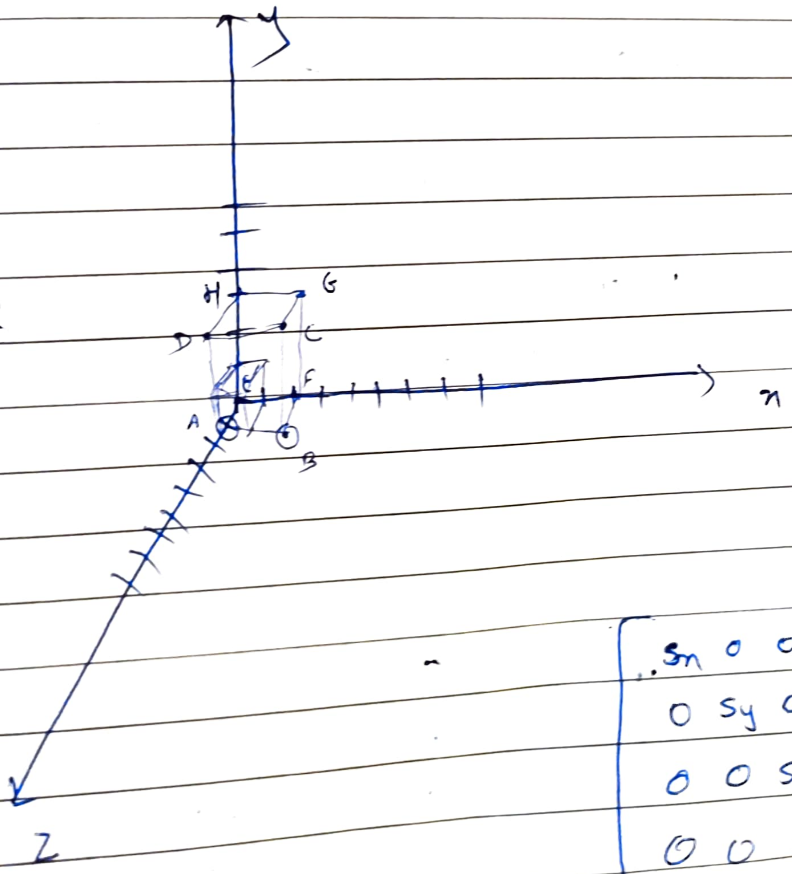
$$\left[\begin{array}{ccc|c} a & b & c & p \\ d & e & f & q \\ g & h & i & r \\ \hline l & m & n & s \end{array}\right]$$

whenever we want to perform a linear transformation on 3D object in the form of rotation reflection and scaling, we need to make changes in upper layer 3x3 sub matrix. Whenever we want to translate 3D object then ~~use~~ we need to make changes in lower left 1x3 matrix. When we want to produce a perspective projection, then we need to make changes in upper right

3×1 sub matrix. ∴ Whenever we want to perform scaling operation we need to make changes in lower right 1×1 matrix.

Q. Consider a 3D object with homogeneous ~~co-ordi~~ position vectors given by $[x] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 2 & 0 & 1 & 1 \\ 2 & 3 & 1 & 1 \\ 0 & 3 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 1 \\ 2 & 3 & 0 & 1 \\ 0 & 3 & 0 & 1 \end{bmatrix}$

Perform local scaling operation on given object by considering scaling factors $\frac{1}{2}, \frac{1}{3}, 1$, along with x, y & z axis resp. Perform the scaling operation on given matrix & show given obj.

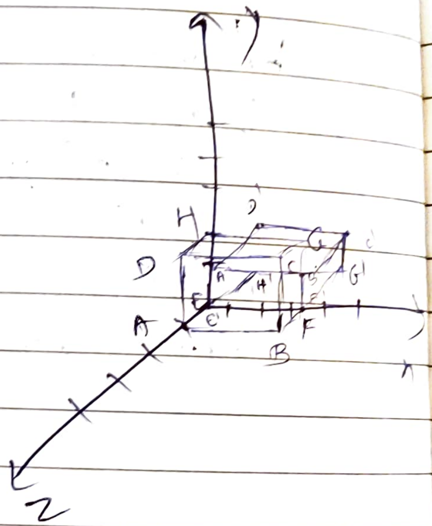


$$\begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & n \end{bmatrix}$$ overall scaling

$$[X] : \begin{bmatrix} 0 & 0 & 1 & 1 \\ 2 & 0 & 1 & 1 \\ 2 & 3 & 1 & 1 \\ 0 & 3 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 1 \\ 2 & 3 & 0 & 1 \\ 0 & 3 & 0 & 1 \end{bmatrix}_{8\times 4} \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.33 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4}$$

$$[Y] : \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$



* 3D rotational operation :

x axis
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

z axis
$$\begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

y axis
$$\begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ +\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Q. Consider the 3D object with the matrix of position vectors $[X] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 3 & 0 & 1 & 1 \\ 3 & 2 & 1 & 1 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 3 & 0 & 0 & 1 \\ 3 & 2 & 0 & 1 \\ 0 & 2 & 0 & 1 \end{bmatrix}$. Rotate the given object about x axis by an angle $\theta = -90°$. Perform the operation and show the resultant object.

$$\therefore [X] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 3 & 0 & 1 & 1 \\ 3 & 2 & 1 & 1 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 3 & 0 & 0 & 1 \\ 3 & 2 & 0 & 1 \\ 0 & 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \overset{0}{\cancel{\cos\theta}} & \overset{-1}{\cancel{\sin\theta}} & 0 \\ 0 & -\overset{1}{\cancel{\sin\theta}} & \overset{0}{\cancel{\cos\theta}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & 1 \\ 3 & 1 & 0 & 1 \\ 3 & 1 & -2 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 \\ 3 & 0 & 0 & 1 \\ 3 & 0 & -2 & 1 \\ 0 & 0 & -2 & 1 \end{bmatrix}$$

* Combined Rotation.

The 3D object position vectors are given by

$$[X] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 2 & 0 & 1 & 1 \\ 2 & 3 & 1 & 1 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 1 \\ 2 & 3 & 0 & 1 \\ 0 & 2 & 0 & 1 \end{bmatrix}$$

Rotate the given object about x axis by angle $\theta = 90^\circ$ and then again rotate about y axis by angle $\theta = 180^\circ$. Perform the transformation on given object and show the resultant object.

$$[R] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 2 & 0 & 1 & 1 \\ 2 & 3 & 1 & 1 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 1 \\ 2 & 3 & 0 & 1 \\ 0 & 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & & & \\ & & & \\ & & & \end{bmatrix}$$

Multiple transformations :-

Consider the matrix form of given object [X]. First translate the object in x, y & z direction by [-1, -1, -1] resp followed succ. by 30° rotation about x axis and -45° rotation about y axis. Perform these ~~object~~ operations on given object and show the resultant matrix.

$$[T] = [0]\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & -1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.9 & 0.5 & 0 \\ 0 & -0.5 & 0.9 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0.7 & 0 & 0.7 & 0 \\ 0 & 1 & 0 & 0 \\ -0.7 & 0 & 0.7 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

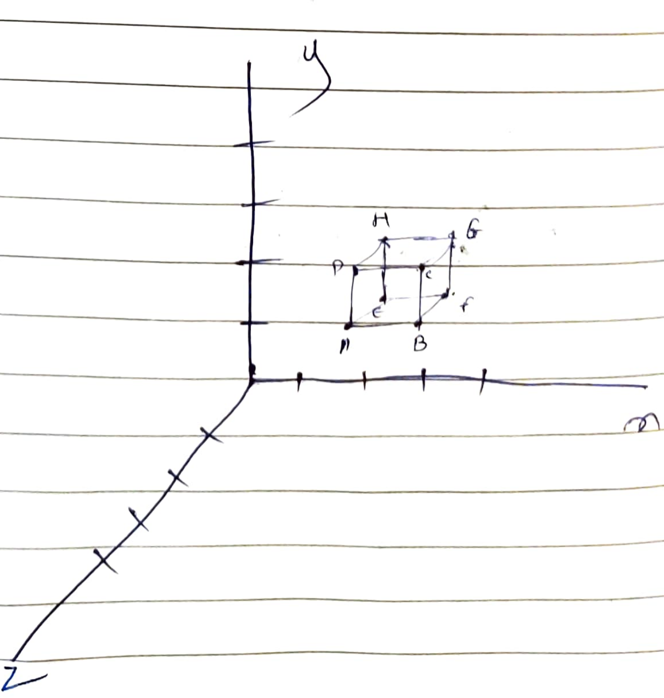
$$= \begin{bmatrix} 0 & 0 & 1 & 1 \\ 2 & 0 & 1 & 1 \\ 2 & 3 & 1 & 1 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 1 \\ 2 & 3 & 0 & 1 \\ 0 & 2 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & -1 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 & 0 & 1 \\ & & & \end{bmatrix}$$

3D Reflection :

To reflect the object through xy plane the matrix T is given by
$$[T] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
To perform reflection through 'YZ plane, the matrix T is given as
$$[T] = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
To perform reflection through 'XZ plane is given by
$$[T] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Q. The block ABCDEFGH has position vectors
$$[X] = \begin{bmatrix} 1 & 0 & -1 & 1 \\ 2 & 0 & -1 & 1 \\ 2 & 1 & -1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 0 & -2 & 1 \\ 2 & 0 & -2 & 1 \\ 2 & 1 & -2 & 1 \\ 1 & 1 & -2 & 1 \end{bmatrix}$$
Reflect the given object through xy plane and show the reflected object.



* Rotation of 3-D object about an axis parallel to main co-ordinate axis :-

To perform this operation we need to go through following steps :-

1. Translate the object such that the local axes of rotation should co-incides with main co-ordinate axis in the same direction.
2. ~~Th~~ Perform the rotation operation by given angle.
3. Translate the rotated object back to its original position.
4. Mathematically this operation is given as $[X^*] = [X][T_r][R][T_r]^{-1}$. so here X^* is final matrix.

5. Consider the object with the pos. vectors is given by

$$[X] = \begin{bmatrix} 1 & 1 & 2 & 1 \\ 2 & 1 & 2 & 1 \\ 2 & 2 & 2 & 1 \\ 1 & 2 & 2 & 1 \\ 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 \\ 2 & 2 & 1 & 1 \\ 1 & 2 & 1 & 1 \end{bmatrix}$$

. Rotate the given object by angle $\theta = 30°$ about the local axis y' where y' local axis is passing through the centroid of the block. The origin of the local axis system is assume to be centroid of block. The centroid of block is $[x_c\ y_c\ z_c\ 1] = [3/2\ 3/2\ 3/2$ ... Perform this rotation operation & find out res. matrix.

$$[X^*] = \begin{bmatrix} 1 & 1 & 2 & 1 \\ 2 & 1 & 2 & 1 \\ 2 & 2 & 2 & 1 \\ 1 & 2 & 2 & 1 \\ 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} \overset{1}{\cancel{\cos\theta}} & 0 & \overset{0}{\cancel{-\sin\theta}} & 0 \\ 0 & 1 & 0 & 0 \\ \overset{0}{\cancel{\sin\theta}} & 0 & \cancel{\cos\theta} & 0 \\ -\frac{3}{2} & 0 & \frac{3}{2} & 1 \end{bmatrix} \begin{bmatrix} 0.9 & 0 & -0.5 & 0 \\ 0 & 1 & 0 & 0 \\ 0.5 & 0 & 0.9 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} [T_r$$

* Rotation of 3D object above an arbitary axis in space.

① It is accomplished with the procedure using translation and simple rotation about the co-ordinate axis.

② Assume an arbitary axis in space is passing through pt. (x_0, y_0, z_0) with the direction cosines (C_x, C_y, C_z).

③ Rotation about arbitary axis by some angle δ is accomplished using following procedure.

a) Translate the object so that the pt. (x_0, y_0, z_0) is at the origin of the main co-ordinate system.

b) Perform appropriate rotations to make the axis of rotation coincident with z axis.

c) Rotate about the z axis by angle δ.

d) Perform the inverse of the combined rotation operations.

e) Perform the inverse of the translation.

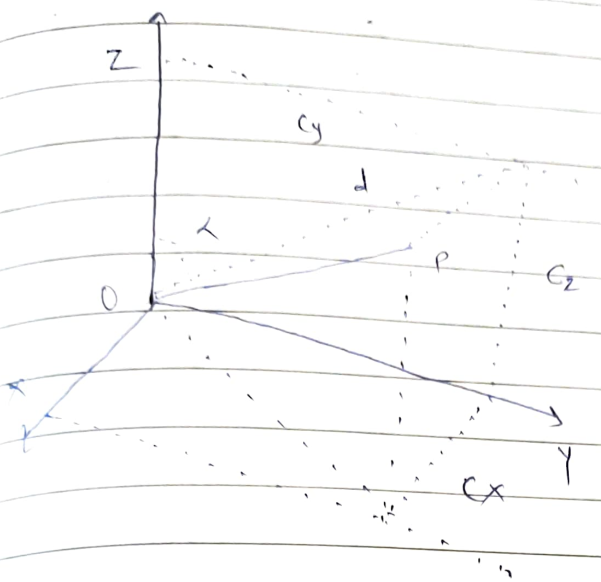
④ In general making an arbitary axis passing through the origin co-incident with one of the co-ordinate axis requires two successive rotations about other two co-ordinate axis.

⑤ To make the arbitary axis co-incident with the z axis first rotate about x axis & then about y axis.

⑥ To determine the rotation angle α about the x axis use to place the arbitary axis in the xz plane. First, project the unit vector along the axis on to the yz plane

⑦ Y & Z component of the projected vector are C_Y and C_Z resp.

First step of Rotation :



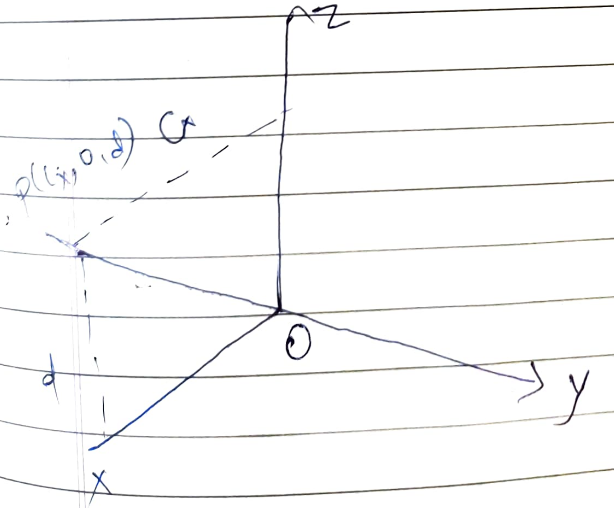
$d = \sqrt{Cy^2 + Cx^2}$

$\cos\alpha = \frac{Cz}{d}$, $\sin\alpha = \frac{Cy}{d}$

After rotation about x axis in xz plane

The z component of unit vector is d, & x component is Cx.

After first step of Rotation :



The length of unit vector is of course 1, thus the rotation angle β about the y axis required to make arbitrary axis coincident with the z axis.

So $\cos\beta = d$ & $\sin\beta = Cx$

$$[T] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -x_0 & -y_0 & -z_0 & 1 \end{bmatrix} \quad R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cz/d & cy/d & 0 \\ 0 & -cy/d & cz/d & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_y = \begin{bmatrix} d & 0 & cx & 0 \\ 0 & 1 & 0 & 0 \\ -cx & 0 & d & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_z = \begin{bmatrix} \cos\delta & \sin\delta & 0 & 0 \\ -\sin\delta & \cos\delta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[R_y^{-1}]\,[R_x^{-1}]\,[T^{-1}]$$