Multiplication Theorem

1. $P(A \cap B) = P(A) \times P(B/A)$ (P of B, A is given) ← for reading
   ↑ conditional probability

2. $P(A \cap B) = P(B) \times P(A/B)$

* For A and B are independant
  then $P(B/A) = P(B)$
  and $P(A/B) = P(A)$

1. $P(A) = P(A \cap B) + P(A \cap B')$

2. $P(B) = P(A \cap B) + P(A' \cap B)$

Q35 $n(S) = {}^{52}C_1$ ← 1 card is selected from a set of 52 cards
$= 52$

Let A = face card
$n(A) = {}^{12}C_1 = 12$

$P(A) = \frac{n(A)}{} = \frac{12}{}$

addition theorem

[1] $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

[2] $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

3. $P(A) + P(A') = 1$

4. $P(A \cup B)' = P(A' \cap B')$ ← Demorgan's

$P(A \cup B)' = 1 - P(A \cup B)$

5. $P(A \cap B)' = P(A' \cup B')$ ← Demorgan's

$P(A \cap B)' = 1 - P(A \cap B)$

6. If A and B are independant events then
$P(A \cap B) = P(A) + P(B)$

7. If A and B are mutually exclusive events
$P(A \cap B) = 0$ (zero)

$P(S) = 1$

8. If A and B are independant events then

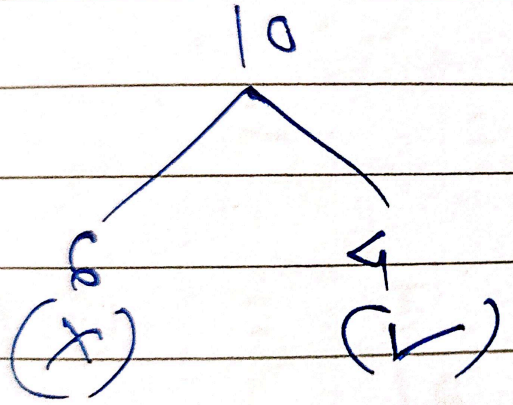
(a) A' and B' are also independant
(b) A' and B are also independant
(c) A and B' are also independant

$P(A_1 \cap A_3) = 1/4 : 1/2 \times 1/2$

$P(A_1 \cap A_3) = 1/4 : 1/2 \times A_{1/2}$

A_1, A_2, A_3 are pair wise independant

Q21



	(✓) 6	(✗) 4
①	2	1
②	1	2
③	0	3

$P(A) = {}^6C_2 \times {}^4C_1 + {}^6C_1 \times {}^4C_2 + {}^6C_0 + {}^4$

Method II

Tuesday
2/4/19

Circular permutation

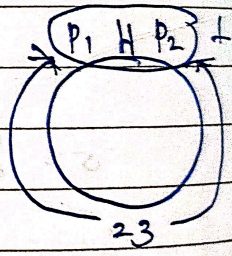
(I) No. of arrangement of 'n' objects around a table $= (n-1)!$

Q10 total persons $= 25 + 1 = 26$
person host

$n(S) = (26-1)! = 25!$

$n(A) = (24-1)! \times {}^2P_2$
$= 23! \times 2!$

P_1 and P_2 can change place



23 + 1 = 24 consider group

Q11 $n(S) = {}^{30}C$

(V) V | others | C

$^3P_1 \times {}^7P_7 \times {}^6P_1$

Remember

confusing don't write $^3P_3$ or $^6P_6$

Q9

For leap year = 366 days

$366 = 7 \times 52 + 2$

$S = \{(Sun, Mon), (Mon, Tue), (Tue, Wed)$

Arrangement of 2 sets (vowels and cons)

internal arrangement of 3 vowels

internal arrangement of 6 consonants

(i) $= {}^{2}P_2 \times {}^{3}P_3 \times {}^{6}P_6$

(ii) LOGARITHM

vowels = O, A, I = 3

consonants = L, G, R, T, H, M = 6

$= {}^{7}P_3 \times {}^{6}P_6$

$7 = 6+1$

V C V C V C V C V C V C V

start with V i.e vowel

write 'C' 6 times

end with vowel

Here for writing C 6 times we writing V 7 times (i.e 6+1 = 7)

(iii)

1 2 3 4 5 6 7 8 9

4 types

$= 4 \times {}^{2}P_2 \times {}^{7}P_7$

arrangement of G and H

other than G and H their arrangements

(iv) 0 [7] T

${}^{1}P_1 \times {}^{7}P_7 \times {}^{1}P_1$

perfect square: {1, 4, 9, 16, 25, ...} (1 is a perfect square)

(i) $P(G) + P(B) + P(Y) = 1$

$\frac{1}{3} + x + \frac{1}{5} = 1$

$x = 1 - \frac{1}{3} - \frac{1}{5}$

$x = \frac{15 - 5 - 3}{15} = \frac{10 - 3}{15}$

$\boxed{x = \frac{7}{15}}$

(ii) $n(G) = \frac{1}{3} \times 15 = 5$

$n(Y) = 3$

$n(B) = 7$

$^{39}C_0$

Q7 $n(S) = {}^{75}C_1 = 75$

(i) $(A) = \{6, 12, 18, \ldots, 60, 66, 72\}$

$n(A) = 12$

$P(A) = \frac{n(A)}{n(S)} = \frac{12}{75}$

Q8 $n(S) = {}^{9}P_9 = 9!$

(1) LOGARITHM

vowels: O, A, I : 3

consonants = L G R T H M : 6

(i) → 3 vowels considered as 1 group

$= 1 + 6 = 7$

$n(A) = {}^{7}P_7 \times {}^{3}P_3$

${}^{3}P_3$ = internal arrangement of vowels

$\star \boxed{{}^{n}C_{n-1} = n}$ $\quad {}^{10}C_9 = 10$, $\quad {}^{20}C_{15} = {}^{20}C_5$

(vii) $= 13 \times {}^{4}C_2$ ← Both card should 2 or Both card should 3

↑ total are there are 13 repeat cards 13 repeat cards 2, 3, 4, 5, ace, jack, queen, king

or

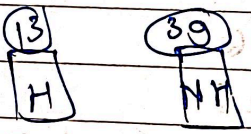
Both should ace cards

Q5 (i) $n(A) = {}^{4}C_3 \times {}^{4}C_1$

(ii) $n(B) = 4 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1$

$n(B) = 52C4$

(iii)



$= {}^{13}C_4 \times {}^{39}C_3 + {}^{13}C_2 \times {}^{39}C_2 + {}^{13}C_3 \times {}^{39}C_1 + {}^{13}C_4$

OR

imp Method – II

(iii) let A = getting at least one heart.

$\therefore$ A' = all non-heart card

$\therefore n(A') = 1 - n(A)$

$= 30$

$\therefore n(A') = {}^{39}C_4 \longrightarrow P(A') = \frac{n(A')}{n(S)} = \frac{{}^{39}C_4}{{}^{52}C_4}$

$\therefore$ ~~$n(A') = {}^{39}$~~

$P(A) = 1 - P(A')$

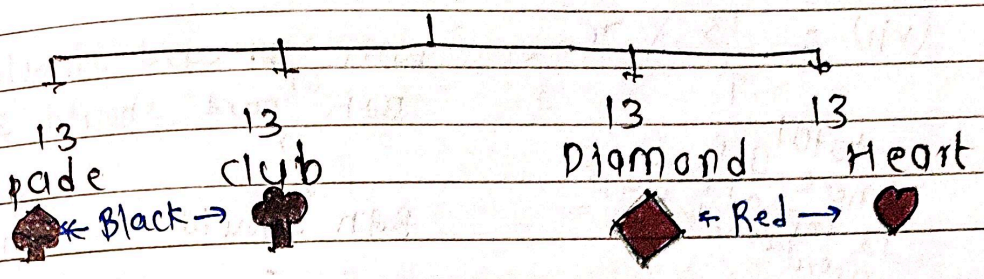
(iv) $= {}^{12}C_3 + {}^{1}C_1$

Q6 $n(S) = {}^{15}C_1 = 15$

$P(G) = 1/3$, $P(Y) = 2/5$

$nC_1 = n$

52 cards



Kings	1	1	1	1	= 4
Queens	1	1	1	1	= 4
Jacks	1	1	1	1	= 4
Aces	1	1	1	1	= 4

Black cards = 26

Red cards = 26

court card / picture cards / face cards = king + queen + jack

= 4+4+4

= 4 × 3 = 12

Non-picture = 52 − 12 = 40

Q4 $n(S) = {}^{52}C_2 = 1326$ ← Remember

(i) $n(A) = {}^{26}C_2$ → $P(A) = n(A)/n(S)$

(ii) $n(B) = {}^{13}C_2$

(iii) $n(C) = {}^{4}C_2$

(iv) $n(D) = {}^{12}C_2$

(v) $n(E) = {}^{13}C_1 \times {}^{39}C_1 = 13 \times 39$

Quick calculation is imp for CET → $\therefore P(E) = \frac{n(E)}{n(S)} = \frac{13 \times 39}{52!/2!50!} = \frac{13 \times 39}{26 \times 51} = \boxed{\frac{39}{102}}$

(vi) $n(F) = {}^{13}C_2 + {}^{13}C_2 + {}^{13}C_2 + {}^{13}C_2 = 4 \cdot {}^{13}C_2$

Q 2 $n(S) = 36$

(i) ~~for~~ $P = \{(1,2), (1,5), (2,1), (2,2), (2,4), (2,6), (3,1) (3,3) (3,6), (3,5) (4,2), (4,4) (4,5), (5,1) (5,4), (5,3), (6,2), (6,3) (6,6)\}$

$n(P) = 20$

(ii) $Q = \{(1,6), \cancel{(6,1)} (2,5) (3,4) (4,3) (5,2) (6,1)\}$

$n(Q) = 6$

(iii) $R = \{(1,1) (1,4) (1,6) (2,1) (2,3) (2,5) (3,2) (3,4) (4,1) (4,3) (5,2) (5,6) (6,1) (6,5)\}$

$n(R) = 14$

(i) P and Q are exclusive, not exhaustive $(P \cup Q) \neq S$

(ii) Q and R are neither exclusive nor exhaustive.

Q 3 $n(S) = 36$

(i) = {

$n(A) = 4$

$P(A) = \frac{n(A)}{n(S)} = \frac{4}{36} = \frac{1}{9}$

$\therefore n(A) = 1$

$\therefore$ Probability of A $= \frac{n(A)}{n(S)} = \frac{1}{4}$

Problems

Q1. (i) $n(S) = {}^{25}C_3$

(ii) $n(S) = {}^{7}C_2$

(iii) $n(S) = {}^{5}P_5 = 5!$ $\quad ({}^{n}P_n = n!)$

* combination (1) ${}^{n}C_r = \frac{n!}{r!\,(n-r)!}$

(2) ${}^{n}P_r = \frac{n!}{(n-r)!}$

→ use PNC formula when there is arrangement.

(iv) $n(S) = {}^{6}P_6 = 6!$

Extra

$0! = 1$

Q Leap year contain 53 Thursday or 53 Friday = 3/7 (Thu & Fri are consecutive
53 monday or 53 friday = 4/7

iv) Number on ticket is divisible by 3 & 5. (mon and Fri are non-conse

Imp Q.8. The letters of the word LOGARITHM are arranged at random. Find the probability that

i) Vowels are always together. ii) Vowels are never together.
iii) exactly 4 letters between G & H iv) Begin with O & end with T
v) Start with vowel & end with consonant. (vi) all vowels and consonants are together

Imp Q.9. Find the probability that a leap year selected at random will contain 53 Mondays.

Q.10. 25 persons were invited for a party by host. What is the probability that two particular persons be seated on either side of the host at a circular table?

Remember method

Q.11. Two numbers p & q are chosen at random from the set of first 30 natural numbers. What is the probability that $p^2 - q^2$ is divisible by 3?

Q.12. If $p(A)=\frac{1}{4}, p(B)=\frac{2}{5}$ & $p(A\cup B)=\frac{1}{2}$. Find the values of the following probabilities

i) $p(A\cap B)$ ii) $p(A\cap B')$ iii) $p(A'\cap B)$ iv) $p(A'\cup B')$ v) $p(A'\cap B')$

Q.13. If p(A') = 0.7, p(B) =0.7, p(B/A) =0.5, find p(A/B) & p(A ∪ B)

Q.14. If A & B are two independent events & P(A) = 3/5, P(B) = 2/3. Find

i) $p(A\cap B)$ ii) $p(A\cap B')$ iii) $p(A'\cap B)$ iv) $p(A'\cap B')$ v) $p(A\cup B)$

Q.15. If A,B & C are mutually exclusive & exhaustive events associated with the random experiment. Find P(A), given that $P(B)=\frac{3}{2}$P(A) & $P(C)=\frac{1}{2}P(B)$

Q.16. A bag contains 50 tickets, numbered from 1 to 50. One ticket is drawn at random. What is the probability that

i) Number on the ticket is perfect square or divisible by 4.
ii) Number on the ticket is a prime number or greater than 30.

Q.17. Two dice are thrown together. What is the probability that

i) Sum of the numbers is divisible by 3 or 4
ii) Sum of the numbers is neither divisible by 3 nor 5.

Q.18. 100 students appeared for two examinations, 60 passed the first, 50 passed the second & 30 passed in both
Find the probability that a student selected at random

i) Passed in at least one examination
ii) Passed in exactly one examination
iii) Failed in both the examinations

Q.19. A computer software company is bidding for computer programs A & B. The probability that the company will get software A is 3/5, the probability that the company will get software B is 1/3 & the company will get both the softwares is 1/8. What is the probability that the company will get at least one software?

Q.20. The probability that a student will solve problem A is 2/3, & the probability that he will not solve problem B is 5/9. If the probability that the student solves at least one problem is 4/5, what is the probability that he will solve both the problems.

Imp Q.21. A room has 3 sockets for lamps. From a collection of 10 light bulbs of which 6 are defective, a person selects 3 bulbs at random & puts them is a socket. What is the probability that the room is lit?

at least one, union, or = +
intesection, and = ×

arranged → use PNC formula

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12th Mathematics, Section II

Probability

Q.1. Find total number of distanct possible outcomes n(s) for the following random experiment

i) From a box containing 25 lottery tickets any 3 tickets are drawn at random

ii) From a group of 4 boys and 3 girls, any two students are selected at random.

iii) 5 balls are randomly placed into 5 cells, such that cell will be occupied. use combination formula

iv) Six students are arranged in row for photograph.

Q.2. Two dice are thrown. Write down the sample space and write favourable outcomes for the following events

i) P : Sum of the numbers on two dice in divisible by 3 or 4

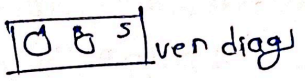
ii) Q: Sum of the numbers on two dice is 7

iii) R: Sum of the numbers on two dice is a prime number

Also check wheter

i) Events P and Q are mutually exclusive and exhaustive

ii) Event Q and R are mutually exclusive and exhaustive



$A \cap B = \phi \longrightarrow n(A \cap B) = 0 \longrightarrow P(A \cap B) = 0$

$A \cup B = S$

$n(A \cup B) = n(S)$

$P(A \cup B) = 1$

Q.3. A fair die thrown two times. What is the chance that

i) Product of the numbers on the uppermost face is 12.

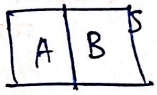
ii) Sum of the numbers on the uppermost face is 10

iii) Sum of the numbers on the uppermost face is at least 10

iv) Sum of the numbers on the uppermost face is at least 4

v) The first throw gives an odd number and second throw gives multiple of 3

vi) Both the times die shows same number (doublests)



ven diagram

Q.4. Two cards are drawn from a pack of 52 cards. What is the probability that

i) Both are black .

ii) Both are diamond

iii) Both are ace cards

iv) Both are face cards

v) One is spade and other is non-spade

vi) Both are from same suit (same type)

vii) Both are from same denomination

and : × = ∩

or : + = ∪

addition

mutiplication

Q.5. Four cards are drawn from a pack of 52 cards. What is the probability that

i) 3 are kings and 1 is jack.

ii) All the cards are from different suit

iii) At least one heart ← Imp

iv) All 4 are clubs and one of them is jack

Q.6. A bag contains 15 balls of three different colour: Green, Black and Yellow. A ball is drawn at random from the bag. The probability of green ball is 1/3. The probability of yellow ball is 1/5.

i) What is the probability of black ball?

ii) How many balls are green, black and yellow?

Q.7. A box contains 75 tickets numbered 1 to 75. A ticket is drawn at random from the box. What is the probability that

'1' is a prefect square

i) Number on ticket is divisible by 6

ii) Number on ticket is a perfect square.

iii) Number on ticket is a perfect square.

perfect square : {1, 4, 9, 16, 25 . . . }