Unit No:- 2

Probability & Probability Distribution.

* Trial or event:

let, an experiment be repeated under initially. let it result in any one of the several possible outcomes, then the experiment is called trial and possible outcomes are called event.

Exaustive events :-

The total no. of all possible outcomes in any trial is known as excustive event.

Favourable event :-

The cases in which the happening of an event are said to be favourable event.

Mutually exclusive event :-

The events are said to be mutually exclusive, if the happening of any one of them rules out happening of others.

Equally like events :-

The events said to be equally like if there is no reason to expect anyone in preference to any one.

Independent event :-

Two or more events are said to be independent if happening or non happening of any one does not depends on the happening or non happening of any others.

* Mutually independent event :
An event 'A' is said to be independendent of an event B, if $P(A/B) = P(A)$.

* Multiplication law of probability :
The prob. of simultaneous occurence of two events is equal to the prob. of one of the events multiplied by the conditional probability of other.
i.e prob. $P(A \cap B) = P(A) \cdot P(B/A)$.,
when $P(B/A)$ represents the conditional prob. of occurence of B when event A has already happen.

Proof :
Suppose trial results in mutually exclusive and equally like outcomes, m of them being favourable to the happening of event A.
∴ prob. of happening of the event A : $P(A)$
$$= \frac{m}{n} \quad \ldots (1)$$

Out of m outcomes favourable to happening of A. Let m_1 be the favourable to happening of the event B.
∴ Conditional prob. of B given that A has happened ~~is eq~~ $P(B/A) = \frac{m_1}{m} \quad \ldots (2)$

Now, out of n exaustive, mutually exclusive & equally like outcomes, m_1 are favourable to the happening of A & B.
∴ prob. of simultaneous occurence of A and B
$$= P(A \cap B) = \frac{m_1}{n}.$$

$$= \frac{m_1}{m} \cdot \frac{m}{n}$$

$$= P(B/A) \cdot P(A)$$

$$\therefore P(A \cap B) = P(A) \cdot P(B/A).$$

Remark 1: Interchanging role of A and B, we get

$$P(A \cap B) = P(B) \cdot P(A/B).$$

2: If A and B are independent events, then

$$P(A \cap B) = P(A) \cdot P(B).$$

Q. A problem in a machine is given to three students A, B, C. Whose chance of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ resp. What is prob. that the problem will be solved?

→ The prob. of A, B, C for solving the problem are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ resp.

The prob of A, B, C not solving the problem are $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$ resp.

$\therefore$ The prob that problem is not solved by any of them $= \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{3}}{4} = \frac{1}{4}$.

$\therefore$ Prob. of solving problem $= 1 - \frac{1}{4} = \frac{3}{4}$.

Q. 'A' can hit a target 4 times in 5 shots. 'B' can hit a target 3 times in 4 shots & 'C' can hit a target twice in 3 shots. What is the prob that at least 2 shots hit.

→ Prob. of A hitting the target = 4/5.
Prob. of B " = 3/4.
" C " = 2/3.

A, B, C all hit the target. the prob for which is $= \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{2}{5}$.

If A, B hit the target, & C miss it, then prob. for which is $\frac{4}{5} \times \frac{3}{4} \times \frac{1}{3} = \frac{1}{5}$.

If A, C hit the target & B miss it, then prob. for which is $\frac{4}{5} \times \frac{1}{4} \times \frac{2}{3} = \frac{2}{15}$.

If B, C hit the target & A miss it, then prob. for which $= \frac{1}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{1}{10}$.

since, these are mutually exclusive events, the required prob. $= \frac{2}{5} + \frac{1}{5} + \frac{2}{15} + \frac{1}{10} = \frac{25}{30} = \frac{5}{6}$

$= 0.83$

'A' has two shares in a lottery in which there are 3 prizes and 5 blanks. 'B' has 3 shares in a lottery in which there are 4 prizes & 6 blanks. Show that A's chance of success is to be as ~~27/35~~ 27/35.

Mathamatical def^n of probability :.

If a trial results in exausive, mutually exclusive and equally like cases and m of them are favourable to the happening of the event 'E', then the prob of happening of event E, is given by

$$p = P(E) = \frac{m}{n}.$$

3* The prob of event will not happen is given by

$$P(\bar{E}) = \frac{n-m}{n}.$$

$\therefore$ ~~$P(E) + P(\bar{E}) =$~~ $p+q = 1$

If n cases are favourable to E and m cases are favourable to $\bar{E}$, then prob. of E $P(\bar{E}) = \frac{m}{n+m}$.

A bag contains seven white, six red & 5 black balls. Two balls are drawn at random. Find prob. that they will both be white.

The total no. of balls = 7W + 6R + 5B = 18

out of 18 balles two can be drawn in

$$^{18}C_2 = 153.$$

$\therefore$ Exaustive no. of cases $= {}^{18}C_2 = \frac{18 \times 17}{2 \times 1}$

$= 153.$

Out of 7 white balls, 2 can be drawn $\therefore$ $^7C_2$.

$$\frac{7 \times 6}{2 \times 1} = 21.$$

∴ Prob. of both balls white = $\frac{21}{153}$

3. Four cards are drawn from a pack of cards. Find the prob. that :
① All are diamonds.
② There are two spades & two hearts.

→ Four cards can be drawn from pack of 52 cards by $^{52}C_4$ ways.
∴ exhaustive no. of cases

$$^{52}C_4 = \frac{52 \times 51 \times 50 \times 49}{4 \times 3 \times 2 \times 1}$$
270725

① There 13 diamonds and 4 can be drawn out of them in $^{13}C_4$ ways.
∴ No. of favourable ~~caes~~ cases

$$\frac{13 \times 12 \times 11 \times 10}{4 \times 3 \times 2 \times 1} = \cancel{770} \cdot 715$$

② Prob. of fav. cases $= \frac{715}{270725}$
$= 0.0026.$

Two spade out of 13 cards can be drawn in $^{13}C_2$ ways and two hearts out of 13 cards are drawn in $^{13}C_2$ ways.

∴ No. of favourable Cases $= {}^{13}C_2 \times {}^{13}C_2$

$= 6084$

$\therefore$ Prob. of favourable cases = $\frac{6084}{270725} = 0.0225$

* Random experiment :-

Occurences in which can be repeated a no. of times, essentially under the same conditions and whose result can not be predicted before hand are known as random experiment.

* Sample space :-

Out of the several possible outcomes of the random experiment one and only one can take place in a trial. The set of ~~out~~ all possible outcomes is called sample space for the perticular experiment. The sample space is denoted by s.

Sample Point :-

The elements of 's' are called sample points.

Event :- Every subset of 's' is called an event.

Since $S \subseteq S$

S itself is an event, then the event is called as certain event. Also, $\emptyset$ is a subset 's' is also an event is called as impossible event.

Axioms :-

① With each event 'E' is associated a real no. betn 0 & 1, called prob. of that event,

(2) The sum of the prob. of all events of the sample space is 1.
(3) Probability of impossible event = 0.
(4) Probability of certain event = 1.

* Probability of complementary ~~unit~~ Event :-

Result 1 :- The probability of impossible event is zero.

Impossible event contains no sample point.

$S \cup \emptyset = S$

$P(S \cup \emptyset) = P(S)$

$\therefore P(S) + P(\emptyset) = P(S)$ $\qquad\qquad$ $P(\emptyset) = 0$

$P(\emptyset) = P(S) - P(S)$

Result 2 :- The prob. of complementary event $\bar{A}$ of A is given by

$$P(\bar{A}) = 1 - P(A).$$

Proof :-

The events A & $\bar{A}$ are disjoint events.

$\therefore$ Also, $A \cup \bar{A} = S$.

$\therefore P(A \cup \bar{A}) = P(S)$.

$\therefore P(A) + P(\bar{A}) = 1$.

$\therefore P(\bar{A}) = 1 - P(A)$.

3 :- For any two events A & B. $P(\bar{A} \cap B) = P(B) - P(A \cap B)$.

~~$\bar{A} \cap B$ - {~~ we know that,

$\bar{A} \cap B = \{x : x \in B \text{ but } x \notin A\}$

$\bar{A} \cap B$ & $A \cap B$ are disjoint event.

Also, $(\bar{A} \cap B) \cup (A \cap B) = B$.

$\therefore$ $P[(\bar{A} \cap B) \cup (A \cap B)] = P(B)$.

$\therefore$ $P(\bar{A} \cap B) + P(A \cap B) = P(B)$.

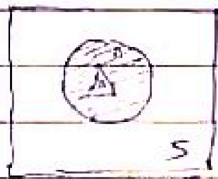
$\therefore$ $P(\bar{A} \cap B) = P(B) - P(A \cap B)$.

Note:- Similarly, we can prove

$$P(A \cap \bar{B}) = P(A) - P(A \cap B).$$

Result 4:- If $B \subseteq A$ then i) $P(A \cap \bar{B}) = P(A) - P(B)$.

ii) $P(B) \leq P(A)$.



$A \cap \bar{B}$

$\rightarrow$ When $B \subseteq A$,

B and $A \cap \bar{B}$ are disjoint.

$\therefore$ $B \cup (A \cap \bar{B}) = A$

$$P[B \cup (A \cap \bar{B})] = P(A)$$

$$P(B) + P(A \cap \bar{B}) = P(A).$$

$\therefore$ $P(A \cap \bar{B}) = P(A) - P(B)$. ... ①.

If E is any event,

$\therefore$ we have

$$0 \leq P(E) \leq 1,$$

$\therefore$ $P(A \cap \bar{B}) \geq 0$.

from eqn ①

$P(A) - P(B) \geq 0$.

$P(A) \geq P(B)$.

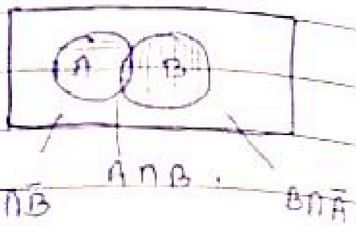
5. Addition therom of probability :-

If A & B are any two events then,

$p(A \cup B) = p(A) + p(B) - p(A \cap B)$.

i.e $p(A \text{ or } B) = p(A) + p(B) - p(A \& B)$.

Proof :-



As $\bar{A}$ & $\bar{A} \cap B$ are disjoint

$\therefore A \cup (\bar{A} \cap B) = A \cup B$.

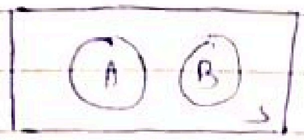
$\therefore p(A \cup (\bar{A} \cap B)) = p(A \cup B)$.

$\therefore p(A) + p(\bar{A} \cap B) = p(A \cup B)$.

$p(A) + p(\bar{A} \cap B) + p(A \cap B) - p(A \cap B) = p(A \cup B)$

$p(A) + p(B) - p(A \cap B) = p(A \cup B)$.

6. Q. If A and B are two mutually disjoint events then prove that $p(A \cup B) = p(A) + p(B)$.



As A & B are disjoint,

$\therefore A \cap B = \emptyset$.

$\therefore P(A \cap B) = P(\emptyset) = 0$

we have,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

i.e $P(A \cup B) = P(A) + P(B)$.

Q. If A, B & C are any three events then P.T.

$P(A \cup B \cup C) = P(A) + p(B) + p(C) - p(A \cap B) - P(B \cap C) - P(A \cap C) + p(A \cap B \cap C)$.

As prob. of

$P(A \cup B) = P(A) + P(B) - P(A \cap C)$.

$\therefore P[(A \cup B) \cup C] = [P(A) + P(B)] + P(C) -$

$\cancel{p(A \cap C) + p(= P(A) + p(B) - P(A \cap C) \mp P(A \cap B \cap C)}$

$\cancel{P(A \cap C) - P(B \cap C) - P(A \cap B) + P(A \cap B \cap C)}$

$P[(P(A) + P(B) - P(A \cap C)) \cap C]$

$= [P(A) + P(B)] + P(C) - P(A \cap B) - P(A \cap C) -$
$P(B \cap C) + P(A \cap B \cap C)$

Q. A card is drawn from a well shephard pack of playing cards, what is the prob. that it is either a spade or Ace

→ Let A denotes the event of drawing is spade and B denotes the event of drawing Ace.

$\therefore$ prob. $P(A) = 13/52$

$P(B) = 4/52$

$\therefore \quad P(A \cap B) = \frac{1}{52}$

$\therefore \; P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$

$= \frac{16}{52} = \frac{4}{13}$

$= 0.308$

* Conditional probability :-

The probability of the happening of event A when another event B is known to you have already happen is called conditional probability, and is denoted by $P(A/B)$