POSTAL Book Package 2021

Computer Science & IT

Objective Practice Sets

Theory of Computation *Contents*

[redacted]

MADE EASY Publications

1 CHAPTER

Grammars, Languages & Automata

Q.1 Suppose $L_1 = \{10, 1\}$ and $L_2 = \{011, 11\}$. How many distinct elements are there in $L = L_1L_2$.
(a) 4 (b) 3
(c) 2 (d) None of these

Q.2 In a string of length n, how many proper prefixes can be generated
(a) 2^n (b) n
(c) $\frac{n(n+1)}{2}$ (d) $n-1$

Q.3 Let $u, v, \in \Sigma^*$ where $\Sigma = \{0, 1\}$. Which of the following are TRUE?
1. $|u.v| = |v.u|$
2. $u.v = v.u$
3. $|u.v| = |u| + |v|$
4. $|u.v| = |u||v|$

(a) 1 and 3 (b) 1, 2 and 3
(c) 2 and 4 (d) 1, 2 and 4

Q.4 How many odd palindromes of length 11 are possible with alphabet $S = \{a, b, c\}$
(a) 3^6 (b) 2^5
(c) 2^6 (d) 3^5

Q.5 The number of distinct subwords present in 'MADEEASY' are _____ .

Q.6 Consider the following statements:
1. Type 0 grammars generate all languages which can be accepted by a Turing machine.
2. Type 1 grammars generate the languages which can all be recognised by a push down automata.
3. Type 3 grammars have one to one correspondence with the set of all regular expressions.
4. There are some languages which are not accepted by a Turing machine.

Which of the above statements are TRUE?
(a) 1, 2 and 3 (b) 1, 2 and 4
(c) 1, 3 and 4 (d) 2, 3 and 4

Q.7 Consider the following table of an FA:

δ	a	b
start	q_1	q_0
q_0	q_1	q_0
q_1	q_2	q_1
q_2	q_3	q_2
q_3	q_4	q_3
q_4	q_4	q_4

If the final state is q_4, the which of the following strings will be accepted?
1. *aaaaa*
2. *aabbaabbbbb*
3. *bbababababbb*

(a) 1 and 2 (b) 2 and 3
(c) 3 and 1 (d) All of these

Q.8 Which of the following statements is correct?
(a) Some finite automatas accept non regular languages.
(b) A grammar with recursion always generates infinite languages.
(c) An infinite language can be generated by a non recursive grammar.
(d) A deterministic push down automata cannot generate all context free languages.

Q.9 The grammer with start symbol S over $\Sigma = \{a, b\}$
$S \to aSbb | abb$ belongs to the class
(a) Type 0 (b) Type 1
(c) Type 2 (d) Type 3

Q.10 What is the language generated by the grammer where S is the start symbol and the set of terminals and non terminals is $\{a\}$ and $\{A, B\}$ respectively?
$S \to Aa$
$A \to B$
$B \to Aa$
(a) Set of strings with atleast one a
(b) Set of strings with even no. of a's
(c) Set of strings with odd no. of a's
(d) Empty language

Q.24 Which of the following conversions is not possible?
(a) Regular grammar to context free grammar
(b) NFA to DFA
(c) Non deterministic PDA to deterministic PDA
(d) Non deterministic Turing machine to deterministic Turing machine

Q.25 If $S = \{ab, ba\}$, which of the following is true?
(a) S^* contains finite no of strings of infinite length.
(b) S^* has no strings having 'aaa' or 'bbb' as substring.
(c) S^* has no strings having aa as substring.
(d) If $T = \{a, b\}$, then $S^* \not\subseteq T^*$,

■■■■

Answers Grammars, Languages & Automata

1. (b)	2. (b)	3. (a)	4. (a)	5. (34)	6. (c)	7. (a)	8. (d)	9. (c)
10. (d)	11. (c)	12. (d)	13. (d)	14. (c)	15. (c)	16. (b)	17. (3)	18. (d)
19. (b)	20. (d)	21. (b)	22. (c)	23. (a)	24. (c)	25. (b)		

Explanations Grammars, Languages & Automata

1. (b)

$$L_1 = \{10, 1\},$$
$$L_2 = \{011, 11\}$$

By concatenation of L_1 and L_2 we get

$$L_1 \cdot L_2 = \{10011, 1011, 1011, 111\}$$

Hence, 3 distinct elements are there.

2. (b)

Suppose, $S = aaab$, $|s| = 4$. The prefixes are $S_p = \{\lambda, a, aa, aaa, aaab\}$. Here aaab is not a proper prefix.

Note: The proper prefix of string S is a prefix, which is not same as string S.

A string of length 4 has 4 proper prefixes. A string of length 5 has 5 proper prefixes. For a string of length n, therefore we can have 'n' proper prefixes.

3. (a)

Let, $u = 1001$ and $v = 001$

$u.v = 1001001$ and $v.u = 001\ 1001$

$|u,v| = |v,u| = |u| + |v|$

But $u.v \neq v.u$

4. (a)

Palindromes can be represented by $\{WW^R \mid W \in \{a, b, c\}^*\}\ U$
$\{WxW^R \mid W \in (a, b, c)^*, x \in (a, b, c)\}$

Since, we need to count the number of odd palindromes of length 11, the number of possible W's of length 5 are $|\Sigma|^5$ i.e. 3^5

Number of possible ways for $x = 3$

$\therefore$ Number of odd palindromes of length 11 $= 3^5 \times 3$
$= 3^6$

Number of odd palindromes of length,

$$\boxed{n = |\Sigma|^{\frac{n-1}{2}} \times |\Sigma| = |\Sigma|^{\frac{n+1}{2}}}$$

5. (34)

Distinct subwords of

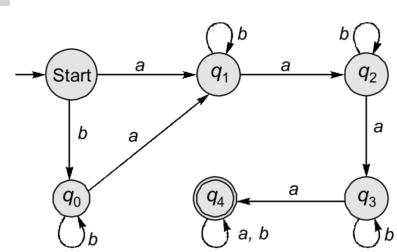
Length 1 = 6	Length 5 = 4
Length 2 = 7	Length 6 = 3
Length 3 = 6	Length 7 = 2
Length 4 = 5	Length 8 = 1

$\therefore$ Total = 34

6. (c)

See Chomoky Hierarchy languages, which are not recursively enumerable are not recognised by any machine.

7. (a)



Drawing the FA we have

we can clearly see that only (i) *aaaaa* and (ii) *aabbaabbbbb* are accepted.

8. (d)

(a) is false since a FA can accept only regular languages as it has finite memory only.

(b) is fase consider the grammar $\{S \to Sa\}$ which a recursive. It generates the empty languages i.e. ϕ which is finite.

(c) is false. To generate an infinite language, the grammar must have recusion.

(d) True DPDA cannot generate all CFLs. It generates a subset of CFLs called DCFLs. DPDA has less recognition power than a PDA.

9. (c)

The given grammer is Type 2 as every rule is restricted as:

$V \to (V \cup T)^*$

where V is the set of non-terminals and T is set of terminals.

10. (d)

Since there is no string which can be generated from the grammar in finite number of steps as there is no termination, (d) is true.

11. (c)

If the sequence has even length say, $n = 2k$, selecting the first k characters completely determines the palindrome since the remaining k characters can be found by repeating the sequence in the reverse roler. Number of palindromes of even length atmost n in alphabet with x characters is

$$x^0 + x^1 + x^2 + \dots x^k = \frac{-1 + x^{k+1}}{x-1}.$$

Here, $x = 3$ and $k = 5$

$\therefore \dfrac{3^6 - 1}{2}$ is the number of palindromes of length atmost 10.

12. (d)

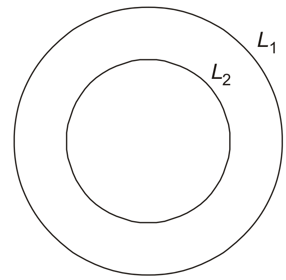
$$L_1^* = \{\phi\}^* = \{\lambda\}$$

$$L_2^* = \{1\}^* = 1^*$$

$$L_1^* \, U L_2^* \, L_1^* = \{\lambda\} U \{\lambda\}.1^* = 1^*$$

13. (d)

L_1 is the set of all strings where any number of a's is followed by an equal number of b's.



L_2 is the set of all strings where an even number of a's is followed by an equal number of b's.

$\therefore \quad L_2 \subseteq L_1$

$L_2 \cap L_1 = L_2$

$L_2 \cup L_1 = L_1$

$L_1 - L_2 =$ (Set of all strings where an odd number of a's is followed by an equal number of b's)

$L_2 - L_1 = \phi$

14. (c)

$$L_1 = \{a^n b^n c^n, n \geq 0\} = \{\lambda, abc, a^2b^2c^2, \dots\}$$

$$L_2 = \{a^{2n} b^{2n} c^{2n}, n \geq 0\} = \{\lambda, a^2b^2c^2, a^4b^4c^4, \dots\}$$

$$L_3 = \{a^{2n} b^{2n} c^n, n \geq 0\} = \{\lambda, a^2b^2c, a^4b^4c^2, \dots\}$$

as we can easily see that

(i) L_1 contains all the words generated by L_2 and also it contains some extra strings.
$\therefore\ L_1 \supseteq L_2$. (or $L_2 \subseteq L_1$)

(ii) Since only λ is common in L_2 and L_3
Hence $L_2 \not\subset L_3$.

15. (c)

L^* is a combination of strings in L.

1. abaabaaabaa = ab aa baa ab aa belongs to L^*.
2. baaaaabaa = baa aa ab aa belongs to L^*.
3. baaaaabaaaab = baa aa ab aa aa b does not belong to L^*.
4. aaaabaaaa = aa aa baa aa belongs to L^*.

16. (b)

Both prefix and suffix consists of ε and L.
However in case of binary alphabet, for instance, prefix (L) = suffix(L)
$\therefore$ Prefix (L) n suffix (L) $\supseteq \{\varepsilon_1 L\}$

17. (3)

L_1 can be represented by a^*b^*

$$L_1^* = (a^*b^*)^* = (a+b)^*$$
$$L_2 = (ba)$$
$$L_1^* \cap L_2 = [(a+b)^*] \cap (ba)$$
$$= (ba)$$
$$\text{Prefix}, (L_3) = (\varepsilon, b, ba)$$

18. (d)

(a) is false as 'aaa' is generated by the grammar.

(b) is false as 'aa' is generated.

(c) is false as 'aaa' is generated.

A generates the language represented by a^* {0 or more a's}

S generate aaa*

19. (b)

L_1 is the set of strings where zero or more a's is followed by zero or more b's.

L_2 is the set of strings where zero or more b's is followed by zero or more a's.

$L_1 \cap L_2$ - Set of strings of only a's or only b's including the NULL string λ.

$$\therefore \quad L_1 \cap L_2 = \{a^* + b^*\}$$

Note: $a^*b^* = a^+b^+ + a^* + b^*$

$b^*a^* = b^+a^+ + a^* + b^*$

20. (d)

$L = \{a^n b^m \mid n, m \geq 0\}$ i.e. the number of a's and number of b's are independent.

$\therefore$ L is a regular language.

$$L_1 = \{\varepsilon, a, aa, aaa, ...\}$$
$$L_2 = \{\varepsilon, b, bb, bbb, ...\}$$
$$L_3 = L_1L_2 = \{\varepsilon, ab, abb, abbb, aab, ...\}$$

21. (b)

1. False

Case (i) L is finite

We know that Σ^* is infinite

$$\overline{L} = \Sigma^* - L$$

$\therefore$ $\overline{L}$ must be infinite as it is obtained by removing a finite number of string from an infinite set.

Case (ii) L is infinite

Σ^* is infinite

$$\overline{L} = \Sigma^* - L$$

$\therefore$ $\overline{L}$ may be finite or infinite

From above, in any case, both L and $\overline{L}$ cannot be finite.

2. False

$$\lambda \in L^*$$
$$\Rightarrow \qquad \lambda \notin \left(\overline{L^*}\right)$$

But $\left(\overline{L}\right)^*$ must contain λ.

$\therefore$ No language satisfies $\left(\overline{L^*}\right) = \left(\overline{L}\right)^*$

3. True

Let $u \in L_1$, $v \in L_2$

$$L_1L_2 = \{uv\}$$
$$(L_1L_2)^R = (uv)^R = v^R u^R$$
$$= (L_2)^R (L_1)^R \ \forall u, v$$

4. True

For all Σ

(i) $L^* \subseteq (L^*)^*$. This is because $L^* = \{w_1, w_2, ...\}$ and therefore $\{w_1, w_2, ...\} \subseteq \{w_1, w_2, ...\}^*$.

(ii) $(L^*)^* \subseteq (L^*)$. For every $w \notin (L^*)^*$, we can decompose it as

$w = w_1 w_2 w_3 ... w_n$ such that each $w_i \in L^*$. Similarly we can decompose w_i such that $w_i = w_{1i} w_{2i} w_{3i} ... w_{iNi}$ where $W_i N_i \in L$. So, $w \in L^*$

Now, $w = w_{11} w_{21} w_{31} .... w_1 N W_{12} .... w_2 N_2 .....$

where $w_{ij} \notin L$

So $w \in L^*$

From (i) and (ii) $L^* = (L^*)^*$

[L^* is the combination of strings in L]

22. (c)

$$L = \{\lambda, a, aa, aaa, ...\}$$

$L^2 = L,L = \{\lambda, a, aa, aaa, aaaa, ... a^n\}$

$\therefore$ L^2 is the set of all strings over Σ

23. (a)

For strings belonging to L^5, they should be a combination of exactly 5 strings $\in L$.

Since L contains λ, the strings in L^5 should be a combination of atmost 5 non null strings which belong to L as the remaining component could be the null string.

(a) 110010 = 1 10 01 0 does not belong to L^5

(b) 101001001 = 10 10 01 001 belong to L^5

(c) 100100 = 10 01 0 0 belongs to L^5

(d) 01101001 = 01 10 10 01 belongs to L^5