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Tutorial no 6

Q1 Define null closure of a set

Let $M = (Q, \Sigma, A, q, s)$ be an NFA NULL let s be any set which is subset of Q $(S \subseteq Q)$ Null closere of s $(\Lambda(S))$ is define as follows

1. every element of s is an element of NULL closure of s
2. For any $q \in \Lambda(s)$, $\delta(q, \Lambda)$ is also $\in \Lambda(s)$

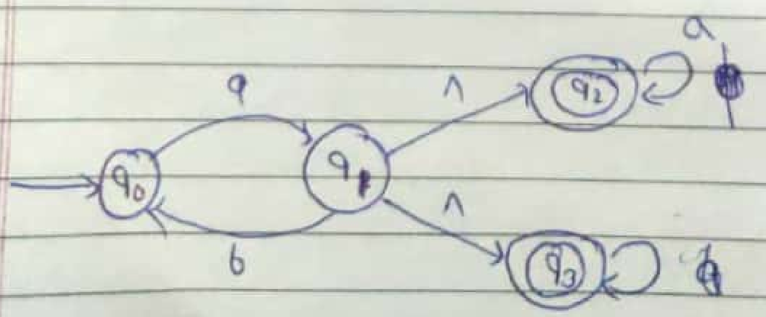
Q 2) Define extended transition function for NFA-Λ

Extenteded Transition function (δ^*) for NFA NULL
Let $M = (Q, \Sigma, q_0, A, \delta)$ be an NFA Null the extended Transition function $\delta^* : Q \times \Sigma^* \to 2^Q$ is define as follows

1) For any $q \in Q$ $\delta^*(q, \Lambda) = \Lambda(\{q\})$
ii) For any $q \in Q$, any $y \in \Sigma^*$, any $a \in \Sigma$

$$\delta^*(q, ya) = \Lambda\left(\bigcup_{r \in \delta^*(q,y)} (r, a)\right)$$

3) for following NFA-Λ :



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i) Find $\delta^*(q_0, aa)$, $\delta^*(q_0, ab)$

ii) convert above NFA-Λ to DFA

i) $\rightarrow$ $\delta^*(q_0, aa) = \Lambda \bigcup_{r \in \delta^*(q_0, a)} (\delta(r, a))$

$= \cancel{\Lambda(r \in \{q_1\})}$ $r \in \{q_1, q_2, q_3\}$

$= \Lambda(\delta(q_1)) \Lambda(\delta(q_1, a) \cup \delta(q_2, a) \delta(q_3, a))$

$= \cancel{\Lambda\{\{q}$

$= \Lambda(\{q_2\})$

$= q_2$

$\delta^*(q_0, a) = \Lambda \bigcup_{r \in \delta^*(q_0, \Lambda)} (\delta(r, a))$

$r \in \{q_0\}$

$= \cancel{\Lambda\{q_0\}}$

$\Lambda(\delta(q_0, a))$

$= \Lambda(\{q_1\}) = \{q_1, q_2, q_3\}$

$=$

$\delta^*(q_0, \Lambda) = \cancel{\Lambda \cup (\delta(r,}\ \Lambda(\{q_0\})$

$= \cancel{\{q_0\}}\ \{q_0\}$

state	Λ	a	b	$\delta^*(q, a)$	$\delta^*(q, b)$
q_0	ϕ	$\{q_1\}$	ϕ	$\{q_1, q_2, q_3\}$	ϕ
q_1	$\{q_2, q_3\}$	ϕ	$\{q_0\}$	$\{q_2\}$	$\{q_3, q_0\}$
q_2	$\{\phi\}$	q_2	ϕ	$\{q_2\}$	ϕ
q_3	ϕ	ϕ	q_3	ϕ	$\{q_3\}$

$\delta^*(q_0, a) = \Lambda\left(\bigcup_{r \in \delta^*(q_0, \Lambda)} \delta(r, a)\right)$

$= \Lambda(\delta(q_0, a))$

$= \Lambda(\{q_1\})$

$= \{q_1, q_2, q_3\}$

$\delta^*(q_0, b) = \Lambda\left(\bigcup_{r \in \delta^*(q_0, \Lambda)} \delta(r, b)\right)$

$= r \in \{q_0\}$

$= \Lambda(\delta(q_0 b))$

$= \Lambda(\phi)$

$= \phi$

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$$\delta^*(q_1,a) = \Lambda\Big(\bigcup_{r \in \delta^*(q_1,\Lambda)} \delta(r,a)\Big)$$

$$= \Lambda(\delta(q_1,a) \cup \delta(q_2,a) \cup \delta(q_3,a))$$

$$= \Lambda(\{q_2\})$$

$$= \{q_2\}$$

$$\delta^*(q_1,b) = \Lambda(\delta(q_1,b) \cup \delta(q_2,b) \cup \delta(q_3,b))$$

$$= \Lambda(\{q_0, q_3\})$$

$$= \{q_0, q_3\}$$

$$\delta^*(q_2,a) = \Lambda\Big(\bigcup_{r \in \delta^*(q_2,\Lambda)} \delta(r,a)\Big) \qquad \delta^*(q_2,b) = \Lambda(\delta(q_2,b))$$

$$r \in q_2 \qquad \phi$$

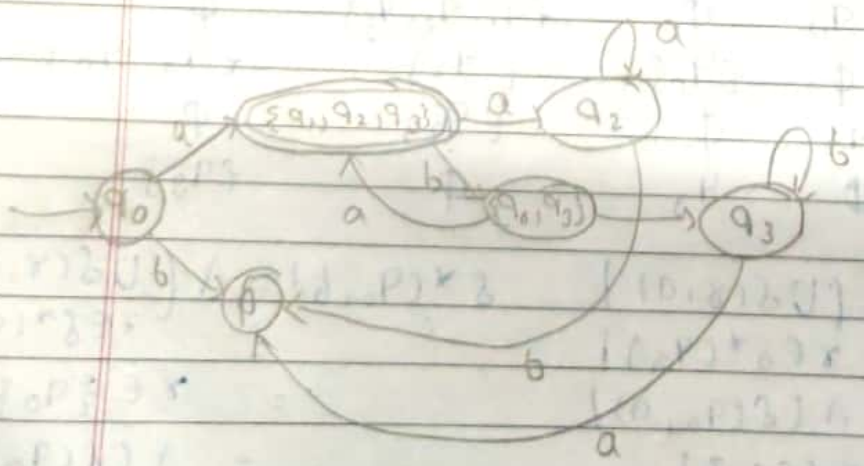
$$= \Lambda(\delta(q_2,a))$$

$$= q_2$$

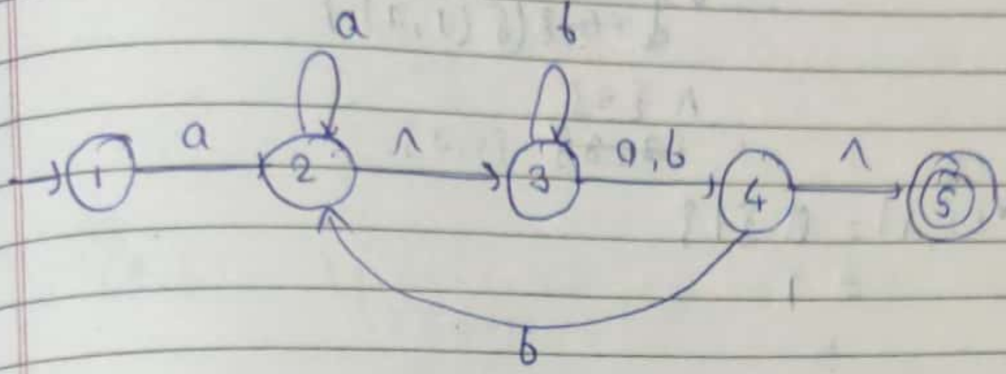
$$\delta^*(q_3,a) = \Lambda(\delta(q_3,a)) \qquad \delta^*(q_3,b) = \Lambda(\delta(q_3,b))$$

$$= \Lambda(\phi) \qquad = \Lambda(q_3)$$

$$= \phi \qquad = q_3$$



1) For following NFA-Λ



i) state whether the above NFA-Λ accepts following string or not
a) aba , abab

ii) convert ~~following OF~~ above NFA to DFA

~~a) $\delta^*(aba) = \Lambda\left(\bigcup_{r \in \delta^*(ab,} \delta(r,a)\right)$~~

a) $\delta^*(1,aba) = \Lambda\left(\bigcup_{r \in \delta^*(1,ab)} (r,a)\right)$

$r \in \{3,4,5\}$

$= \Lambda(\delta(3,a) \cup \delta(4,a) \cup \delta(5,a))$

$= \Lambda(4\}$

$= \{4,5\}$

$\delta^*(1,ab) = \Lambda\left(\bigcup_{r \in \delta^*(1,a)} (r,b)\right)$

$= \Lambda(r \in \{2,3\}$

$= \Lambda(\delta(2,b) \cup \delta(3,b))$

$= \Lambda(\{3,4\})$

$= \{3,4,5\}$

$\delta^*(1,a) = \Lambda\left(\bigcup_{r \in \delta(1,\Lambda)} \delta(r,a)\right)$

$= \Lambda\left(\bigcup_{r \in \{1\}} \delta(1,a)\right)$

$\Lambda\{2\}$

$= \cancel{\{2,3\}}\ \{2,3\}$

$\delta^*(1,\Lambda) = \Lambda\{1\}$

$= 1$

$\delta^*(1,aba) = \{4,5\}$

∴ string is accepted

ii) abab

$\delta^*(1,abab) = \Lambda\left(\bigcup_{r \in \delta^*(1,aba)} \delta(r,b)\right)$

$r \in \{4,5\}$

$= \Lambda(\delta(4,b) \cup \delta(5,b))$

$= \Lambda(\{2\})$

$= \{2,3\}$

$\delta^*(1,aba) = \{4,5\}$

states	Λ	a	b	$\delta^*(q,a)$	$\delta^*(q,b)$
1	ϕ	{2}	ϕ	{2,3}	ϕ
2	{3}	{2}	ϕ	{2,4}	{3,4}
3	ϕ	{4}	{3,4}	{4,5}	{3,4,5}
4	{5}	ϕ	{2}	{ϕ}	{2,3}
5	ϕ	ϕ	ϕ	ϕ	ϕ

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$\delta^*(1,a) = \Lambda\left(\bigcup_{r \in \delta^*(1,\Lambda)} \delta(r,a)\right)$

$r \in \{1\}$

$= \Lambda(\delta(1,a))$

$= \Lambda(2)$

$= \{2,3\}$

$\delta^*(1,\Lambda) = \Lambda(\phi)$

$= \phi\ \{1\}$

$\delta^*(1,b) = \Lambda(\delta(1,b))$

$= \Lambda(\phi)$

$= \phi$

$\delta^*(2,b) = \Lambda\left(\bigcup_{r \in \delta^*(2,\Lambda)} \delta(r,b)\right)$

$= r \in \{2,3\}$

$\delta(2,b) \cup \delta(3,b)$

$= \{3,4\}$

$\delta^*(2,\Lambda) = \{2,3\}$

$\cancel{\delta^*(2,a)} =$

$(\delta(2,a) \cup \delta(3,a))$

$= \{2,4\}$

$\delta^*(3,a) = \Lambda\left(\bigcup_{r \in \delta^*(3,\Lambda)} \delta(r,a)\right)$

$r \in \{3\}$

$= \Lambda(\delta(3,a))$

$= \Lambda(4)$

$\{4,5\}$

$\delta^*(3,\Lambda) = \Lambda\{3\}$

$\{3\}$

$\delta^*(3,b) = \Lambda(\delta(3,b))$

$= \Lambda(\{3,4\})$

$= \{3,4,5\}$

$\delta^*(4,a) = \Lambda\left(\bigcup_{r \in \delta^*(4,\Lambda)} \delta(r,a)\right)$

$r \in \{4,5\}$

$= \Lambda(\delta(4,a) \cup \delta(5,a))$

$= \Lambda(\{\phi\})$

$= \phi$

$\delta^*(4,\Lambda) = \{4,5\}$

$\delta^*(4,b) = \Lambda(\delta(4,b) \cup \delta(5,b))$

$= \Lambda(2)$

$= \{2,3\}$

$\delta^*(5,a) = \Lambda\left(\bigcup_{r \in \delta^*(5,\Lambda)} \delta(r,a)\right)$

$- r \in \{5\}$

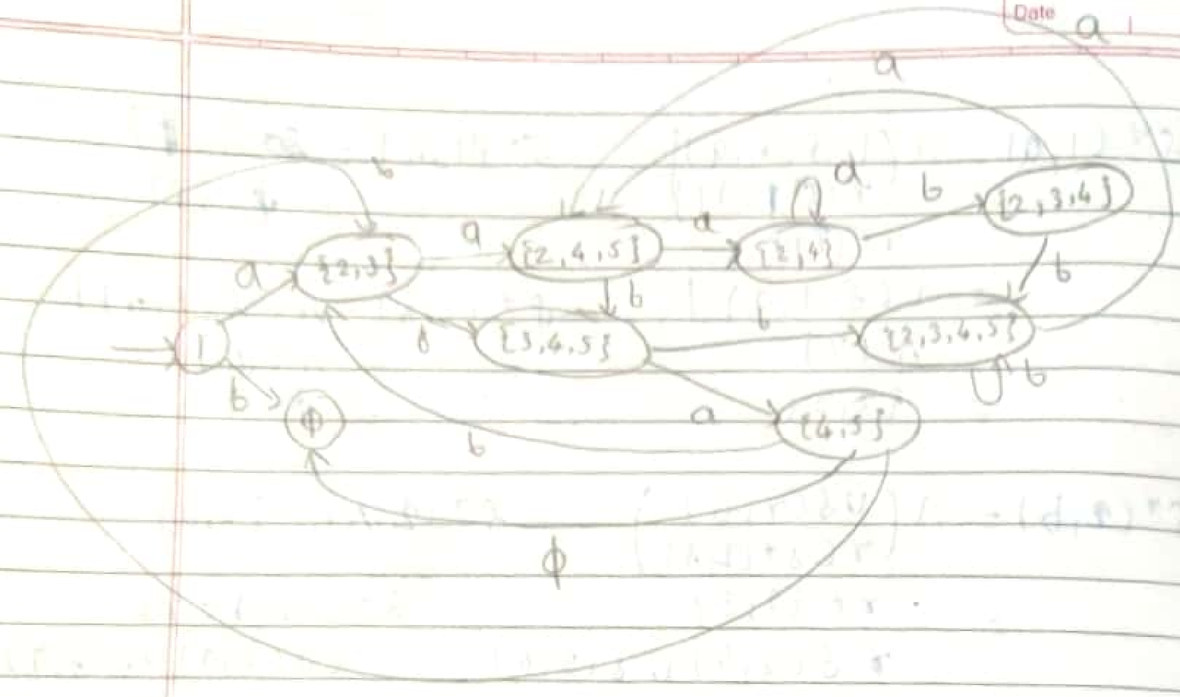
$= \delta(5,a)$

$= \phi$

$\delta^*(5,\Lambda) = \Lambda\{5\}$

$= \phi\ 5$

$\delta^*(5,b) = \phi$



Q6 prove ~~p~~ Theorem part I & II

Kleene's theorem

Any regular language can be accepted by a finite automata

Proof :- For this Theorem it is sufficient to show that every regular language can be accepted by NFA Null

This Basic step of proof is to show that three basic language ~~can~~ be accepted by NFA Null L_1 & L_2 are two languages that can be accepted by NFA Null and we are going to show that $L_1 \cup L_2$, $L_1 L_2$ & L^* can also be accepted by NFA NULL

suppose L_1 & L_2 are recognized by M_1 & M_2 NFA Null resp

$$M_1 = (Q_1, \Sigma, q_1, A_1, \delta_1)$$

$$M_2 = (Q_2, \Sigma, q_2, A_2, \delta_2)$$

we are assumming some basic step

$Q_1 \cap Q_2 = \phi$

we are going to construct three NFA NULL M_u, C_{aa}, M_k recognizing the languages $L_1 \cup L_2$, L_1L_2 & L_1^*

Part I

construction of NFA Null for union of the languages

let $M_u = (Q_u, \Sigma, q_u, A_u, \delta_u)$ is NFA Null for union of Language

→ let q_u be a new state ~~not~~ present in Q_1 or Q_2

$Q_u = Q_1 \cup Q_2 \cup \{q_u\}$

$A_u = A_1 \cup A_2$

q_u = initial state

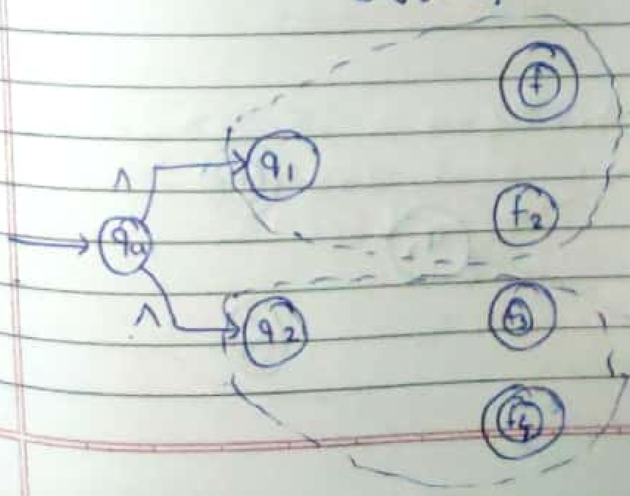
Now we define δ_u so that M_u can move from its initial state to either q_1 or q_2 by a Null transition

$\delta_u(q_u, \Lambda) = \{q_1, q_2\}$

$\delta_u(q_u, a) = \phi \quad a \in \Sigma$

For each $q \in Q_1$ or Q_2

$$\delta_u(q, a) = \begin{cases} \delta_1(q, a) & q \in Q_1 \\ \delta_2(q, a) & q \in Q_2 \end{cases}$$



part II

Construction of Mca

~~$M_1 = (Q_1, \Sigma$~~

Let $M_{ca} = (Q_c, \Sigma, q_c, A_c, \delta_c)$ is an NFA Null for concatination of $L_1 L_2$

$Q_c = Q_1 \cup Q_2$

$q_c = \{q_1\}$

$A_c = A_2$

$\delta_c = (q_F, \Lambda)$

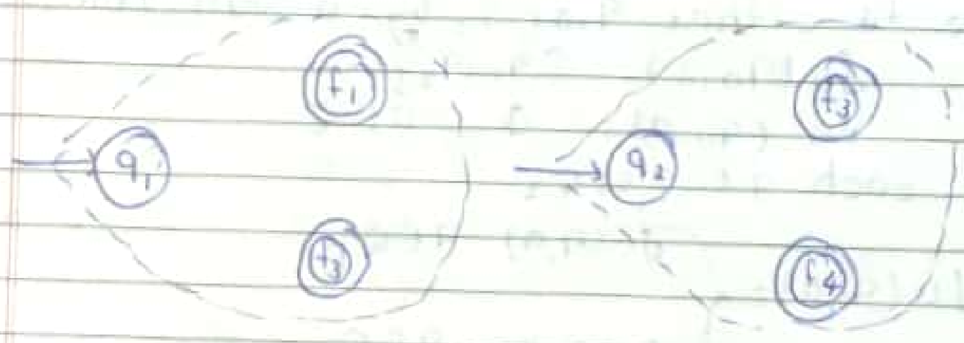
where q_F is all final states of M

& q_2 is initial state of M_2

For other

$$\delta_c(q, a) = \begin{cases} \delta_1(q, a) & \text{if } q \in Q_1 \\ \delta_2(q, a) & \text{if } q \in Q_2 \end{cases}$$

eg



then m_3