

## Tutorial 3

Q.1. Explain encryption and decryption in DES.

Ans: DES Encryption:

- 64 bit plaintext pass through initial permutation rearrange bit to produce permuted input.
- followed by 16 rounds of same function which involves permutation and substitution functions.
- The output of last ~~four~~ round swapped to produce pre-output.
- Preoutput pass through a permutation ( $IP^{-1}$ )

DES Decryption:

- As with any Feistel cipher, decryption uses the same algorithm as encryption
- The subkeys are reversed.

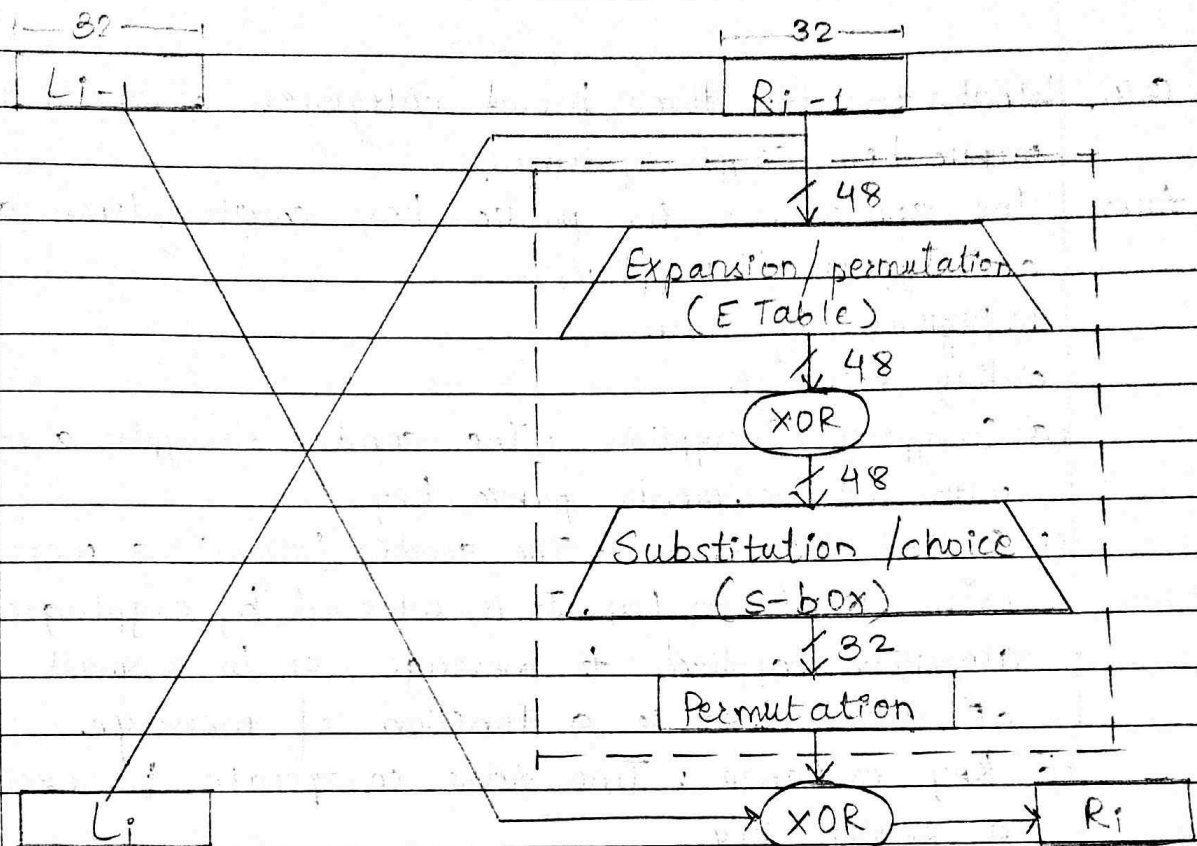
Q.2. Explain single round in DES.

Ans: Left and right halves of 64 bits are separated into  $2 \times 32$  bit parts  $L_i, R_i$

$$L_i = R_{i-1}$$

$$R_i = L_{i-1} \oplus F(R_{i-1}, K_i)$$

- $R$  is expanded to 48 bits using permutation  $E$
- Resulting 48 bits are ~~XORED~~ XORed with  $K_i$
- 48 bit result passes thru substitution function  $f$  (8-S-boxes) producing 32-bit output.
- Output is permuted using permutation function.



Q.3. What are the principle elements of public key cryptosystems?

- Ans:
- Plaintext**: The data to be protected.
  - Encryption Algorithm**: It takes plaintext and encryption key as input and produces ciphertext.
  - Ciphertext**: The coded message produced by encryption algorithm using the encryption key.
  - Decryption Algorithm**: It produces a unique plaintext for any given ciphertext and decryption key.
  - Encryption key**: It is a value known to the sender. The sender uses it to compute ciphertext.
  - Decryption key**: It is a value known to the receiver. The receiver uses it with ciphertext to compute plaintext.



Q.4. What are the three broad categories of application of public key crypto systems?

Ans: The applications for public-key cryptosystems are-

a) Encryption / Decryption.

b) Digital Signature

c) Key exchange.

(a) Encryption / Decryption : The sender encrypts a message with the recipient's public key.

(b) Digital Signature : The sender 'signs' a message with its private key. It is achieved by cryptographic algorithm applied to message or to a small block of data that is a function of message.

(c) Key exchange : Two sides co-operate to exchange a session key.

Q.5. Explain RSA algorithm.

Ans: - Plaintext is encrypted in blocks

- Block's binary value  $2^k \leq n$

- Equivalently, block size  $k \leq \log_2(n)$

- Encryption (plaintext block  $M$ , ciphertext  $C$ )

$$C = M^e \bmod n$$

- Decryption

$$M = C^d \bmod n = (M^e)^d \bmod n = M^{ed} \bmod n$$

- To encrypt a message  $M$ , the sender:

• obtains public key of recipient  $K_U = \{e, N\}$

• computes:  $C = M^e \bmod N$ , where  $0 \leq M < N$

- To decrypt the ciphertext  $C$  the owner:

• uses their private key  $K_R = \{d, p, q\}$

• computes:  $M = C^d \bmod N$

- The message  $M$  must be smaller than modulus  $N$ .



Q.6. Perform encryption and decryption using the RSA algorithm for the following.

a.  $p=3, q=11, e=7, M=5$

Ans:

$$n = p \times q$$

$$= 3 \times 11 = 33$$

$$\phi(N) = (p-1)(q-1) = 20$$

$$d = e^{-1} \bmod \phi(N)$$

$$= 1/7 \bmod 20 = 3$$

$$C = M^e \bmod n = 5^7 \bmod 33 = 14 \rightarrow \text{Encryption}$$

$$M = C^d \bmod n = 14^3 \bmod 33 = 5 \rightarrow \text{Decryption}$$

b.  $p=5, q=11, e=3, m=9$

Ans:

$$n = p \times q = 55$$

$$\phi(N) = (p-1)(q-1) = 4 \times 10 = 40$$

$$d = e^{-1} \bmod \phi(N) = 27$$

$$C = M^e \bmod n = 9^3 \bmod 55 = 14 \rightarrow \text{Encryption}$$

$$M = C^d \bmod n = 14^{27} \bmod 55 = 9 \rightarrow \text{Decryption}$$

c.  $p=11, q=13, e=11, M=7$

Ans:

$$n = p \times q = 143$$

$$\phi(N) = (p-1)(q-1) = 120$$

$$d = e^{-1} \bmod \phi(N) = 1/11 \bmod 120 = 11$$

$$\text{public key, } K_{EU} = \{e, n\} = \{11, 143\}$$

$$\text{private key, } K_{PR} = \{d, n\} = \{11, 143\}$$

$$C = M^e \bmod n = 7^{11} \bmod 143 = 106 \rightarrow \text{Encryption}$$

$$M = C^d \bmod n = 106^{11} \bmod 143 = 7 \rightarrow \text{Decryption}$$

d.  $p=17, q=31, e=7, M=2$

Ans:

$$n = p \times q = 17 \times 31 = 527$$

$$\phi(N) = (p-1)(q-1) = 480$$

$$\begin{aligned}
 d &= e^{-1} \bmod \phi(N) \\
 &= 1/7 \bmod 480 \\
 &= 343
 \end{aligned}$$

$$\begin{aligned}
 C &= M^e \bmod n = 2^7 \bmod 527 = 128 \rightarrow \text{Encryption} \\
 M &= C^d \bmod n = 128^{343} \bmod 527 = 2 \rightarrow \text{Decryption}
 \end{aligned}$$

Q.7. In a public key system using RSA you intercept the ciphertext  $C=10$  sent to a user whose public key is  $e=5$ ,  $n=35$  what is  $M$ ?

Ans:  $C = M^e \bmod n$   
 $10 = M^5 \bmod 35$

Let  $p=5$ ,  $q=7$  then

$$n = p \times q = 5 \times 7 = 35$$

$$\phi(N) = (p-1)(q-1) = 24$$

$$\begin{aligned}
 \gcd(\phi(N), e) &= \gcd(24, 5) \\
 &= 1 \text{ and } 1 < e < \phi(N)
 \end{aligned}$$

$$\text{Now } d = e^{-1} \bmod \phi(N)$$

$$ed = \bmod \phi(N)$$

$$e=5, \phi(N)=24$$

$$\therefore d=5$$

$$KR = \{d, n\} = \{5, 35\}$$

$$\text{RSA : } M = C^d \bmod n$$

$$= 10^5 \bmod 35 = 5$$

To verify correctness,

$$C = M^e \bmod n = 5^5 \bmod 35$$

$$\therefore C = 10$$

$$\therefore \text{Plaintext } M=5$$

Q.8. In RSA, system the public key of given user is  $e=31$ ,  $n=3599$ . What is private key of the user?

Ans:  $n=3599 = 3600 - 1$   
 $= (60)^2 - 1^2$   
 $= (60-1)(60+1)$

i.e. 59, 61

$$p=59, q=61 \text{ and } e=31$$

$$\begin{aligned}\phi(N) &= (p-1)(q-1) \\ &= 58 \times 60 \\ &= 3480\end{aligned}$$

$$d = e^{-1} \bmod \phi(n)$$

$$ed = \bmod \phi(n)$$

$$ed \bmod \phi(N) = 1$$

$$e=31, \phi(N)=3480$$

So,

$$31 \cdot d \bmod 3480 = 1$$

$$\therefore d = 3031$$

Private key  $KR = \{d, n\} = \{3031, 3599\}$