Experiment No.: 08

Title: Write a program for implementation of logistic regression.

Objectives: To learn logistic regression

Theory:

Logistic regression is a method for classifying data into discrete outcomes. For example, we might use logistic regression to classify an email as spam or not spam. Logistic regression is a statistical model that in its basic form uses a logistic function to model a binary dependent variable, although many more complex extensions exist. In regression analysis, logistic regression (or logit regression) is estimating the parameters of a logistic model (a form of binary regression). Mathematically, a binary logistic model has a dependent variable with two possible values, such as pass/fail which is represented by an indicator variable, where the two values are labeled "0" and "1". In the logistic model, the log-odds (the logarithm of the odds) for the value labeled "1" is a linear combination of one or more independent variables ("predictors"); the independent variables can each be a binary variable (two classes, coded by an indicator variable) or a continuous variable (any real value). The corresponding probability of the value labeled "1" can vary between 0 (certainly the value "0") and 1 (certainly the value "1"), hence the labeling; the function that converts log-odds to probability is the logistic function, hence the name. The unit of measurement for the log-odds scale is called a logit, from logistic unit, hence the alternative names. Analogous models with a different sigmoid function instead of the logistic function can also be used, such as the probit model; the defining characteristic of the logistic model is that increasing one of the independent variables multiplicatively scales the odds of the given outcome at a constant rate, with each independent variable having its own parameter; for a binary dependent variable this generalizes the odds ratio.

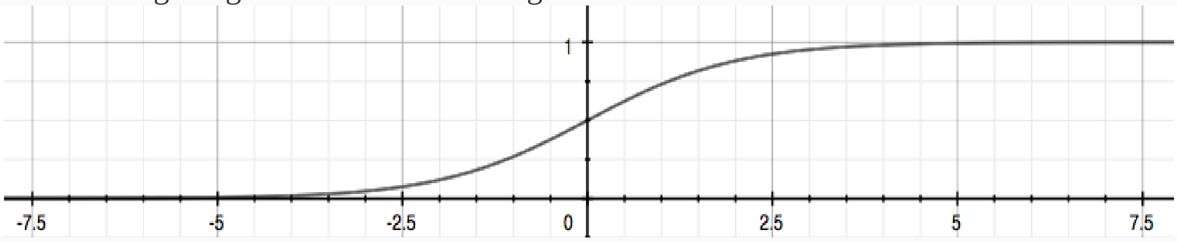
Hypothesis Representation

We could approach the classification problem ignoring the fact that y is discrete-valued, and use our old linear regression algorithm to try to predict y given x. However, it is easy to construct examples where this method performs very poorly. Intuitively, it also doesn't make sense for $h_\theta(x)$ to take values larger than 1 or smaller than 0 when we know that $y \in \{0, 1\}$. To fix this, let's change the form for our hypotheses $h_\theta(x)$ to satisfy $\mathbf{0 \le h_\theta(x) \le 1}$. This is accomplished by plugging $\boldsymbol{\theta^T . x}$ into the Logistic Function.

Our new form uses the "Sigmoid Function," also called the "Logistic Function":

$$h_\theta(x) = g(\theta^T.x)$$
$$z = \theta^T.x$$
$$g(z) = \frac{1}{1+e^{-z}}$$

The following image shows us what the sigmoid function looks like:



The function g (z), shown here, maps any real number to the (0, 1) interval, making it useful for transforming an arbitrary-valued function into a function better suited for classification.

h_θ **(x)** will give us the **probability** that our output is 1. For example h_θ **(x)** = 0.7 gives us a probability of 70% that our output is 1. Our probability that our prediction is 0 is just the complement of our probability that it is 1 (e.g. if probability that it is 1 is 70%, then the probability that it is 0 is 30%).

$$h_\theta(x) = P(y=1 \mid x;\theta) = 1 - P(y=0 \mid x;\theta)$$

$$P(y=0 \mid x;\theta) + P(y=1 \mid x;\theta) = 1$$

Hypothesis Function

- Hypothesis Function for Logistic regression is,
- $h_\theta(x) = g((\theta^T x))$

$$h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$$

i.e.

Cost Function

- Cost Function for Logistic regression is

$$J(\theta) = -\frac{1}{m}\left[\sum_{i=1}^{m} y^{(i)} \log h_\theta(x^{(i)}) + (1-y^{(i)}) \log (1-h_\theta(x^{(i)}))\right]$$

Gradient Descent

- Gradient Descent Function for logistic regression is

Repeat {

$$\theta_j := \theta_j - \alpha \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

(simultaneously update all θ_j)

}