CSC447: Digital Image Processing

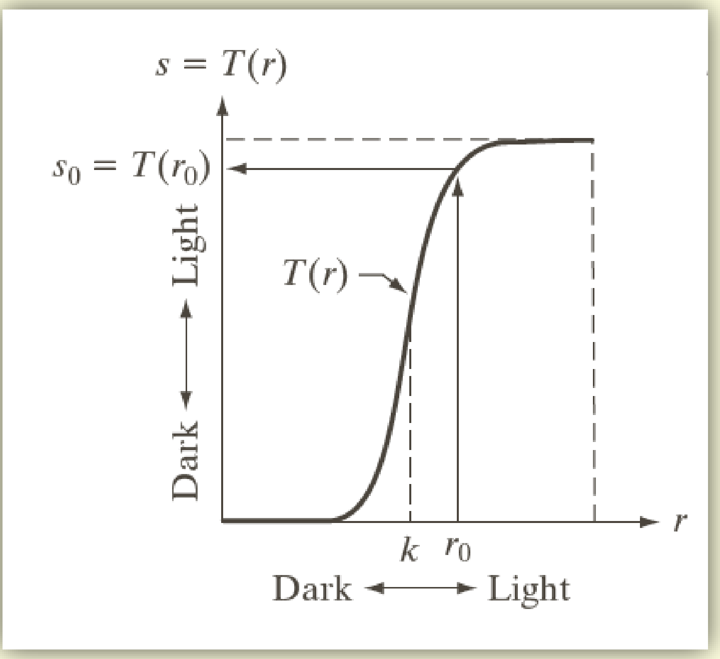
Chapter 3: Image Enhancement in the Spatial Domain

Pixel (Point) Operations

- The intensity transformation operations always obey the equation:

$$s = T(r)$$

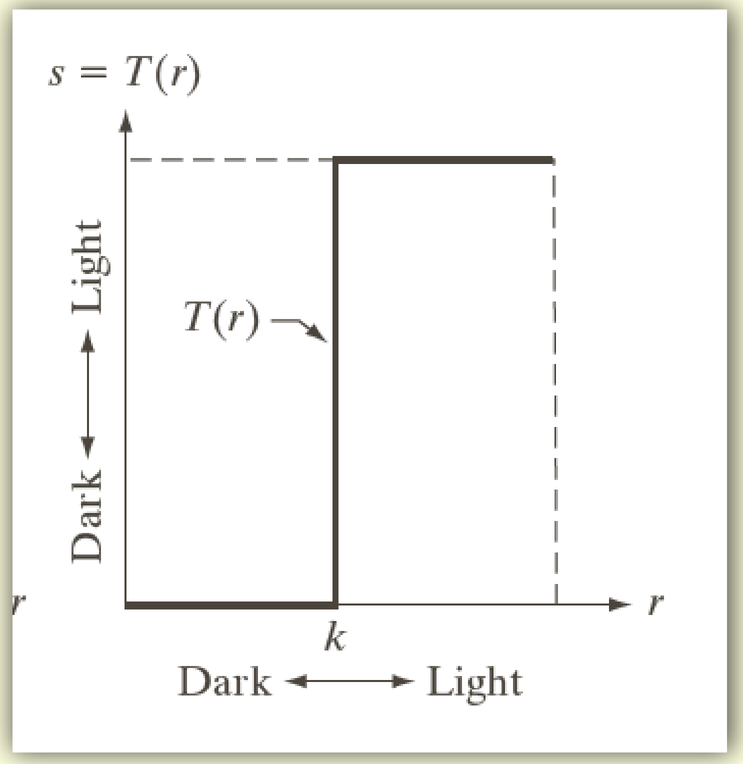
- r: the input pixel intensity
- s: the output pixel intensity
- T: the operation
- For example:
  - Image *Thresholding*



Some Point Transforms

- Image Binarisation
  - Converting image to Black & White
  - Image *Thresholding*

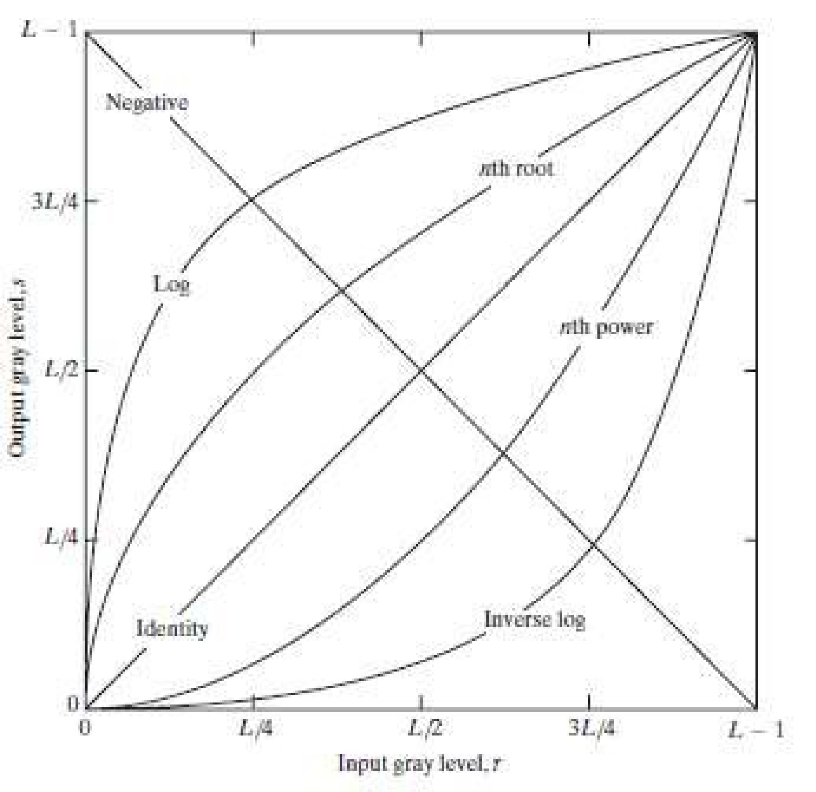
  - S = ?



Some Point Transforms

- **Image Negatives**

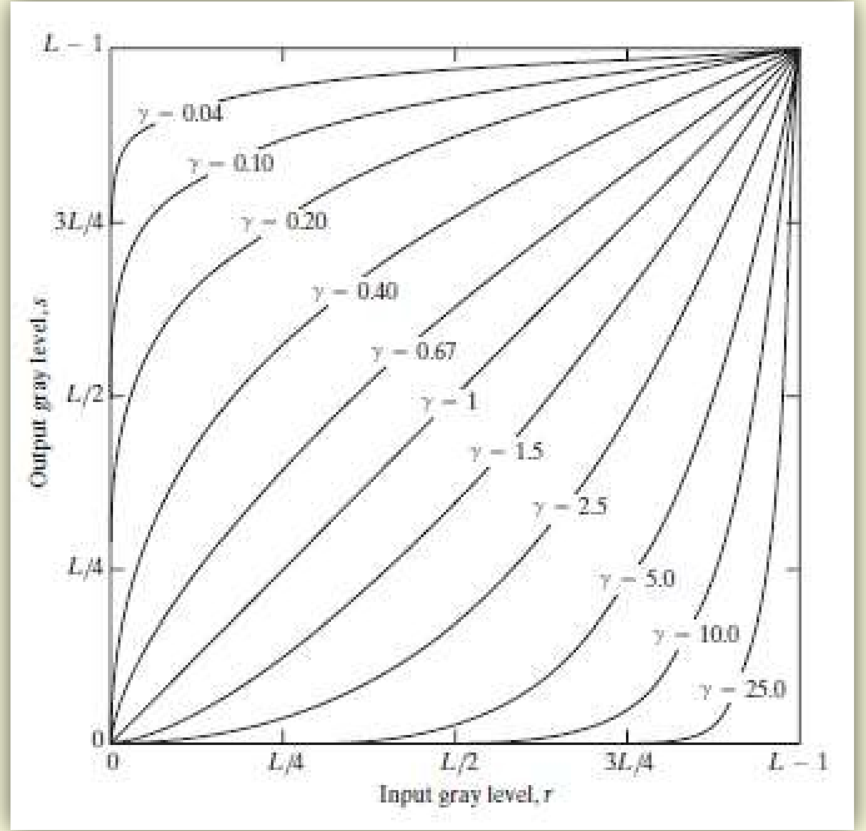
  $s = L - 1 - r$



Some Point Transforms

- **Power-Law Transformations**

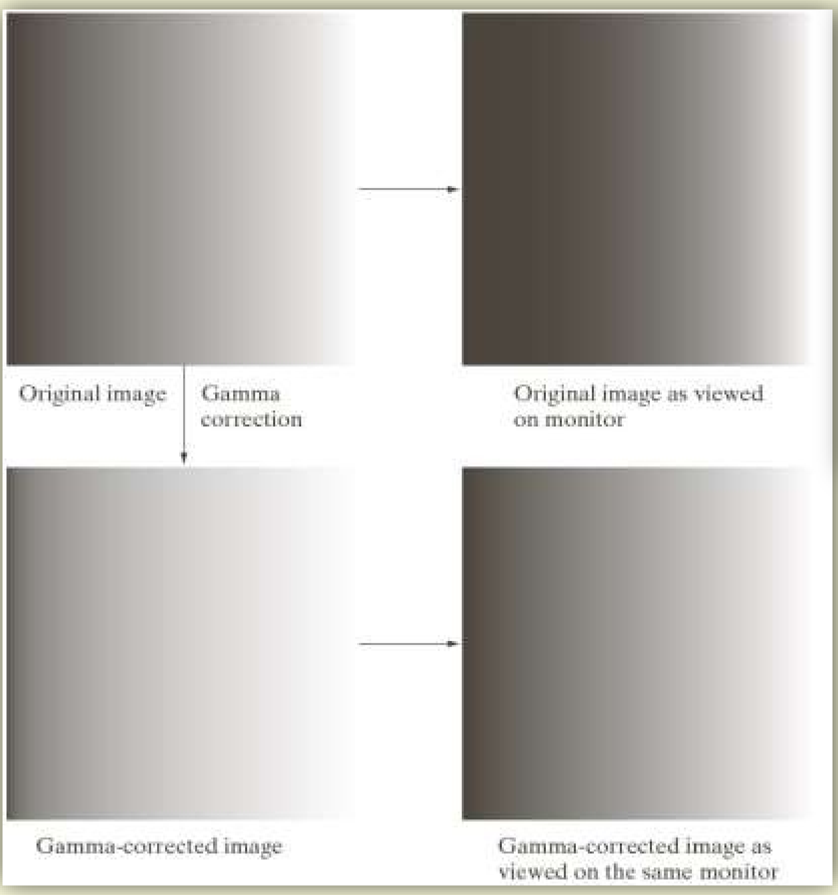
$$s = c\, r^{\gamma}$$



Some Point Transforms

- **Gamma Correction**

$$s = c\, r^{\gamma}$$



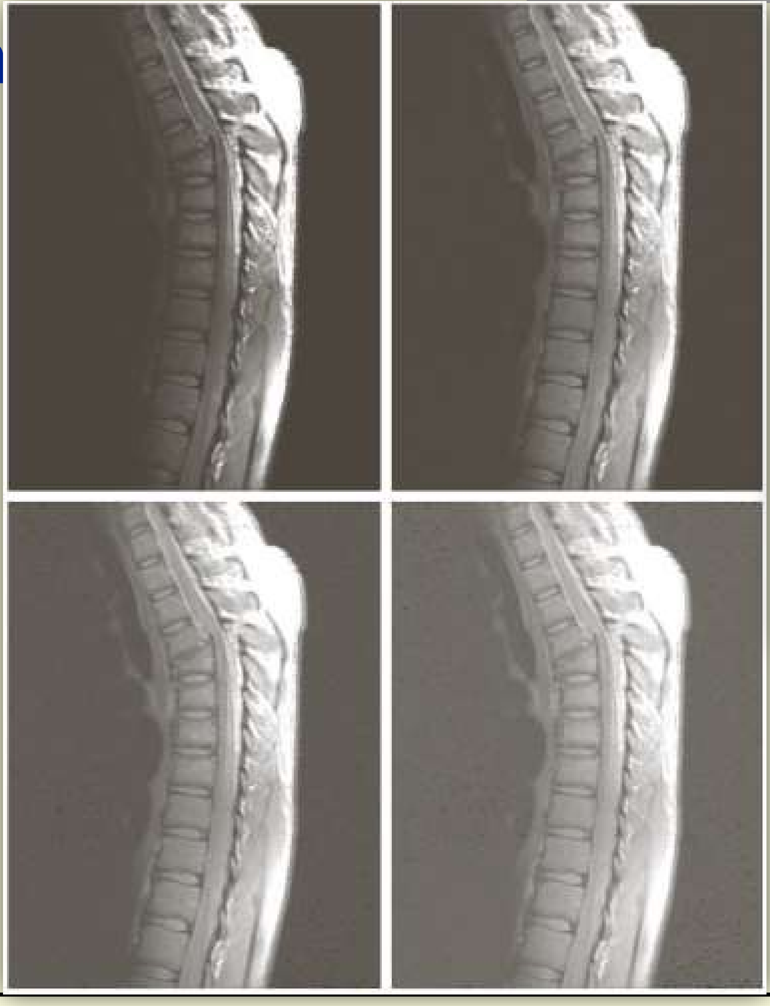
a b
c d

FIGURE 3.7 (a) Intensity ramp image. (b) Image as viewed on a simulated monitor with a gamma of 2.5. (c) Gamma-corrected image. (d) Corrected image as viewed on the same monitor. Compare (d) and (a).

Some Point Transforms

- **Gamma Correction**

$$s = c\, r^{\gamma}$$



a b
c d

FIGURE 3.8 (a) Magnetic resonance image (MRI) of a fractured human spine. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 0.6, 0.4,$ and 0.3, respectively. (Original image courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

Some Point Transforms

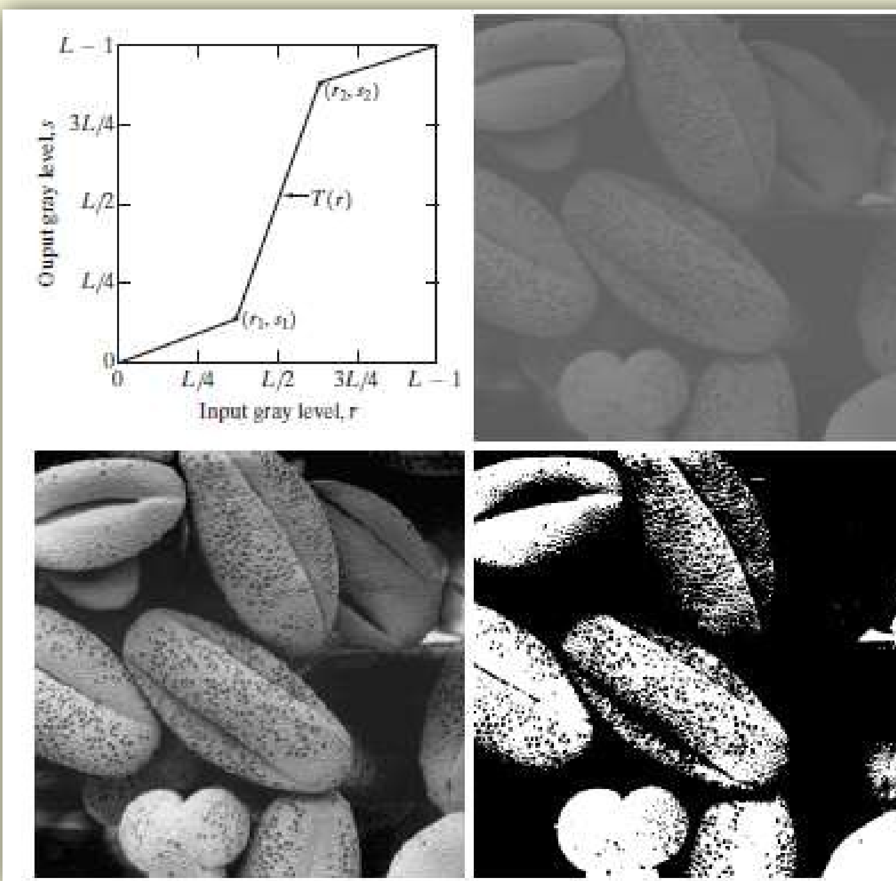
- **Power-Law Transformations**

a b
c d

FIGURE 3.9
(a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0, 4.0,$ and 5.0, respectively. (Original image for this example courtesy of NASA.)

Some Point Transforms

- **Piecewise-Linear Transformation Functions**

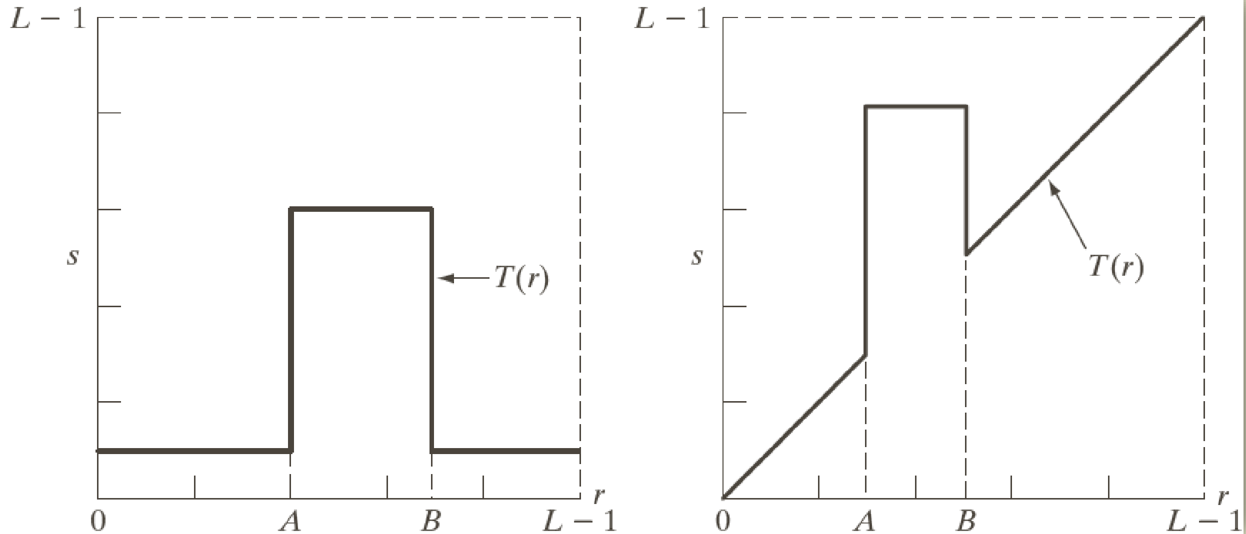


a b
c d

FIGURE 3.10 Contrast stretching. (a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

Some Point Transforms

- **Transformation Functions for Intensity Range**

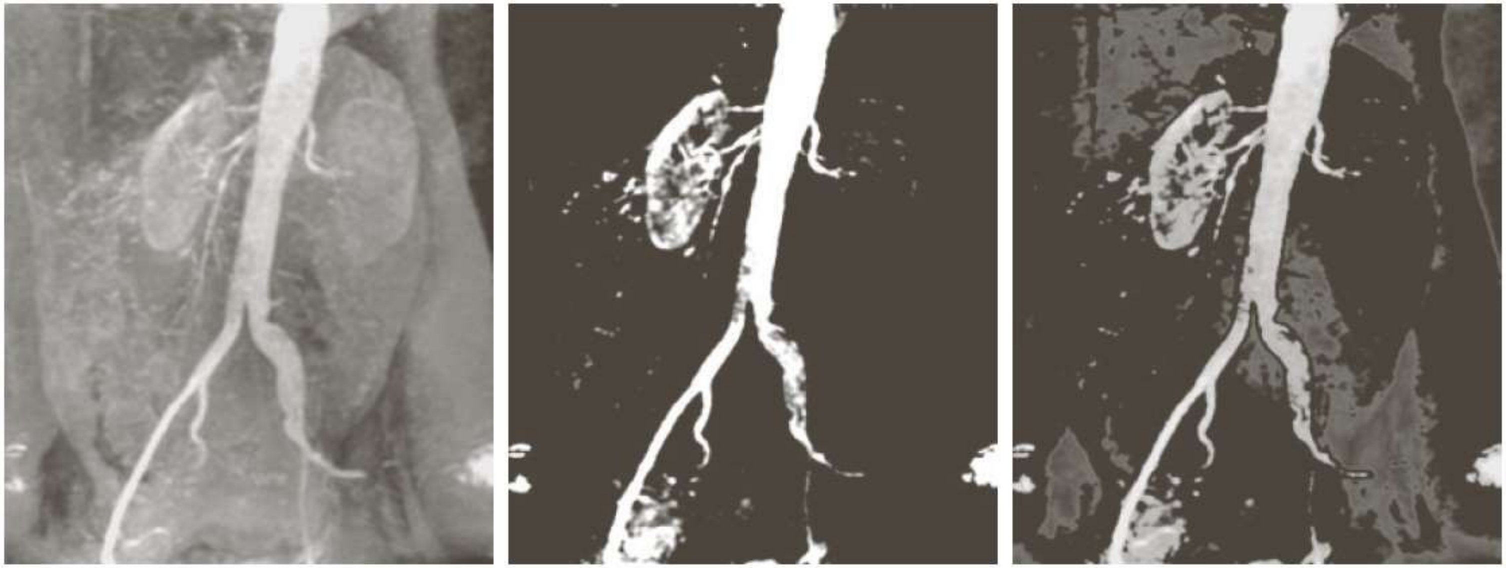


a b

FIGURE 3.11 (a) This transformation highlights intensity range $[A, B]$ and reduces all other intensities to a lower level. (b) This transformation highlights range $[A, B]$ and preserves all other intensity levels.

Some Point Transforms

- **Transformation Functions for Intensity Range**



a b c

FIGURE 3.12 (a) Aortic angiogram. (b) Result of using a slicing transformation of the type illustrated in Fig. 3.11(a), with the range of intensities of interest selected in the upper end of the gray scale. (c) Result of using the transformation in Fig. 3.11(b), with the selected area set to black, so that grays in the area of the blood vessels and kidneys were preserved. (Original image courtesy of Dr. Thomas R. Gest, University of Michigan Medical School.)

Contrast Stretching

- **Contrast Stretching:**

Contrast stretching is another way to enhance the image contrast by stretching the gray levels in the image over the dynamic range of the gray scale.

Contrast Stretching

1. **Contrast stretching.**

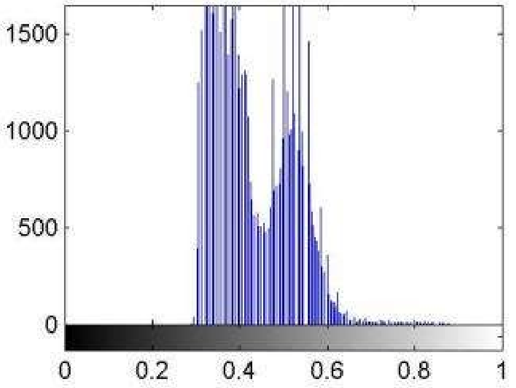
$$b[m,n] = \left(2^B - 1\right) \bullet \frac{a[m,n] - \text{minimum}}{\text{maximum} - \text{minimum}}$$

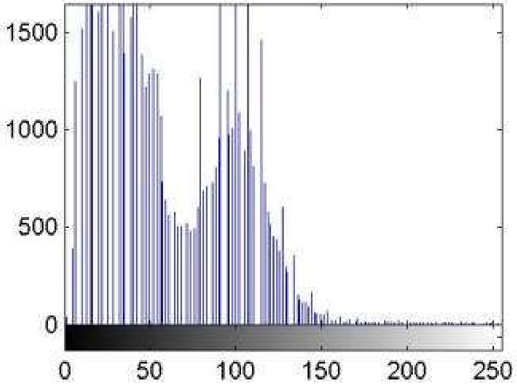
or

$$b[m,n] = \begin{cases} 0 & a[m,n] \le p_{\text{low}}\% \\ \left(2^B - 1\right) \bullet \dfrac{a[m,n] - p_{\text{low}}\%}{p_{\text{high}}\% - p_{\text{low}}\%} & p_{\text{low}}\% < a[m,n] < p_{\text{high}}\% \\ \left(2^B - 1\right) & a[m,n] \ge p_{\text{high}}\% \end{cases}$$

Contrast Stretching

1. **Contrast stretching.**



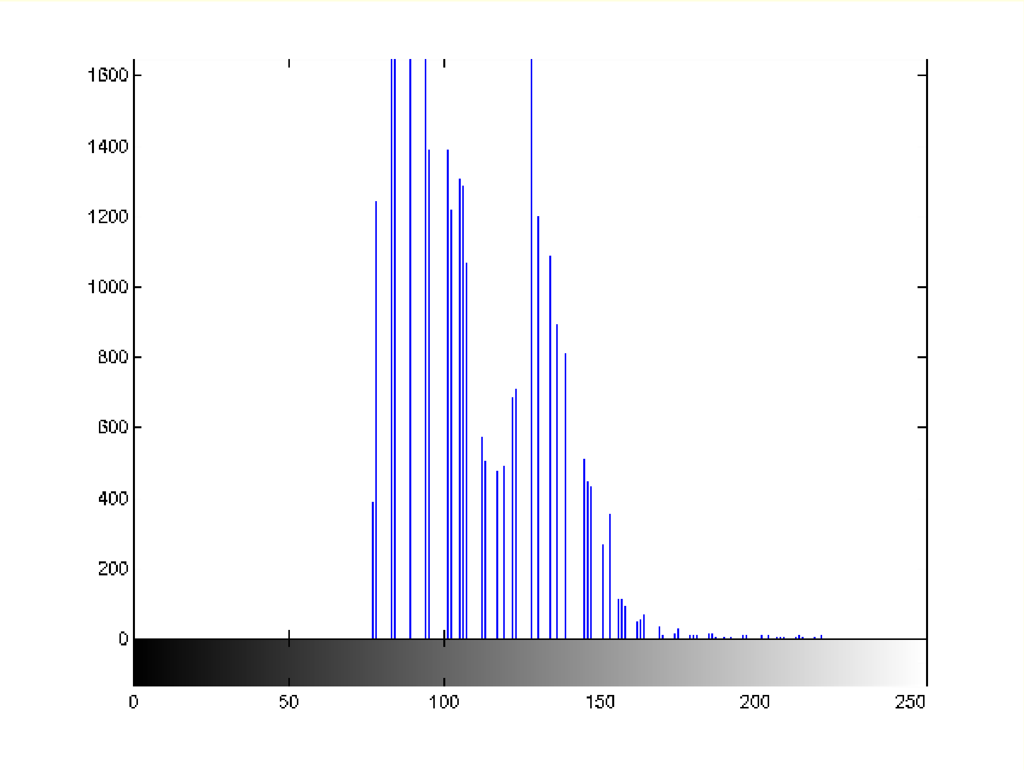


Histogram-based Operations

- What is image histogram?
  - The *histogram* of an image or a region, $h(r)$ is a function whose *domain* is the gray levels and its *codomain* is the frequency of occurrence of those gray levels in the image or the region.
  - The histogram is computed by counting the number of times that each brightness (gray level) occurs in the image or the region.
  - That is: $h(r) = n_r$ = no. of pixels with intensity g
  - The ***normalized histogram*** *is* $h(r) = n_r/n$*; where* n *is the total number of pixels in the image*

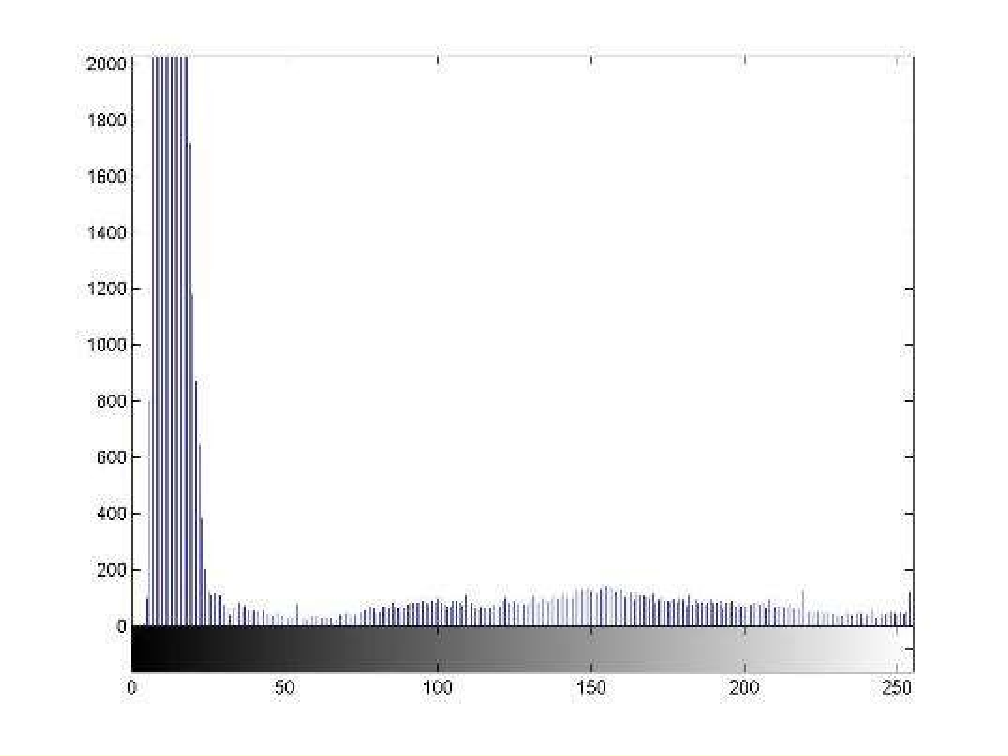
Histogram-based Operations

- Examples of image histograms:



Histogram-based Operations

- Examples of image histograms:



Histogram-based Operations

- Examples of image histograms:

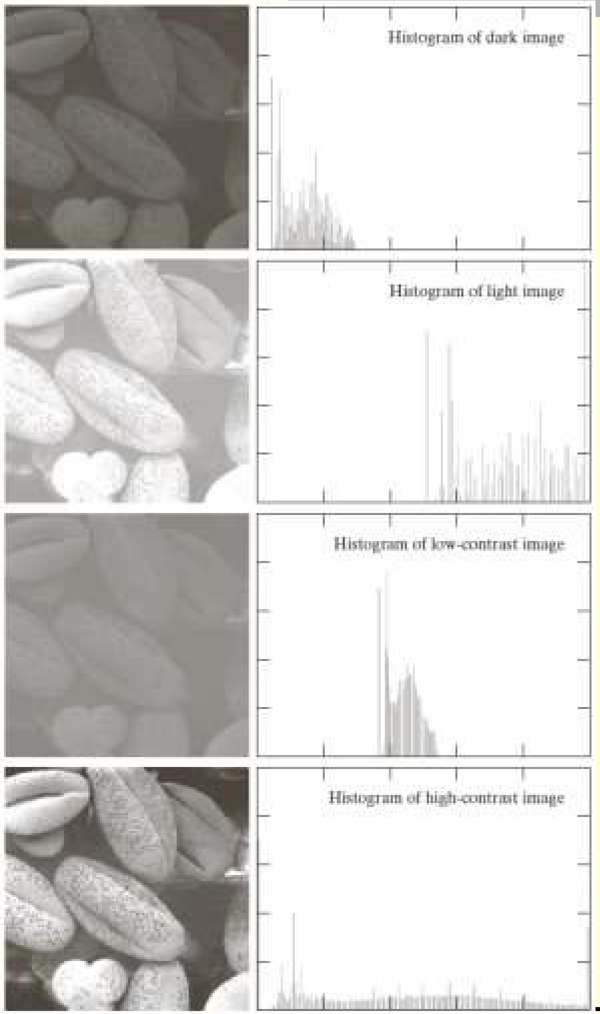


FIGURE 3.16 Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms.

Histogram-based Operations

1. **Histogram Equalization:**

   Histogram equalization is a way to enhance the image contrast by extending the image intensity over the full dynamic range of the gray scale.

Histogram-based Operations

1. **(Continuous) Histogram Equalization.**

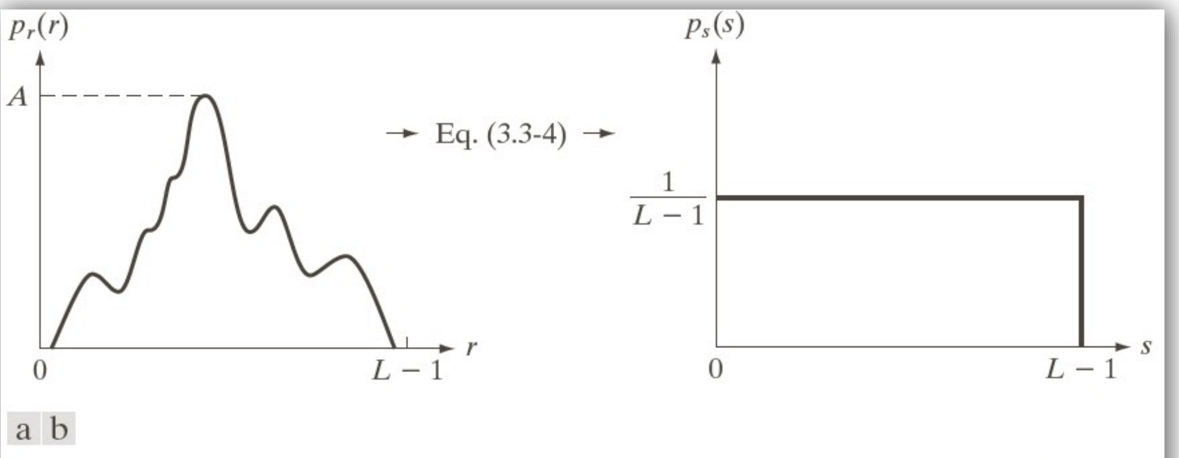


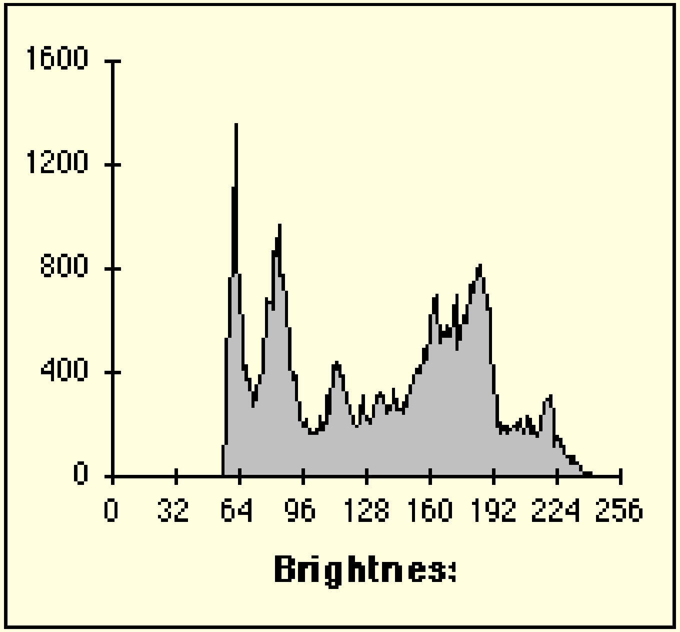
FIGURE 3.18 (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels, r. The resulting intensities, s, have a uniform PDF, independently of the form of the PDF of the r's.

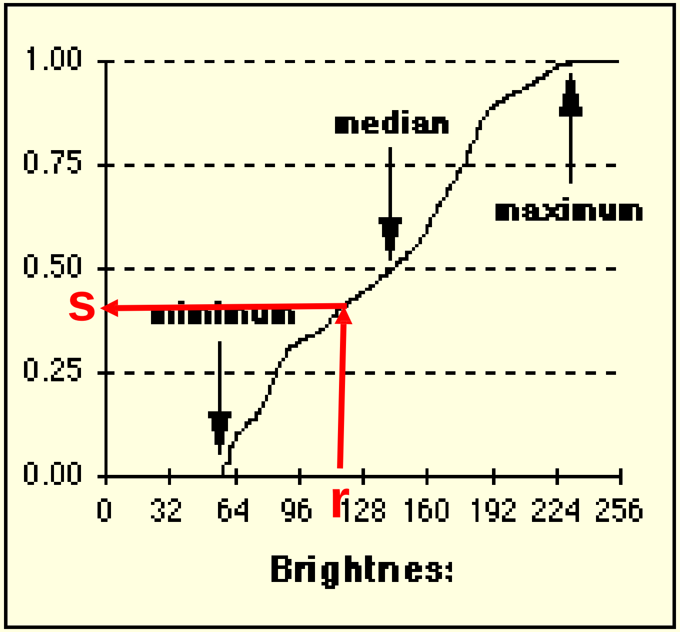
$$s = T(r) = \int_0^r p_r(w)\,dw$$

$$p_s(s) = \begin{cases} 1 & \text{for } 0 \le s \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Histogram-based Operations

1. **(Discrete) Histogram Equalization.**





$$s = P(r) = (255) \cdot \frac{1}{n} \sum_{g=0}^{r} h(g)$$

Histogram-based Operations

1. **Histogram Equalization.**

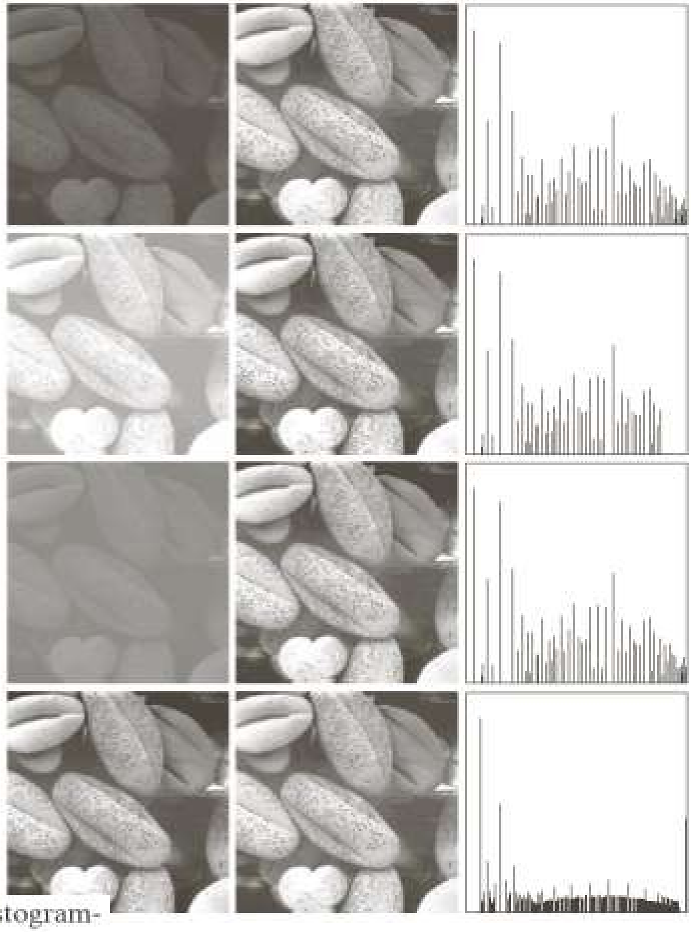
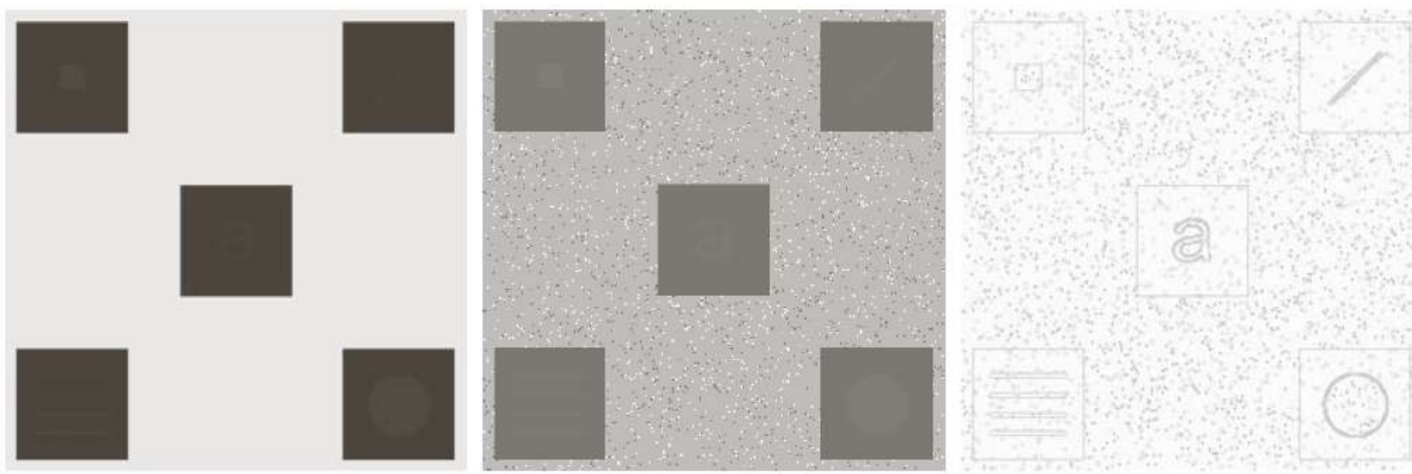


FIGURE 3.20 Left column: images from Fig. 3.16. Center column: corresponding histogram-equalized images. Right column: histograms of the images in the center column.

Histogram-based Operations

1. **Local Histogram Equalization.**

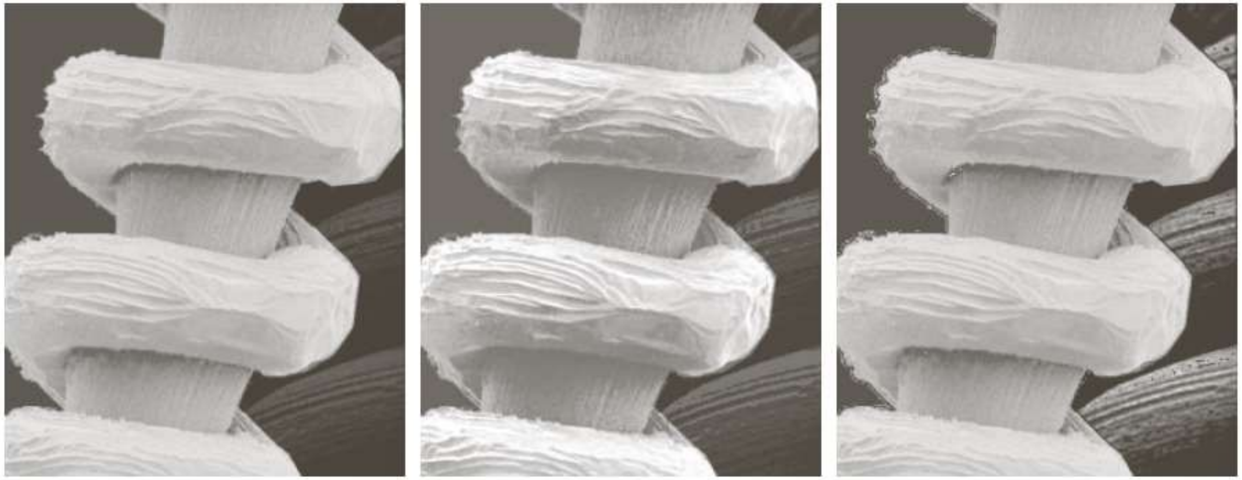


a b c

FIGURE 3.26 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization applied to (a), using a neighborhood of size 3×3.

Histogram-based Operations

1. **Local Histogram Equalization.**

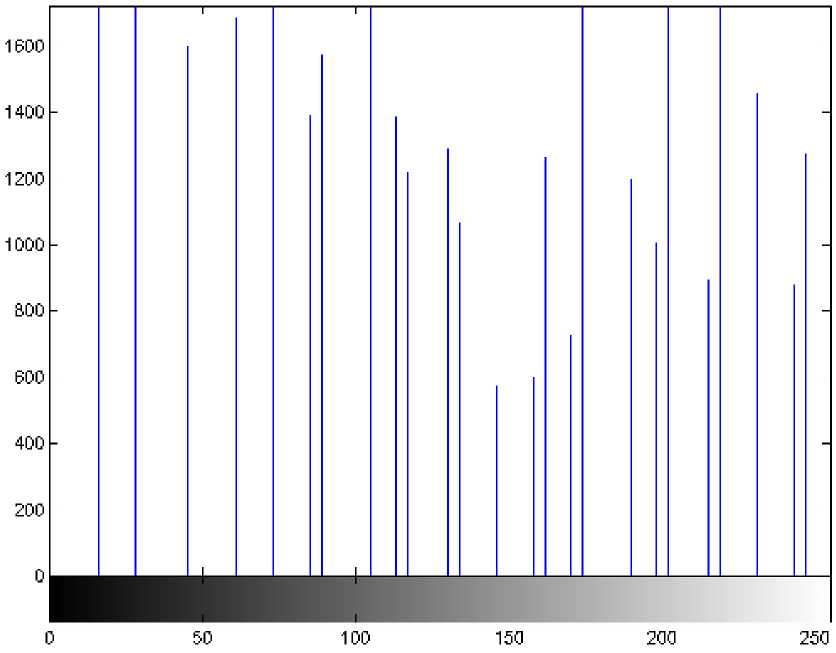


a b c

FIGURE 3.27 (a) SEM image of a tungsten filament magnified approximately 130×. (b) Result of global histogram equalization. (c) Image enhanced using local histogram statistics. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

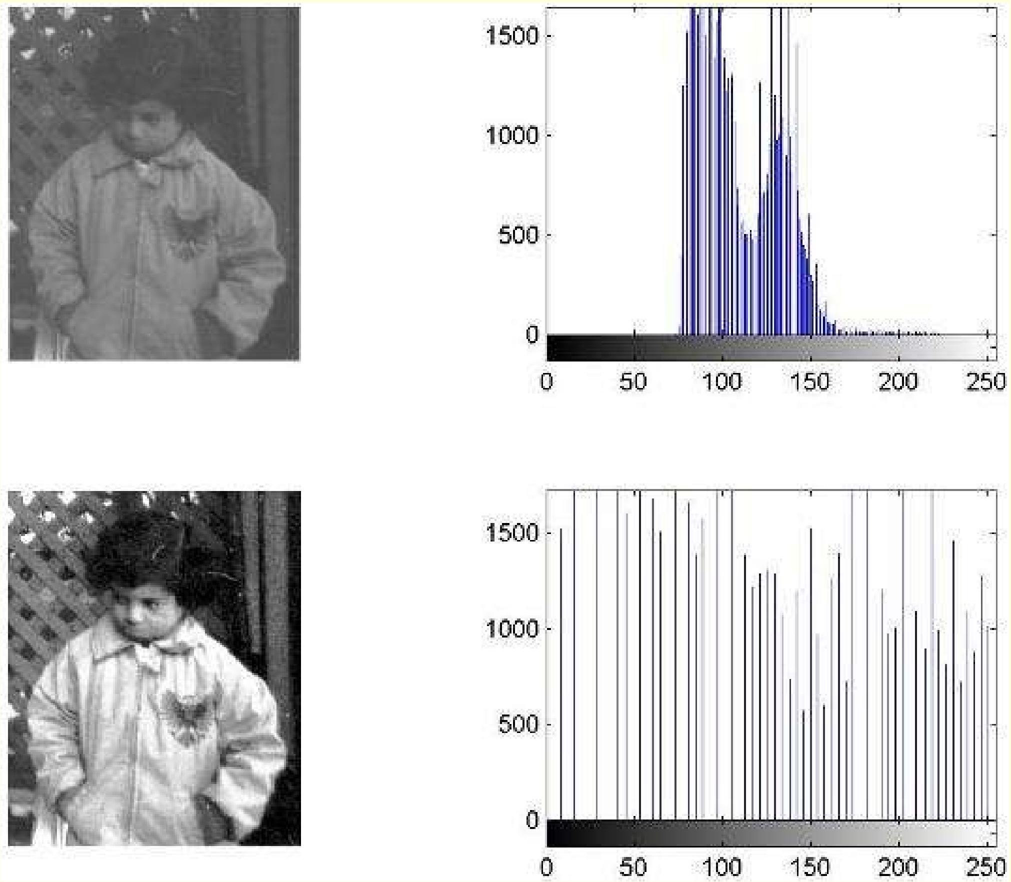
Histogram-based Operations

1. **Histogram Equalization.**



Histogram-based Operations

- Histogram equalization (Matlab)



Histogram-based Operations

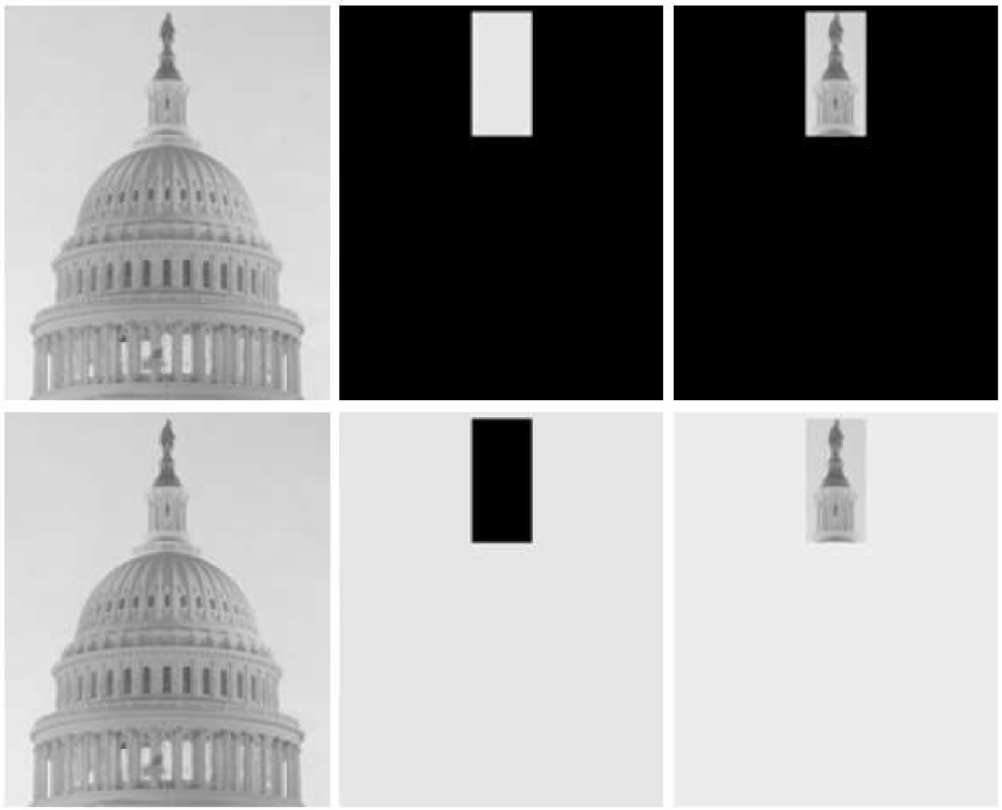
Original

Histogram Equalization

Contrast stretching

Arithmetic/Logic Operations

Logical Operators



a b c
d e f

FIGURE 3.27 (a) Original image. (b) AND image mask. (c) Result of the AND operation on images (a) and (b). (d) Original image. (e) OR image mask. (f) Result of operation OR on images (d) and (e).

Arithmetic/Logic Operations

Image Averaging

- Consider a noisy image g(x, y) formed by the addition of noise η(x, y) to an original image f(x, y); That is:

$$g(x, y) = f(x, y) + \eta(x, y)$$

- If the noise is uncorrelated, we can remove the noise by averaging K noisy images

$$\bar{g}(x, y) = \frac{1}{K} \sum_{i=1}^{K} g_i(x, y)$$

where

$$E\{\bar{g}(x, y)\} = f(x, y)$$

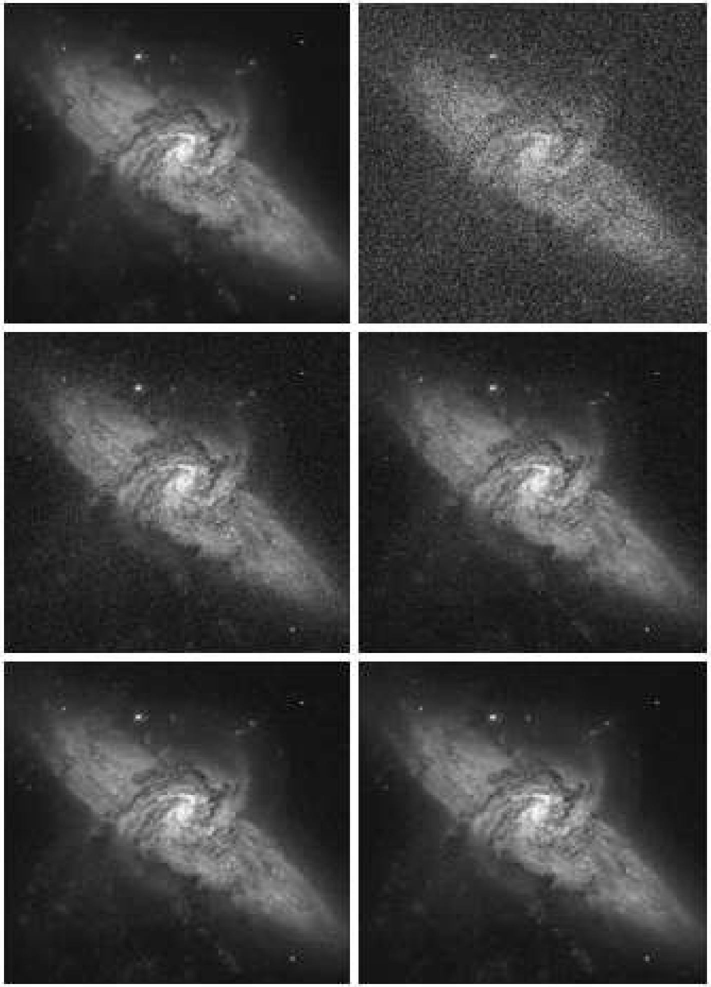
Arithmetic/Logic Operations

Image Averaging

- where E{g(x, y)} is the expected of g(x, y) at coordinates (x, y).
- The standard deviation at any point in the average image is

$$\sigma_{\bar{g}(x,y)} = \frac{1}{\sqrt{K}}\sigma_{\eta(x,y)}.$$

- As K increase the variability (noise) in the pixel value decrease



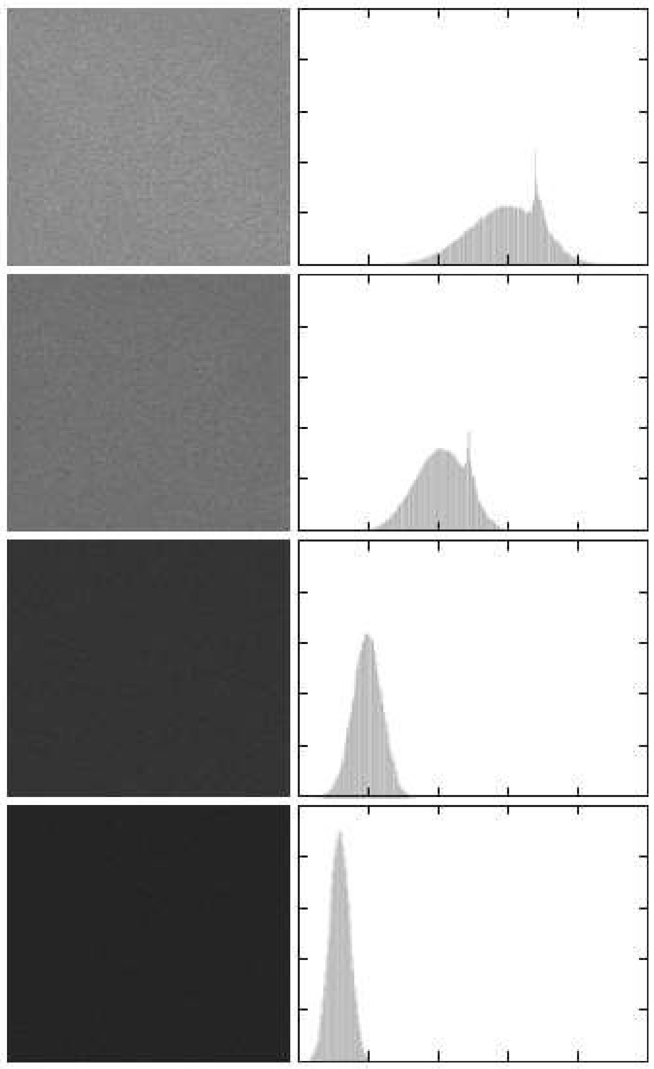
a b
c d
e f

FIGURE 3.30 (a) Image of Galaxy Pair NGC 3314. (b) Image corrupted by additive Gaussian noise with zero mean and a standard deviation of 64 gray levels. (c)–(f) Results of averaging $K = 8, 16, 64$, and 128 noisy images. (Original image courtesy of NASA.)

Arithmetic/Logic Operations

Image Averaging

- where E{g(x, y)} is the expected value of g(x, y) at coordinates (x, y).
- The standard deviation at any point in the average image is



a b

FIGURE 3.31 (a) From top to bottom: Difference images between Fig. 3.30(a) and the four images in Figs. 3.30(c) through (f), respectively. (b) Corresponding histograms.

Local Operations

Convolution and

Image Filtering

Spatial Filtering: Foundations

- **Linear Filtering:**

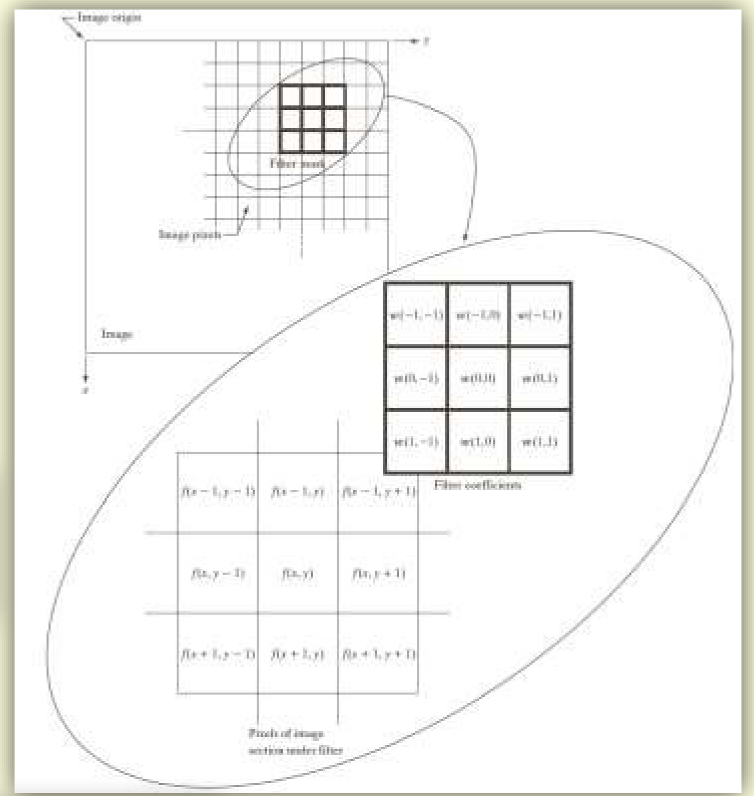
  Using *convolution*

  Filter*, *mask*, *filter

  Mask*, *kernel*, *window

$$g(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x + s, y + t)$$

a=(m-1)/2 and b=(n-1)/2



$$\begin{aligned} R = {} & w(-1,-1)f(x-1, y-1) + w(-1,0)f(x-1, y) + \cdots \\ & + w(0,0)f(x, y) + \cdots + w(1,0)f(x+1, y) + w(1,1)f(x+1, y+1), \end{aligned}$$

Spatial Filtering : Smoothing

- **Smoothing/Average Filtering:**
  Also called *lowpass* filter

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \qquad \frac{1}{16} \times \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

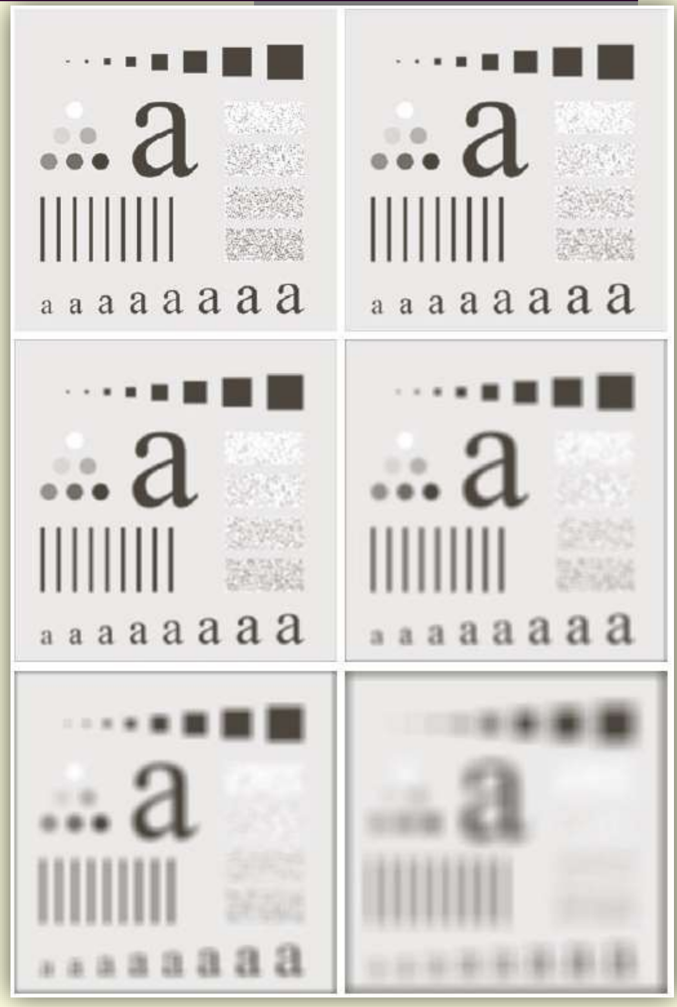
Average　　　　Weighted average

a b

FIGURE 3.32 Two 3×3 smoothing (averaging) filter masks. The constant multiplier in front of each mask is equal to 1 divided by the sum of the values of its coefficients, as is required to compute an average.

Spatial Filtering: Smoothing

- **Smoothing/Average Filtering:**

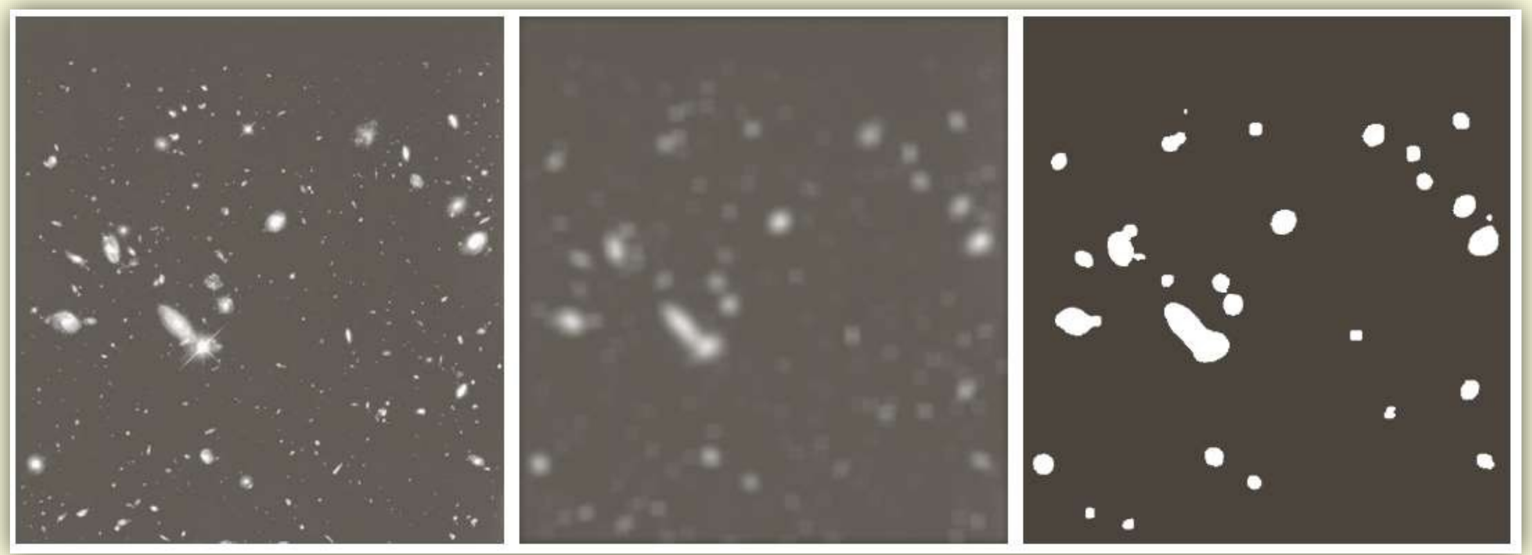


a b
c d
e f

FIGURE 3.33 (a) Original image, of size 500 × 500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $m = 3, 5, 9, 15$, and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their intensity levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50 × 120 pixels.

Spatial Filtering: Smoothing

- **Smoothing/Average Filtering:**

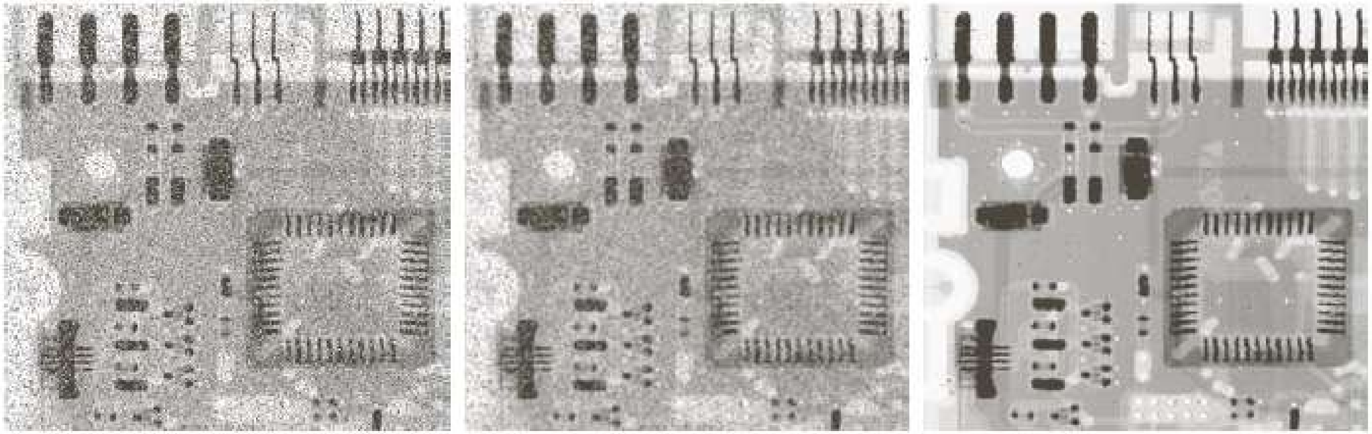


a b c

FIGURE 3.34 (a) Image of size 528 × 485 pixels from the Hubble Space Telescope. (b) Image filtered with a 15 × 15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

Spatial Filtering: Median

- **Order-Statistics Filtering:**
  - **Median (Nonlinear) filter**



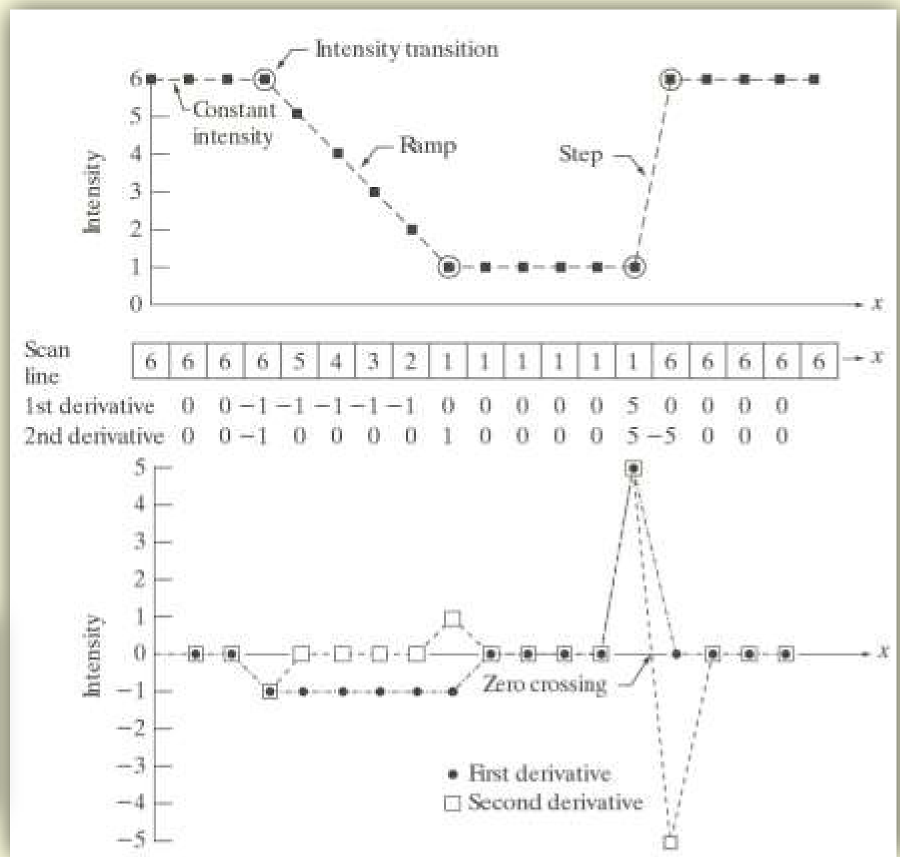
a b c

FIGURE 3.35 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Spatial Filtering: Sharpening

- **Foundation: Image Derivative**

$$\frac{\partial f}{\partial x} = f(x+1) - f(x).$$



a
b
c

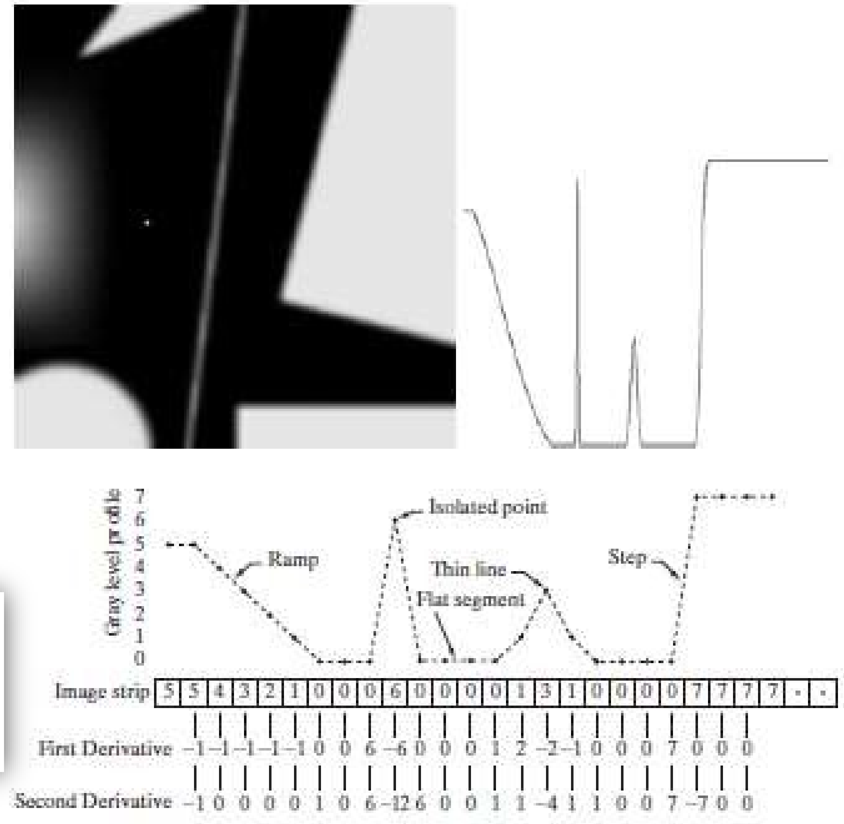
FIGURE 3.36 Illustration of the first and second derivatives of a 1-D digital function representing a section of a horizontal intensity profile from an image. In (a) and (c) data points are joined by dashed lines as a visualization aid.

Spatial Filtering: Sharpening

- **Foundation: Image Derivative**

a b
c

FIGURE 3.38 (a) A simple image. (b) 1-D horizontal gray-level profile along the center of the image and including the isolated noise point. (c) Simplified profile (the points are joined by dashed lines to simplify interpretation).



$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x).$$

Spatial Filtering: Sharpening

- **Sharpening using the Laplacian filter**

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}.$$

$$\frac{\partial^2 f}{\partial^2 x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial^2 y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

$$\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] - 4f(x, y).$$

$$g(x, y) = \begin{cases} f(x, y) - \nabla^2 f(x, y) & \text{if the center coefficient of the Laplacian mask is negative} \\ f(x, y) + \nabla^2 f(x, y) & \text{if the center coefficient of the Laplacian mask is positive.} \end{cases} \quad (3.7\text{-}5)$$

Spatial Filtering: Sharpening

- **The Laplacian Mask**

(a)

0	1	0
1	−4	1
0	1	0

(b)

1	1	1
1	−8	1
1	1	1

(c)

0	−1	0
−1	4	−1
0	−1	0

(d)

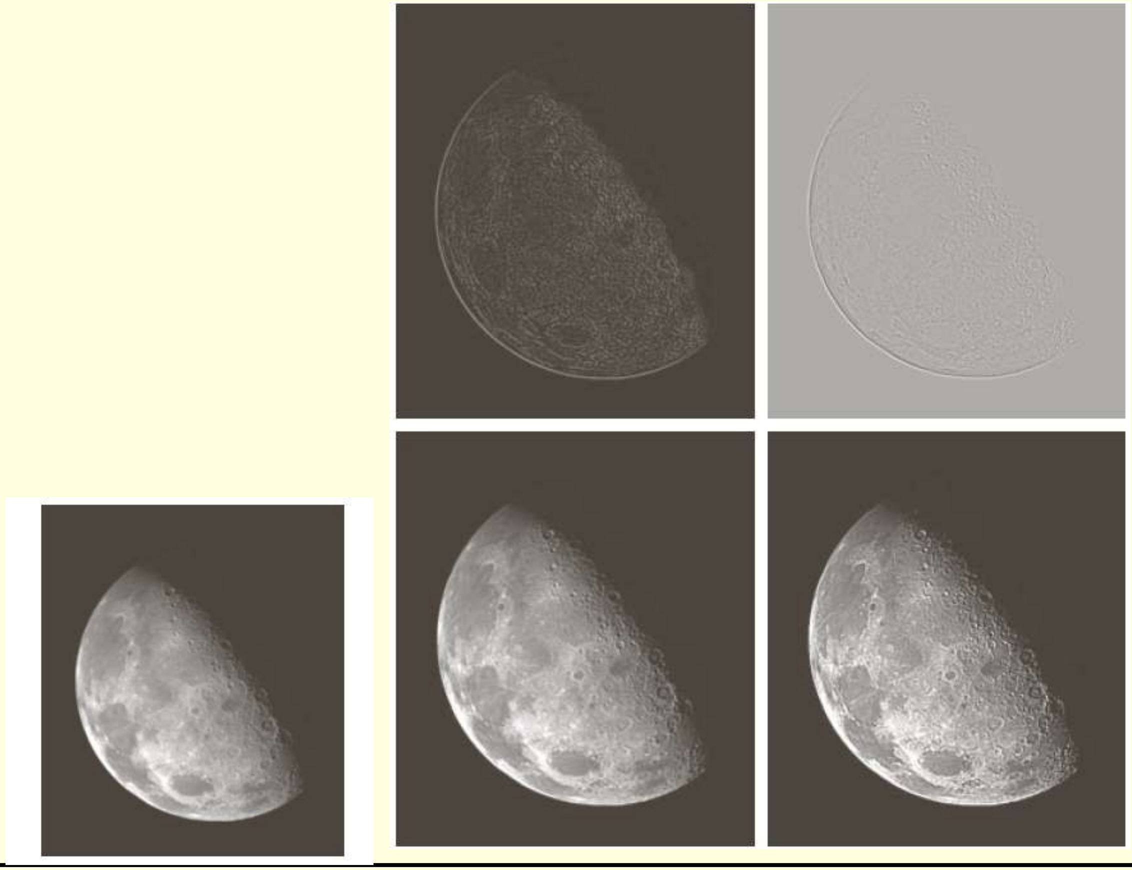
−1	−1	−1
−1	8	−1
−1	−1	−1

a b
c d

FIGURE 3.37
(a) Filter mask used to implement Eq. (3.6-6). (b) Mask used to implement an extension of this equation that includes the diagonal terms. (c) and (d) Two other implementations of the Laplacian found frequently in practice.

Spatial Filtering: Sharpening

- **Sharpening Using The Laplacian Filters**

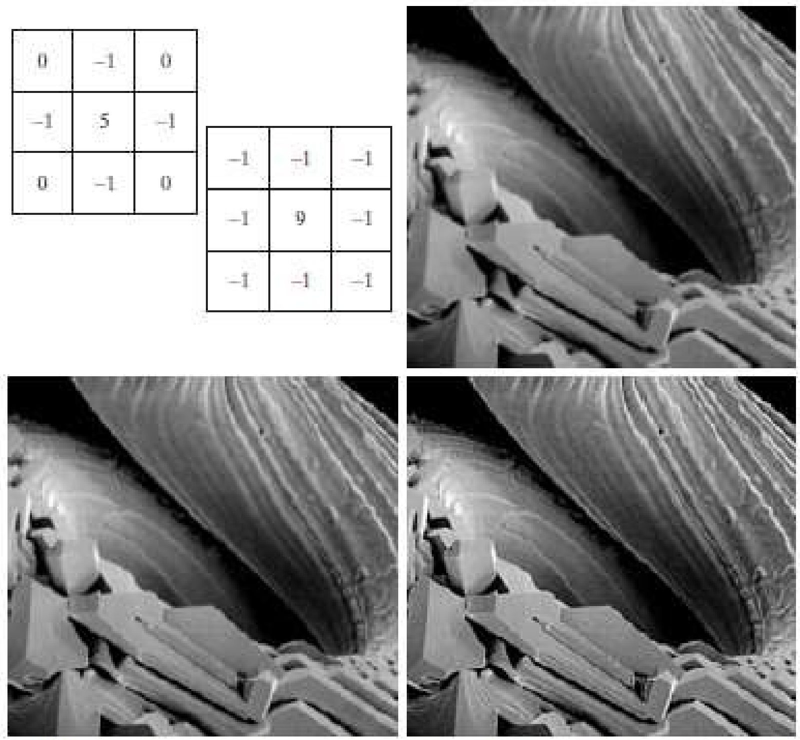


a
b c
d e

FIGURE 3.38 (a) Blurred image of the North Pole of the moon. (b) Laplacian without scaling. (c) Laplacian with scaling. (d) Image sharpened using the mask in Fig. 3.37(a). (e) Result of using the mask in Fig. 3.37(b). (Original image courtesy of NASA.)

Spatial Filtering: Sharpening

- **Sharpening Using The Laplacian Filters**



a b c
d e

FIGURE 3.41 (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Spatial Filtering: Sharpening

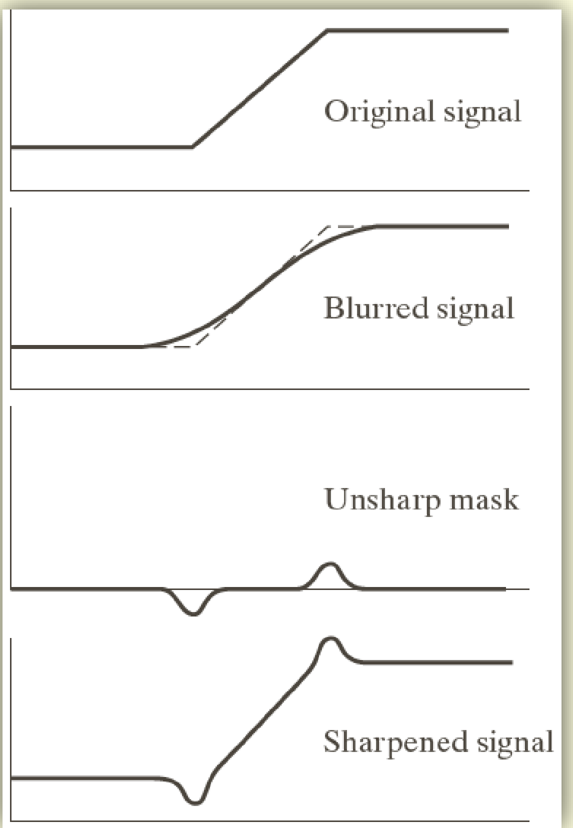
- **Unsharp Masking**

$$f_s(x, y) = f(x, y) - \bar{f}(x, y)$$

Blurred Image

- **High Boost Filters**

$$f_{hb}(x, y) = Af(x, y) - \bar{f}(x, y)$$



a
b
c
d

FIGURE 3.39 1-D illustration of the mechanics of unsharp masking. (a) Original signal. (b) Blurred signal with original shown dashed for reference. (c) Unsharp mask. (d) Sharpened signal, obtained by adding (c) to (a).

Spatial Filtering: Sharpening

- **High-Boost Filters**

a

0	–1	0
–1	$A + 4$	–1
0	–1	0

b

–1	–1	–1
–1	$A + 8$	–1
–1	–1	–1

FIGURE 3.42 The high-boost filtering technique can be implemented with either one of these masks, with $A \geq 1$.

Spatial Filtering: Sharpening

- **High-Boost Filters**

a b
c d

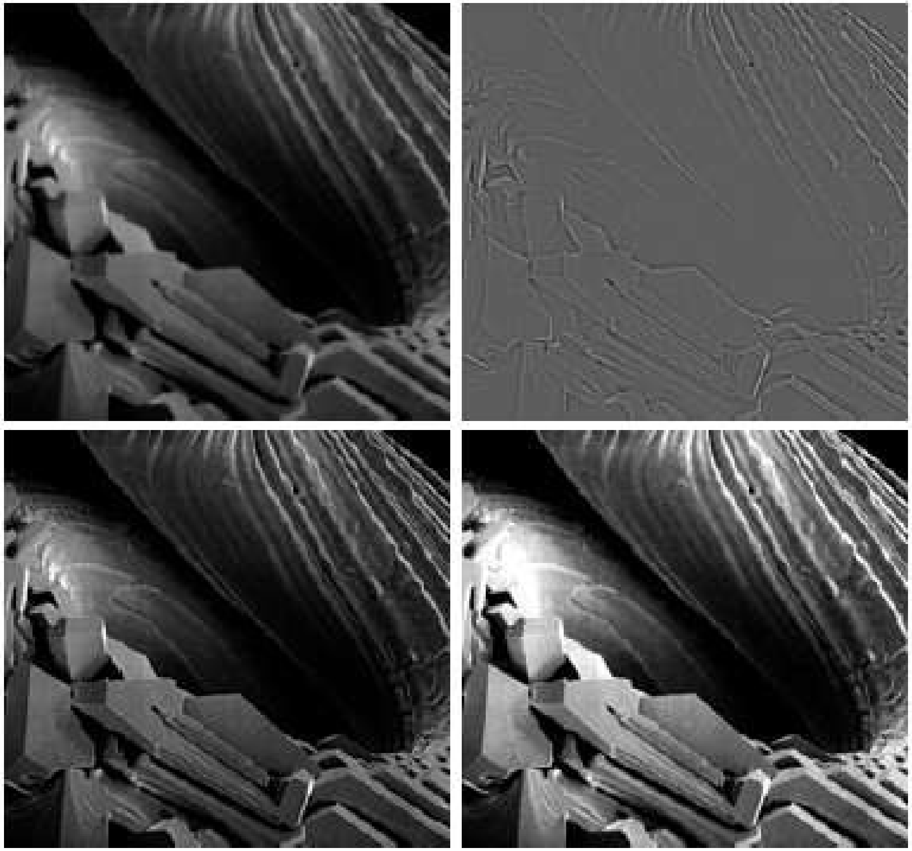


FIGURE 3.43 (a) Same as Fig. 3.41(c), but darker. (a) Laplacian of (a) computed with the mask in Fig. 3.42(b) using $A = 0$. (c) Laplacian enhanced image using the mask in Fig. 3.42(b) with $A = 1$. (d) Same as (c), but using $A = 1.7$.

Spatial Filtering: Sharpening

- **High-Boost Filters**

FIGURE 3.40 (a) Original image. (b) Result of blurring with a Gaussian filter. (c) Unsharp mask. (d) Result of using unsharp masking. (e) Result of using highboost filtering.

Spatial Filtering: Edge Detection

- **Robert & Sobel Filters**

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

−1	0
0	1

0	−1
1	0

−1	−2	−1
0	0	0
1	2	1

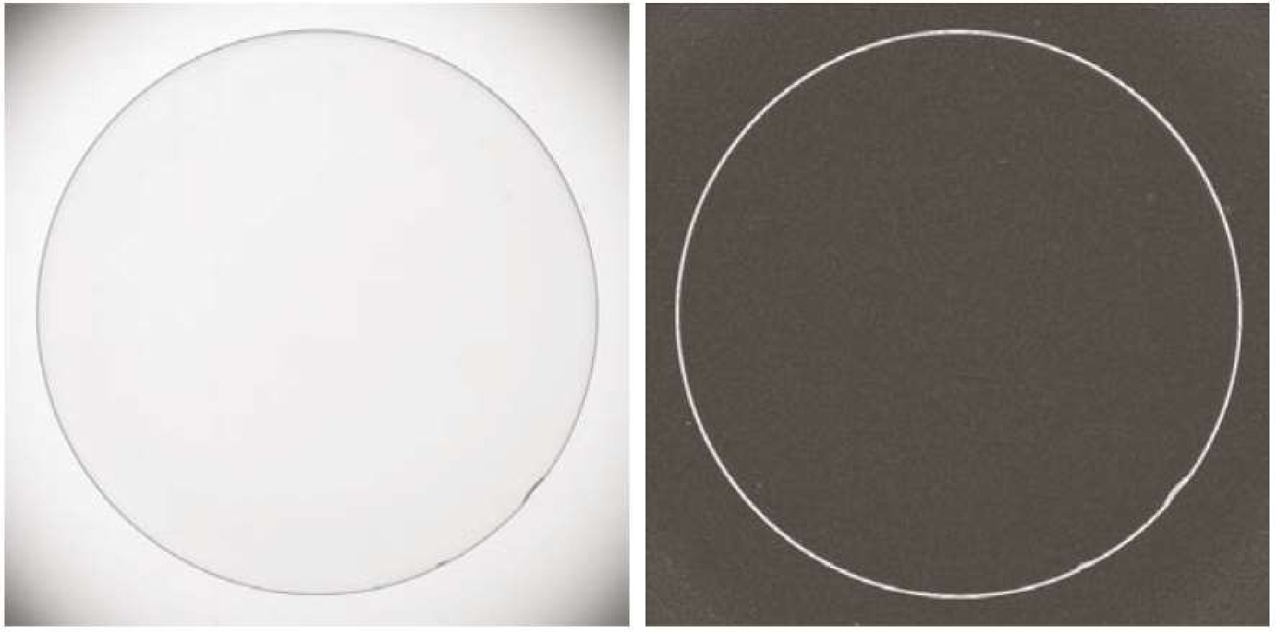
−1	0	1
−2	0	2
−1	0	1

a	
b	c
d	e

FIGURE 3.41 A 3 × 3 region of an image (the zs are intensity values). (b)–(c) Roberts cross gradient operators. (d)–(e) Sobel operators. All the mask coefficients sum to zero, as expected of a derivative operator.

Spatial Filtering: Edge Detection

- **Robert & Sobel Filters**



a b

FIGURE 3.42 (a) Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock). (b) Sobel gradient. (Original image courtesy of Pete Sites, Perceptics Corporation.)

HW2

- **3.2, 3.4, 3.7, 3.8, and 3.17**