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Bachelor of Information Technology (BIT)

BSM 101

Exercise on Matrix Operations and Solving the System of Equations

In [1-7] perform the operation as indicated:

1. If $A = \begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 5 \\ 7 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 7 \\ 1 & 5 \end{bmatrix}$,
   (a) Find $|A|, |B|$ and $|C|$
   (b) Show that $AB = AC$.
   (c) Show that $|AB| = |A||B|$

2. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, with I is the 2×2 identity matrix, and $A^2 = A \times A$, then show that $A^2 - 3I = 2A$,

3. Given that A $= \begin{bmatrix} 2 & -3 \\ 5 & 4 \end{bmatrix}$ and B $= \begin{bmatrix} -1 & 1 \\ 5 & 7 \end{bmatrix}$, verify that AB $\neq$ BA.

4. A $= \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$ and B $= \begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}$,
   (a) Find $|A|$ and $|B|$
   (b) Verify that AB $\neq$ BA.

5. If A $= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & -2 \end{bmatrix}$, B $= \begin{bmatrix} 1 & 1 & -1 \\ 2 & 0 & 3 \\ 3 & -1 & 2 \end{bmatrix}$ and C $= \begin{bmatrix} 1 & 3 \\ 0 & 2 \\ -1 & 4 \end{bmatrix}$. Show that A(BC) = (AB)C.

6. Find the product of
   a. $\begin{bmatrix} 3 & 4 \\ 4 & -2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 3 & 2 \\ 1 & 2 & 3 & 0 \end{bmatrix}$.
   b. $\begin{bmatrix} 1 & 2 \\ 0 & 3 \\ 5 & -1 \end{bmatrix} \begin{bmatrix} 4 & -2 & 1 \\ 2 & -3 & 0 \end{bmatrix}$.

7. Find the inverse matrix for each matrix that has exist an inverse.
   a. $\begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 5 & 6 \\ -1 & 3 & 3 \end{bmatrix}$ c. $\begin{bmatrix} 1 & 3 & 5 \\ -1 & -1 & 2 \\ 1 & 5 & 12 \end{bmatrix}$

d. $\begin{bmatrix} 1 & -1 & 4 \\ -1 & 0 & -2 \\ -1 & -3 & 4 \end{bmatrix}$ e. $\begin{bmatrix} 1 & 2 & 4 \\ 1 & -1 & -3 \\ 2 & 1 & 1 \end{bmatrix}$ f. $\begin{bmatrix} 3 & 4 & -1 \\ 4 & 2 & 2 \\ 2 & 6 & -4 \end{bmatrix}$

8. Given that $A = \begin{bmatrix} 1 & -2 & 7 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 11 \\ 5 \end{bmatrix}$ and $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$. If $X = AB$, find x_1, x_2 and x_3.

9. Solve the following systems of equations by using inverse of matrix and Cramer's rule.

   (a) $x + 3y = 4$
   $2x + 5y = 9$

   (b) $x - 3y - 5$
   $3x - y = -1$

   (c) $3x - \frac{4}{y} - 10$
   $-2x + \frac{3}{y} = -1$

   (d) $x - 2y + 3z = -1$
   $x - y + 2z = 0$
   $2x + 2y + z = -3$

   (e) $2x - 3y - z = 4$
   $x - 2y - z = 1$
   $x - y + 2z = 9$

   (f) $x + 2y - z = -5$
   $2x - y + z = 6$
   $x - y - 3z = -3$

10. Use row operations on augmented matrices to solve the given systems of linear equations.

    a. $\begin{cases} x + y - z = 0 \\ x + 2y + 3z = -5 \\ 2x - y - 13z = 17 \end{cases}$

    b. $\begin{cases} x + 2y - z = 3 \\ 2x + 5y - 2z = 7 \\ -x + y + 5z = -12 \end{cases}$

    c. $\begin{cases} 2x - 6y - 12z = 6 \\ 3x - 10y - 20z = 5 \\ 2x - 17z = -4 \end{cases}$

    d. $\begin{cases} -3x + 6y - 9z = 3 \\ x - y - 2z = 0 \\ 5x + 5y - 7z = 63 \end{cases}$

11. Solve the following system of equation by using Gaussian elimination method.

(a) $2x - 2y + z = 3$
$3x + y - z = 7$
$x - 3y + 2z = 0$

(b) $3x + y - 2z = 2$
$x - 2y + z = 3$
$2x - y - 3z = 3$

(c) $2x_1 - 4x_2 + x_3 = -4$
$4x_1 - 8x_2 + 7x_3 = 2$
$-2x_1 + 4x_2 - 3x_3 = 5$

(d) $2x_1 - 4x_2 - x_3 = -8$
$4x_1 - 8x_2 + 3x_3 = 4$
$-2x_1 + 4x_2 + x_3 = 11$

12. Use inverse matrices to find the solution of the systems of equations.

a. $\begin{cases} x + y + z = 3 \\ 2x + y + z = 4 \\ 2x + 2y + z = 5 \end{cases}$

b. $\begin{cases} 2x - y - 2z = 2 \\ 3x - y + z = -3 \\ x + y - z = 7 \end{cases}$

c. $\begin{cases} x + y + 2z = 8 \\ 2x + y + z = 7 \\ 2x + 2y + z = 10 \end{cases}$

d. $\begin{cases} x - 2y + z = 0 \\ 2x + y - 2z = 2 \\ 3x + 2y - 3z = 2 \end{cases}$