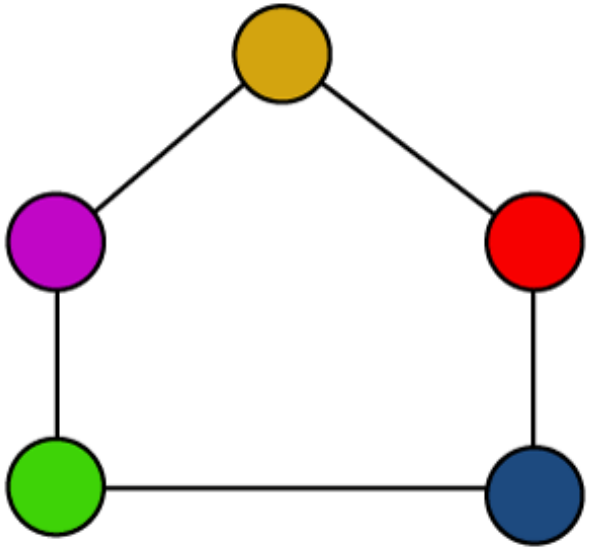
Graph Theory

Graph Coloring and its application

Graph coloring

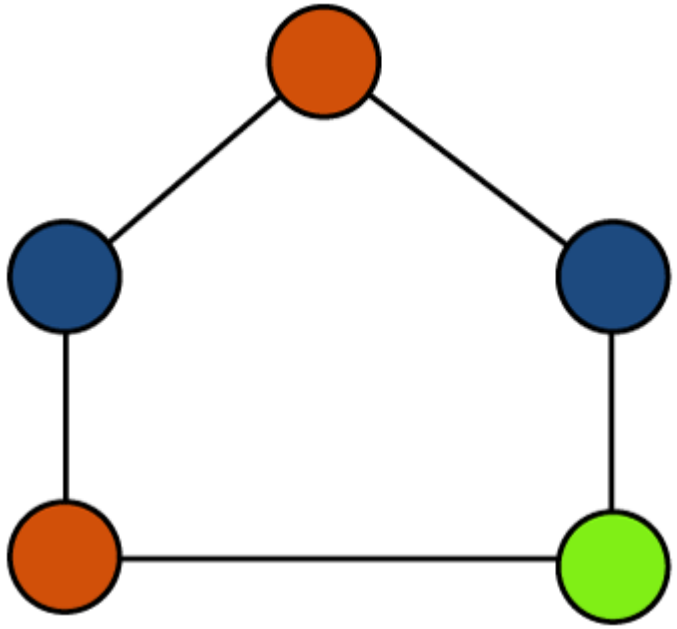
- Graph coloring can be described as a process
  of assigning colors to the vertices of a graph.
- In this, the same color should not be used to
  fill the two adjacent vertices.
- We can also call graph coloring
  as Vertex Coloring.



Chromatic Number

- The chromatic number can be described as the minimum number of colors required to properly color any graph.
- In other words, the chromatic number can be described as a minimum number of colors that are needed to color any graph in such a way that no two adjacent vertices of a graph will be assigned the same color.

Example of Chromatic number



The given graph contains some points, which are described as follows:

- The same color is not used to color the two adjacent vertices.
- The minimum number of colors of this graph is 3, which is needed to properly color the vertices.
- Hence, in this graph, the chromatic number = 3

Graph Coloring Algorithm-

Greedy Algorithm-

Step-01:

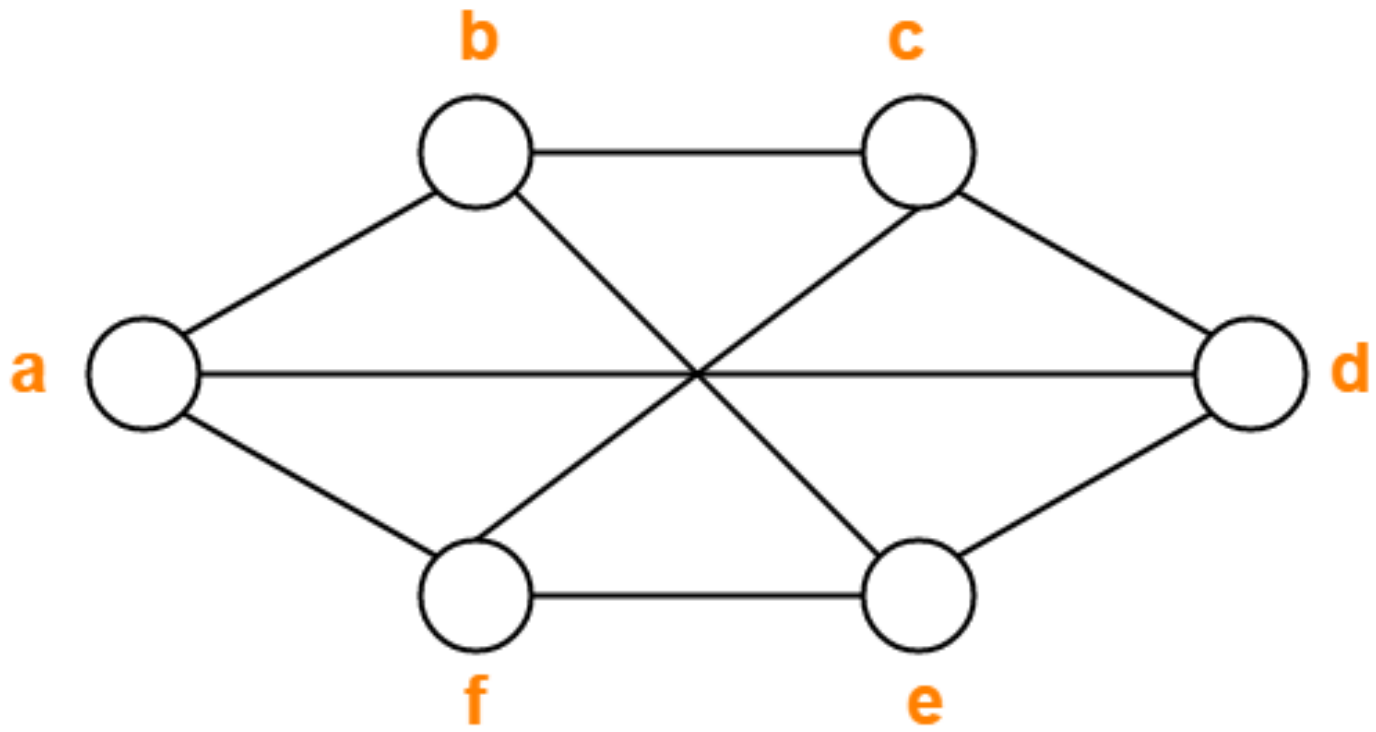
- Color first vertex with the first color.

Step-02:

- Now, consider the remaining (V-1) vertices one by one and do the following-
- Color the currently picked vertex with the lowest numbered color if it has not been used to color any of its adjacent vertices.
- If it has been used, then choose the next least numbered color.
- If all the previously used colors have been used, then assign a new color to the currently picked vertex.

Problem-01

Find chromatic number of the following graph-



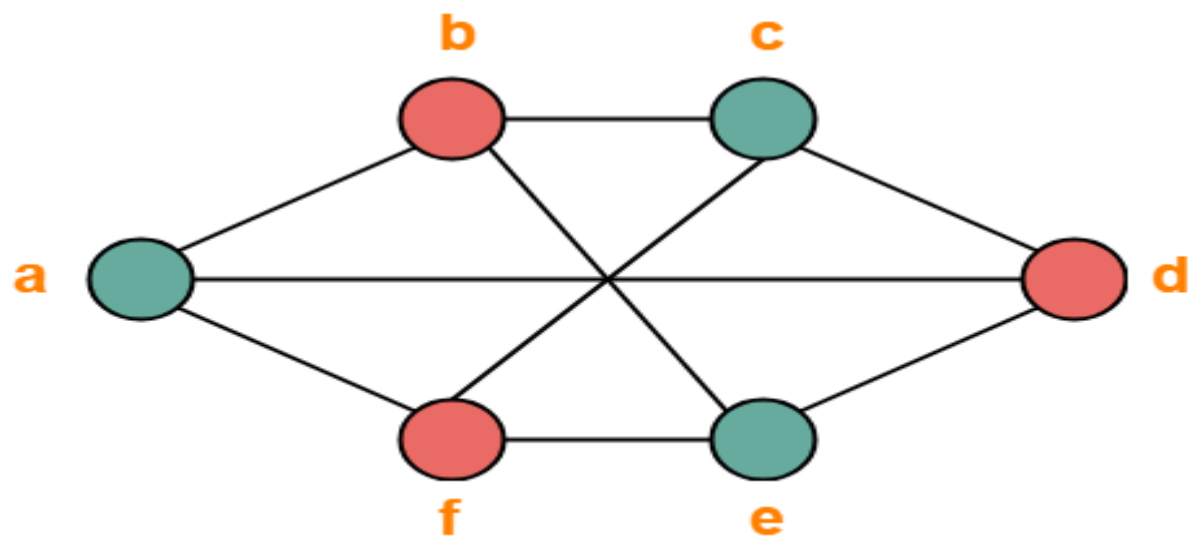
Applying Greedy Algorithm, we have-

Vertex	a	b	c	d	e	f
Color	C1	C2	C1	C2	C1	C2

From here,

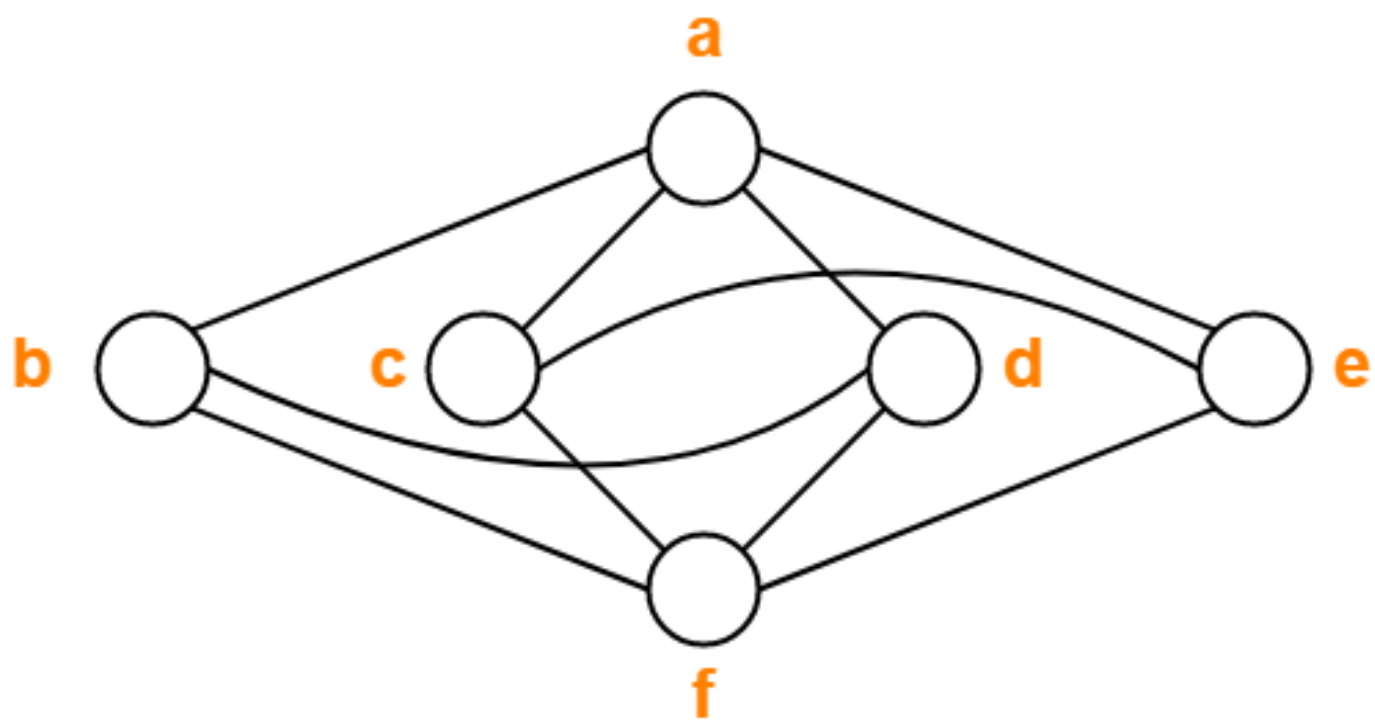
- Minimum number of colors used to color the given graph are 2.
- Therefore, Chromatic Number of the given graph = 2.

The given graph may be properly colored using 2 colors as shown below-



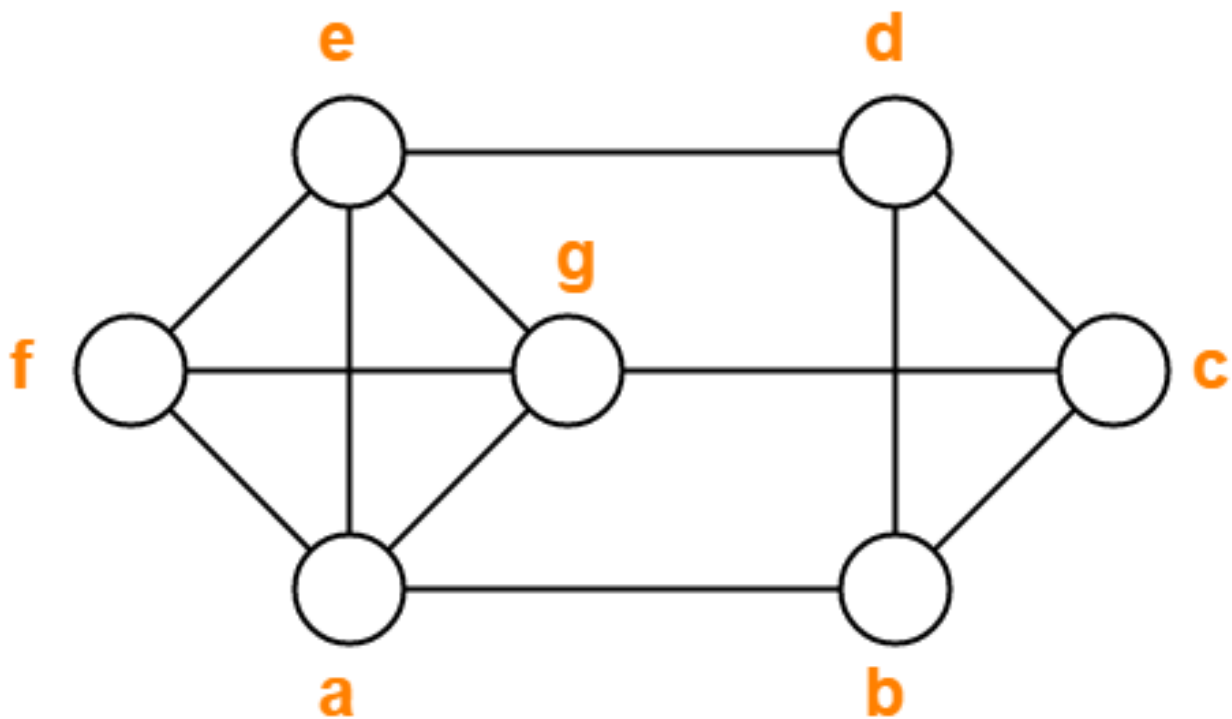
Problem-02:

Find chromatic number of the following graph-



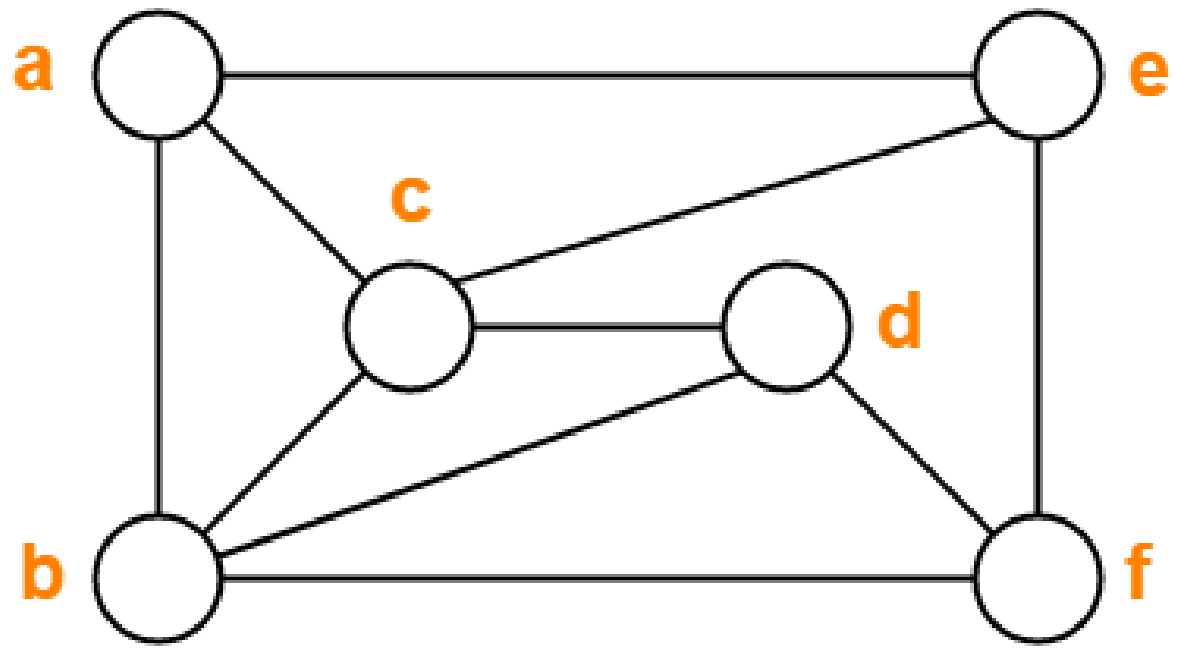
Problem-03:

Find chromatic number of the following graph-



Problem-04:

Find chromatic number of the following graph-



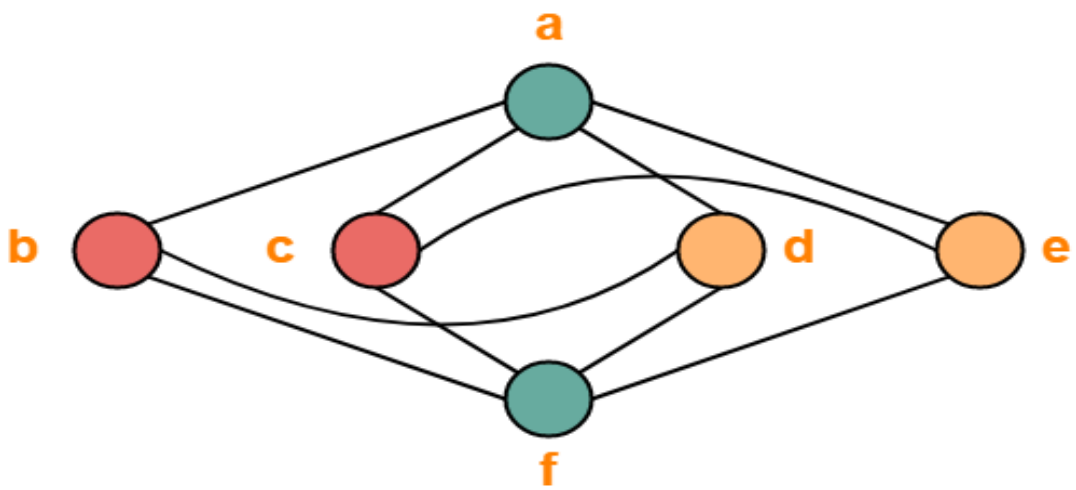
Solution-(problem-02)

Vertex	a	b	c	d	e	f
Color	C1	C2	C2	C3	C3	C1

From here,

- Minimum number of colors used to color the given graph are 3.
- Therefore, Chromatic Number of the given graph = 3.

The given graph may be properly colored using 3 colors as shown below-



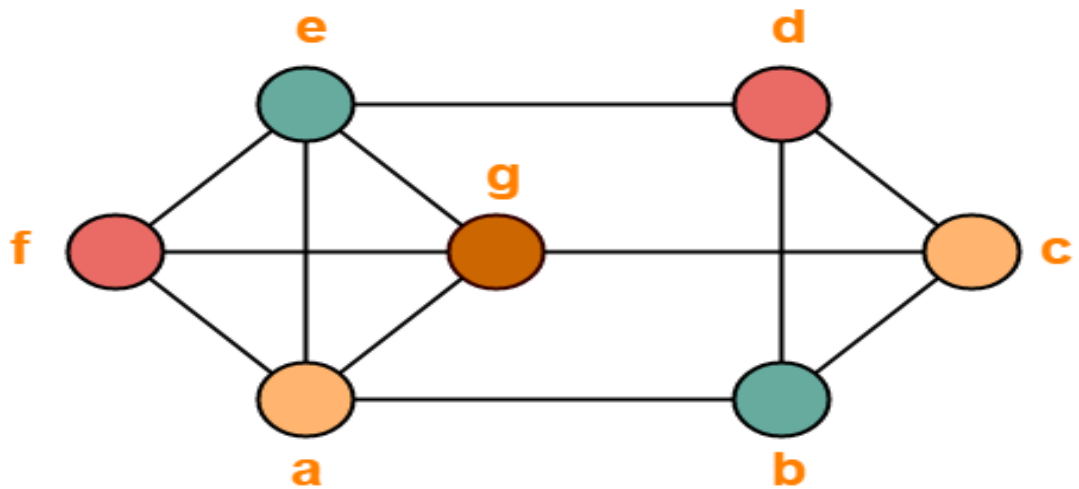
Solution-(problem-03)

Vertex	a	b	c	d	e	f	g
Color	C1	C2	C1	C3	C2	C3	C4

From here,

- Minimum number of colors used to color the given graph are 4.
- Therefore, Chromatic Number of the given graph = 4.

The given graph may be properly colored using 4 colors as shown below-



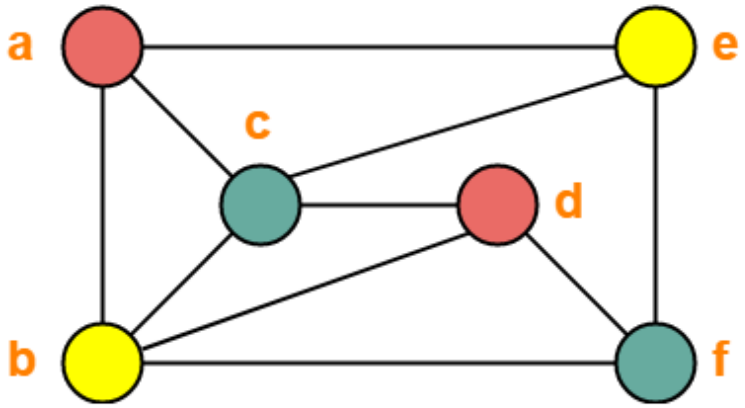
Solution-(problem-04)

Vertex	a	b	c	d	e	f
Color	C1	C2	C3	C1	C2	C3

From here,

- Minimum number of colors used to color the given graph are 3.
- Therefore, Chromatic Number of the given graph = 3.

The given graph may be properly colored using 3 colors as shown below-



Applications of Graph coloring

There are various applications of graph coloring. Some of their important applications are described as follows:

- Assignment
- Map coloring
- Scheduling the tasks
- Prepare time table

Application of graph theory