Poisson distribution

A discrete random variable *X* is said to have a Poisson distribution, with parameter $\lambda>0$, if it has a probability mass function given by

$$P(X=x) = \begin{cases} \dfrac{e^{-\lambda}\lambda^{x}}{x!}; & x = 0, 1, 2, \ldots.. \\ 0 & otherwise \end{cases}$$

Conditions:

- The number of trails '*n'* is indefinitely large i.e., $n \to \infty$
- The probability of a success '*p*' for each trial is very small i.e., $p \to 0$
- $np = \lambda$ is finite
- Events are Independent

Characteristics of Poisson Distribution

- Poisson distribution is a discrete distribution i.e., X can take values 0, 1, 2, …
- p is small, q is large, and n is indefinitely large i.e., $p \to 0$ $q \to 1$ and $n \to 3$ and np is finite
- Values of constants: (a) Mean = λ = variance (b) Standard deviation = $\sqrt{\lambda}$ (c) Skewness = $1/\sqrt{\lambda}$ (iv) Kurtosis $=1/\lambda$
- It may have one or two modes
- If X and Y are two independent Poisson variates, X+Y is also a Poisson variate.
- If X and Y are two independent Poisson variates, X-Y need not be a Poisson variate.
- Poisson distribution is positively skewed.
- It is leptokurtic.

1)If 2% of electric bulbs manufactured by a certain company are defective find the probability that in a sample of 200 bulbs (i) less than 2 bulbs are defective (ii) more than 3 bulbs are defective. [$e^{-4} = 0.0183$]

Solution:

Let X denote the number of defective bulbs

$X \sim P(\lambda)$

$$\therefore \quad P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!} \qquad x = 0, 1, 2, \ldots\infty$$

Given $p = P$ (a defective bulb) $= 2\% = \frac{2}{100} = 0.02$

$$n = 200$$

$$\therefore \quad \lambda = np = 200 \times 0.02 = 4$$

$$\therefore \quad P(X = x) = \frac{e^{-4}4^x}{x!}, \ x = 0, 1, 2, \ldots\infty$$

(i) P (less than 2 bulbs are defective)

$$= P(X < 2)$$

$$= P(x = 0) + P(x = 1)$$

$$= \frac{e^{-4}.4^0}{0!} + \frac{e^{-4},4^1}{1!}$$

$$= e^{-4}(1 + 4)$$

$$= 0.0183 \text{ x } 5$$

$$= 0.0915$$

(ii) P (more than 3 defectives)

$$= P(X > 3)$$

$$= 1 - P(X \leq 3)$$

$$= 1 - \{P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3)\}$$

$$= 1 - \left\{\frac{e^{-4}.4^0}{0!} + \frac{e^{-4}.4^1}{1!} + \frac{e^{-4}.4^2}{2!} \frac{e^{-4}.4^3}{3!}\right\}$$

$$= 1 - e^{-4}\{1 + 4 + 8 + 10.667\}$$

$$= 1 - 0.0183 \times 23.667$$

$$= 0.567$$

2)In a Poisson distribution 3*P* (*X*= 2) = *P*(*X* = 4). Find its parameter 'λ

Solution:

The *pmf* of Poisson distribution is $P\ (X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!}, \quad x = 0, 1, 2, \ldots\infty,$

Given $3\ P\ (X = 2) = P\ (X = 4)$

$$3.\frac{e^{-\lambda}\lambda^{2}}{2!} = \frac{e^{-\lambda}.\lambda^{4}}{4!}$$

$$\lambda^{2} = \frac{3 \times 4!}{2!} = 36$$

$$\therefore \lambda = 6 \text{ as } \lambda > 0$$

3)Find the skewness and kurtosis of a Poisson variate with parameter 4.

Solution:

$$\lambda = 4$$

$$\text{Skewness} = \frac{1}{\sqrt{\lambda}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$$

$$\text{Kurtosis} = \frac{1}{\lambda} = \frac{1}{4}$$

4)If there are 400 errors in a book of 1000 pages, find the probability that a randomly chosen page from the book has exactly 3 errors.

Solution:

Let X denote the number of errors in pages

$X \sim P(\lambda)$

$$\therefore P\ (X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!} \qquad x = 0, 1, 2, \ldots\infty$$

The average number of errors per page $= \frac{400}{1000}$

i.e., $\lambda = \frac{400}{1000} = 0.4$

$$P(X=3) = \frac{e^{-\lambda}\lambda^{3}}{3!} = \frac{e^{-0.4}(0.4)^{3}}{3 \times 2 \times 1}$$

$$= \frac{0.6703 \times 0.064}{6}$$

$$= 0.00715$$

5)If X is a Poisson variate with $P(X=0) = 0.2725$, find $P(X=1)$

Solution:

$X \sim P(\lambda)$

$\therefore P(X = x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$ $\quad x = 0, 1, 2, \ldots\infty$

$P(X=0) = 0.2725$

$\dfrac{e^{-\lambda}\lambda^0}{0!} = 0.2725$

$e^{-\lambda} = 0.2725$

$\lambda = 1.3$ (from the table of values of e^{-m})

$$P(x=1) = \frac{e^{-\lambda}\lambda^1}{1!} = \frac{e^{-1.3} \times 1.3^1}{1!}$$

$$= 0.2725 \times 1.3$$

$$= 0.3543$$

6)The probability of safety pin manufactured by a firm to be defective is 0.04. (i) Find the probability that a box containing 100 such pins have one defective pin. (ii) Among 200 such boxes, how many boxes will have no defective pin

Solution:

Let X denote the number boxes with defective pins

$X \sim P(\lambda)$

$\therefore \quad P(X = x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$ $\quad x = 0, 1, 2, \ldots\infty$

$p = 0.04$

$n = 100$

$\lambda = np = 4$

(i) $\quad P(X=1) = \dfrac{e^{-\lambda}\lambda}{1!} = e^{-4}(4) = 0.0183 \times 4$

$= 0.0732$

(ii) $\quad P(X=0) = \dfrac{e^{-\lambda}\lambda^0}{0!} = e^{-4} = 0.0183$

Number of boxes having no defective pin $= 200 \times 0.0183$

$= 3.660$

$= 4$