

Poisson Distribution

1. If the prices of new cars increase an average of four times every 3 years, find the probability of
 - a) No price hikes in a randomly selected period of 3 years.
 - b) Two price hikes.
 - c) Five or more price hikes.
2. The number of accidents that occurs on an assembly line with an average of three accidents per week, what is the probability that:
 - a) A particular week will be accident free.
 - b) Exactly five accidents will occur in a week.
 - c) At least three accidents in a week.
3. The number of accidents in a year attributed to taxi driver in a city, follows Poisson distribution with mean 3. Out of 1000 taxi drivers, find approximately the number of drivers with
 - a) No accident in a year.
 - b) More than 3 accidents in a year.
4. The probability of getting no misprint in a page of book is e^{-4} . What is the probability that a page contains more than 2 misprints?
5. Calculate mean and variance of a Poisson variable X, if $P(X=4) = P(X=5)$.
6. The standard deviation of a Poisson distribution is 2. Find the probability that $X=3$.
7. If mean and variance of a distribution is 2, find the $P(X<3)$.

Poisson is a Good Approximation to Binomial

Poisson approximation to binomial when $np < 10$, $n \geq 20$ and $p \leq 0.05$

$$\begin{aligned}\text{Now, } P(X=x) &= \frac{e^{-\lambda} \lambda^x}{x!} \\ &= \frac{e^{-np} (np)^x}{x!}, x=0, 1, 2, \dots\end{aligned}$$

$$\lambda = np = \text{mean}$$

1. If 3% of electric bulbs manufactured by a company are defective, use Poisson distribution to find the probability that in a sample of 100 bulbs
 - a) None is defective
 - b) 3 bulbs are defective.

2. In a certain manufacturing process 5% of the tools produced turn out to be defective. Find the probability that in a sample of 40 tools, at most 2 will be defective.

3. Assuming that the probability of a fatal accident in a factory during the year is 1/120. Calculate the probability that in a factory employing 30 workers there will be

- Exactly 2 fatal accidents in a year.
- At least 2 fatal accidents in a year.
- At most 2 fatal accidents in a year.

Fitting of Poisson Distribution

If a series of trials is repeated N times and satisfied the condition of Poisson distribution, then expected or theoretical frequency for x successes is given by,

$$f(x) = N P(X=x)$$

$$= N \frac{e^{-\lambda} \lambda^x}{x!}, x=0, 1, 2, \dots$$

The following steps are generally followed to fit Poisson distribution.

1. First of all, we calculate the mean of given frequency distribution by using following formula.

$$\bar{X} = \frac{\sum fX}{N}$$

if λ is known (given), it is not necessary to find mean.

2. The computed mean is equating to the parameter of Poisson distribution (λ) i.e. $\bar{X} = \lambda$.

3. Calculate the expected frequencies by using the formula

$$f(x) = N \frac{e^{-\lambda} \lambda^x}{x!}, x=0, 1, 2, \dots$$

Practical problem

1. Fit a Poisson distribution to the following frequency distribution.

No. of heads	0	1	2	3	4	Total
Frequency	28	62	46	20	4	160

2. Fit a Poisson distribution to the following frequency distribution.

X	0	1	2	3	4	5	Total
f	1	4	10	31	26	13	85