Probability

Introduction

The concept of probability is difficult to define in precise terms. In ordinary language, the word probable means likely or chance. The probability theory is an important branch of mathematics. Generally, the word, probability, is used to denote the happening of a certain event, and the likelihood of the Occurrence of that event, based on past experiences. By looking at the clear sky, one will say that there will not be any rain today. On the other hand, by looking at the cloudy sky or overcast sky, one will say that there will be rain today. In the earlier sentence, we aim that there will not be rain, and, in the latter, we expect rain. On the other hand, a mathematician says that the probability of rain is 0 in the first case and that the probability of rain is 1 in the second case. In between 0 and 1, there are fractions denoting the chance of the event occurring.

If a coin is tossed, the coin falls dawn. The coin has two sides: head and tail. On tossing a coin, the coin may fall down either with the head up or tail up. A coin, on reaching the ground, will not stand on its edge or rather, we assume; so, the probability of the coin coming down is 1. The probability of the head coming up is 50% and the tail coming up is 50%; in other words, we can say the probability of the head or the tail coming up is 1/2, 1/ 2 Ince 'head' and 'tail' share equal chances. The probability that it will come down head or tail is unity.

Some useful terms

Before discussing the theory of probability, let us have an understanding of the following terms:

Random Experiment:

If an experiment or trial can be repeated under the same conditions, any number of times and it is possible to count the total number of outcomes, but individual result i.e., individual outcome is not predictable. Suppose we toss a coin. It is not possible to predict exactly the outcomes. The outcome may be either head up or tail up. Thus, an action or an operation which can produce any result or outcome is called a random experiment.

Event and Trials:

Any possible outcome of a random experiment is called an event. Performing an experiment is called trial and outcomes are termed as events. An event whose occurrence is inevitable when a certain random experiment is performed, is called a sure event or certain event. At the same time, an event which can never occur when a certain random experiment is performed is called an impossible event. The events may be simple or composite. An event is called simple if it corresponds to a single possible outcome. For example, in rolling a die, the chance of getting 2 is a simple event. Further in tossing a die, chance of getting event numbers (1, 3, 5) are compound event.

Sample space:

The set or aggregate of all possible outcomes is known as sample space. For example, when we roll a die, the possible outcomes are 1, 2, 3, 4, 5, and 6; one and only one face come upwards. Thus, all the outcomes— 1, 2, 3, 4, 5 and 6 are sample space. And each possible outcome or element in a sample space called sample point.

Mutually exclusive events or cases:

Two events are said to be mutually exclusive if the occurrence of one of them excludes the possibility of the occurrence of the other in a single observation. The occurrence of one event prevents the occurrence of the other event. As such, mutually exclusive events are those events, the occurrence of which prevents the possibility of the other to occur. All simple events are mutually exclusive. Thus, if a coin is tossed, either the head can be up, or tail can be up; but both cannot be up at the same time.

Similarly, in one throw of a die, an even and odd number cannot come up at the same time. Thus, two or more events are considered mutually exclusive if the events cannot occur together.

Equally likely events:

The outcomes are said to be equally likely when one does not occur more often than the others. That is, two or more events are said to be equally likely if the chance of their happening is equal. Thus, in a throw of a die the coming up of 1, 2, 3, 4, 5 and 6 is equally likely. For example, head and tail are equally likely events in tossing an unbiased coin.

Exhaustive events

The total number of possible outcomes of a random experiment is called exhaustive events. The group of events is exhaustive, as there is no other possible outcome. Thus, tossing a coin, the possible outcomes are head or tail; exhaustive events are two. Similarly throwing a die, the outcomes are 1, 2, 3, 4, 5 and 6. In case of two coins, the possible number of outcomes are 4 i.e. (2^2), i.e., HH, HT TH and TT. In case of 3 coins, the possible outcomes are 23=8 and so on. Thus, in a throw of n" coin, the exhaustive number of cases is 2^n.

Independent events:

Events are said to be independent if the happening of one event does not affect the happening of the other event. In tossing of a coin, the occurrence of head in first tossing is independent of the occurrence of head in the second tossing.

Dependent events:

Events are said to be dependent if the occurrence of one event affects the probability of the occurrence of the other event.

For example, the probability of drawing a king from a pack of 52 cards is 4/52; the card is not put back; then the probability of drawing a king again is 3/51. Thus, the outcome of the first event affects the outcome of the second event, and they are dependent. But if the card is put back, then the probability of drawing a king is 4/52 and is an independent event.

Favorable Cases

The number of outcomes which result in the happening of a desired event are called favorable cases to the event. For example, in drawing a card from a pack of cards, the cases favorable to "getting a diamond" are 13 and to "getting an ace of spade" is only one. Take another example, in a single throw of a dice the number of favorable cases of getting an odd number are three -1,3 &5.

MEASUREMENT OF PROBABILITY

The origin and development of the theory of probability dates back to the seventeenth century. Ordinarily speaking the probability of an event denotes the likelihood of its happening. A value of the probability is a number ranges between 0 and 1. Different schools of thought have defined the term probability differently. The various schools of thought which have defined probability are discussed briefly.

Classical Approach (Priori Probability)

The classical approach is the oldest method of measuring probabilities and has its origin in gambling games. According to this approach, the probability is the ratio of favorable events to the total number of equally likely events. If we toss a coin, we are certain that the head or tail will come up. The probability of the coin coming down is 1, of the head coming up is 1/2 and of the tail coming up is 1/2.

$$P = \frac{\text{Number of favorable cases}}{\text{total number of cases}}$$

If an event can occur in 'a' way and fail to occur in 'b' ways and these are equally to occur, then the probability of the event occurring, $\frac{a}{a+b}$is denoted by P. Such probabilities are also known as unitary or theoretical or mathematical probability. P is the probability of the event happening and q is the probability of its not happening.

$$P = \frac{a}{a+b} \text{ and } q = \frac{b}{a+b}$$

$$\text{Hence } p + q = \frac{a}{a+b} + \frac{b}{a+b} = 1$$

Therefore, $p + q = 1$

Probability can be expressed either as ratio, fraction or percentage, such as $\frac{1}{2}$ or 0.5 or 50%.

Limitations of Classical Approach:

- We cannot apply this method when the total number of cases cannot be calculated.
- When the outcomes of a random experiment are not equally likely, this method cannot be applied.

Relative Frequency Theory of probability:

Classical approach is useful for solving problems involving game of chances—throwing dice, coins, etc. but if applied to other types of problems it does not provide answers. For instance, if a man jumps from a height of 300 feet, the probability of his survival will, not be 50%, since survival and death are not equally alike.

Similarly, the prices of shares of a Joint Stock Company have three alternatives i.e., the prices may remain constant, or prices may go up or prices may go down. Thus, the classical approach fails to answer questions of these type.

If we toss a coin 20 times, the classical probability suggests that we; should have heads ten times. But in practice it may not be so. This empirical approach suggests, that if a coin is tossed a large number of times, say, 1,000 times, we can expect 50% heads and 50% tails. Vor Miscs explained, "If the experiment be repeated a large number of times under essentially identical conditions, the limiting value of the ratio of the number of times the event **A** happens to the total, number of trials of the experiments as the number of trials increases indefinitely, is called the probability of the occurrence of **A**".

Thus, $P(A) = \lim_{n \to \infty} \frac{m}{n}$

The happening of an event is determined on the basis of past experience or on the basis of relative frequency of success in the past.

(i) the relative frequency obtained on the basis of past experience can be shown to come very close to the classical probability. For example, as said earlier, a coin is tossed for 6 times, we may not get exactly 3 heads and 3 tails. But the coin is tossed for larger number of times, say 10,000 times, we can expect heads and tails very close to 50%.

(ii) There are certain laws, according to which the 'occurrence' or 'non-occurrence of the events take place. Posterior probabilities, also called Empirical Probabilities are based on experiences of the past and on experiments conducted. Thus, relative frequency can be termed as a measure of probability, and it is calculated on the basis of empirical or statistical findings. For instance, if a machine produces 100 articles in the past, 2 particles were found to be defective, then the probability of the defective articles is 2/100 or 2%.

Subjective approach

This approach of probability is completely based on the personal belief of personal discretion of a person. Since different persons may assign different probabilities, one cannot arrive at objective conclusions using probabilities assigned by this subjective method.