Mathematics II (BSM 102)

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BSM 102

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Concept of Partial Derivatives

Production Function: Output Q at a factory is often regarded as a function of the amount K of capital investment and the size L of the labor force. Output functions of the form

$$Q(K, L) = AK^{\alpha}L^{\beta}$$

where A, α, and β are positive constants with $\alpha + \beta = 1$, have proved to be especially useful in economic analysis and are known as Cobb-Douglas production functions.

Example

For the Cobl-Douglas production function $f(K, L) = 40K^{1/3}L^{2/3}$,
a) Find the rate at which production changes with respect to capital K, called the marginal productivity of capital, when $K = 27$ units and $L = 8$ units.
b) Find the rate at which production changes with respect to labor L, called the marginal productivity of Labor, when $K = 27$ units and $L = 8$ units.
Solution: The marginal productivity of the capital is the partial derivative of function with respect to K. $\frac{\partial f}{\partial K} = \frac{\partial}{\partial K}\left(40K^{1/3}L^{2/3}\right)$

$$= 40 \times \frac{1}{3} \times K^{-2/3}L^{2/3} = \frac{40}{3}K^{-2/3}L^{2/3}$$

When $K = 27$ unts and $L = 8$ units,

$$\frac{\partial f}{\partial K}(27, 8) = \frac{40}{3}(27)^{-2/3}(8)^{2/3} = \frac{40}{3} \times \frac{(8)^{2/3}}{(27)^{2/3}} = \frac{40}{3} \times \frac{4}{9} = \frac{160}{27}$$

Example Continue

This means that an increase in capital from level $K = 27$ units to level $K = 28$ units will result in an increase of approximately $160/27$ units of production.

(b) The marginal productivity of the labor is the partial derivative of a function with respect to L

$$\begin{aligned}\frac{\partial f}{\partial L} &= \frac{\partial}{\partial L}\left(40K^{1/3}L^{2/3}\right)\\ &= 40 \times \frac{2}{3} \times K^{1/3}L^{-1/3} = \frac{80}{3}K^{1/3}L^{-1/3}\end{aligned}$$

When K = 27 units and L = 8 units

$$\frac{\partial f}{\partial L}(27, 8) = \frac{80}{3}(27)^{1/3}(8)^{-1/3} = \frac{80}{3} \times \frac{(27)^{1/3}}{(8)^{1/3}} = \frac{80}{3} \times \frac{3}{2} = 40$$

This means that an increase in labor from level $L = 8$ units to level $L = 9$ units will result in an increase of approximately 40 units of production.

Application Problem

1. Suppose the Cobb-Douglas production function for a company is given by

$$z = 300x^{2/3}y^{1/3}$$

where x is the company's capital investment and y is the size of the labor force (in work-hours).
(a) Find the marginal productivity of x.
(b) Find the marginal productivity of y,

Application Problem

2. Suppose the Cobb-Douglas production function for a company is given by

$$z = 400x^{3/5}y^{2/5}$$

where x is the company's capital investment and y is the size of the labor force (in work-hours).
(a) Find the marginal productivity of x, evaluate at $x = 243, y = 1024$.
(b) Find the marginal productivity of y, evaluate at $x = 243, y = 1024$.

Thank You