Gandaki University

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Bachelor of Information Technology

BSM 102

Exercise on Complex Variable Functions

1. Are the following functions analytic?
   (i) $f(z) = e^{-x}(\cos y - i \sin y)$
   (ii) $f(z) = \operatorname{Re} z + \operatorname{Im} z$
   (iii) $f(z) = e^{2x}(\cos y + i \sin y)$
2. Are the following functions harmonic? If your answer is yes, find a harmonic conjugate and corresponding analytic function $f(z) = u(x, y) + iv(x, y)$.
   (i) $u = x^3 - 3xy^2$
   (ii) $v = xy$
   (iii) $u = xy$
   (iv) $v = 2xy$
   (v) $u = x^2 - y^2$
3. Prove that $\cos z, \sin z, \cosh z, \sinh z$ are entire functions.
4. Verify by differentiation that $\operatorname{Re}(\cos z)$ and $\operatorname{Im}(\sin z)$ are harmonic.
5. Use exponential form of trigonometric and hyperbolic trigonometric functions to prove the following identity.
   (i) $\cosh z = \cosh x \cos y + i \sinh x \sin y$
   (ii) $\sinh z = \sinh x \cos y + i \cosh x \sin y$.
   (iii) $\cosh (z_1 + z_2) = \cosh z_1 \cosh z_2 + \sinh z_1 \sinh z_2$
   (iv) $\sinh (z_1 + z_2) = \sinh z_1 \cosh z_2 + \cosh z_1 \sinh z_2$
   (v) $\sin z = \sin x \cosh y + i \cos x \sinh y$
   (vi) $\cosh^2 z - \sinh^2 z = 1$
   (vii). $\cosh^2 z + \sinh^2 z = \cosh 2z$
6. Compute (in the form $u + iv$)
   (i) $\cos(1 + i)$
   (ii) $\sin(1 + i)$