Gandaki University

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Bachelor of Information Technology

BSM 102

Exercise on Functions of Several Variables

1. Find the domain of each of the following functions:

   (a) $f(x, y, z) = \dfrac{3x - 4y + 2z}{\sqrt{9 - x^2 - y^2 - z^2}}$

   (b) $g(x, y, t) = \dfrac{\sqrt{2t - 4}}{x^2 - y^2}$

2. Find first order partial derivatives:

   (a) $f(x, y) = 1/(x + y)$

   (b) $f(x, y) = e^{(x+y+1)}$

   (c) $f(x, y) = e^{-x} \sin(x + y)$

   (d) $f(x, y) = \ln(x + y)$

   (e) $f(x, y) = e^{xy} \ln y$

   (f) $f(x, y) = \sin^2(x - 3y)$

   (g) $f(x, y) = x^y$

   (h) $f(x, y) = y^x$

   (i) $f(x, y) = \dfrac{1}{(x + y)}$

   (j) $f(x, y) = \dfrac{x}{(x^2 + y^2)}$

   (k) $f(x, y) = \dfrac{x + y}{xy - 1}$

   (l) $f(x, y, z) = \dfrac{2xyz}{x^2 + y^2 + z^2}$

3. Verify the mixed derivative theorem (Euler's Theorem) for the following functions.

   (a) $w = \ln(2x + 3y)$

   (b) $w = e^x + x \ln y + y \ln x$

   (c) $w = xy^2 + x^2y^3 + x^3y^4$

   (d) $w = x \sin y + y \sin x + xy$

4. Find f_x, f_y, f_z from the following functions:

   (a) $f(x, y, z) = 1 + xy^2 - 2z^2$

   (b) $f(x, y, z) = xy + yz + xz$

(c) $f(x, y, z) = x - \sqrt{y^2 + z^2}$

(d) $f(x, y, z) = \dfrac{1}{\sqrt{(x^2 + y^2 + z^2)}}$

(e) $f(x, y, z) = \ln(x + 2y + 3z)$

(f) $f(x, y, z) = yz \ln(xy)$

(g) $f(x, y, z) = e^{-(x^2+y^2+z^2)}$

(h) $f(x, y, z) = e^{-xyz}$

5. Given $f(x, y) = x^2 + x - 3xy + y^3 - 5$, find all points at which $f_x(x, y) = f_y(x, y) = 0$ simultaneously.

6. Given $f(x, y) = 2x^2 + 2xy + y^2 + 2x - 3$, find all points at which $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$ simultaneously.

7. Given $f(x, y) = y^3 - 3yx^2 - 3y^2 - 3x^2 + 1$, find all points on f at which $f_x(x, y) = f_y(x, y) = 0$ simultaneously.

8. If $z = 4y \ln x + e^{xy}$, find z_{xy} and z_{yx}.

9. Find all the second order partial derivatives of the functions

(a) $f(x, y) = x + y + xy$

(b) $f(x, y) = \sin xy$

(c) $g(x, y) = x^2 y + \cos y + y \sin x$

(d) $h(x, y) = xe^y + y + 1$

(e) $r(x, y) = \ln(x + y)$