Mathematics I (BSM 101)

Manju Subedi

Gandaki University
Bachelor in Information Technology(BIT)
BSM 101

manjusubedi.061@gmail.com

March 24, 2023

Properties of Determinants

- **Reflection Property:** The determinant remains unaltered if its rows are changed into columns and the columns into rows. This is known as the property of reflection i.e $|A| = |A^T|$
- **All-zero Property:** If all the elements of a row (or column) are zero, then the determinant is zero.
- **Proportionality (Repetition) Property:** If the all elements of a row (or column) are proportional (identical) to the elements of some other row (or column), then the determinant is zero.
- **Switching Property:**The interchange of any two rows (or columns) of the determinant changes its sign.
- **Scalar Multiple Property:** If all the elements of a row (or column) of a determinant are multiplied by a non-zero constant, then the determinant gets multiplied by the same constant.

- **Sum Property:**

$$\begin{vmatrix} a_1+b_1 & c_1 & d_1 \\ a_2+b_2 & c_2 & d_2 \\ a_3+b_3 & c_3 & d_3 \end{vmatrix} = \begin{vmatrix} a_1 & c_1 & d_1 \\ a_2 & c_2 & d_2 \\ a_3 & c_3 & d_3 \end{vmatrix} + \begin{vmatrix} b_1 & c_1 & d_1 \\ b_2 & c_2 & d_2 \\ b_3 & c_3 & d_3 \end{vmatrix}$$

- **Triangle Property:** If all the elements of a determinant above or below the main diagonal consist of zeros, then the determinant is equal to the product of diagonal elements. That is,

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ 0 & b_2 & b_3 \\ 0 & 0 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & 0 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1b_2c_3$$

Rank of a Matrix

- Row rank
- Column rank

The maximum number of its linearly independent columns (or rows) of a matrix is called the rank of a matrix. The rank of a matrix cannot exceed the number of its rows or columns.

If we consider a square matrix, the columns (rows) are linearly independent only if the matrix is nonsingular. In other words, the rank of any nonsingular matrix of order m is m. Rank of a matrix A is denoted by $\rho(A)$.

The rank of a null matrix is zero. A null matrix has no non-zero rows or columns. So, there are no independent rows or columns. Hence the rank of a null matrix is zero.

Rank of a Matrix?

- The rank of a unit matrix of order m is m.
- If A matrix is of order $m \times n$, then $\rho(A) \leq \min\{m, n\} =$ minimum of m, n.
- If A is of order $n \times n$ and $|A| \neq 0$, then the rank of $A = n$.
- If A is of order $n \times n$ and $|A| = 0$, then the rank of A will be less than n.

  Note: To determine the rank of the matrix first transform the given into row-echelon form.

Row-echelon Form

A non-zero matrix A is said to be in a **row-echelon form** if:

- All zero rows of A occur below every non-zero row of A.
- The first non-zero element in any row i of A occurs in the j^{th} column of A, then all other elements in the j^{th} column of A below the first non-zero element of row i are zeros.
- The first non-zero entry in the i^{th} row of A lies to the left of the first non-zero entry in $(i+1)^{\text{th}}$ row of A.

Note: A non-zero matrix is said to be in a row-echelon form, if all zero rows occur as bottom rows of the matrix and if the first non-zero element in any lower row occurs to the right of the first non-zero entry in the higher row.
If a matrix is in row-echelon form, then all elements below the leading diagonal are zeros.

Example

Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$

Solution: Given $\mathrm{A} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$

Now we transform the matrix A to echelon form by using elementary transformation.

$\mathrm{R}_2 \to \mathrm{R}_2 - 2\mathrm{R}_1 \quad \& \quad \mathrm{R}_3 \to \mathrm{R}_3 - 3\mathrm{R}_1 \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & -1 & -2 \end{bmatrix}$

$\mathrm{R}_3 \to \mathrm{R}_3 - \mathrm{R}_2 \qquad \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$

Number of non-zero rows $= 2$

Hence, the rank of matrix $A = 2$

Example

Find the rank of the given matrix: $M = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

Solution:

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \to \mathrm{R}_3 \to \mathrm{R}_3 - \mathrm{R}_1 \to \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & -1 \end{bmatrix} \to \mathrm{R}_3 \to \mathrm{R}_3 - \mathrm{R}_2$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{bmatrix}$$

Divide R_3 by -2

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Since there are three non zero rows, rank $= 3$

Thank You