Gandaki University

Pokhara

Exercise on Ordinary Differential Equations (ODE)

A Find a general solution. Show the steps of derivation. Check your answer by substitution.

vc 1. $y' + (x+2)y^2 = 0$

2. $y' = 2\sec 2y$

3. $y' = (y+9x)^2$ (suppose $y + 9x = v$)

4. $yy' + 36x = 0$

5. $y' = \left(4x^2 + y^2\right)/(xy)$

6. $y' \sin \pi x = y \cos \pi x$

7. $xy' = \frac{1}{2}y^2 + y$

8. $y'e^{\pi x} = y^2 + 1$

9-15 INITIAL VALUE PROBLEMS Find the particular solution. Show the steps of derivation, beginning with the general solution. (L, R, b are constants.)

9. $yy' + 4x = 0, y(0) = 3$

10. $\dfrac{dr}{dt} = -2tr, r(0) = r_0$

11. $2xyy' = 3y^2 + x^2, y(1) = 2$

12. $y' = y/x + \left(2x^3/y\right)\cos\left(x^2\right), y(\sqrt{\pi/2}) = \sqrt{\pi}$

13. $e^{2x}y' = 2(x+2)y^3, y(0) = 1/\sqrt{5} \approx 0.45$

14. $xy' = y + 4x^6\cos^2(y/x), y(2) = 0$

15. $y'x\ln x = y, y(3) = \ln 81$

B Test for exactness. If exact, solve. If not, use an integrating factor as given or find it by inspection or from the theorems in the text. Also, if an initial condition is given, determine the corresponding particular solution.

1. $x^3dx + y^3dy = 0$

2. $(x-y)(dx-dy) = 0$

3. $-\pi \sin \pi x \sinh y dx + \cos \pi x \cosh y dy = 0$

4. $(e^y - ye^x)\,dx + (xe^y - e^x)\,dy = 0$

5. $9xdx + 4ydy = 0$

6. $e^x(\cos y dx - \sin y dy) = 0$

7. $e^{-2\theta}dr - 2re^{-2\theta}d\theta = 0$

8. $\left(2x + 1/y - y/x^2\right)dx + \left(2y + 1/x - x/y^2\right)dy = 0$

9. $\left(-y/x^2 + 2\cos 2x\right)dx + (1/x - 2\sin 2y)dy = 0$

10. $-2xy\sin\left(x^2\right)dx + \cos\left(x^2\right)dy = 0$

11. $-ydx + xdy = 0$

12. $(e^{x+y} - y)\,dx + (xe^{x+y} + 1)\,dy = 0$
13. $-3ydx + 2xdy = 0, \quad F(x, y) = y/x^4$
14. $\left(x^4 + y^2\right)dx - xydy = 0, \quad y(2) = 1$
15. $e^{2x}(2\cos ydx - \sin ydy) = 0, \quad y(0) = 0$
16. $-\sin xy(ydx + xdy) = 0, \quad y(1) = \pi$
17. $(\cos \omega x + \omega \sin \omega x)dx + e^x dy = 0, y(0) = 1$
18. $(\cos xy + x/y)dx + (1 + (x/y)\cos xy)dy = 0$
19. $e^{-y}dx + e^{-x}\left(-e^{-y} + 1\right)dy = 0, \quad F = e^{x+y}$
20. $\left(\sin y \cos y + x\cos^2 y\right)dx + xdy = 0$
21. Under what conditions for the constants A, B, C, D is $(Ax + By)dx + (Cx + Dy)dy = 0$ exact? Solve the exact equation.

C Find the general solution. If an initial condition is given, find also the corresponding particular solution. (Show the details of your work.)

1. $y' + 3.5y = 2.8$
2. $y' = 4y + x$
3. $y' + 1.25y = 5, \quad y(0) = 6.6$
4. $x^2y' + 3xy = 1/x, \quad y(1) = -1$
5. $y' + ky = e^{2itx}$
6. $y' + 2y = 4\cos 2x, \quad y\left(\frac{1}{4}\pi\right) = 2$
7. $y' = 6(y - 2.5)\tanh 1.5x$
8. $y' + 4x^2y = \left(4x^2 - x\right)e^{-x^2/2}$
9. $y' + 2y\sin 2x = 2e^{\cos 2x}, \quad y(0) = 0$
10. $y'\tan x = 2y - 8, \quad y\left(\frac{1}{2}\pi\right) = 0$
11. $y' + 4y\cot 2x = 6\cos 2x, y\left(\frac{1}{4}\pi\right) = 2$
12. $y' + y\tan x = e^{-0.01x}\cos x, \quad y(0) = 0$
13. $y' + y/x^2 = 2xe^{1/x}, \quad y(1) = 13.86$
14. $y'\cos^2 x + 3y = 1, \quad y\left(\frac{1}{4}\pi\right) = \frac{4}{3}$
15. $x^3y' + 3x^2y = 5\sinh 10x$