

Unit 5

Graph Theory

Graph

Definition:

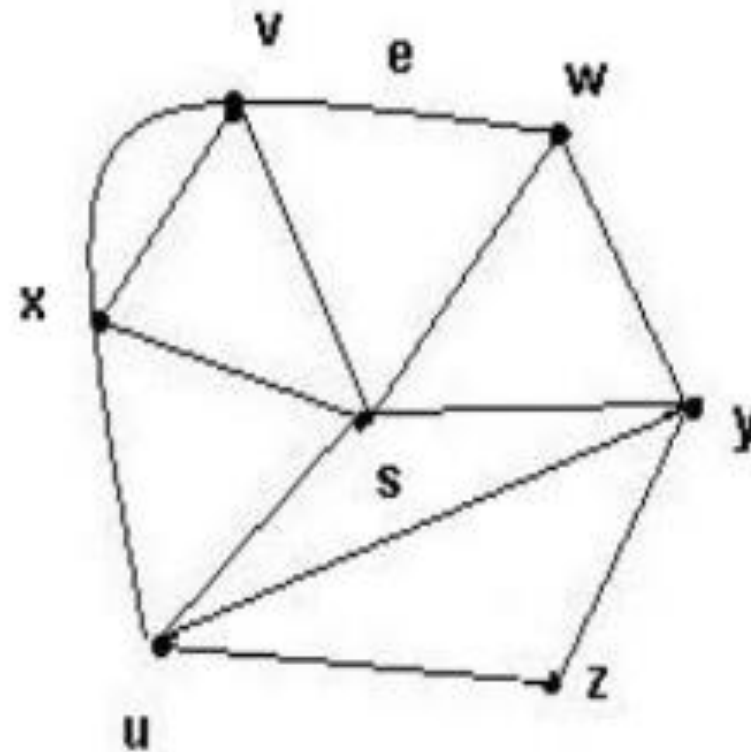
A **simple graph** $G = (V, E)$ consists of V , a nonempty set of vertices, and E , a set of **unordered pairs** of distinct elements of V called edges.

Graph G consists of two things:

1. A set $V = V(G)$ whose elements are called vertices, points or nodes of G .
2. A set $E = E(G)$ of an unordered pair of distinct vertices called edges of G .
3. We denote such a graph by $G(V, E)$.

Example

- What is a graph G ?
- It is a pair $G = (V, E)$, where
 - $V = V(G)$ = set of vertices
 - $E = E(G)$ = set of edges
- **Example:**
 - $V = \{s, u, v, w, x, y, z\}$
 - $E = \{(x,s), (x,v)_1, (x,v)_2, (x,u), (v,w), (s,v), (s,u), (s,w), (s,y), (w,y), (u,y), (u,z), (y,z)\}$



Types of Graphs

1. Null Graph: A **null graph** is a graph in which there are no edges between its vertices.

A null graph is also called empty graph.

Example: The graph shown in fig is a null graph, and the vertices are isolated vertices.

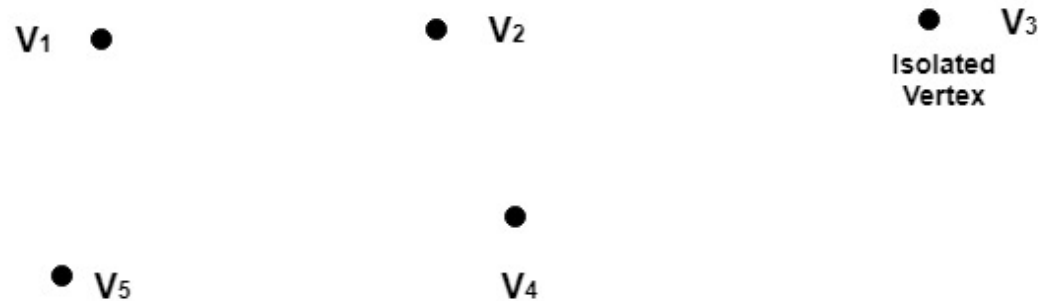
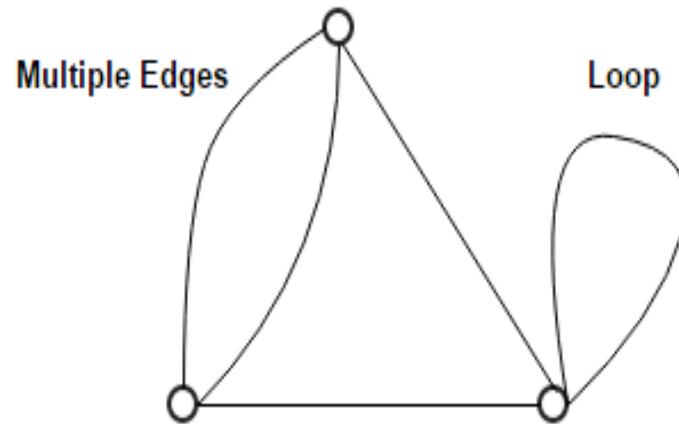


Fig:Null Graph

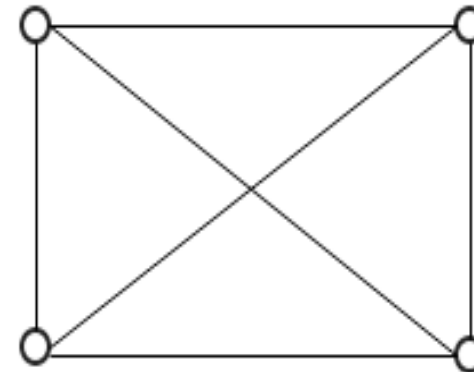
Types of Graphs

2. Simple Graph

A **simple graph** is the undirected graph with **no parallel edges** and **no loops**.



Not a Simple Graph



Simple Graph

Types of Graphs

3. Multigraph: If in a graph multiple edges between the same set of vertices are allowed, it is known as Multigraph.

In other words, it is a graph having at least one loop or multiple edges.

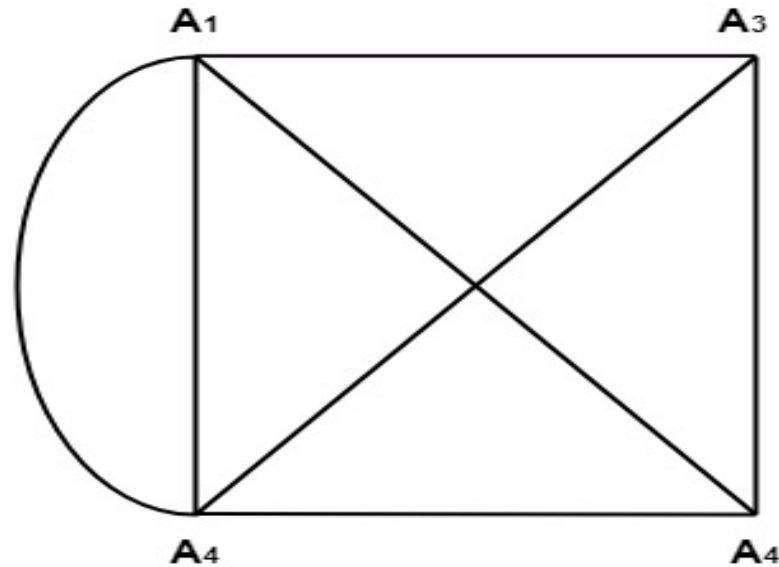
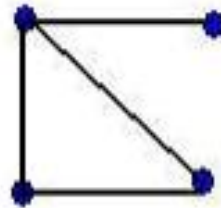


Fig:Multigraph

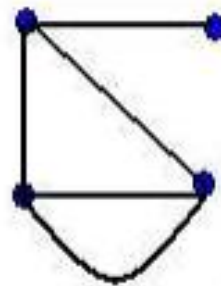
Types of Graphs

4. Pseudograph: A pseudograph is a non-simple graph in which both graph loops and multiple edges are permitted.

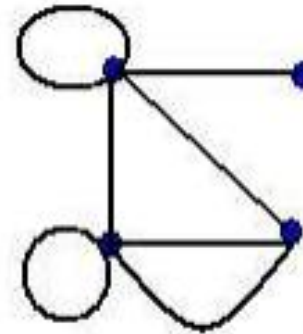
In other words, it is a graph having at least one loop.



simple graph



multigraph



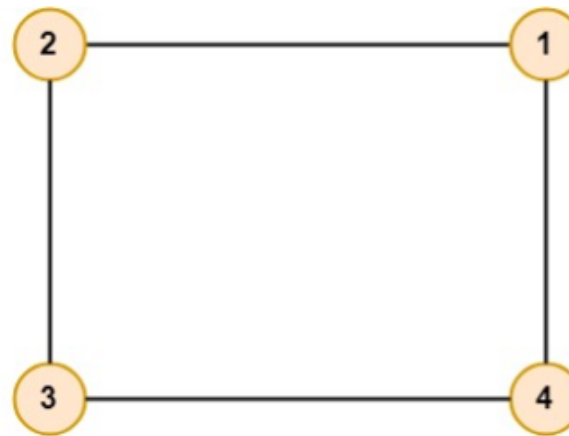
pseudograph

Types of Graphs

5. Undirected Graphs: An Undirected graph G consists of a set of vertices, V and a set of edge E .

An **undirected graph** is a graph whose edges are **not directed**.

Example: Let $V = \{1, 2, 3, 4\}$ and $E = \{(1, 2), (1, 4), (3, 4), (2, 3)\}$. Draw the graph.

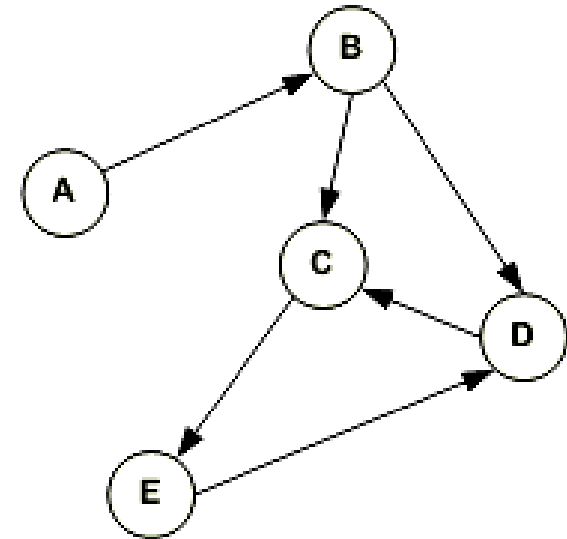


Types of Graphs

5. Directed Graph:

A **directed graph** is a graph in which the **edges are directed** by arrows. Directed graph is also known as **digraphs**.

In the above graph, each edge is directed by the arrow. A directed edge has an arrow from A to B, means A is related to B, but B is not related to A.



Types of Graphs

6. Complete Graph:

- A graph in which every pair of vertices is joined by exactly one edge is called **complete graph**. It contains all possible edges.
- It is represented by K_n .

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K_1

K_2

K_3

K_4

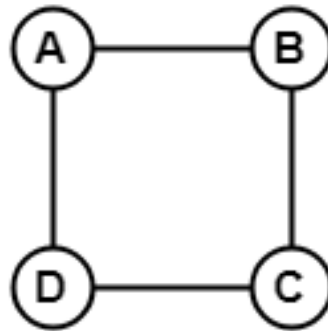
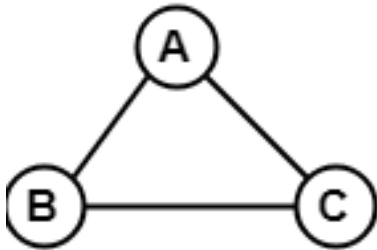
K_5

K_6

Types of Graphs

7. Regular Graph:

- A **Regular graph** is a graph in which degree of all the vertices is same.
- If the degree of all the vertices is k , then it is called k -regular graph.

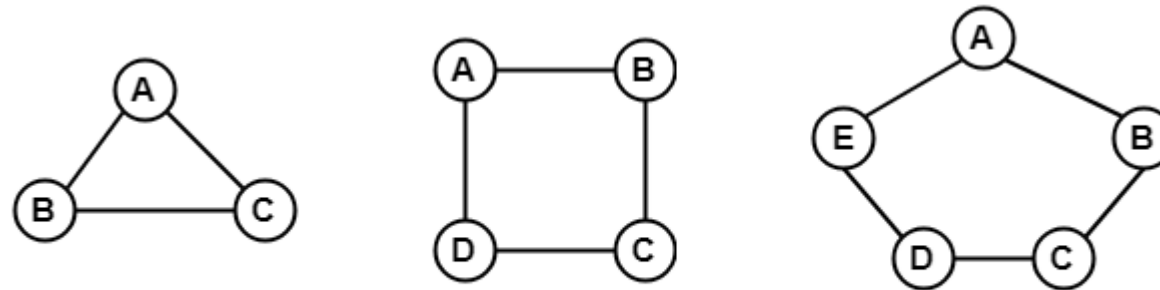


In the above example, all the vertices have degree 2. Therefore they are called 2- **Regular graph**.

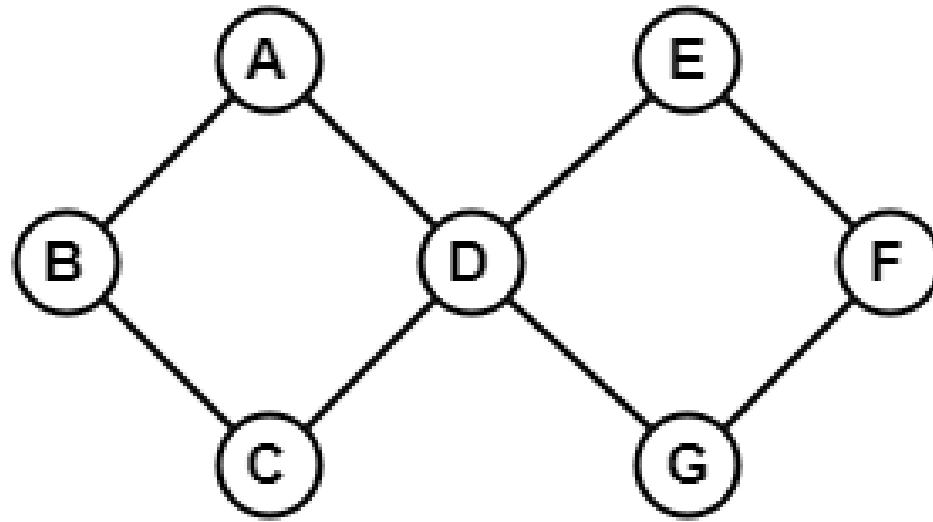
Types of Graphs

8. Cyclic Graph:

- A graph with ' n ' vertices (where, $n \geq 3$) and ' n ' edges forming a cycle of ' n ' with all its edges is known as **cycle graph**.
- A graph containing at least one cycle in it is known as a **cyclic graph**.
- In the cycle graph, degree of each vertex is 2.
- The cycle graph which has n vertices is denoted by C_n .



In the above example, all the vertices have degree 2. Therefore they all are cyclic graphs.

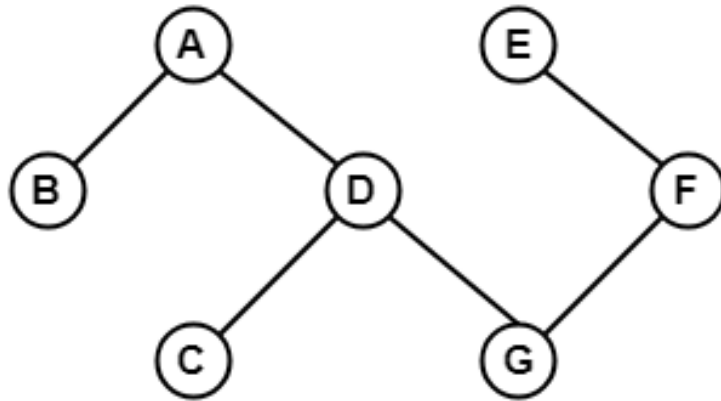


Since, the above graph contains two cycles in it therefore, it is a cyclic graph.

Types of Graphs

9. Acyclic Graph:

A graph which does not contain any cycle in it is called as an **acyclic graph**.



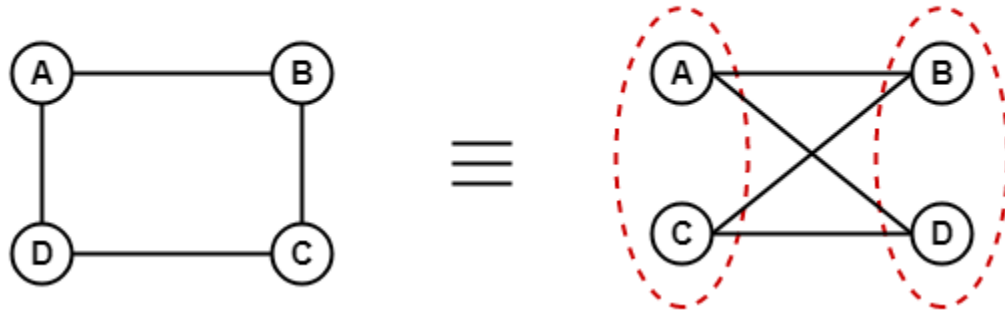
Since, the above graph does not contain any cycle in it therefore, it is an acyclic graph.

Types of Graphs

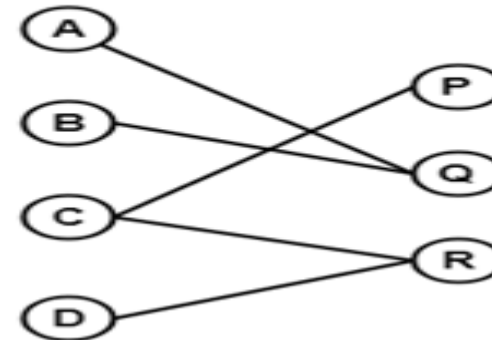
10. Bipartite Graph:

A **bipartite graph** is a graph in which the vertex set can be partitioned into two sets such that edges only go between sets, not within them.

Example 1



Example 2

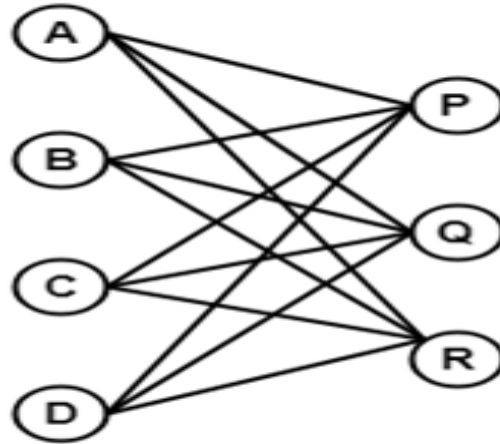


Types of Graphs

11. Complete Bipartite Graph:

A **complete bipartite graph** is a bipartite graph in which each vertex in the first set is joined to each vertex in the second set by exactly one edge.

Example



The above graph is known as $K_{4,3}$.

Types of Graphs

12. Weighted Graph:

A weighted graph is a graph whose edges have been labeled with some weights or numbers.

