Indian Institute of Technology Mandi

IC-252

Mid Semester Exam

Duration: 2 Hours **28.03.2023** **Total: 30 Marks**

1. Let A and B be two events with P(A)>0, P(B|A)=0.3, and $P(A \cap B^c) = 0.2$. Then find P(A). [1 Mark]

2. A system with m components functions if and only if at least one of m components functions. Suppose all the m components of the system functions independently, each with probability 3/4. If the probability of functioning of the system is 63/64, then find the value of m. [1 Mark]

3. Consider a telephone operator who, on average, handles five calls every three minutes. What is the probability there will be no call in next minute? [1 Mark]

4. Let X be the random variable with density function $f(x) = 7e^{-7x}, 0 < x < \infty$. Let Y = 4X+3, then find the density function of Y. [1 Mark]

5. Consider the discrete bivariate random vector with the joint probability mass function is given by
f(10,1) = f(20,1) = f(20,2) = 1/10, f(10,2) = f(10,3) = 1/5, and f(20,3) = 3/10. Show that random variables X and Y are not independent. [1 Mark]

6. Let the joint probability mass function of (X, Y) is given by f(0,10) = f(0,20) = 2/18, f(1,10) = f(1,30) = 3/18, f(1,20) = 4/18, and f(2,30) = 4/18 and zero elsewhere. Then find f(y=10|x=0). [1 Mark]

7. Let A be the partitions of the sample space, and let B be any set. Then, for each i=1,2,3,… write the Bayes' rule. [1 Mark]

8. Write the necessary and sufficient condition for cumulative distribution function. [1 Mark]

9. Suppose we are experiencing an extreme heatwave and there are two weather phenomena that can occur: Heatstroke (H) and Dehydration (D). Let's assume that the probability of experiencing a Heatstroke is 80% and the probability of experiencing Dehydration is 65%. We also know that the probability of

experiencing both Heatstroke and Dehydration is 60%. If someone is experiencing Heatstroke, what is the probability that they are also experiencing Dehydration? [1 Mark]

10. Suppose we have a team of 10 scientists who are conducting research in Antarctica during an extreme cold wave. The team needs to select two scientists from their group to be the lead researcher and the assistant lead researcher. How many ways can this be done? [1 Mark]

11. What is the probability that a committee of 10 people chosen from a group consisting of 40 principals, 35 teachers, and 25 students, will include three principals, five teachers, and two students? [2 Marks]

12. A catalyst producer produces a device for testing defects in a certain electrocatalyst (EC). The catalyst producer claims that the test is 97% reliable if the EC is defective and 99% reliable when it is flawless. However, 4% of said EC may be expected to be defective upon delivery. What is the probability that EC found flawless given that it is tested defective? [2 Marks]

13. Suppose that 5 people, including you and a friend, line up at random. Let the random variable X denote the number of people standing between you and a friend. Determine the probability mass function of X in tabular form. Also, verify that the p.m.f. is a valid p.m.f. [2 Marks]

14. Let Jim and his fiance stay together. Let

C = Event that Jim had Covid
T = Event that Jim tests positive
F = Event that his fiance had Covid
$P(C)= 0.1, \quad P(T|\bar{C})= 0.005, \quad P(\bar{T}|C)= 0, \quad P(F|\bar{C})= 0, \quad P(F|C)=0.95$

Find the probability that Jim had Covid given that Jim tests positive and his fiance had not covid. [2 Marks]

15. How many different ways can the letters of the word TRIANGLE be arranged [2 Marks]

a) If the order of the vowels IAE cannot be changed, though their placement may (IAETRNGL and TRIANGEL are acceptable but EIATRNGL and TRIENGLA are not)?
b) If the order of the vowels IAE can be changed, though their placement may not?

16. The joint probability of random variable X, Y is given below [3 Marks]

$$f_{x,y}(x,y) = \begin{cases} xy\, e^{-\frac{(x^2+y^2)}{2}} & x \geq 0, y \geq 0 \\ 0 & elsewhere \end{cases}$$

(a) Find $f_x(x)$.
(b) Find $f_{Y/X}(y/x)$.

17. The two random variables X and Y have a joint CDF. [3 Marks]

$$F_{x,y}(x,y) = \begin{cases} \left(1-\frac{1}{x^2}\right)\left(1-\frac{1}{y^2}\right) & x \geq 1, y \geq 1 \\ 0 & elsewhere \end{cases}$$

(a) Find the marginal CDF of X and Y.
(b) Are X and Y independent?
(c) Find the probability for {X≤5, Y≤5}.
(d) Using the results obtained in above parts, find probability for {X>3, Y>3}

18. Let Y be a random variable having the density function as

$$f(y; y_0, \beta) = \frac{\beta y_0^{\beta}}{y^{(\beta+1)}} \; if \; y > y_0.$$

Where $\beta > 0, y_0 > 0$. If $X = \log\left(\frac{Y}{y_0}\right)$, then find range of X, density function of X i.e., f(x), P(X>3), and P(X=3). [4 Marks]