Suppose a fair coin is tossed 10 times. What is the probability that we get exactly 5 Heads? What happened for at leat 5 heads or at most 5 heads?

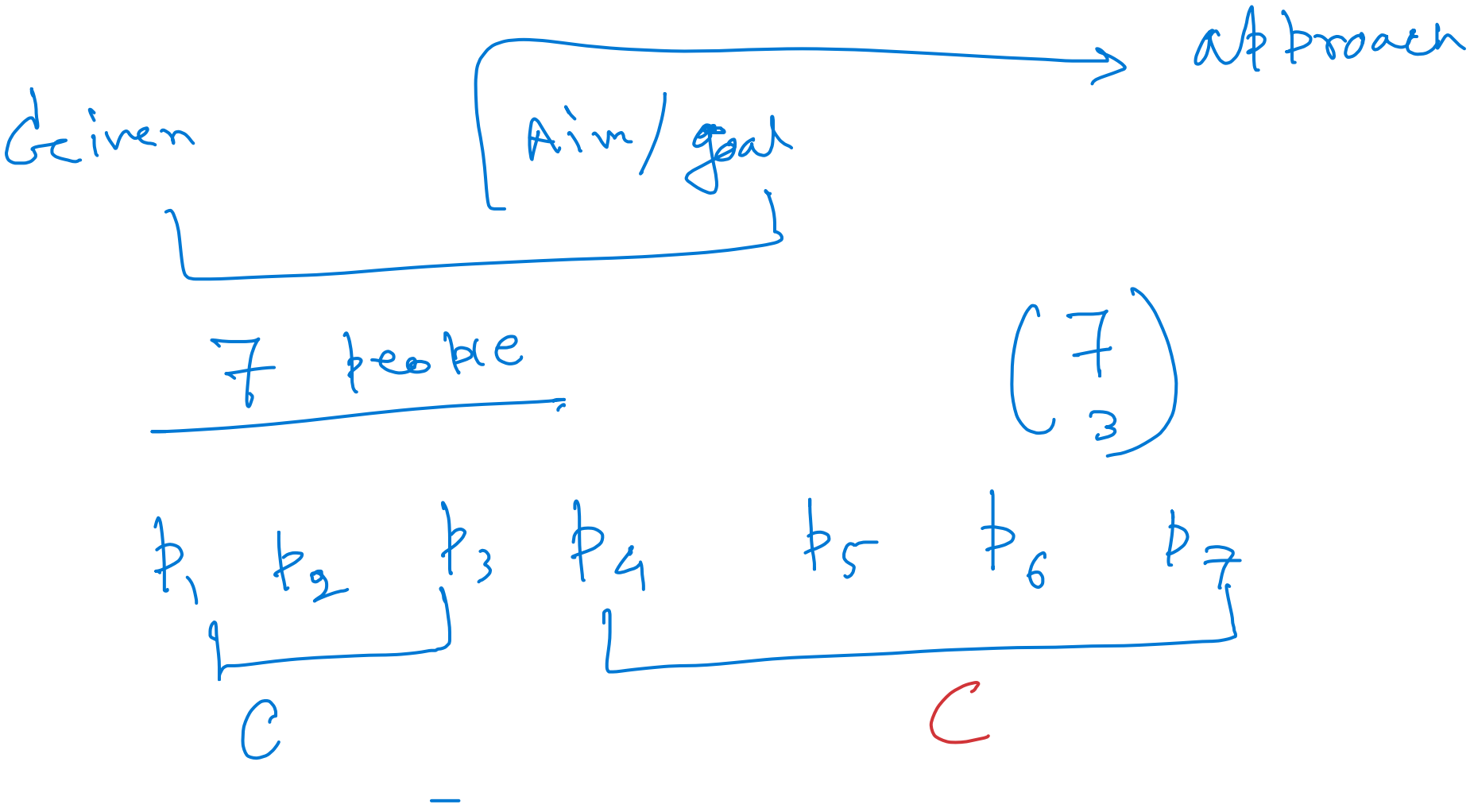
$P(X=5)$ ← $P(X=j) = \binom{n}{j} p^j (1-p)^{n-j}$

$P(X \geq 5)$ → at least → $1 - P(X \leq 4)$

$P(X \leq 5)$ → at most → $P(X \leq x) = \sum_{x} \binom{n}{x} p^x (1-p)^{n-x}$

$1 - P(X > 5)$

Suppose, a middle row on a plane seats 7 people. Three of them order chowmein (C) and the remaining four pasta (P). The flight attendant returns with the meals, but has forgotten who ordered what and discovers that they are all asleep, so she puts the meals in front of them at random. What is the probability that they all receive correct meals?



Sit 3 men and 4 women at random in a row. What is the probability that either all the men or all the women end up sitting together?

E_M E_W

$$P(E_M \cup E_W) = P(E_M) + P(E_W) - P(E_M \cap E_W)$$

A large company with n employees has a scheme according to which each employee buys a Christmas gift and the gifts are then distributed at random to the employees. What is the probability that someone gets his or her own gift?

Read matching problem carefully.

Pick an integer in [1, 1000] at random. Compute the probability that it is divisible neither by 12 nor by 15.

$\#n = \left\lfloor \frac{1000}{n} \right\rfloor$

From an urn containing 5 white and 5 black balls, 5 balls are transferred at random into an empty second urn from which one ball is drawn and it is found to be white. What is the probability that all balls are transferred from the first urn are white?

Bayes theorem

$A_1, A_2, \ldots, A_n$ from some RE E

mutually exclusive & exhaustive, $X \to E$

$P(A_i|X)$ / $P(X)$

$P(X|A_j)$ $\forall j = 1, \ldots, n$ are known to us

$$P(A_i|X) = \frac{P(A_i)\,P(X|A_i)}{\sum_{j=1}^{n} P(A_j)\,P(X|A_j)} = \frac{P(A_i)\,P(X|A_i)}{P(X)}$$

Check whether the followings are PMF?

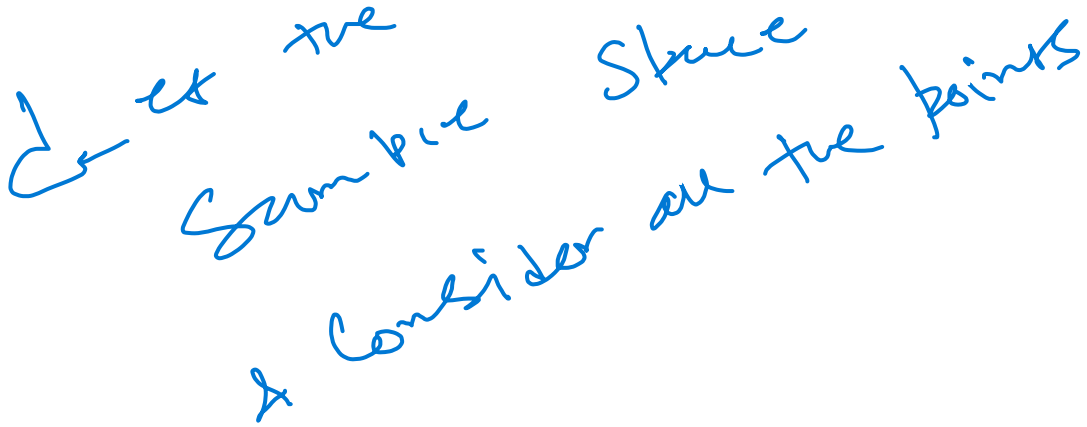
$$f(x) = \begin{cases} 2, & \text{for } x = 1/2 \\ 1, & \text{for } x = 1/4 \\ -1, & \text{for } x = 3/4 \\ 0, & \text{elsewhere} \end{cases}$$

$$g(x) = \begin{cases} 1/8, & \text{for } x = 1 \\ 2/8, & \text{for } x = 2 \\ 3/8, & \text{for } x = 3 \\ 0, & \text{elsewhere} \end{cases}$$

Checking whether $f(x)$ & $g(x)$ are PMF or not).

Flip a fair coin. If you toss Heads, roll 1 die. If you toss Tails, roll 2 dice. Compute the probability that you roll exactly one 6.

An urn contains 11 balls, 3 white, 3 red, and 5 blue balls. Take out 3 balls at random, without replacement. You win $1 for each red ball you select and lose a $1 for each white ball you select. Determine the p. m. f. of X, the amount you win.



Generate Samples

will be updated after mid-term examination.