

**SECTION – I**  
**(INTEGER ANSWER TYPE )**

This section contains 8 questions. The answer is a single digit integer ranging from 0 to 9 (both inclusive).

**Marking scheme +4 for correct answer , 0 if not attempted and 0 in all other cases.**

41. 10 identical balls are to be distributed in 5 different boxes kept in a row and labeled A, B, C, D and E. If the number of ways in which the balls can be distributed in the boxes, such that no two adjacent boxes remain empty is  $k$ , then find the remainder when  $k$  is divided by 10.
42. Sixteen players  $s_1, s_2, \dots, s_{16}$  play in a tournament. They are divided into eight pairs at random. From each pair a winner is decided on the basis of a game played between the two players of the pair. Assume that all the players are of equal strength. Find the greatest integer less than or equal to reciprocal of the probability that “exactly one of the two players  $s_1$  and  $s_2$  is among the eight winners”.
43. Three numbers are chosen at random without replacement from set of first 10 natural numbers. If the probability that the minimum of the chosen numbers is 3 or the maximum of the chosen numbers is 7 is  $\frac{a}{b}$ , where  $a$  and  $b$  are co-prime natural numbers, then number of ciphers at the end in  $(b-a)!$  is
44. There are  $n$  different gift coupons, each of which can be placed in  $N$  ( $N > n$ ) different envelopes, with the same probability  $1/N$ . Consider following:  
 $P_1$ : The probability that there will be one gift coupon in each of  $n$  particular envelopes out of  $N$  given envelopes.  
 $P_2$ : The probability that there will be one gift coupon in each of  $n$  arbitrary envelopes out of  $N$  given envelopes.

Given that each envelope can have any number of coupons.

Find the number of correct statements from the following :

- |                                      |                             |                                       |
|--------------------------------------|-----------------------------|---------------------------------------|
| (i) $P_1 = P_2$                      | (ii) $P_1 = \frac{n!}{N^n}$ | (iii) $P_2 = \frac{N!}{N^n (N - n)!}$ |
| (iv) $P_2 = \frac{n!}{N^n (N - n)!}$ | (v) $P_1 = \frac{N!}{N^n}$  |                                       |

45. A determinant is chosen at random from the set of all determinants of order 2 with elements 0 or 1 only. The probability that the determinant chosen has non-negative value is  $\frac{a}{b}$ , where  $a$  and  $b$  are co-prime natural numbers, then find the number of positive integral factors of  $|a - b|$ .
46. 5 different marbles are placed in 5 different boxes randomly. Given that each box can hold any number of marbles. If  $p$  is the probability that exactly two boxes remain empty, then find the value of  $\left\lceil \frac{1}{p} \right\rceil$ , where  $\lceil . \rceil$  represents GIF.
47.  $m$  red socks and  $n$  blue socks ( $m > n$ ) in a cupboard are well mixed up, where  $m + n \leq 101$ . Let two socks are taken out at random and the probability that they have the same colour is  $\frac{1}{2}$ . If the largest value of  $m$  is  $k$ , then find the remainder obtained, when  $k$  is divided by 8.
48. Integers  $a, b, c$  and  $d$  not necessarily distinct, are chosen independently and at random from the set  $S = \{1, 2, 3, \dots, 2006, 2007, 2008\}$ . If the probability that  $|ad - bc|$  is even (zero included) is  $\frac{p}{q}$ , where  $p$  and  $q$  are relatively prime, then find the sum of digits of  $(p + q)$ .

## SECTION – II

### (MULTIPLE CORRECT ANSWER TYPE)

This section contains 8 multiple choice questions. Each question has 4 options (A), (B), (C) and (D) for its answer, out of which ONE OR MORE than ONE option can be correct.

**Marking scheme: +4 for correct answer, 0 if not attempted and -2 in all other cases.**

49. The number of ways in which 8 non-identical apples can be distributed among 3 boys, such that every boy should get atleast 1 apple and atmost 4 apples is  $(K \cdot {}^7P_3)$ , then sum of all prime divisors of  $K$  is greater than or equal to
- A) 5                      B) 11                      C) 13                      D) 17

50. The number of ordered triplets  $(p, q, r)$ , where  $1 \leq p, q, r \leq 10$ , such that  $(2^p + 3^q + 5^r)$  is a multiple of 4, is  $k$ , then  $k$  is a factor of (where  $p, q, r \in \mathbb{N}$ )
- A)  $60^5$                       B)  $150^3$                       C)  $36^{10}$                       D)  $20^{10}$
51. Let  $mn$  distinct coins have been distributed into  $m$  different purses,  $n$  coins into each purse. If  $p$  is probability that two specified coins will be found in the same purse, then
- A)  $p < \frac{n+1}{mn-1}$               B)  $p > \frac{n+1}{mn-1}$               C)  $p > \frac{n-1}{mn+1}$               D)  $p < \frac{n-1}{mn+1}$
52. An unbiased dice with faces numbered 1, 2, 3, 4, 5 and 6 is thrown 'n' times and the list of 'n' numbers showing up is noted. If the probability that among the numbers 1, 2, 3, 4, 5 and 6 only three numbers appear in the list is  $\frac{20}{6^n} f(n)$ , then
- A)  $f(4) = 36$               B)  $f(4) = 72$               C)  $f(5) = 75$               D)  $f(5) = 150$
53. Consider all functions  $f : \{1, 2, 3, 4\} \longrightarrow \{1, 2, 3, 4\}$  which are one-one, onto and satisfy the following property: "If  $f(k)$  is odd, then  $f(k+1)$  is even,  $k = 1, 2, 3$ ". If  $n$  is the number of such functions, then  $n$  is divisible by
- A) 2                      B) 3                      C) 4                      D) 5
54. Suppose that there are 5 red points and 4 blue points on a circle. If  $\frac{m}{n}$  (where  $m$  and  $n$  are relatively prime) be the probability that a convex polygon whose vertices are among the 9 points has at least one blue vertex, then  $(n-m)$  is less than or equal to
- A) 5                      B) 50                      C) 100                      D) 200
55. A is a set containing  $n$  distinct elements. A non-zero subset  $P$  of  $A$  is chosen at random. The set  $A$  is reconstructed by replacing the elements of  $P$ . A non-zero subset  $Q$  of  $A$  is again chosen at random. If  $f(n)$  is the probability that  $P$  and  $Q$  have no common elements, then
- A)  $f(4) = \frac{2}{9}$               B)  $f(5) = \frac{180}{961}$               C)  $f(4) = \frac{4}{9}$               D)  $f(5) = \frac{360}{961}$

56. A coin is tossed  $(m + n)$  times, where  $(m > n)$ . If  $f(m, n)$  be the probability of getting “atleast  $m$  consecutive heads”, then

A)  $f(10, 6) = \frac{1}{256}$     B)  $f(10, 4) = \frac{3}{1024}$     C)  $f(9, 5) = \frac{7}{1024}$     D)  $f(9, 6) = \frac{1}{128}$

**SECTION – III**  
**(PARAGRAPH TYPE)**

This section contains **2 groups of questions**. Each group has 2 multiple choice questions based on a paragraph. Each question has 4 choices A), B), C) and D) for its answer, out of which **ONE OR MORE** is/are correct.

**Marking scheme: +4 for correct answer, 0 if not attempted and -2 in all other cases.**

**Paragraph for Question Nos. 57 to 58:**

Suppose  $E_1, E_2, E_3$  be three mutually exclusive events such that  $P(E_i) = p_i$ , where  $0 < p_i < 1$  for  $i = 1, 2, 3$

57. If  $p_1 = \frac{1}{2}(1 - p)$ ,  $p_2 = \frac{1}{3}(1 + 2p)$  and  $p_3 = \frac{1}{5}(2 + 3p)$ , then which of the following intervals is contained in the set of values of  $p$  :

A)  $\left(-\frac{1}{2}, 0\right)$     B)  $\left(-\frac{1}{3}, \frac{2}{3}\right)$     C)  $\left(-\frac{1}{2}, \frac{7}{23}\right)$     D)  $\left(-\frac{2}{5}, -\frac{1}{3}\right)$

58. If  $p_1, p_2, p_3$  are the roots of equation  $27x^3 - 27x^2 + ax - 1 = 0$ , then value of  $a$  is
- A) 9    B) 6    C) 3    D) 12

**Paragraph for Question Nos. 59 to 60:**

Box 1 contains three cards bearing numbers 1, 2, 3; box 2 contains five cards bearing numbers 1, 2, 3, 4, 5 and box 3 contains seven cards bearing numbers 1, 2, 3, 4, 5, 6, 7. A card is drawn from each of the boxes. Let  $x_i$  be the number on the card drawn from the  $i^{\text{th}}$  box,  $i = 1, 2, 3$ .

59. The probability that  $x_1 + x_2 + x_3$  is odd, is

A)  $\frac{29}{105}$     B)  $\frac{53}{105}$     C)  $\frac{57}{105}$     D)  $\frac{1}{2}$

60. The probability that  $x_1, x_2, x_3$  are in an arithmetic progression is  $\frac{k}{105}$ , then sum of digits of ‘ $k$ ’ is divisible by

A) 2    B) 3    C) 5    D) 11