Machine Learning with R and Python

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About

This book is a companion to the Machine Learning Class. It will be a repository of R and Python Code. The chapters will be updated as we progress through the class.

1 Introduction

1.1 Introduction to Machine Learning

Machine Learning (ML) is a branch of artificial intelligence (AI) that focuses on building systems that **learn patterns from data** and make predictions or decisions without being explicitly programmed to perform specific tasks. Instead of writing rules by hand, a machine learning algorithm "learns" from examples in the data.

1.2 What is Machine Learning?

Formally, machine learning is the process of using data to **train a model** to make accurate predictions or decisions. The model identifies patterns in the training data and generalizes these patterns to new, unseen data.

ML can be thought of as:

- **Predictive modeling** – learning a function that maps inputs (features) to outputs (targets).
- **Automated decision-making** – systems that improve their performance over time without explicit reprogramming.

1.3 Major Types of Machine Learning

Machine learning is commonly categorized into several types based on the **nature of the task** and **availability of labeled data**.

1.3.1 1. Supervised Learning

- Uses **labeled data**, meaning that each training example has an input and a known output.
- The goal is to **learn a function** that maps inputs to outputs accurately.
- Typical tasks:
  - **Regression**: Predict a continuous variable (e.g., house prices).
  - **Classification**: Predict a categorical variable (e.g., spam vs. non-spam email).

1.3.2 2. Unsupervised Learning

- Works with **unlabeled data**, where the output is unknown.
- The goal is to **discover patterns or structure** in the data.
- Typical tasks:
  - **Clustering**: Group similar observations together (e.g., customer segmentation).
  - **Dimensionality reduction**: Reduce the number of features while retaining important information (e.g., PCA).

1.3.3 3. Semi-Supervised Learning

- Combines a small amount of labeled data with a large amount of unlabeled data.
- Useful when labeling data is expensive or time-consuming.

1.3.4 4. Reinforcement Learning

- Focuses on **learning through trial and error**.
- An agent learns to take actions in an environment to **maximize cumulative reward**.
- Examples: Robotics, game AI, recommendation systems.

1.4 Objectives of Machine Learning

The main objective of machine learning is to **build models that generalize well** from historical data to make accurate predictions or decisions on **new, unseen data**. Key considerations include:

1. **Accuracy** – How well does the model predict outcomes?

2. **Generalization** – Does the model perform well on unseen data, or is it overfitting the training data?
3. **Interpretability** – Can humans understand how the model makes predictions?

1.5 Bias-Variance Tradeoff

A fundamental concept in machine learning is the **bias-variance tradeoff**, which explains the balance between:

- **Bias**: Error due to overly simplistic models that **underfit** the data.
- **Variance**: Error due to overly complex models that **overfit** the training data.

The goal is to find a model that **minimizes the total prediction error**:

[Total Error = Bias^2 + Variance + Irreducible Error]

- **Underfitting** → High bias, low variance
- **Overfitting** → Low bias, high variance

The optimal model balances bias and variance for the best predictive performance.

1.6 The Machine Learning Process

A typical workflow in machine learning consists of the following steps:

1. **Data Collection**
   - Gather data from experiments, databases, or external sources.
2. **Data Preprocessing**
   - Handle missing values, outliers, and feature engineering.
3. **Model Selection**
   - Choose an appropriate algorithm (e.g., linear regression, random forest, neural networks).
4. **Model Training**

- Fit the model to the training data.

5. **Model Evaluation**
   - Assess performance using metrics such as RMSE, accuracy, precision, recall, or AUC.
6. **Model Tuning**
   - Adjust hyperparameters to improve performance.
7. **Model Deployment**
   - Use the trained model to make predictions on new data.
8. **Monitoring and Maintenance**
   - Continuously track model performance and retrain if necessary.

1.7 Summary

Machine learning allows us to build predictive models that **learn from data**. Understanding the different types of ML, their objectives, and the tradeoffs between bias and variance is critical to applying ML effectively. The remainder of this book explores practical algorithms, their implementation in **R and Python**, and strategies for model selection and interpretation.

2 Model Selection and Regularization - R Version

2.1 Overview

In this section, we'll use the **Ames Housing** dataset to demonstrate model selection and regularization.

We'll cover:

1. Splitting data into training and testing sets
2. Performing **stepwise regression** with cross-validation
3. Running **Ridge** and **Lasso** regression, and visualizing how affects RMSE and model complexity
4. Exploring **Elastic Net**, varying to balance Ridge and Lasso
5. Comparing all models on the test set

2.2 Step 0: Setup and Data Preparation

We'll use a reduced set of variables for speed and clarity.

```
# Suppress messages and warnings globally
knitr::opts_chunk$set(echo = TRUE, message = FALSE, warning = FALSE)

library(tidyverse)
library(caret)
library(glmnet)
library(AmesHousing)
```

```r
library(GGally)

set.seed(123)

# Load data and split into 70% training, 30% testing
ames <- make_ordinal_ames() %>% mutate(id = row_number())
train <- ames %>% sample_frac(0.7)
test  <- anti_join(ames, train, by = "id")

# Select a manageable subset of predictors
keep <- c("Sale_Price", "Bedroom_AbvGr", "Year_Built", "Mo_Sold", "Lot_Area",
          "Street", "Central_Air", "First_Flr_SF", "Second_Flr_SF", "Full_Bath",
          "Half_Bath", "Fireplaces", "Garage_Area", "Gr_Liv_Area", "TotRms_AbvGrd")

train <- train %>% select(all_of(keep))
test  <- test  %>% select(all_of(keep))

# Convert categorical variables to factors
train <- train %>% mutate(across(c(Street, Central_Air), as.factor))
test  <- test %>% mutate(across(c(Street, Central_Air), as.factor))
```

2.3 Step 1: Stepwise Regression with 10-Fold Cross Validation

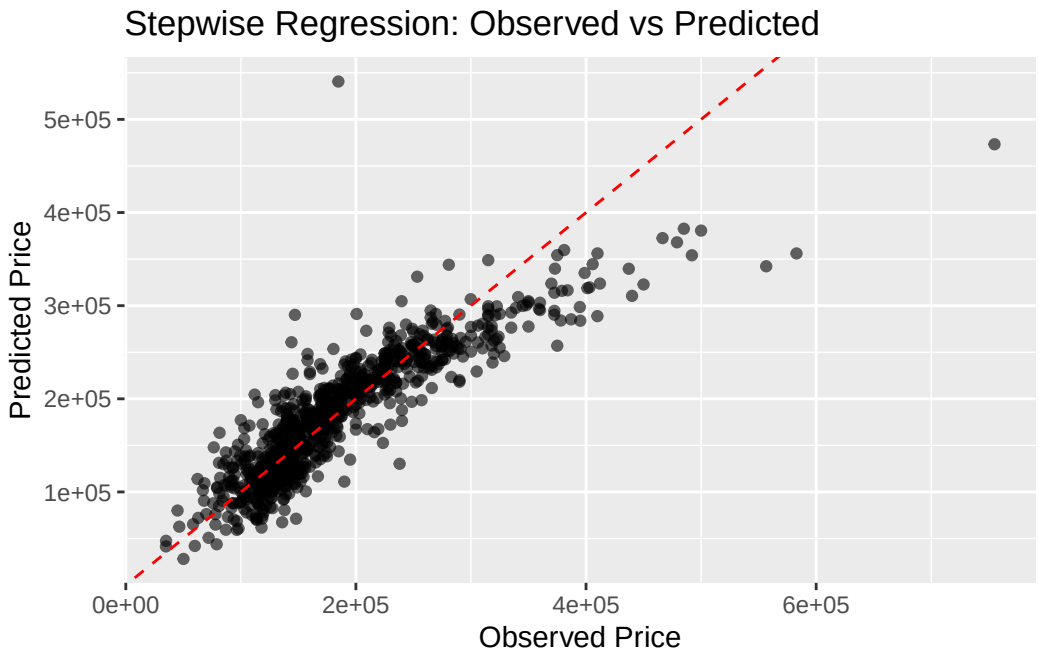
We'll use backward stepwise regression as a traditional benchmark.

```r
ctrl <- trainControl(method = "cv", number = 10)

step_model <- train(
  Sale_Price ~ ., data = train,
  method = "lmStepAIC",
  trControl = ctrl,
  trace = FALSE,
  direction = "backward"
)

# Evaluate on the test set
step_pred <- predict(step_model, newdata = test)
```

```r
step_perf <- postResample(step_pred, test$Sale_Price)
step_perf
```

```
        RMSE     Rsquared          MAE
3.820776e+04 7.670269e-01 2.613098e+04
```

```r
# Visualize predicted vs observed
ggplot(data.frame(obs = test$Sale_Price, pred = step_pred), aes(obs, pred)) +
  geom_point(alpha = 0.6) +
  geom_abline(linetype = "dashed", color = "red") +
  labs(title = "Stepwise Regression: Observed vs Predicted",
       x = "Observed Price", y = "Predicted Price")
```



Note:
Stepwise regression is intuitive and fast, but it can be unstable.

Stepwise regression (forward, backward, or both) selects predictors one at a time based on how much they improve a criterion (like AIC or adjusted R^2).

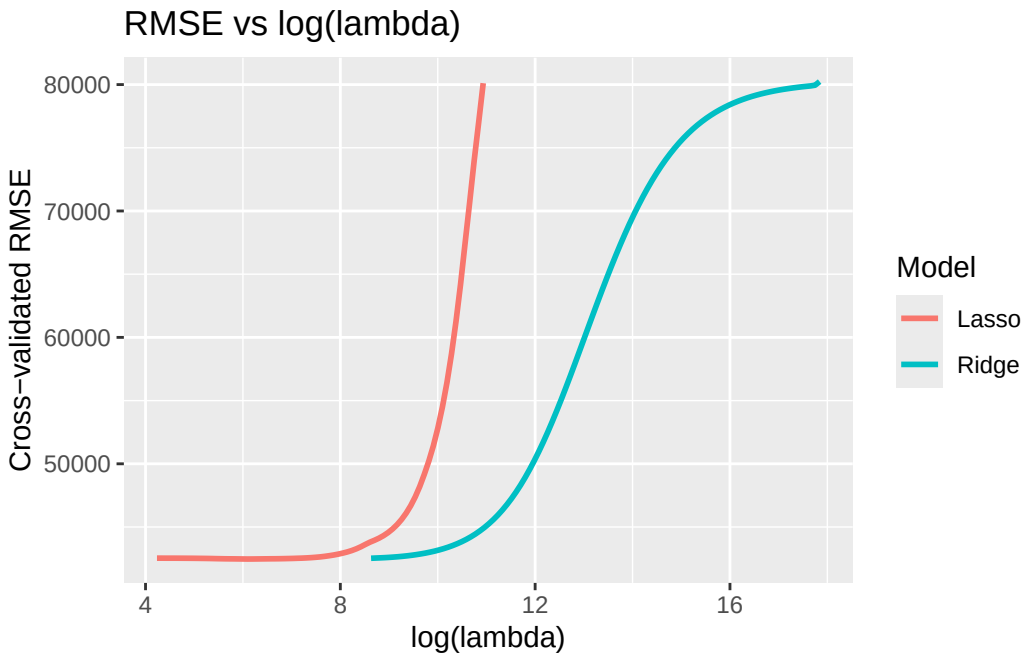
2.4 Step 2: Ridge and Lasso Regression

Ridge and Lasso both shrink coefficients, but in different ways: - Ridge shrinks all coefficients toward zero. - Lasso can set some coefficients *exactly* to zero, performing variable selection.

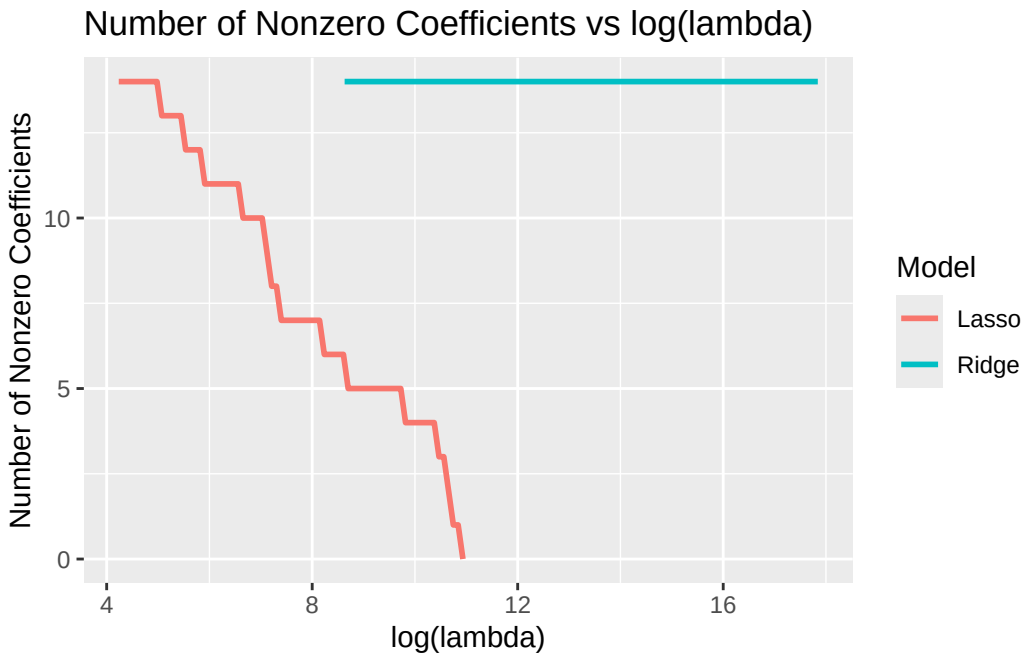
```
x_train <- model.matrix(Sale_Price ~ ., train)[, -1]
y_train <- train$Sale_Price
x_test  <- model.matrix(Sale_Price ~ ., test)[, -1]
y_test  <- test$Sale_Price

set.seed(123)
cv_ridge <- cv.glmnet(x_train, y_train, alpha = 0, nfolds = 10)
cv_lasso <- cv.glmnet(x_train, y_train, alpha = 1, nfolds = 10)
```

2.4.1 Visualizing RMSE and Model Complexity

```
build_path_df <- function(cvfit, label) {
  fit <- cvfit$glmnet.fit
  tibble(
    lambda = fit$lambda,
    log_lambda = log(fit$lambda),
    RMSE = sqrt(cvfit$cvm),
    nonzero = colSums(abs(fit$beta) > 0),
    Model = label
  )
}

ridge_df <- build_path_df(cv_ridge, "Ridge")
lasso_df <- build_path_df(cv_lasso, "Lasso")
df <- bind_rows(ridge_df, lasso_df)

ggplot(df, aes(log_lambda, RMSE, color = Model)) +
  geom_line(size = 1) +
  labs(title = "RMSE vs log(lambda)", y = "Cross-validated RMSE", x = "log(lambda)")
```




```
ggplot(df, aes(log_lambda, nonzero, color = Model)) +
  geom_line(size = 1) +
  labs(title = "Number of Nonzero Coefficients vs log(lambda)",
       y = "Number of Nonzero Coefficients", x = "log(lambda)")
```



Note:
- Increasing increases regularization.
- Ridge never eliminates variables, Lasso can.
- There's a sweet spot where RMSE is minimized.

2.5 Step 3: Elastic Net (Balancing Ridge and Lasso)

Elastic Net introduces to control the mix between Ridge (= 0) and Lasso (= 1).

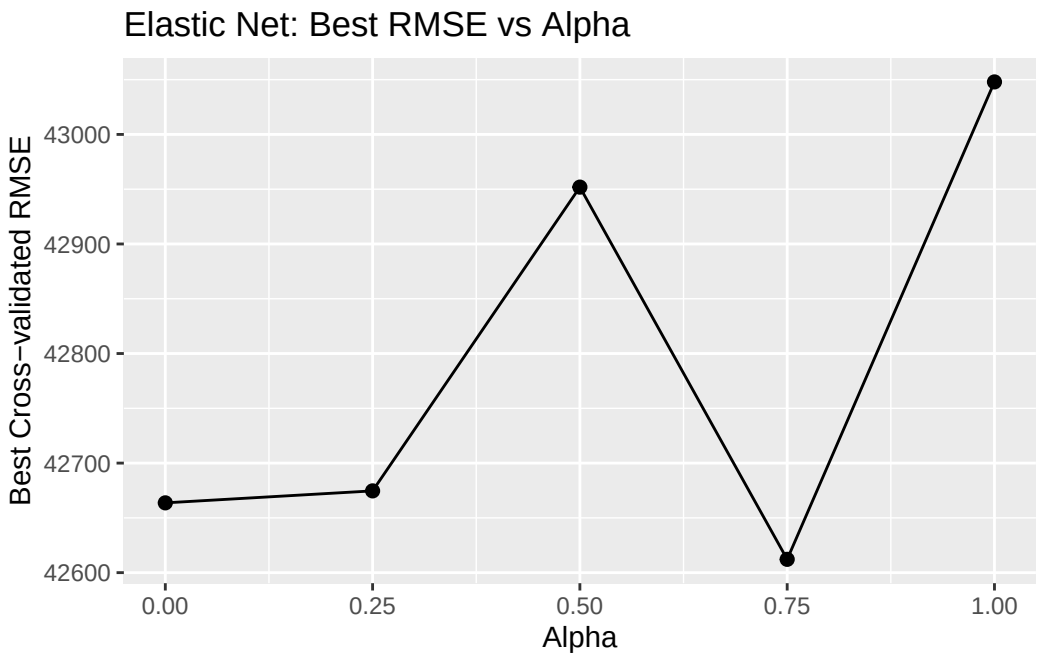
```
alpha_grid <- seq(0, 1, by = 0.25)

elastic_results <- map_df(alpha_grid, function(a) {
  cv_fit <- cv.glmnet(x_train, y_train, alpha = a, nfolds = 10)
  tibble(
    alpha = a,
    best_lambda = cv_fit$lambda.min,
    best_RMSE = sqrt(min(cv_fit$cvm)),
    nonzero = sum(abs(coef(cv_fit, s = "lambda.min")[-1]) > 0)
  )
```

```
})

elastic_results
```

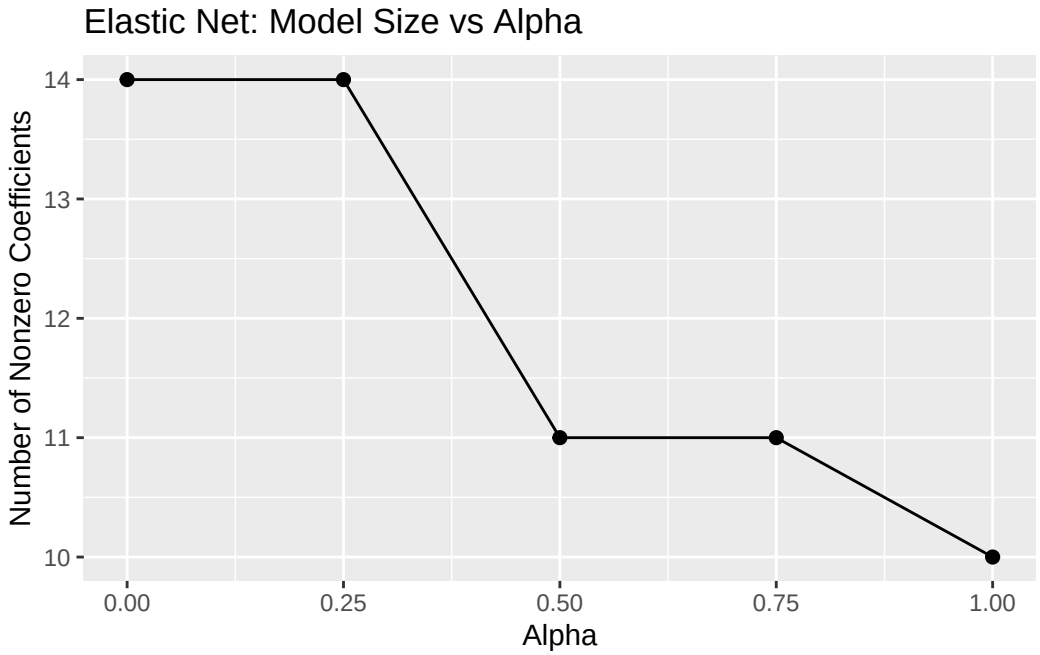
```
# A tibble: 5 x 4
  alpha best_lambda best_RMSE nonzero
  <dbl>       <dbl>     <dbl>   <int>
1  0          5603.    42664.      14
2  0.25       1224.    42675.      14
3  0.5        1552.    42952.      11
4  0.75        650.    42612.      11
5  1          1126.    43048.      10
```

2.5.1 Visualizing Effects

```
ggplot(elastic_results, aes(alpha, best_RMSE)) +
  geom_line() + geom_point(size = 2) +
  labs(title = "Elastic Net: Best RMSE vs Alpha",
       y = "Best Cross-validated RMSE", x = "Alpha")
```



```r
ggplot(elastic_results, aes(alpha, nonzero)) +
  geom_line() + geom_point(size = 2) +
  labs(title = "Elastic Net: Model Size vs Alpha",
       y = "Number of Nonzero Coefficients", x = "Alpha")
```



Note:
Elastic Net combines the strengths of Ridge and Lasso: - Ridge for stability. - Lasso for sparsity. - Often performs best at intermediate values.

2.6 Step 4: Comparing All Models on the Test Set

We'll compare Stepwise, Ridge, Lasso, and the best Elastic Net.

```r
ridge_pred <- predict(cv_ridge, newx = x_test, s = "lambda.min")
lasso_pred <- predict(cv_lasso, newx = x_test, s = "lambda.min")

best_alpha <- elastic_results %>% arrange(best_RMSE) %>% slice(1) %>% pull(alpha)
cv_en_best <- cv.glmnet(x_train, y_train, alpha = best_alpha, nfolds = 10)
elastic_pred <- predict(cv_en_best, newx = x_test, s = "lambda.min")
```

```r
compare_tbl <- bind_rows(
  Stepwise = as_tibble_row(postResample(step_pred, test$Sale_Price)),
  Ridge = as_tibble_row(postResample(ridge_pred, y_test)),
  Lasso = as_tibble_row(postResample(lasso_pred, y_test)),
  Elastic = as_tibble_row(postResample(elastic_pred, y_test)),
  .id = "Model"
)

colnames(compare_tbl) <- c("Model", "RMSE", "Rsquared", "MAE")
compare_tbl <- compare_tbl %>% arrange(RMSE)
compare_tbl
```

```
# A tibble: 4 x 4
  Model      RMSE Rsquared    MAE
  <chr>     <dbl>    <dbl>  <dbl>
1 Stepwise 38208.    0.767 26131.
2 Lasso    38297.    0.767 26045.
3 Elastic  38301.    0.767 26039.
4 Ridge    38412.    0.766 26022.
```

```r
# ------------------------------------------------------------

# Display the best cross-validated Lasso, Ridge, and Elastic Net models


# ------------------------------------------------------------

library(glmnet)
library(knitr)

# Helper function to summarize a fitted cv.glmnet model

summarize_glmnet <- function(cvfit, X_train, y_train, X_test, y_test, model_name) {
  cat("\n==========================================\n")
  cat("Model:", model_name, "\n")
  cat("==========================================\n")

  # Extract best lambda
  best_lambda <- cvfit$lambda.min
```

```r
  cat("Best Lambda (lambda.min):", round(best_lambda, 6), "\n")

  # Coefficients (non-zero)
  coefs <- as.matrix(coef(cvfit, s = "lambda.min"))
  nonzero <- coefs[coefs != 0, , drop = FALSE]

  # Print nonzero coefficients
  cat("\nNonzero Coefficients:\n")
  print(knitr::kable(as.data.frame(nonzero), digits = 4))

  # Predictions and performance
  pred_train <- predict(cvfit, newx = X_train, s = "lambda.min")
  pred_test  <- predict(cvfit, newx = X_test,  s = "lambda.min")

  rmse <- function(y, yhat) sqrt(mean((y - yhat)^2))
  rsq  <- function(y, yhat) 1 - sum((y - yhat)^2) / sum((y - mean(y))^2)

  train_rmse <- rmse(y_train, pred_train)
  test_rmse  <- rmse(y_test, pred_test)
  test_r2    <- rsq(y_test, pred_test)

  # Return one-row summary
  data.frame(
    Model = model_name,
    Lambda = round(best_lambda, 6),
    Train_RMSE = round(train_rmse, 2),
    Test_RMSE = round(test_rmse, 2),
    Test_R2 = round(test_r2, 3),
    Nonzero_Coeff = nrow(nonzero)
  )
}

# Summarize and display all three models
results <- rbind(
  summarize_glmnet(cv_lasso, x_train, y_train, x_test, y_test, "Lasso"),
  summarize_glmnet(cv_ridge, x_train, y_train, x_test, y_test, "Ridge"),
  summarize_glmnet(cv_en_best, x_train, y_train, x_test, y_test, "Elastic Net")
)
```

```
==========================================
Model: Lasso
```

```
==========================================
Best Lambda (lambda.min): 444.0297

Nonzero Coefficients:


|              |    lambda.min|
|:-------------|-------------:|
|(Intercept)   | -1536712.4502|
|Bedroom_AbvGr |   -12871.7248|
|Year_Built    |      786.0097|
|Lot_Area      |        0.3320|
|StreetPave    |    14782.6929|
|Central_AirY  |     3463.4742|
|First_Flr_SF  |       22.9872|
|Half_Bath     |    -1509.7005|
|Fireplaces    |    11647.3233|
|Garage_Area   |       62.2549|
|Gr_Liv_Area   |       74.6167|
|TotRms_AbvGrd |     1301.9029|

==========================================
Model: Ridge
==========================================
Best Lambda (lambda.min): 5603.019

Nonzero Coefficients:


|              |    lambda.min|
|:-------------|-------------:|
|(Intercept)   | -1398127.9251|
|Bedroom_AbvGr |   -13051.9562|
|Year_Built    |      706.9534|
|Mo_Sold       |      105.9352|
|Lot_Area      |        0.4001|
|StreetPave    |    22908.4375|
|Central_AirY  |     6951.0203|
|First_Flr_SF  |       45.3168|
|Second_Flr_SF |       21.9030|
|Full_Bath     |     4203.4612|
|Half_Bath     |      -26.1950|
|Fireplaces    |    12666.1199|
```

```
|Garage_Area   |       64.5695|
|Gr_Liv_Area   |       42.3845|
|TotRms_AbvGrd |     3067.4622|

==========================================
Model: Elastic Net
==========================================
Best Lambda (lambda.min): 592.0396

Nonzero Coefficients:


|              |    lambda.min|
|:-------------|-------------:|
|(Intercept)   | -1532932.2050|
|Bedroom_AbvGr |   -12808.7530|
|Year_Built    |      783.9250|
|Lot_Area      |        0.3319|
|StreetPave    |    14821.6671|
|Central_AirY  |     3497.9772|
|First_Flr_SF  |       23.2544|
|Half_Bath     |    -1319.3051|
|Fireplaces    |    11686.0135|
|Garage_Area   |       62.4230|
|Gr_Liv_Area   |       73.9778|
|TotRms_AbvGrd |     1388.0576|
```

```r
# Combined performance table
cat("\n\n***Model Performance Comparison***\n")
```

```
***Model Performance Comparison***
```

```r
kable(results, caption = "Performance of Cross-Validated Regularized Models")
```

Table 2.1: Performance of Cross-Validated Regularized Models

Model	Lambda	Train_RMSE	Test_RMSE	Test_R2	Nonzero_Coeff
Lasso	444.0297	41745.42	38296.94	0.764	12

Model	Lambda	Train_RMSE	Test_RMSE	Test_R2	Nonzero_Coeff
Ridge	5603.0194	41830.59	38411.78	0.763	15
Elastic Net	592.0396	41748.11	38300.92	0.764	12

2.7 Key Takeaways

- Stepwise selection is simple but can lead to unstable models.
- Ridge adds stability via coefficient shrinkage.
- Lasso enforces sparsity by removing weak predictors.
- Elastic Net balances both effects.
- Regularization often produces models that generalize better than purely stepwise ones.

3 Model Selection and Regularization - Python Version

3.1 Overview

This section parallels the R version of Model Selection and Regularization.
We will:

1. Load the Ames Housing data
2. Split into training and test sets
3. Perform Ridge, Lasso, and Elastic Net regressions with 10-fold CV
4. Visualize RMSE and coefficient shrinkage patterns
5. Compare test set performance

3.2 Step 0: Setup and Data

We'll use scikit-learn for modeling.
If the CSVs `ames_training.csv` and `ames_testing.csv` are not present, you can load Ames Housing via the `fetch_openml` function.

```
import warnings
warnings.filterwarnings("ignore")

import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from sklearn.datasets import fetch_openml
from sklearn.model_selection import train_test_split, KFold, cross_val_score
```

```
from sklearn.preprocessing import OneHotEncoder, StandardScaler
from sklearn.compose import ColumnTransformer
from sklearn.pipeline import Pipeline
from sklearn.linear_model import RidgeCV, LassoCV, ElasticNetCV
from sklearn.metrics import mean_squared_error

# Load Ames dataset directly
ames = fetch_openml(name="house_prices", as_frame=True)
df = ames.frame

# Subset variables to match the R version
keep = [
    "SalePrice","BedroomAbvGr","YearBuilt","MoSold","LotArea","Street","CentralAir",
    "1stFlrSF","2ndFlrSF","FullBath","HalfBath","Fireplaces","GarageArea",
    "GrLivArea","TotRmsAbvGrd"
]
df = df[keep].copy()

# Clean up names to match Python variable rules
df.columns = ["Sale_Price","Bedroom_AbvGr","Year_Built","Mo_Sold","Lot_Area","Street",
              "Central_Air","First_Flr_SF","Second_Flr_SF","Full_Bath","Half_Bath",
              "Fireplaces","Garage_Area","Gr_Liv_Area","TotRms_AbvGrd"]

# Drop missing rows
df = df.dropna()

# Train/test split (70/30)
train, test = train_test_split(df, test_size=0.3, random_state=123)

# Identify predictors and target
y_train = train["Sale_Price"]
y_test  = test["Sale_Price"]
X_train = train.drop(columns="Sale_Price")
X_test  = test.drop(columns="Sale_Price")

cat_vars = ["Street","Central_Air"]
num_vars = [c for c in X_train.columns if c not in cat_vars]

# Preprocess: scale numeric, one-hot encode categorical
preprocessor = ColumnTransformer([
    ("num", StandardScaler(), num_vars),
    ("cat", OneHotEncoder(drop="first"), cat_vars)
```

```
])
```

3.3 Step 1: Ridge Regression

We'll use cross-validation to tune (alpha).
Then we visualize RMSE vs log(lambda) and the number of nonzero coefficients.

```
alphas = np.logspace(4, -4, 80)
ridge = Pipeline([
    ("prep", preprocessor),
    ("model", RidgeCV(alphas=alphas, scoring="neg_mean_squared_error", cv=10))
])
ridge.fit(X_train, y_train)

ridge_best = ridge.named_steps["model"].alpha_
print("Best Ridge alpha:", ridge_best)

# Evaluate RMSE on test set
ridge_pred = ridge.predict(X_test)
ridge_rmse = np.sqrt(mean_squared_error(y_test, ridge_pred))
print("Test RMSE (Ridge):", ridge_rmse)
```

```
Best Ridge alpha: 94.33732216299774
Test RMSE (Ridge): 37055.84511170759
```

3.4 Step 2: Lasso Regression

Lasso adds variable selection — some coefficients go exactly to zero.

```
lasso = Pipeline([
    ("prep", preprocessor),
    ("model", LassoCV(alphas=alphas, cv=10, max_iter=20000))
])
lasso.fit(X_train, y_train)
```

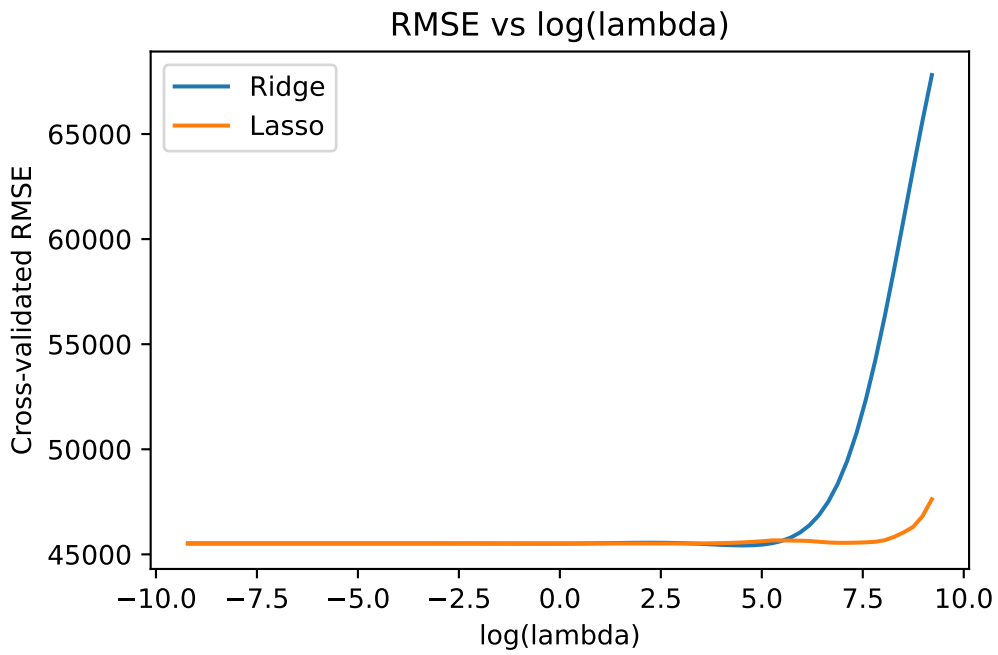
```
lasso_best = lasso.named_steps["model"].alpha_
print("Best Lasso alpha:", lasso_best)

lasso_pred = lasso.predict(X_test)
lasso_rmse = np.sqrt(mean_squared_error(y_test, lasso_pred))
print("Test RMSE (Lasso):", lasso_rmse)
```

```
Best Lasso alpha: 11.568875283162821
Test RMSE (Lasso): 37074.74978063246
```

3.5 Step 3: Visualize Coefficient Shrinkage and RMSE

Let's visualize how RMSE and model sparsity change with .

```
# Compute RMSE path manually
def cv_rmse(model_class, X, y, alphas):
    rmse = []
    for a in alphas:
        model = Pipeline([
            ("prep", preprocessor),
            ("model", model_class(alpha=a, max_iter=20000))
        ])
        scores = cross_val_score(model, X, y, scoring="neg_mean_squared_error", cv=KFold(10,
        rmse.append(np.sqrt(-scores.mean()))
    return np.array(rmse)

from sklearn.linear_model import Ridge, Lasso
ridge_rmse_path = cv_rmse(Ridge, X_train, y_train, alphas)
lasso_rmse_path = cv_rmse(Lasso, X_train, y_train, alphas)

plt.figure()
plt.plot(np.log(alphas), ridge_rmse_path, label="Ridge")
plt.plot(np.log(alphas), lasso_rmse_path, label="Lasso")
plt.xlabel("log(lambda)")
plt.ylabel("Cross-validated RMSE")
plt.title("RMSE vs log(lambda)")
plt.legend()
plt.show()
```



3.6 Step 4: Elastic Net

Elastic Net blends Ridge and Lasso.
We'll vary (the mix) and visualize the tradeoff.

```
l1_ratios = np.linspace(0, 1, 6)
en_cv = Pipeline([
    ("prep", preprocessor),
    ("model", ElasticNetCV(l1_ratio=l1_ratios, alphas=alphas, cv=10, max_iter=50000))
])
en_cv.fit(X_train, y_train)

print("Best Elastic Net alpha:", en_cv.named_steps["model"].alpha_)
print("Best Elastic Net l1_ratio:", en_cv.named_steps["model"].l1_ratio_)

en_pred = en_cv.predict(X_test)
en_rmse = np.sqrt(mean_squared_error(y_test, en_pred))
print("Test RMSE (Elastic Net):", en_rmse)
```

```
Best Elastic Net alpha: 0.55825862688627
Best Elastic Net l1_ratio: 0.8
Test RMSE (Elastic Net): 37134.497858434275
```

3.7 Step 5: Compare Models

```python
results = pd.DataFrame({
    "Model": ["Ridge", "Lasso", "Elastic Net"],
    "Test_RMSE": [ridge_rmse, lasso_rmse, en_rmse]
})
print(results.sort_values("Test_RMSE"))
```

```
         Model     Test_RMSE
0        Ridge  37055.845112
1        Lasso  37074.749781
2  Elastic Net  37134.497858
```

3.8 Summary

- Ridge stabilizes coefficients by shrinking them.
- Lasso enforces sparsity by zeroing weak predictors.
- Elastic Net blends both for balance.
- Regularization often beats pure stepwise regression on unseen data.

4 GAM – Piecewise, MARS, LOESS, and Splines - R Version

Goal: Predict `Sale_Price` from a focused set of Ames features using increasingly flexible methods in the GAM framework.

4.1 1. Setup and Data

```
set.seed(4321)

library(tidyverse)
library(dplyr)
library(tidyr)
library(AmesHousing)   # make_ordinal_ames()
library(earth)         # MARS
library(segmented)     # piecewise (segmented) regression
library(splines)       # regression splines (bs, ns) - kept for reference
library(mgcv)          # smoothing splines via GAM
library(caret)         # data splitting / utilities
library(Metrics)       # rmse, mse helpers (plus we'll compute R2 manually)
theme_set(theme_minimal())
```

```
# Limit to requested columns
cols <- c("Sale_Price","Bedroom_AbvGr","Year_Built","Mo_Sold","Lot_Area","Street",
          "Central_Air","First_Flr_SF","Second_Flr_SF","Full_Bath","Half_Bath",
          "Fireplaces","Garage_Area","Gr_Liv_Area","TotRms_AbvGrd")

ames <- make_ordinal_ames() |>
  dplyr::select(dplyr::all_of(cols)) |>
  tidyr::drop_na()

# Train/test split
set.seed(4321)
```

```r
idx <- sample.int(nrow(ames), size = floor(0.7*nrow(ames)))
train <- ames[idx,]
test  <- ames[-idx,]

glimpse(train)
```

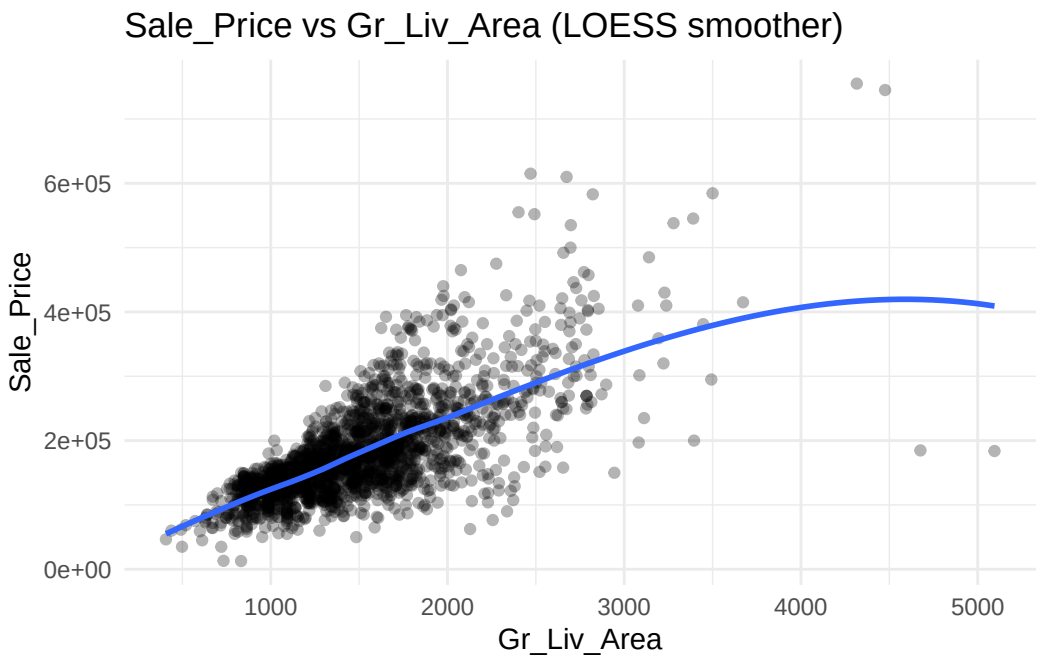
```
Rows: 2,051
Columns: 15
$ Sale_Price    <int> 274000, 75200, 329900, 145400, 108000, 184000, 176000, 1~
$ Bedroom_AbvGr <int> 3, 2, 4, 3, 2, 2, 3, 3, 3, 3, 3, 3, 3, 4, 3, 2, 3, 3, 2,~
$ Year_Built    <int> 2001, 1922, 2005, 1926, 1949, 1999, 1962, 1915, 1999, 19~
$ Mo_Sold       <int> 1, 9, 8, 5, 5, 6, 5, 6, 9, 6, 8, 5, 7, 7, 4, 8, 5, 11, 5~
$ Lot_Area      <int> 9720, 3672, 11643, 7000, 8777, 5858, 19296, 8094, 3768, ~
$ Street        <fct> Pave, Pave, Pave, Pave, Pave, Pave, Pave, Pave, Pave, Pa~
$ Central_Air   <fct> Y, Y, Y, Y, Y, Y, Y, Y, Y, Y, Y, Y, Y, Y, Y, Y, Y, Y, Y,~
$ First_Flr_SF  <int> 1366, 816, 1544, 861, 1126, 1337, 1382, 1048, 713, 792, ~
$ Second_Flr_SF <int> 581, 0, 814, 424, 0, 0, 0, 720, 739, 725, 0, 1151, 695, ~
$ Full_Bath     <int> 2, 1, 2, 1, 2, 2, 1, 2, 2, 1, 1, 2, 2, 2, 1, 2, 2, 1, 1,~
$ Half_Bath     <int> 1, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 0, 1, 0, 1, 0, 0,~
$ Fireplaces    <int> 1, 0, 1, 0, 0, 1, 1, 0, 0, 2, 0, 1, 2, 1, 0, 1, 0, 0, 1,~
$ Garage_Area   <dbl> 725, 100, 784, 506, 520, 511, 884, 576, 506, 400, 0, 434~
$ Gr_Liv_Area   <int> 1947, 816, 2358, 1285, 1126, 1337, 1382, 1768, 1452, 151~
$ TotRms_AbvGrd <int> 7, 5, 10, 6, 5, 5, 6, 8, 6, 7, 5, 8, 7, 5, 6, 7, 6, 6, 5~
```

```r
summary(train$Sale_Price)
```

```
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  12789  130000  161000  180897  215000  755000
```

4.2 2. Quick Visual: Nonlinearity is Common

```r
ggplot(train, aes(Gr_Liv_Area, Sale_Price)) +
  geom_point(alpha=.3) +
  geom_smooth(method="loess", se=FALSE) +
  labs(title="Sale_Price vs Gr_Liv_Area (LOESS smoother)")
```



Takeaway: The relationship bends—nonlinear methods can help.

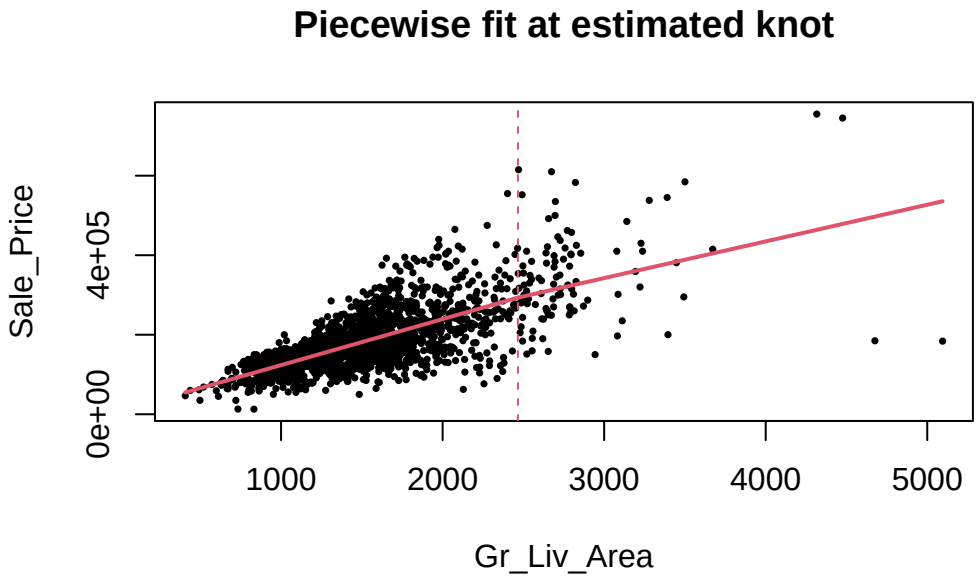
4.3 3. Piecewise Regression

```
m_lin <- lm(Sale_Price ~ Gr_Liv_Area, data=train)
m_seg_init <- lm(Sale_Price ~ Gr_Liv_Area, data=train)
m_seg <- segmented(m_seg_init, seg.Z = ~ Gr_Liv_Area, psi = list(Gr_Liv_Area = 2000))
summary(m_seg)
```

```
    ***Regression Model with Segmented Relationship(s)***

Call:
segmented.lm(obj = m_seg_init, seg.Z = ~Gr_Liv_Area, psi = list(Gr_Liv_Area = 2000))

Estimated Break-Point(s):
                  Est.  St.Err
psi1.Gr_Liv_Area 2466 332.881
```

```
Coefficients of the linear terms:
               Estimate Std. Error t value Pr(>|t|)
(Intercept)    7901.186   4698.641   1.682   0.0928 .
Gr_Liv_Area     115.705      3.171  36.488   <2e-16 ***
U1.Gr_Liv_Area  -23.479     12.436  -1.888       NA
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 55110 on 2047 degrees of freedom
Multiple R-Squared: 0.5124,  Adjusted R-squared: 0.5116

Boot restarting based on 6 samples. Last fit:
Convergence attained in 1 iterations (rel. change 0)
```

```r
plot(train$Gr_Liv_Area, train$Sale_Price, pch=16, cex=.5,
     xlab="Gr_Liv_Area", ylab="Sale_Price", main="Piecewise fit at estimated knot")
plot(m_seg, add=TRUE, col=2, lwd=2)
abline(v = m_seg$psi[,"Est."], lty=2, col=2)
```



Interpretation (piecewise): Two linear slopes before/after a knot. Useful when you expect a threshold effect.

4.4 4. MARS (earth) – From Simple to Interactions

MARS = *Multivariate Adaptive Regression Splines*. It builds **hinge** basis functions like `h(x - c) = max(0, x - c)` and can **interact** them.
- A term such as `h(Gr_Liv_Area - 1500)` means *once* `Gr_Liv_Area` exceeds 1500, the fitted line's slope can change.
- An interaction like `h(Gr_Liv_Area - 1500) * Central_AirY` means the slope change applies when `Central_Air == "Y"`.

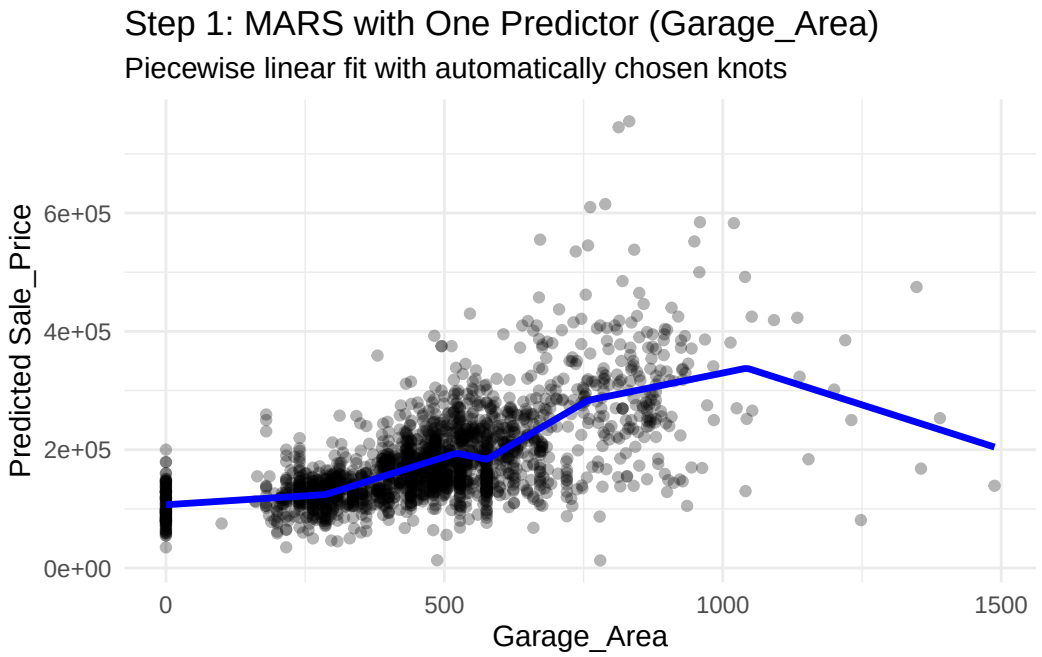
4.4.1 4A. Univariate MARS (Garage_Area)

```
mars1 <- earth(Sale_Price ~ Garage_Area, data=train)
summary(mars1)
```

```
Call: earth(formula=Sale_Price~Garage_Area, data=train)

                    coefficients
(Intercept)           124159.039
h(286-Garage_Area)       -60.257
h(Garage_Area-286)       297.277
h(Garage_Area-521)      -483.642
h(Garage_Area-576)       733.859
h(Garage_Area-758)      -356.460
h(Garage_Area-1043)     -490.873

Selected 7 of 7 terms, and 1 of 1 predictors
Termination condition: RSq changed by less than 0.001 at 7 terms
Importance: Garage_Area
Number of terms at each degree of interaction: 1 6 (additive model)
GCV 3427475346    RSS 6.94092e+12    GRSq 0.4492014    RSq 0.4556309
```

```
grid <- tibble(Garage_Area = seq(min(train$Garage_Area), max(train$Garage_Area), length.out=1
grid$pred <- predict(mars1, newdata=grid)

ggplot(train, aes(Garage_Area, Sale_Price)) +
  geom_point(alpha=.3) +
  geom_line(data=grid, aes(Garage_Area, pred), color="blue", linewidth=1.2) +
```

```
  labs(title="Step 1: MARS with One Predictor (Garage_Area)",
       subtitle="Piecewise linear fit with automatically chosen knots",
       y="Predicted Sale_Price")
```



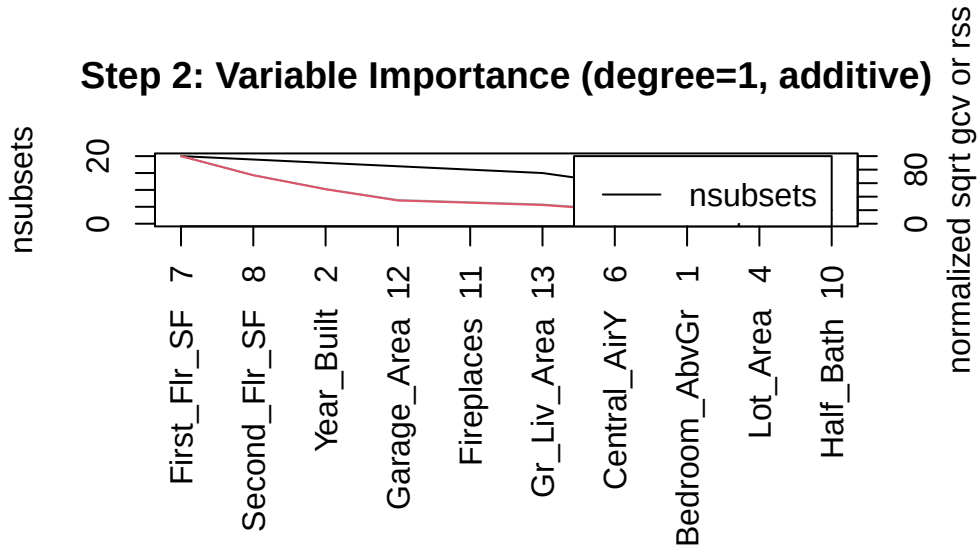
4.4.2 4B. Additive MARS (degree = 1; no interactions)

```
mars2 <- earth(Sale_Price ~ Bedroom_AbvGr + Year_Built + Mo_Sold + Lot_Area +
                 Street + Central_Air + First_Flr_SF + Second_Flr_SF + Full_Bath +
                 Half_Bath + Fireplaces + Garage_Area + Gr_Liv_Area + TotRms_AbvGrd,
               data=train, degree=1, nfold=5)
summary(mars2)
```

```
Call: earth(formula=Sale_Price~Bedroom_AbvGr+Year_Built+Mo_Sold+Lot_Ar...),
            data=train, degree=1, nfold=5)

                      coefficients
(Intercept)             319493.46
Central_AirY             20289.49
h(4-Bedroom_AbvGr)        9214.66
h(Bedroom_AbvGr-4)      -23009.05
```

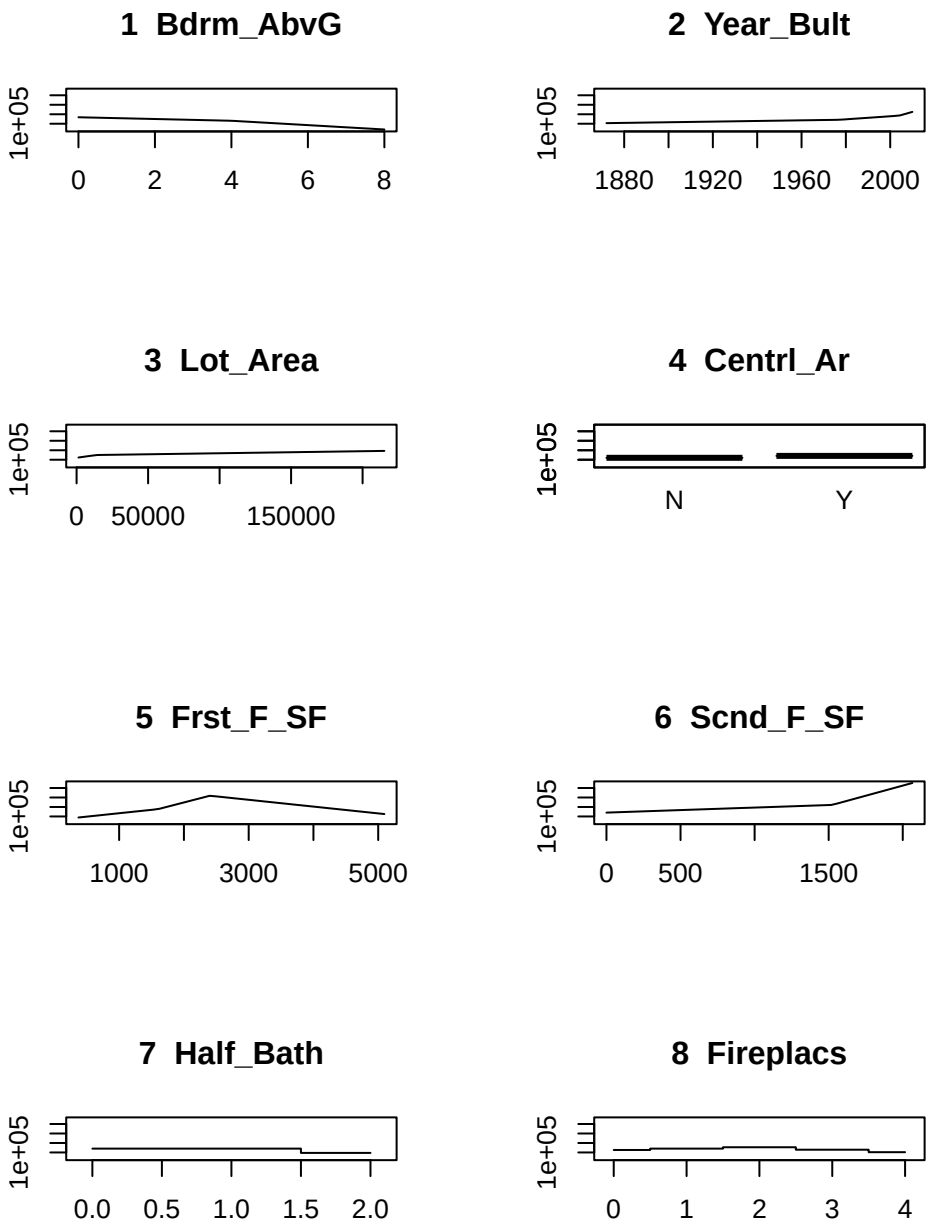
```
h(Year_Built-1977)         1275.57
h(2004-Year_Built)         -336.64
h(Year_Built-2004)         5315.57
h(13869-Lot_Area)            -2.09
h(Lot_Area-13869)             0.22
h(First_Flr_SF-1600)        104.91
h(2402-First_Flr_SF)        -71.56
h(First_Flr_SF-2402)       -176.61
h(1523-Second_Flr_SF)       -53.13
h(Second_Flr_SF-1523)       426.63
h(Half_Bath-1)           -45378.31
h(2-Fireplaces)          -14408.56
h(Fireplaces-2)          -26072.58
h(Garage_Area-539)           101.97
h(Garage_Area-1043)         -294.30
h(Gr_Liv_Area-2049)           65.21
h(Gr_Liv_Area-3194)         -159.79

Selected 21 of 24 terms, and 10 of 14 predictors
Termination condition: Reached nk 29
Importance: First_Flr_SF, Second_Flr_SF, Year_Built, Garage_Area, ...
Number of terms at each degree of interaction: 1 20 (additive model)
GCV 1033819964  RSS 2.036439e+12  GRSq 0.8338641  RSq 0.8402842  CVRSq 0.7978671

Note: the cross-validation sd's below are standard deviations across folds

Cross validation:   nterms 21.00 sd 0.71    nvars 9.20 sd 0.84

     CVRSq    sd     MaxErr     sd
     0.798 0.043    -428941 282429
```

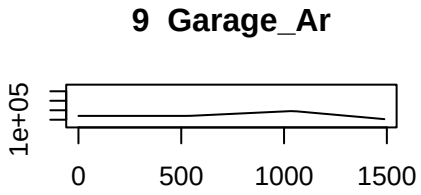
```r
ev2 <- evimp(mars2); plot(ev2, main="Step 2: Variable Importance (degree=1, additive)")
```

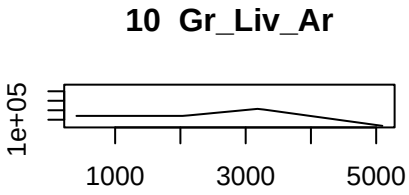



```
par(mfrow=c(2,2)); plotmo(mars2, type="response", nresponse=1, do.par=FALSE); par(mfrow=c(1,1
```

```
plotmo grid:    Bedroom_AbvGr Year_Built Mo_Sold Lot_Area Street Central_Air
                            3       1974       6     9480   Pave           Y
First_Flr_SF Second_Flr_SF Full_Bath Half_Bath Fireplaces Garage_Area
        1092             0         2         0          1         480
Gr_Liv_Area TotRms_AbvGrd
       1442             6
```


1 Bdrm_AbvG
1e+05
0
2
4
6
8
2 Year_Built
1e+05
1880
1920
1960
2000
3 Lot_Area
1e+05
0
50000
150000
4 Centrl_Ar
1e+05
N
Y
5 Frst_F_SF
1e+05
1000
3000
5000
6 Scnd_F_SF
1e+05
0
500
1500
7 Half_Bath
1e+05
0.0
0.5
1.0
1.5
2.0
8 Fireplacs
1e+05
0
1
2
3
4





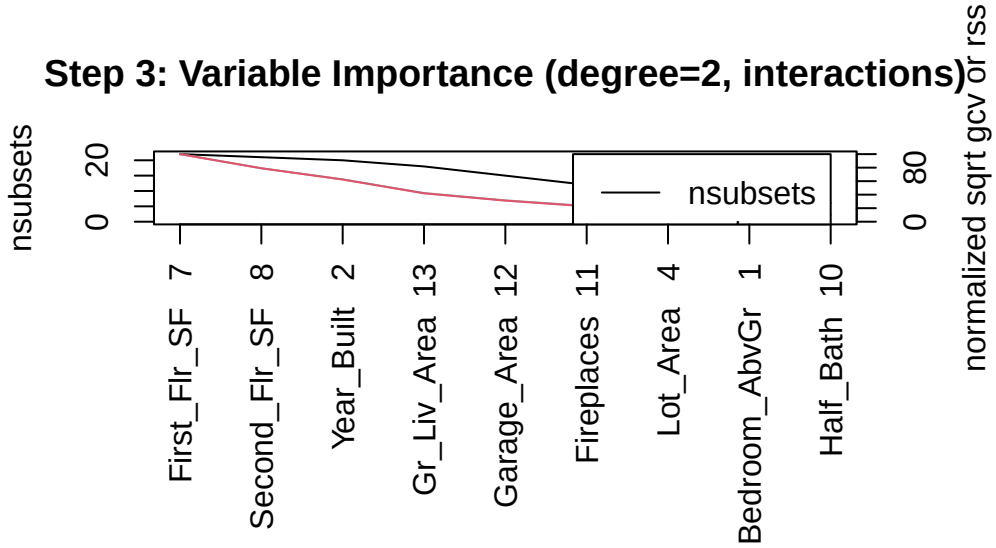
4.4.3 4C. MARS with 2-way Interactions (degree = 2)

```
mars3 <- earth(Sale_Price ~ Bedroom_AbvGr + Year_Built + Mo_Sold + Lot_Area +
                 Street + Central_Air + First_Flr_SF + Second_Flr_SF + Full_Bath +
                 Half_Bath + Fireplaces + Garage_Area + Gr_Liv_Area + TotRms_AbvGrd,
               data=train, degree=2, nfold=5)
summary(mars3)
```

```
Call: earth(formula=Sale_Price~Bedroom_AbvGr+Year_Built+Mo_Sold+Lot_Ar...),
            data=train, degree=2, nfold=5)

                                        coefficients
(Intercept)                               411949.27
h(Year_Built-2004)                        169869.37
h(14803-Lot_Area)                             -1.75
h(Lot_Area-14803)                              0.48
h(1570-First_Flr_SF)                        -150.79
h(First_Flr_SF-1570)                         224.90
h(1523-Second_Flr_SF)                        -86.91
h(Second_Flr_SF-1523)                        156.02
h(1-Half_Bath)                             -7743.19
```

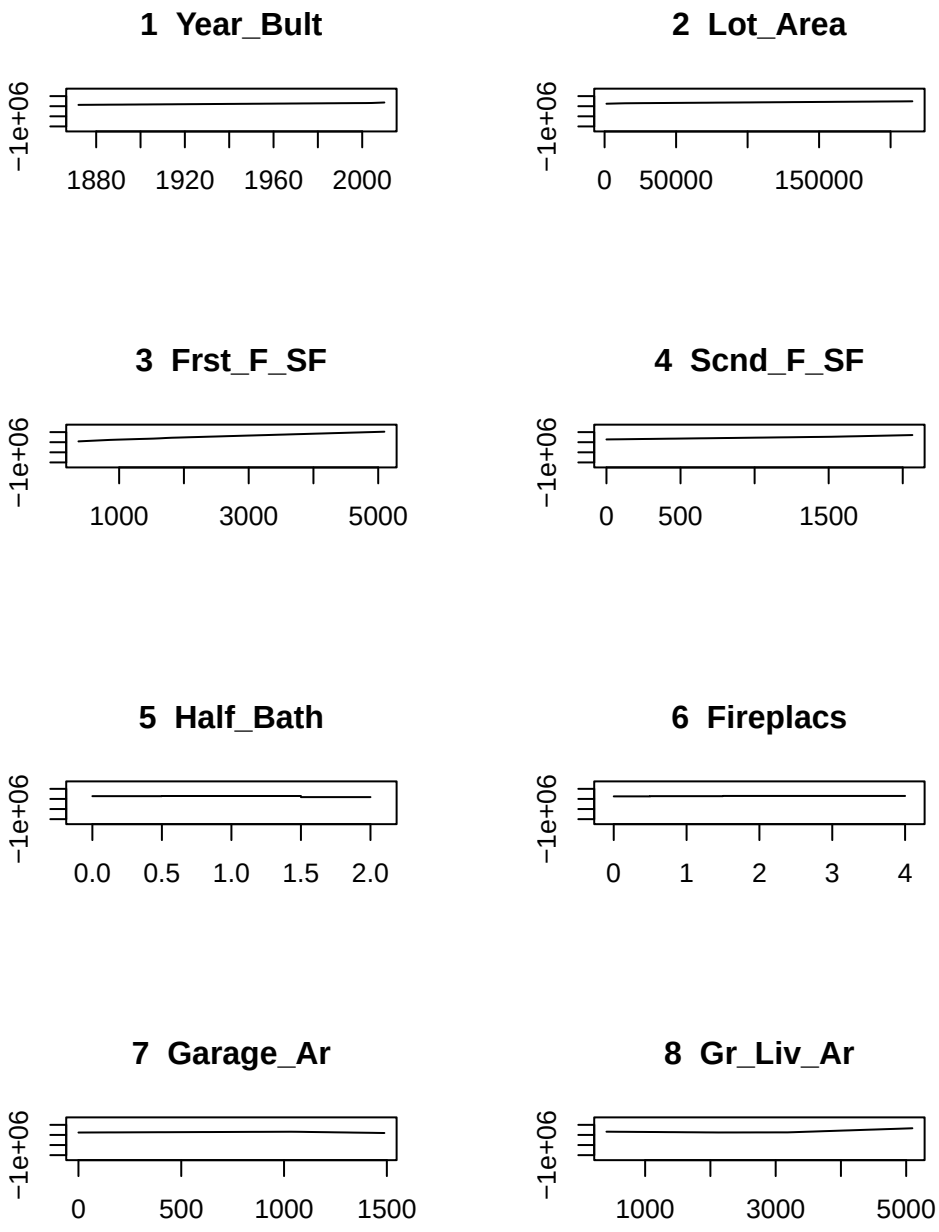
```
h(Half_Bath-1)                                  -54819.24
h(2-Fireplaces)                                 -10516.92
h(1043-Garage_Area)                                -70.97
h(Garage_Area-1043)                               -141.86
h(Gr_Liv_Area-3194)                                106.27
h(3-Bedroom_AbvGr) * h(1523-Second_Flr_SF)           7.01
h(Bedroom_AbvGr-3) * h(1523-Second_Flr_SF)          -9.61
h(2004-Year_Built) * h(First_Flr_SF-876)            -1.86
h(2004-Year_Built) * h(3194-Gr_Liv_Area)            -0.17
h(Year_Built-2004) * h(Gr_Liv_Area-2320)          -159.80
h(Year_Built-2004) * h(2320-Gr_Liv_Area)           156.64
h(Year_Built-2004) * h(3194-Gr_Liv_Area)          -173.05
h(First_Flr_SF-1778) * h(2-Fireplaces)             -78.19
h(1043-Garage_Area) * h(2122-Gr_Liv_Area)            0.05

Selected 23 of 29 terms, and 9 of 14 predictors
Termination condition: Reached nk 29
Importance: First_Flr_SF, Second_Flr_SF, Year_Built, Gr_Liv_Area, ...
Number of terms at each degree of interaction: 1 13 9
GCV 795873462  RSS 1.544416e+12  GRSq 0.8721024  RSq 0.8788731  CVRSq 0.7824362

Note: the cross-validation sd's below are standard deviations across folds

Cross validation:   nterms 22.60 sd 1.82    nvars 9.40 sd 1.14

     CVRSq    sd     MaxErr     sd
     0.782 0.114    -945502 422348
```

```r
ev3 <- evimp(mars3); plot(ev3, main="Step 3: Variable Importance (degree=2, interactions)")
```

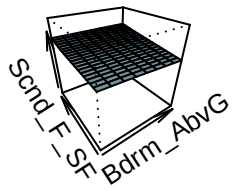



```
par(mfrow=c(2,2)); plotmo(mars3, type="response", nresponse=1, do.par=FALSE); par(mfrow=c(1,1
```

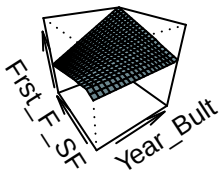
```
plotmo grid:    Bedroom_AbvGr Year_Built Mo_Sold Lot_Area Street Central_Air
                            3       1974       6     9480   Pave           Y
First_Flr_SF Second_Flr_SF Full_Bath Half_Bath Fireplaces Garage_Area
        1092             0         2         0          1         480
Gr_Liv_Area TotRms_AbvGrd
       1442             6
```


1 Year_Bult
−1e+06
1880
1920
1960
2000
2 Lot_Area
−1e+06
0
50000
150000
3 Frst_F_SF
−1e+06
1000
3000
5000
4 Scnd_F_SF
−1e+06
0
500
1500
5 Half_Bath
−1e+06
0.0
0.5
1.0
1.5
2.0
6 Fireplacs
−1e+06
0
1
2
3
4
7 Garage_Ar
−1e+06
0
500
1000
1500
8 Gr_Liv_Ar
−1e+06
1000
3000
5000

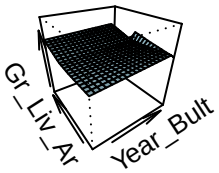
1 Bdrm_AbvG: Scnd_F_SF



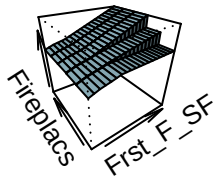
2 Year_Bult: Frst_F_SF



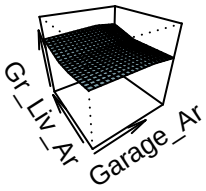
3 Year_Bult: Gr_Liv_Ar



4 Frst_F_SF: Fireplacs



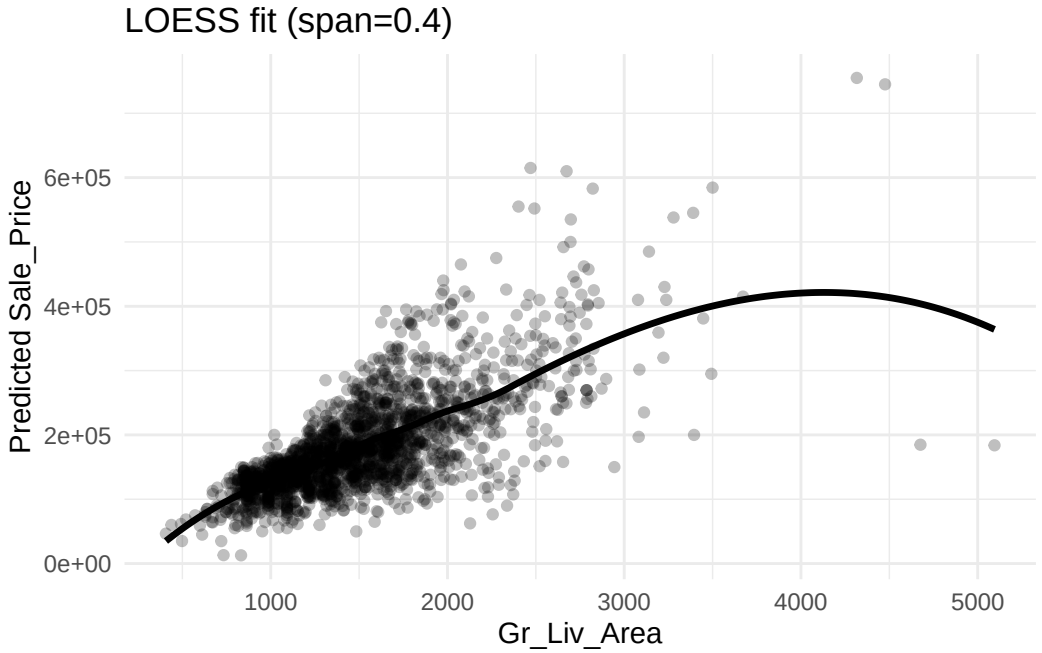
5 Garage_Ar: Gr_Liv_Ar



Interpreting MARS terms:
Look for `h()` pieces and products. Coefficients on `h(x-c)` indicate how slope changes after the knot `c`. Interaction products mean "the slope change depends on another variable".

4.5 5. LOESS (Visual local regression)

```
loess_fit <- loess(Sale_Price ~ Gr_Liv_Area, data=train, span=0.4)
grid_lo <- tibble(Gr_Liv_Area = seq(min(train$Gr_Liv_Area), max(train$Gr_Liv_Area), length.ou
grid_lo$pred <- predict(loess_fit, newdata=grid_lo)

ggplot() +
  geom_point(data=train, aes(Gr_Liv_Area, Sale_Price), alpha=.25) +
  geom_line(data=grid_lo, aes(Gr_Liv_Area, pred), linewidth=1.2) +
  labs(title="LOESS fit (span=0.4)", y="Predicted Sale_Price")
```



We keep LOESS as a visualization-oriented smoother (not used in the multivariate comparison below).

4.6 6. GAM Splines with `mgcv` – Simple → Complex

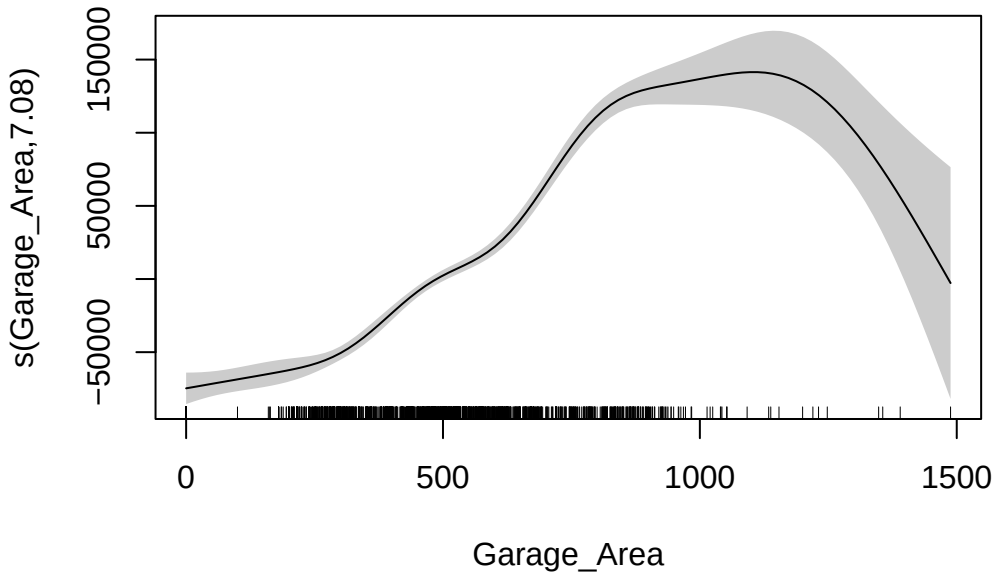
GAM smooths are written `s(x)` and estimated with penalized splines. The **edf** (effective degrees of freedom) in the summary tells you the smooth's complexity (larger edf wigglier function).

4.6.1 6A. Univariate GAM: `Sale_Price ~ s(Garage_Area)`

```
gam1 <- gam(Sale_Price ~ s(Garage_Area, k=9), data=train, method="REML")
summary(gam1)  # inspect edf and significance
```

```
Family: gaussian
Link function: identity

Formula:
Sale_Price ~ s(Garage_Area, k = 9)

Parametric coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   180897       1292     140   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Approximate significance of smooth terms:
                edf Ref.df     F p-value
s(Garage_Area) 7.078  7.719 217.1  <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

R-sq.(adj) =   0.45   Deviance explained = 45.2%
-REML =  25418  Scale est. = 3.4232e+09  n = 2051
```

```
plot(gam1, pages=1, shade=TRUE, scheme=1, scale=0)
```



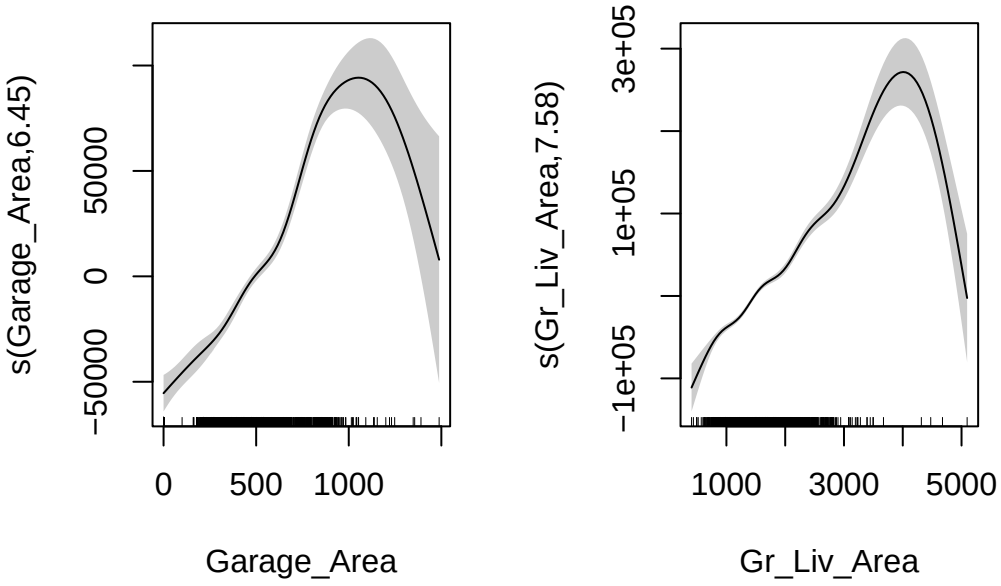
4.6.2 6B. Two-smooth Additive GAM: `s(Garage_Area) + s(Gr_Liv_Area)`

```
gam2 <- gam(Sale_Price ~ s(Garage_Area, k=9) + s(Gr_Liv_Area, k=9), data=train, method="REML"
summary(gam2)
```

```
Family: gaussian
Link function: identity

Formula:
Sale_Price ~ s(Garage_Area, k = 9) + s(Gr_Liv_Area, k = 9)

Parametric coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   180897       1026   176.2   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Approximate significance of smooth terms:
                 edf Ref.df     F p-value
s(Garage_Area) 6.446  7.293 103.3  <2e-16 ***
```

```
s(Gr_Liv_Area) 7.585  7.941 152.6  <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

R-sq.(adj) =  0.653   Deviance explained = 65.5%
-REML =  24949  Scale est. = 2.1608e+09  n = 2051
```

```r
plot(gam2, pages=1, shade=TRUE, scheme=1, scale=0)
```



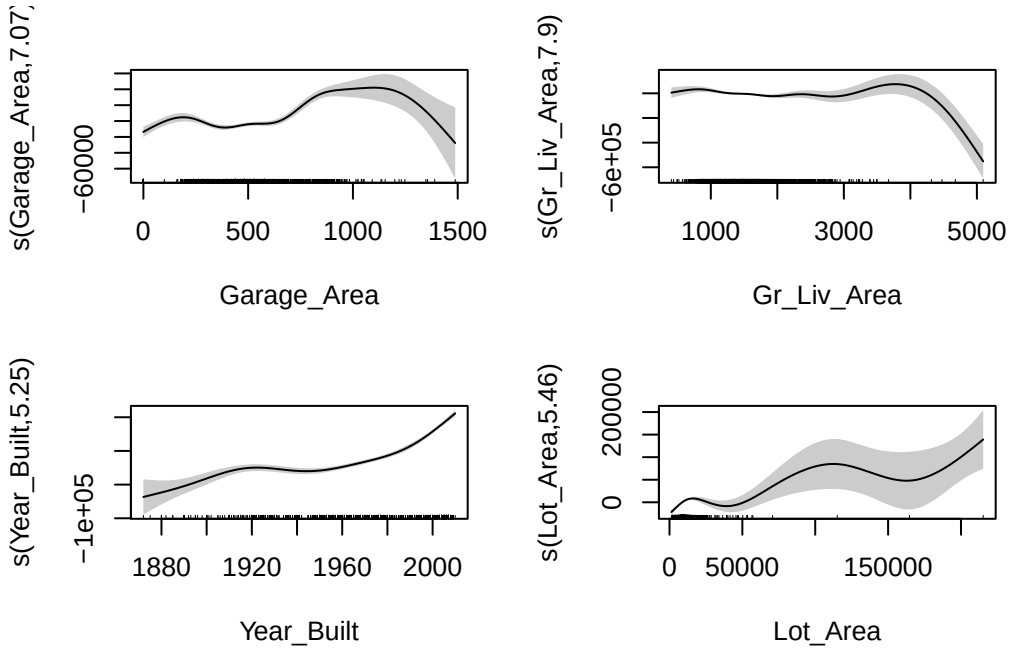
4.6.3 6C. Full Multivariate GAM (selected smooths + factors)

```r
gam3 <- gam(Sale_Price ~
              s(Garage_Area, k=9) +
              s(Gr_Liv_Area, k=9) +
              s(Year_Built, k=7) +
              s(Lot_Area, k=7) +
              Bedroom_AbvGr + Mo_Sold +
              Street + Central_Air +
              First_Flr_SF + Second_Flr_SF +
              Full_Bath + Half_Bath +
```

```
              Fireplaces + TotRms_AbvGrd,
            data=train, method="REML")
summary(gam3)
```

```
Family: gaussian
Link function: identity

Formula:
Sale_Price ~ s(Garage_Area, k = 9) + s(Gr_Liv_Area, k = 9) +
    s(Year_Built, k = 7) + s(Lot_Area, k = 7) + Bedroom_AbvGr +
    Mo_Sold + Street + Central_Air + First_Flr_SF + Second_Flr_SF +
    Full_Bath + Half_Bath + Fireplaces + TotRms_AbvGrd

Parametric coefficients:
               Estimate Std. Error t value Pr(>|t|)
(Intercept)   -21237.92   30106.84  -0.705   0.4806
Bedroom_AbvGr -10350.20    1382.38  -7.487 1.05e-13 ***
Mo_Sold         -409.12     270.24  -1.514   0.1302
StreetPave     30589.21   13486.61   2.268   0.0234 *
Central_AirY   17189.68    3366.03   5.107 3.59e-07 ***
First_Flr_SF     140.27      17.44   8.044 1.47e-15 ***
Second_Flr_SF    101.93      17.36   5.871 5.06e-09 ***
Full_Bath      -4548.94    2211.69  -2.057   0.0398 *
Half_Bath       2355.98    2175.40   1.083   0.2789
Fireplaces     13066.61    1371.59   9.527  < 2e-16 ***
TotRms_AbvGrd  -1640.87     948.67  -1.730   0.0838 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Approximate significance of smooth terms:
                 edf Ref.df      F p-value
s(Garage_Area) 7.067  7.712  24.96  <2e-16 ***
s(Gr_Liv_Area) 7.899  7.996  46.44  <2e-16 ***
s(Year_Built)  5.255  5.764 128.05  <2e-16 ***
s(Lot_Area)    5.465  5.883  16.44  <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

R-sq.(adj) =  0.827   Deviance explained =   83%
-REML =  24167  Scale est. = 1.0786e+09  n = 2051
```

```r
plot(gam3, pages=1, shade=TRUE, scheme=1, scale=0)
```



Interpreting GAM splines:
A smooth `s(x)` shows how the expected outcome varies with `x` *holding other terms constant.* The edf indicates complexity; wide confidence bands suggest greater uncertainty in those regions.

4.7 7. Model Comparison (R^2, RMSE, MSE)

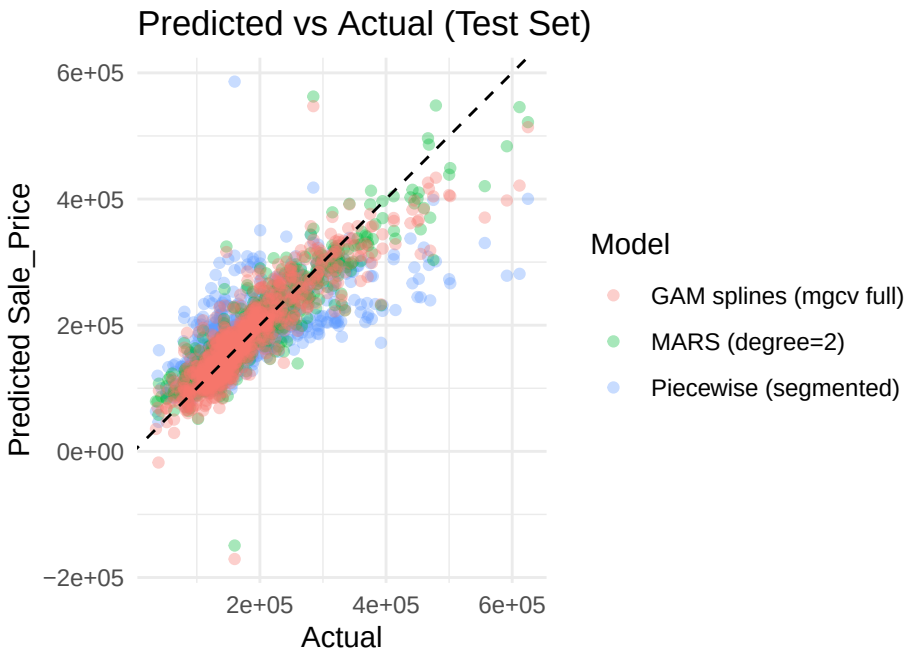
```r
# Helper for R^2 on test
rsq <- function(y, yhat) {
  1 - sum((y - yhat)^2) / sum((y - mean(y))^2)
}

# Predictions
p_piece <- predict(m_seg, newdata=test)
p_mars  <- predict(mars3, newdata=test)   # final MARS with interactions
p_gam   <- predict(gam3, newdata=test)    # full multivariate GAM
```

```r
# Metrics
tbl <- tibble(
  Model = c("Piecewise (segmented)", "MARS (degree=2)", "GAM splines (mgcv full)"),
  RMSE  = c(rmse(test$Sale_Price, p_piece),
            rmse(test$Sale_Price, p_mars),
            rmse(test$Sale_Price, p_gam)),
  MSE   = c(mse(test$Sale_Price, p_piece),
            mse(test$Sale_Price, p_mars),
            mse(test$Sale_Price, p_gam)),
  R2    = c(rsq(test$Sale_Price, p_piece),
            rsq(test$Sale_Price, p_mars),
            rsq(test$Sale_Price, p_gam))
) |> arrange(desc(R2))

tbl
```

```
# A tibble: 3 x 4
  Model                     RMSE         MSE    R2
  <chr>                    <dbl>       <dbl> <dbl>
1 MARS (degree=2)         33028. 1090840972. 0.839
2 GAM splines (mgcv full) 36796. 1353918715. 0.800
3 Piecewise (segmented)   59034. 3484963067. 0.484
```

4.8 8. Final Plot: Predicted vs Actual (Test Set)

```r
plot_df <- bind_rows(
  tibble(Model="Piecewise (segmented)", Actual=test$Sale_Price, Pred=p_piece),
  tibble(Model="MARS (degree=2)", Actual=test$Sale_Price, Pred=p_mars),
  tibble(Model="GAM splines (mgcv full)", Actual=test$Sale_Price, Pred=p_gam)
)

ggplot(plot_df, aes(Actual, Pred, color=Model)) +
  geom_point(alpha=.35) +
  geom_abline(intercept=0, slope=1, linetype="dashed") +
  labs(title="Predicted vs Actual (Test Set)", y="Predicted Sale_Price") +
  coord_equal()
```



4.9 9. How to Explain

- **Piecewise**: "There's a threshold where the effect changes." Simple story, limited flexibility.
- **MARS**: "The model finds breakpoints and sometimes *when* they matter (interactions)." Explain `h(x-c)` as *extra slope after c*.
- **GAM (splines)**: "We model smooth curves for key variables; edf shows wiggliness. More edf more flexibility."
- **Tradeoff**: More flexibility usually improves error (RMSE/MSE) and R^2, but reduce interpretability. Validate with cross-validation and keep the story aligned with business intuition.

5 GAM— Piecewise, LOESS, and GAM Splines - Python Version

Note: The original MARS section used the `sklearn-contrib-py-earth` package, which is **no longer actively maintained** and may fail to install on modern Python versions.
The MARS code is left **commented out** for reference — you can un-comment it if you successfully install `py-earth` (try `conda install -c conda-forge sklearn-contrib-py-earth`).
Equivalent flexibility can be achieved through **GAM** or **spline-based models**, which are shown below.

5.1 1. Setup and Data

```
import sys
print("Active Python interpreter:", sys.executable)

import numpy as np, pandas as pd
import matplotlib.pyplot as plt

from sklearn.model_selection import train_test_split
from sklearn.compose import ColumnTransformer
from sklearn.preprocessing import OneHotEncoder, StandardScaler
from sklearn.pipeline import Pipeline
from sklearn.metrics import mean_squared_error, r2_score
from sklearn.linear_model import LinearRegression, Ridge
import inspect

# --- Fix for SciPy >= 1.13 removing .A from sparse matrices ---
import scipy.sparse as sp

if not hasattr(sp.spmatrix, "A"):
    def _toarray(self):
```

```
        return self.toarray()
    sp.spmatrix.A = property(_toarray)
# -----------------------------------------------------------------

np.random.seed(4321)
```

```
Active Python interpreter: /opt/anaconda3/bin/python
```

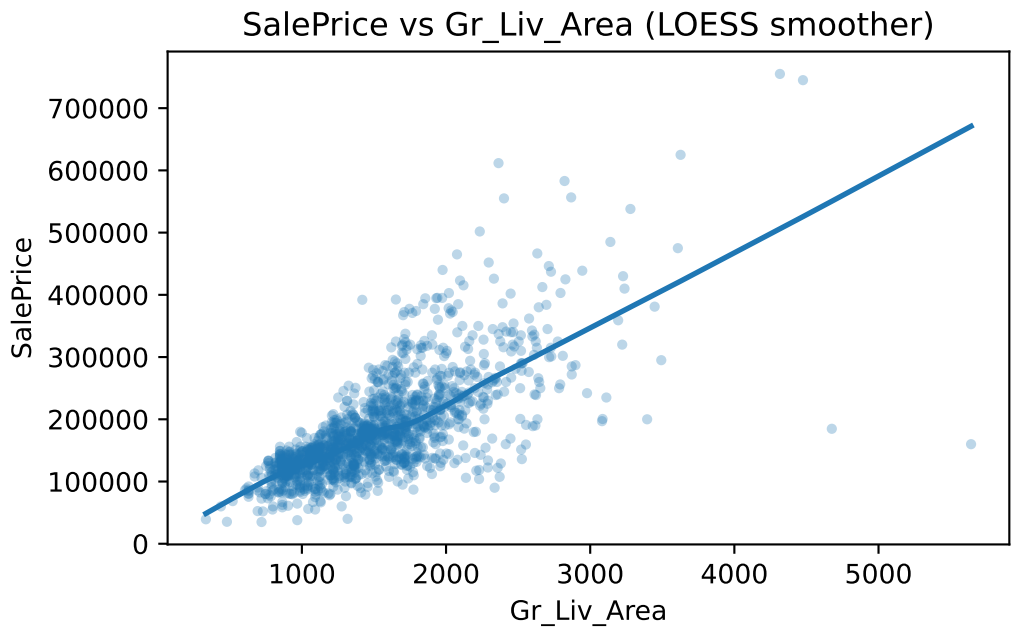
```
from sklearn.datasets import fetch_openml

ames = fetch_openml(name="house_prices", as_frame=True).frame

keep = {
    "SalePrice":"SalePrice",
    "BedroomAbvGr":"Bedroom_AbvGr",
    "YearBuilt":"Year_Built",
    "MoSold":"Mo_Sold",
    "LotArea":"Lot_Area",
    "Street":"Street",
    "CentralAir":"Central_Air",
    "1stFlrSF":"First_Flr_SF",
    "2ndFlrSF":"Second_Flr_SF",
    "FullBath":"Full_Bath",
    "HalfBath":"Half_Bath",
    "Fireplaces":"Fireplaces",
    "GarageArea":"Garage_Area",
    "GrLivArea":"Gr_Liv_Area",
    "TotRmsAbvGrd":"TotRms_AbvGrd"
}

df = ames[list(keep.keys())].rename(columns=keep).dropna().copy()

X = df.drop(columns=["SalePrice"])
y = df["SalePrice"].values

num_cols = X.select_dtypes(include=[np.number]).columns.tolist()
cat_cols = [c for c in X.columns if c not in num_cols]

# Handle OneHotEncoder version changes and ensure dense output
if "sparse_output" in inspect.signature(OneHotEncoder).parameters:
    encoder = OneHotEncoder(drop="first", handle_unknown="ignore", sparse_output=False)
```

```
else:
    encoder = OneHotEncoder(drop="first", handle_unknown="ignore", sparse=False)

pre = ColumnTransformer([
    ("num", StandardScaler(), num_cols),
    ("cat", encoder, cat_cols)
], sparse_threshold=0)

X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.3, random_state=4321)
df.head()
```

	SalePrice	Bedroom_AbvGr	Year_Built	Mo_Sold	Lot_Area	Street	Central_Air	First_Flr_SF
0	208500	3	2003	2	8450	Pave	Y	856
1	181500	3	1976	5	9600	Pave	Y	1262
2	223500	3	2001	9	11250	Pave	Y	920
3	140000	3	1915	2	9550	Pave	Y	961
4	250000	4	2000	12	14260	Pave	Y	1145

5.2 2. Visual Check for Nonlinearity

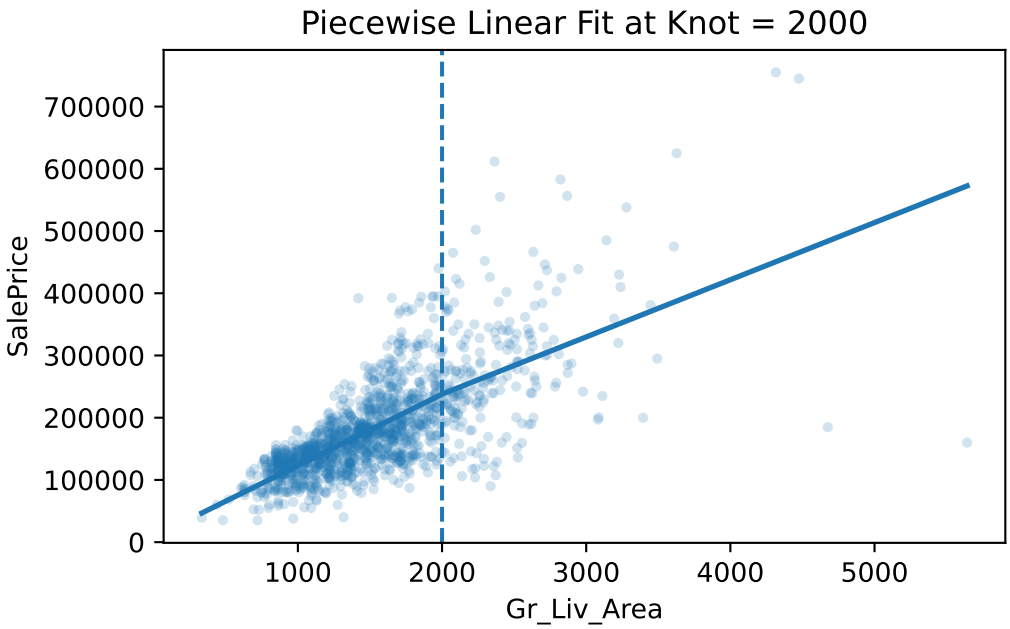
```
import statsmodels.api as sm
fig = plt.figure()
plt.scatter(df["Gr_Liv_Area"], df["SalePrice"], s=8, alpha=0.3)
low = sm.nonparametric.lowess(df["SalePrice"], df["Gr_Liv_Area"], frac=0.3, return_sorted=Tru
plt.plot(low[:,0], low[:,1], linewidth=2)
plt.title("SalePrice vs Gr_Liv_Area (LOESS smoother)")
plt.xlabel("Gr_Liv_Area"); plt.ylabel("SalePrice")
plt.tight_layout()
```



5.3 3. Piecewise Regression Example

```
class PiecewiseLinear:
    def __init__(self, knot=2000.0):
        self.knot = knot
        self.lin = LinearRegression()

    def _transform(self, x):
        x1 = np.asarray(x).reshape(-1,1)
        hx = np.maximum(0, x1 - self.knot)
        return np.hstack([x1, hx])

    def fit(self, x, y):
        Z = self._transform(x)
        self.lin.fit(Z, y)
        return self

    def predict(self, x):
        Z = self._transform(x)
```

```
        return self.lin.predict(Z)

pw = PiecewiseLinear(knot=2000).fit(df["Gr_Liv_Area"].values, df["SalePrice"].values)

grid = np.linspace(df["Gr_Liv_Area"].min(), df["Gr_Liv_Area"].max(), 200)
pred = pw.predict(grid)

plt.figure()
plt.scatter(df["Gr_Liv_Area"], df["SalePrice"], s=8, alpha=0.2)
plt.plot(grid, pred, linewidth=2)
plt.axvline(2000, linestyle="--")
plt.title("Piecewise Linear Fit at Knot = 2000")
plt.xlabel("Gr_Liv_Area"); plt.ylabel("SalePrice")
plt.tight_layout()
```

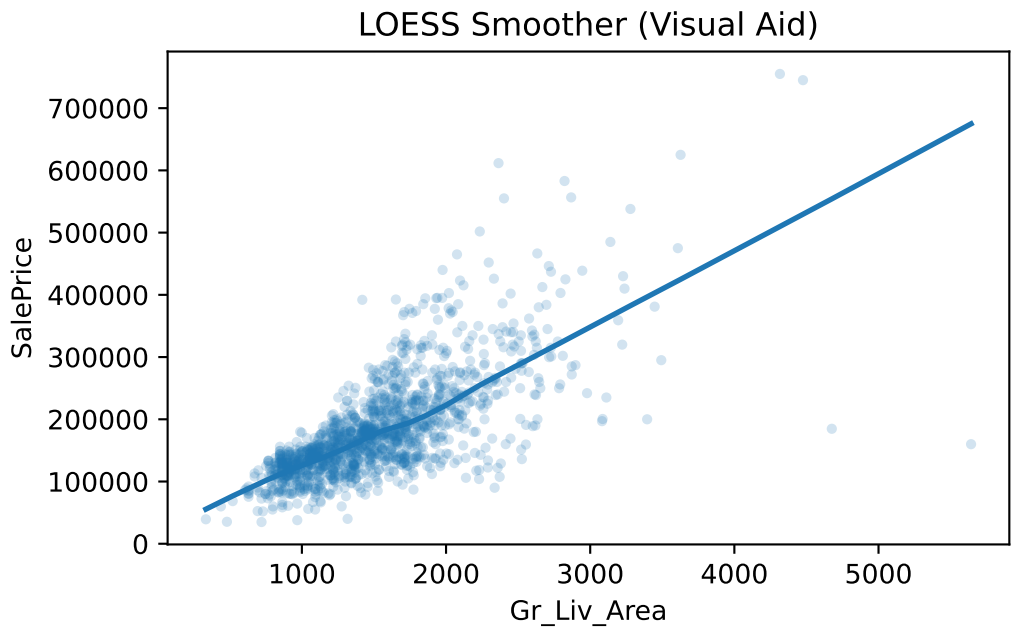


5.4 4. MARS (Commented Out — For Reference Only)

```
# The MARS implementation (py-earth) is not actively maintained and may fail to install on mo
# If you wish to try it, install from conda-forge:
#     conda install -c conda-forge sklearn-contrib-py-earth
#
# Example (commented out):
#
# from pyearth import Earth
#
# mars = Pipeline([("prep", pre), ("model", Earth(max_degree=2))]).fit(X_train, y_train)
# print(mars.named_steps["model"].summary())
#
# xs = np.linspace(X["Garage_Area"].min(), X["Garage_Area"].max(), 200)
# yhat = mars.predict(pd.DataFrame({"Garage_Area": xs}))
#
# plt.figure()
# plt.scatter(X_train["Garage_Area"], y_train, s=8, alpha=0.2)
# plt.plot(xs, yhat, linewidth=2)
# plt.title("MARS with Garage_Area (Commented Out Example)")
# plt.xlabel("Garage_Area"); plt.ylabel("SalePrice")
# plt.tight_layout()
```

5.5 5. LOESS Visualization

```
fig = plt.figure()
plt.scatter(df["Gr_Liv_Area"], df["SalePrice"], s=8, alpha=0.2)
low = sm.nonparametric.lowess(df["SalePrice"], df["Gr_Liv_Area"], frac=0.4, return_sorted=Tru
plt.plot(low[:,0], low[:,1], linewidth=2)
plt.title("LOESS Smoother (Visual Aid)")
plt.xlabel("Gr_Liv_Area"); plt.ylabel("SalePrice")
plt.tight_layout()
```



5.6 6. GAM and Spline Approaches (Modern Alternatives)

```
from pygam import LinearGAM, s
gam1 = LinearGAM(s(0)).fit(X_train[["Garage_Area"]].values, y_train)
print(gam1.summary())
gam2 = LinearGAM(s(0) + s(1)).fit(X_train[["Garage_Area","Gr_Liv_Area"]].values, y_train)
print(gam2.summary())
num_train = X_train[num_cols].values
gam3 = LinearGAM().fit(num_train, y_train)
print(gam3.summary())
```

```
LinearGAM
============================================= ==========================================
Distribution:                        NormalDist Effective DoF:
Link Function:                     IdentityLink Log Likelihood:
23263.1874
Number of Samples:                         1022 AIC:
                                                AICc:
```

```
                                                GCV:                                3
                                                Scale:                              3
                                                Pseudo R-Squared:
=============================================================================================
Feature Function                  Lambda               Rank         EDoF         P > x
================================= ==================== ============ ============ ============
s(0)                              [0.6]                20           11.7         1.11e-
16     ***
intercept                                              1            0.0          1.11e-
16     ***
=============================================================================================
Significance codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

WARNING: Fitting splines and a linear function to a feature introduces a model identifiabilit
         which can cause p-values to appear significant when they are not.

WARNING: p-values calculated in this manner behave correctly for un-penalized models or model
         known smoothing parameters, but when smoothing parameters have been estimated, the p
values
         are typically lower than they should be, meaning that the tests reject the null too
None
LinearGAM
============================================== ==========================================
Distribution:                        NormalDist Effective DoF:
Link Function:                     IdentityLink Log Likelihood:
22770.8323
Number of Samples:                         1022 AIC:
                                                AICc:
                                                GCV:                                19
                                                Scale:                               1
                                                Pseudo R-Squared:
=============================================================================================
Feature Function                  Lambda               Rank         EDoF         P > x
================================= ==================== ============ ============ ============
s(0)                              [0.6]                20           12.6         1.11e-
16     ***
s(1)                              [0.6]                20           8.3          1.11e-
16     ***
intercept                                              1            0.0          1.11e-
16     ***
=============================================================================================
Significance codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
WARNING: Fitting splines and a linear function to a feature introduces a model identifiabilit
         which can cause p-values to appear significant when they are not.

WARNING: p-values calculated in this manner behave correctly for un-penalized models or model
         known smoothing parameters, but when smoothing parameters have been estimated, the p
values
         are typically lower than they should be, meaning that the tests reject the null too
None
LinearGAM
============================================= ==========================================
Distribution:                        NormalDist Effective DoF:
Link Function:                     IdentityLink Log Likelihood:
21995.395
Number of Samples:                         1022 AIC:
                                                AICc:
                                                GCV:                                    10
                                                Scale:                                   8
                                                Pseudo R-Squared:
=================================================================================================
Feature Function                  Lambda               Rank         EDoF         P > x
================================= ==================== ============ ============ ============
s(0)                              [0.6]                20           8.7          6.58e-
07     ***
s(1)                              [0.6]                20           12.8         1.11e-
16     ***
s(2)                              [0.6]                20           11.0         5.15e-
01
s(3)                              [0.6]                20           8.4          1.11e-
16     ***
s(4)                              [0.6]                20           8.6          1.11e-
16     ***
s(5)                              [0.6]                20           10.1         2.17e-
03     **
s(6)                              [0.6]                20           3.2          9.76e-
02     .
s(7)                              [0.6]                20           2.2          4.19e-
01
s(8)                              [0.6]                20           2.9          3.16e-
10     ***
s(9)                              [0.6]                20           9.0          1.11e-
16     ***
s(10)                             [0.6]                20           5.7          1.11e-
16     ***
```

```
s(11)                              [0.6]                20           6.5          3.78e-
01
intercept                                               1            0.0          1.11e-
16     ***
=============================================================================================
Significance codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

WARNING: Fitting splines and a linear function to a feature introduces a model identifiabilit
         which can cause p-values to appear significant when they are not.

WARNING: p-values calculated in this manner behave correctly for un-penalized models or model
         known smoothing parameters, but when smoothing parameters have been estimated, the p
values
         are typically lower than they should be, meaning that the tests reject the null too
None
```

5.7 7. Regression Spline Example (Fallback)

```
from patsy import dmatrix

design_tr = dmatrix("bs(Garage_Area, df=6) + bs(Gr_Liv_Area, df=6)", data=X_train, return_typ
design_te = dmatrix("bs(Garage_Area, df=6) + bs(Gr_Liv_Area, df=6)", data=X_test, return_type

ridge = Ridge(alpha=1.0).fit(design_tr, y_train)
pred_spline = ridge.predict(design_te)
```

5.8 8. Model Comparison (Piecewise vs GAM vs Spline)

```
def rmse(y, yhat):
    try:
        return mean_squared_error(y, yhat, squared=False)
    except TypeError:
        return np.sqrt(mean_squared_error(y, yhat))
```

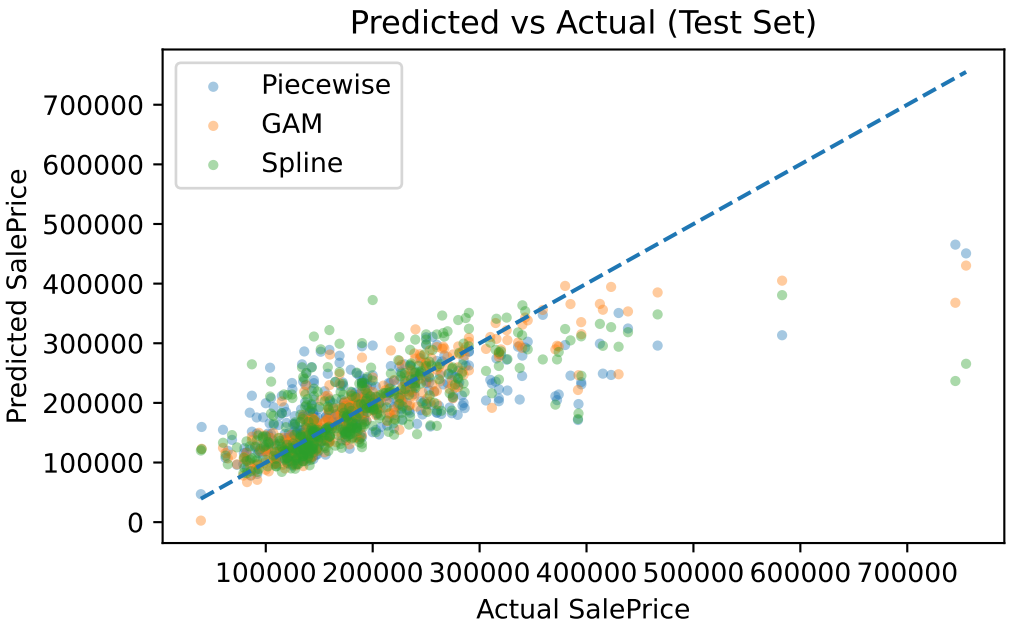
```
def mse(y, yhat): return mean_squared_error(y, yhat)
def rsq(y, yhat): return r2_score(y, yhat)

p_piece = pw.predict(X_test["Gr_Liv_Area"].values)

try:
    p_gam = gam3.predict(X_test[num_cols].values)
except Exception:
    p_gam = np.full_like(y_test, np.nan, dtype=float)

p_spline = pred_spline

cmp = pd.DataFrame({
    "Model": ["Piecewise", "GAM (pyGAM)" if not np.isnan(p_gam).all() else "GAM (not availabl
    "RMSE": [rmse(y_test, p_piece), rmse(y_test, p_gam) if not np.isnan(p_gam).all() else np
    "MSE": [mse(y_test, p_piece), mse(y_test, p_gam) if not np.isnan(p_gam).all() else np.nan
    "R2": [rsq(y_test, p_piece), rsq(y_test, p_gam) if not np.isnan(p_gam).all() else np.nan
}).sort_values("R2", ascending=False)

cmp
```

	Model	RMSE	MSE	R2
1	GAM (pyGAM)	40805.867162	1.665119e+09	0.766556
0	Piecewise	58108.537553	3.376602e+09	0.526612
2	Regression Spline	60289.000524	3.634764e+09	0.490419

5.9 9. Predicted vs Actual Comparison

```
plt.figure()
plt.scatter(y_test, p_piece, s=8, alpha=0.4, label="Piecewise")
if not np.isnan(p_gam).all():
    plt.scatter(y_test, p_gam, s=8, alpha=0.4, label="GAM")
plt.scatter(y_test, p_spline, s=8, alpha=0.4, label="Spline")
plt.plot([y_test.min(), y_test.max()], [y_test.min(), y_test.max()], linestyle="--")
plt.legend()
```

```
plt.title("Predicted vs Actual (Test Set)")
plt.xlabel("Actual SalePrice"); plt.ylabel("Predicted SalePrice")
plt.tight_layout()
```



5.10 10. Interpretation Summary

- **Piecewise:** adds simple thresholds, easy to explain.
- **GAMs:** provide smooth, interpretable nonlinearities.
- **Splines:** flexible approximations that behave like local polynomials.
- **MARS (optional):** similar conceptually but less maintained in Python; consider using R's `earth` instead.

6 tree-based-models

7 neural-network-models

8 naive-bayes-models

9 model-agnostic-interpretability

10 support-vector-machines