Extended Dimensionality Reduction MUSIC Method for Signal-Selective Direction Estimation

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***Abstract*—This paper presents a novel extension to the subspace-based direction-of-arrival (DOA) algorithm Dimensionality Reduction Multiple Signal Classification (DR-MUSIC). The extension operates with a single six-component vector sensor collecting multiple temporally displaced data sets. The temporal displacements invoke a series of phase-shifts specific to the carrier frequencies of each signal to be located and are arranged into a matrix. This matrix of phase-shifts is combined via tensor product with the arrival angle function steering matrix used in DR-MUSIC's iterative search to create a larger, more robust steering matrix. This procedure allows the proposed method to selectively resolve signals greatly exceeding the number of antenna elements while rejecting any present interference signals with impressive accuracy. Simulations demonstrate that the method performs extremely well under sub-zero SNR and SIR conditions.**

***Index Terms*—Antenna arrays, array signal processing, direction-of-arrival estimation, eigenvalues and eigenfunctions, numerical simulation**

I. Introduction

A. Summary of Relevant Literature

THERE exist many algorithms today that estimate the direction of arrival (DOA) of signals derived from sensor data. Among the most well-known and effective of these include so-called "eigenstructure-based" methods, such as multiple signal classification (MUSIC) [8] and estimation of signal parameters via rotational invariance techniques (ESPRIT) [9]. These two popular methods have both inspired many additional techniques which introduce additional signal processing methods, as well as more advanced sensor array configurations. Many methods explore the use of electromagnetic vector sensor arrays over scalar sensor arrays due to their distinct advantages, including inherent frequency-independence and more effective polarization estimation. Nehorai and Paldi [7] created the idea of using a vector sensor to recreate the DOA information of a signal by estimating its magnetic and electric-field components first and then taking their vector cross-product, while Li [5] adapted ESPRIT to a vector sensor array. Uni-Vector-Sensor ESPRIT, developed by Wong and Zoltowski [2], is a method that uses ESPRIT with a single vector sensor and collects a pair of temporally displaced data sets to perform direction estimation. MUSIC was optimized for use in a vector-sensor by Wang et. al. [1] by decreasing its required four-dimensional search to a two-dimensional search. Signal-selective algorithms have been developed that reject interfering signals by exploiting the cyclostationarity of each signal [12] - [14]. Many signals exhibit cyclostationarity, or periodic correlation, and can be classified according to such properties. These methods utilize the cyclic correlation matrix instead of the covariance matrix used in MUSIC and ESPRIT methods.

B. Summary of the Proposed Method

The method proposed here is a DOA algorithm that is intended for use in an environment where both target signals to be located and unwanted interference signals are present. It is built upon the DR-MUSIC method [1], meaning that it is a MUSIC-based subspace method that functions with, but is not limited to, a single electromagnetic vector sensor and is capable of high-resolution DOA estimation of multiple concurrent signals possessing the same carrier frequency. The extension described in this paper grants DR-MUSIC the ability to locate any number of signals with distinct frequencies while successfully rejecting any interfering signals with a single six-component vector sensor. The method accomplishes these goals while providing performance increases under very low signal-to-noise (SNR) and signal-to-interference (SIR) levels.

The method can be summarized as follows. First, it is required that a priori knowledge of the carrier frequency of every signal to be located is known, as this information is key to the method's ability to selectively locate target signals under high noise and interference. The first main addition to DR-MUSIC is to record multiple instances of temporally displaced data sets. The data sets are distinguished from each other via phase-shifts specific to the frequency of each signal to be located. Because the time-delays between data sets are determined by the algorithm itself, the exact phase-shifts that occur for the target signals are calculated and recorded in a matrix. This matrix of phase-shifts is then combined via a tensor product with the arrival angle function steering matrix used in DR-MUSIC's iterative search. This enables the method to specifically focus on target signals in the surrounding electromagnetic spectrum and further allows the method to resolve signals in lower SNR environments.

In Section 2, the signal model that a six-component vector sensor collects is presented. Section 3 presents the data model for the new method as well as for DR-MUSIC itself. Section 4 presents the theory behind DR-MUSIC, while Section 5 presents the extension to DR-MUSIC. Simulation results are given in Section 7 and conclusions are drawn in Section 8.

II. Array Model

This method involves the use of an electromagnetic vector sensor - three spatially colocated identical but orthogonally oriented, electrically short dipoles and magnetically small loops measuring all three electric field components and all three magnetic components of incident signals.

Suppose that the kth $(1 \leq k \leq K)$ narrow-band, transverse, polarized electromagnetic signal impinges upon the vector sensor. For an incoming unit-power signal, the vector sensor manifold can be expressed as

$$\begin{aligned}
\mathbf{a}_k &= \begin{bmatrix} e_x(\theta_k, \phi_k, \gamma_k, \eta_k) \\ e_y(\theta_k, \phi_k, \gamma_k, \eta_k) \\ e_z(\theta_k, \phi_k, \gamma_k, \eta_k) \\ h_x(\theta_k, \phi_k, \gamma_k, \eta_k) \\ h_y(\theta_k, \phi_k, \gamma_k, \eta_k) \\ h_z(\theta_k, \phi_k, \gamma_k, \eta_k) \end{bmatrix} \\
&= \underbrace{\begin{bmatrix} \cos\theta_k \cos\phi_k & -\sin\phi_k \\ \cos\theta_k \sin\phi_k & \cos\phi_k \\ -\sin\theta_k & 0 \\ -\sin\phi_k & -\cos\theta_k\cos\phi_k \\ \cos\phi_k & -\cos\theta_k \sin\phi_k \\ 0 & \sin\theta_k \end{bmatrix}}_{\mathbf{\Omega}(\theta_k, \phi_k)} \underbrace{\begin{bmatrix} \sin\gamma_k e^{j\eta_k} \\ \cos\gamma_k \end{bmatrix}}_{\mathbf{g}(\gamma_k, \eta_k)}
\end{aligned}$$

where $\theta_k \in [0, \pi]$ is the signal's elevation measured from the z-axis, $\phi_k \in [0, 2\pi]$ is the azimuth angle, $\gamma_k \in [0, \pi/2]$ is the auxiliary polarization angle and $\eta \in [-\pi, \pi]$ is the polarization phase difference.

III. Data Model

K narrow-band transverse electromagnetic signals, consisting of D target signals and $K-D$ interfering signals impinge upon the vector-sensor. A total of L time-delayed sets of data are collected by the vector sensor, each consisting of N snapshots. Thus, the kth monochromatic signal creates L $6 \cdot N$ data sets

$$\begin{gathered}
\mathbf{a}_k s(t_n, f_k), \quad n = 1, ..., N \\
\mathbf{a}_k s(t_n + \Delta_1, f_k) = \mathbf{a}_k s(t_n, f_k) e^{j2\pi f_k \Delta_1} \\
\mathbf{a}_k s(t_n + \Delta_2, f_k) = \mathbf{a}_k s(t_n, f_k) e^{j2\pi f_k \Delta_2} \\
\vdots \\
\mathbf{a}_k s(t_n + \Delta_{L-1}, f_k) = \mathbf{a}_k s(t_n, f_k) e^{j2\pi f_k \Delta_{L-1}}
\end{gathered}$$

where f_k is the kth signal's frequency, $s(t_n, f_k)$ is the signal itself, and $\{\Delta_1, ..., \Delta_{L-1}\}$ is a set of uniformly distributed random time delays. Given sufficiently small time-delays, the data sets can be overlapped. Note that when f_k and Δ_i are known, all $L-1$ phase differences are exactly known and is key to the method's selective process.

Given any K signals impinging on the array with additive zero-mean Gaussian white noise, the data taken in by the vector sensor takes the form

$$\mathbf{z}(t_n) = \sum_{k=1}^{K} \begin{bmatrix} \mathbf{a}_k s(t_n, f_k) \\ \mathbf{a}_k s(t_n, f_k) e^{j2\pi f_k \Delta_1} \\ \vdots \\ \mathbf{a}_k s(t_n, f_k) e^{j2\pi f_k \Delta_{L-1}} \end{bmatrix} + \mathbf{n}(t_n).$$

With the following definitions,

$$\begin{aligned}
\mathbf{A}_1 &\triangleq [\mathbf{a}_1, ..., \mathbf{a}_K], \\
\mathbf{A}_2 &\triangleq [\mathbf{a}_1 e^{j2\pi f_1 \Delta_1}, ..., \mathbf{a}_K e^{j2\pi f_K \Delta_1}], \\
&\vdots \\
\mathbf{A}_L &\triangleq [\mathbf{a}_1 e^{j2\pi f_1 \Delta_{L-1}}, ..., \mathbf{a}_K e^{j2\pi f_K \Delta_{L-1}}], \\
\mathbf{s}(t_n) &\triangleq \begin{bmatrix} s(t_n, f_1) \\ \vdots \\ s(t_n, f_K) \end{bmatrix}, \ \mathbf{n}(t_n) \triangleq \begin{bmatrix} n_1(t_n) \\ \vdots \\ n_{6\times L}(t_n) \end{bmatrix}
\end{aligned}$$

we can compactly write the data collected by the vector sensor as

$$\begin{aligned}
\mathbf{z}(t_n) &= \begin{bmatrix} \mathbf{A}_1 \\ \vdots \\ \mathbf{A}_L \end{bmatrix} \mathbf{s}(t_n) + \mathbf{n}(t_n), \\
\mathbf{Z} &\triangleq [\mathbf{z}(t_1), ..., \mathbf{z}(t_N)].
\end{aligned}$$

Here, $\mathbf{Z}$ is a $(6 \cdot L) \times N$ matrix of data.

IV. DR-MUSIC Method

It is important to keep in mind that DR-MUSIC strictly collects one set of data and is not meant for use when interference signals are present, meaning that $L = 1$ and $K = D$. The covariance matrix of the data $\mathbf{Z}(t)$ is given by

$$\mathbf{R}_Z = \mathrm{E}[\mathbf{Z}\mathbf{Z}^{\mathrm{H}}] = \mathbf{A}\mathbf{R}_s\mathbf{A}^{\mathrm{H}} + \sigma^2 \mathbf{I}$$

where $(\cdot)^{\mathrm{H}}$ denotes complex conjugate transpose, σ^2 is the white noise power, and $\mathbf{R}_s = \mathrm{E}[\mathbf{s}^{\mathrm{H}}(t)\mathbf{s}(t)]$ is the source covariance matrix. Assuming ergodic noise, $\mathbf{R}_Z$ can be estimated as

$$\hat{\mathbf{R}}_Z = \frac{1}{N}\mathbf{Z}\mathbf{Z}^{\mathrm{H}}.$$

DR-MUSIC begins by executing an eigendecomposition on the matrix $\hat{\mathbf{R}}_Z$. The resulting eigenvectors decompose the matrix into a K-dimensional signal subspace and a $(6-K)$ dimensional noise subspace. Let $\mathbf{U}_n$ represent the $6 \times (6 - K)$ matrix of noise eigenvectors associated with the $6-K$ smallest eigenvalues of $\hat{\mathbf{R}}_Z$. The DR-MUSIC spectrum is then given as

$$\mathbf{P} = \frac{1}{\mathbf{a}^{\mathrm{H}}\mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}\mathbf{a}}$$

The peaks of this spectrum will correspond to the desired angles to be found. Thus, the searching involved in DR-MUSIC equates to

$$\mathbf{P}_{\mathrm{MU}}(\hat{\theta_k}, \hat{\phi_k}, \hat{\gamma_k}, \hat{\eta_k}) = \left[\min_{\theta,\phi,\gamma,\eta} \mathbf{a}^{\mathrm{H}}\mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}\mathbf{a}\right]^{-1}. \tag{1}$$

This four-dimensional search is very computationally expensive. DR-MUSIC shrinks this search to only two dimensions via use of the Rayleigh-Ritz theorem. Since

$$\mathbf{a}(\theta, \phi, \gamma, \eta) = \mathbf{\Omega}(\theta, \phi)\mathbf{g}(\gamma, \eta),$$

and $\mathbf{g}^{\mathrm{H}}\mathbf{g} = 1$, (1) can be rewritten as

$$\mathbf{P}_{\mathrm{MU}}(\hat{\theta_k}, \hat{\phi_k}, \hat{\gamma_k}, \hat{\eta_k}) = \left[\min_{\theta,\phi,\gamma,\eta} \frac{\mathbf{g}^{\mathrm{H}}\mathbf{\Omega}^{\mathrm{H}}\mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}\mathbf{\Omega}\mathbf{g}}{\mathbf{g}^{\mathrm{H}}\mathbf{g}}\right]^{-1}. \tag{2}$$

Letting $\mathbf{B}(\theta,\phi) = \mathbf{\Omega}^{\mathrm{H}}\mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}\mathbf{\Omega}$, (2) becomes

$$\mathbf{P}_{\mathrm{MU}}(\hat{\theta}_k, \hat{\phi}_k, \hat{\gamma}_k, \hat{\eta}_k) = \left[\min_{\theta,\phi,\gamma,\eta} \frac{\mathbf{g}^{\mathrm{H}}\mathbf{B}(\theta,\phi)\mathbf{g}}{\mathbf{g}^{\mathrm{H}}\mathbf{g}}\right]^{-1}. \tag{3}$$

Since $\mathbf{B}$ is Hermitian and $\mathbf{g}^{\mathrm{H}}\mathbf{g} = 1$, the right-hand side of (3) is a perfect candidate for the Rayleigh-Ritz theorem, which then transforms the minimization problem to finding the minimum eigenvalue of $\mathbf{B}$, i.e.,

$$\mathbf{P}_{\mathrm{DR}}(\hat{\theta}_k, \hat{\phi}_k, \hat{\gamma}_k, \hat{\eta}_k) = \left[\min_{\theta,\phi} \lambda_{\min}(\mathbf{B}(\theta,\phi))\right]^{-1}.$$

Since $\mathbf{B}$ only depends on θ and ϕ, the 4-dimensional search has been decreased to 2 dimensions.

V. DR-MUSIC WITH SIGNAL SELECTION

Now a modification is made to DR-MUSIC, granting it the ability to locate D target signals and reject $K-D$ interference signals with a single vector sensor. The first step is to record $L > 1$ sets of data as described in the data model in order to gain more information about the electromagnetic spectrum. This causes the matrix $\mathbf{R}$ to grow to a $(6\cdot L)\times(6\cdot L)$ matrix and $\mathbf{U}$ to increase to size $(6\cdot L)\times[(6\cdot L)-K]$. For DR-MUSIC, $\mathbf{U}$ is a $6\times(6-K)$ matrix, meaning that if at least one noise vector is to be used for direction-finding, then no more than $K = 5$ signals can be located at the same time. By taking $L > 1$ data sets, the requirement for having at least one noise eigenvector for direction finding now depends on L, namely, $L \geq (1+K)/6$ (although it is recommend that L be greater). Thus, for a sufficiently large enough number of data sets L, any number K of signals with distinct frequencies can be located or rejected simultaneously. The method is still limited to 5 simultaneous signals of the same carrier frequency. However, any number of frequency groups can still be located, e.g., 5 signals at 30 MHz and 5 signals at 31 MHz can still be located simultaneously.

With $\mathbf{U}_n$ now a $(6\cdot L)\times[(6\cdot L)-K]$ matrix, this means that $\mathbf{\Omega}$ must have at least $6\cdot L$ rows in order to be compatible with (2). We construct a new matrix to be used in searching by combining $\mathbf{\Omega}$ with phase-shift information invoked from the L time-delayed data sets. For the frequencies of the D target signals $\{f_1,...,f_D\}$, discard any duplicate frequencies to obtain a distinct set of frequencies $\{f_1,...,f_d\}$. Now, construct the matrices

$$\mathbf{\Psi} \triangleq \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \mathrm{e}^{j2\pi f_1\Delta_1} & \mathrm{e}^{j2\pi f_2\Delta_1} & \cdots & \mathrm{e}^{j2\pi f_d\Delta_1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{e}^{j2\pi f_1\Delta_{L-1}} & \mathrm{e}^{j2\pi f_2\Delta_{L-1}} & \cdots & \mathrm{e}^{j2\pi f_d\Delta_{L-1}} \end{bmatrix}$$

$$\text{and} \qquad \widehat{\mathbf{\Omega}} \triangleq \mathbf{\Psi}\otimes\mathbf{\Omega}.$$

$\widehat{\mathbf{\Omega}}$ is now a $(6\cdot L)\times(2\cdot D)$ matrix that contains all the phase information of each signal relative at every time-delay and is suitable for searching. It is important to note here that the set of frequencies $\{f_1,...f_d\}$ present in $\mathbf{\Psi}$ is key to the method's ability to focus on target signals and ignore everything else. It should be clear now that including only distinct frequencies from the D signals is essential to ensure that $\mathbf{\Psi}$ and $\widehat{\mathbf{\Omega}}$ remain full rank. With $\widehat{\mathbf{B}}(\theta,\phi) = \widehat{\mathbf{\Omega}}^{\mathrm{H}}\mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}\widehat{\mathbf{\Omega}}$, the new spectrum becomes

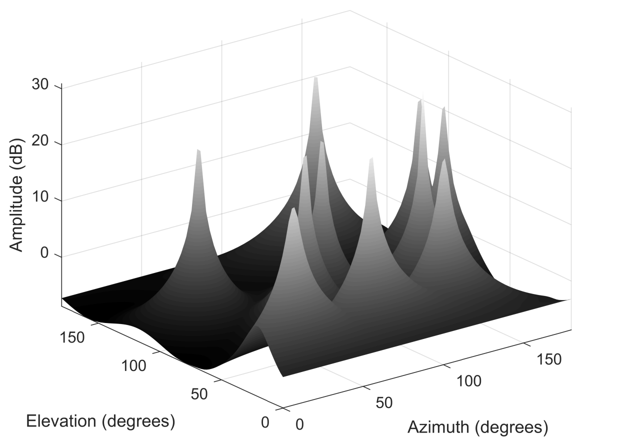

Fig. 1. Resolving azimuth and elevation of 10 target signals while rejecting 10 interference signals.

$$\mathbf{P}_{\mathrm{SDR}}(\hat{\theta}_k, \hat{\phi}_k, \hat{\gamma}_k, \hat{\eta}_k) = \left[\min_{\theta,\phi} \lambda_{\min}(\widehat{\mathbf{B}}(\theta,\phi))\right]^{-1}.$$

With this modification, the proposed method effectively and selectively finds the directions of of all D signals when provided with frequency information.

VI. SIMULATION RESULTS

Several simulations are conducted here to evaluate the DOA estimation abilities of the proposed algorithm. For all simulation experiments, a single vector sensor is used while each data set taken consists of 1024 snapshots. For the first experiment, a total of 20 uncorrelated equal-powered monochromatic electromagnetic sources impinge on the vector sensor with the objective of resolving exactly ten of the sources and rejecting the remaining 10. SNR is taken to be 15 dB and is relative to unity signal power. Figure 1 shows the spatial spectrum and illustrates that the method effectively finds all 10 signals despite the imposed limitations. For all following experiments, 100 independent trials are held under a variety of SNR and SIR levels to observe the method's performance under these varying conditions. The following experiments use $L = 5$ data sets when involving the proposed algorithm. In the second experiment, we task the algorithm with locating a signal with the parameter values $(\phi,\theta,\gamma,\eta) = (30, 60, 20, 60)$ with no interference signals and an SNR that ranges between -15 and 15 dB. The results for our method with comparisons to DR-MUSIC and Uni-Vector-Sensor ESPRIT are shown in Figures 2 and 3. The DR-MUSIC and Uni-Vector-Sensor ESPRIT comparisons are conducted using one data set with 5120 snapshots.

In the third experiment, we repeat the previous simulation while adding 5 additional interference sources to be rejected. The target signal is given a frequency of 75 MHz while the interference signals are assigned random frequencies for each trial ranging between 67.5 and 82.5 MHz. The DOA and polarization parameters of the interfering signals are also randomly assigned and the SIR is held constant at 0 dB. The

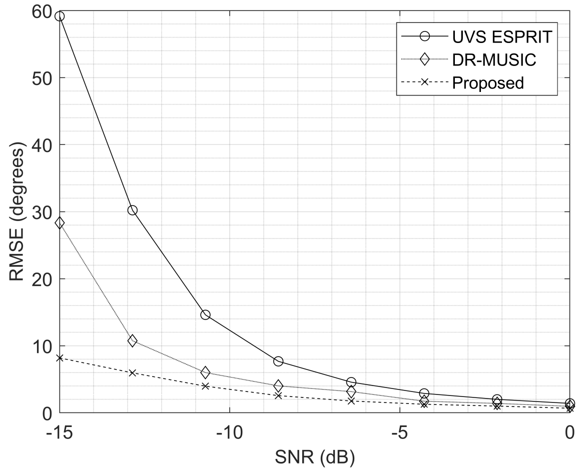

Fig. 2. RMSE of azimuth under low SNR conditions.

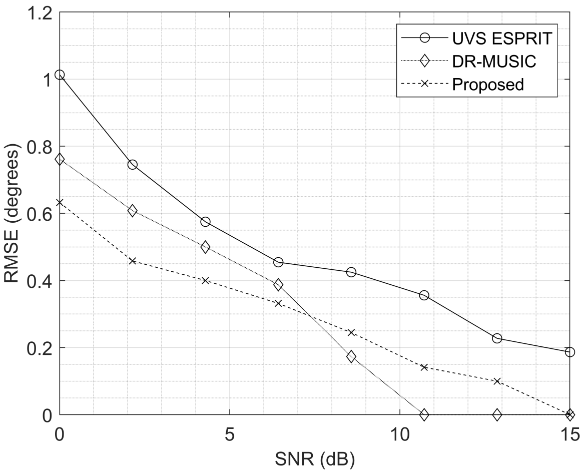

Fig. 3. RMSE of azimuth under high SNR.

results are shown in Figure 4.

The fourth experiment reveals the strength at which the method can reject interference signals. The parameters remain the same as in the previous experiment, but SNR is held constant at 0 dB while SIR ranges from -30 to 0 dB and is illustrated in Figure 5. The results show that low SIR conditions have no critical effect on the method's DOA estimation abilities.

VII. Conclusion

In this paper, a novel extension to the DR-MUSIC method applied to a single vector sensor has been proposed which, through collecting more data and performing more computation, increases the algorithm's performance in estimating the direction of signals in very low SNR and SIR conditions. In addition to this, the algorithm also has the ability to locate more signals than there are antenna components - a limitation that all other DOA algorithms currently possess.

References

[1] L. Wang, L. Yang, G. Wang, Z. Chen, M. Zou, "Uni-Vector-Sensor Dimensionality Reduction MUSIC Algorithm for DOA and Polarization Estimation"

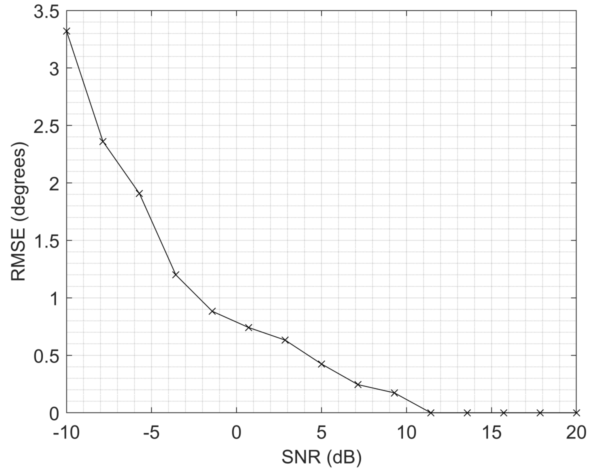

Fig. 4. RMSE of azimuth with interference signals.

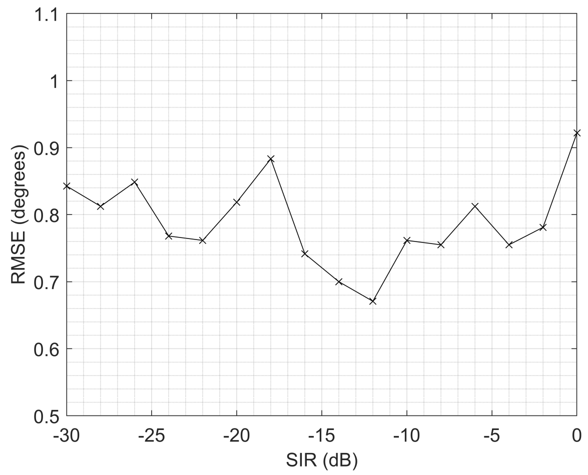

Fig. 5. RMSE of azimuth under very low SIR conditions.

[2] K. T. Wong & M. D. Zoltowski, "Uni-Vector-Sensor ESPRIT for Multisource Azimuth, Elevation, and Polarization Estimation", *IEEE Transactions on Antennas and Propagation*, vol. 45, no. 10, pp. 1467-1474, Oct. 1997

[3] M. D. Zoltowski and K. T. Wong, "ESPRIT-based 2-D direction finding with a sparse uniform array of electromagnetic vector sensors, *IEEE Transactions on Signal Processing*, vol. 48, no. 8, pp. 2195-2204, 2000.

[4] K. T. Wong and M. D. Zoltowski, "Closed-form direction finding and polarization estimation with arbitrarily spaced electromagnetic vector-sensors at unknown locations, *IEEE Transactions on Antennas and Propagation*, vol. 48, no. 5, pp. 671681, 2000.

[5] J. Li, "Direction and polarization estimation using arrays with small loops and short dipoles, *IEEE Transactions on Antennas and Propagation*, vol. 41, no. 3, pp. 379386, 1993.

[6] J. Li and R. T. Compton Jr., "Angle and polarization estimation using ESPRIT with a polarization sensitive array, *IEEETransactions on Antennas and Propagation*, vol. 39, no. 9, pp. 13761383, 1991

[7] A. Nehorai and E. Paldi, "Vector-sensor array processing for electromagnetic source localization, *IEEE Transactions on Signal Processing*, vol. 42, no. 2, pp. 376398, 1994.

[8] R. O. Schmidt, "Multiple emitter location and signal parameter estimation, *IEEE Transactions on Antennas and Propagation*, vol. AP-34, no. 3, pp. 276280, 1986.

[9] R. Roy and T. Kailath, "ESPRIT-estimation of signal parameters via rotational invariance techniques, *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 37, pp. 984995, July 1989.

[10] K. T. Wong and M. D. Zoltowski, "Self-Initiating MUSIC-Based direction finding and polarization estimation in spatio-polarizational beamspace, *IEEE Transactions on Antennas and Propagation*, vol. 48,

no. 8, pp. 12351245, 2000.
[11] K. T. Wong, L. Li, and M. D. Zoltowski, "Root-MUSIC-based direction-finding and polarization estimation using diversely polarized possibly collocated antennas, *IEEE Antennas and Wireless Propagation Letters*, vol. 3, no. 1, pp. 129132, 2004.
[12] B. V. Schell, R. A. Calabretta, W. A. Gardner and B. G. Agee, "Cyclic MUSIC Algorithms for Signal-Selective Direction Estimation," *Acoustics, Speech, and Signal Processing*, ICASSP-88., International Conference June 1989.
[13] H. Yan and H. Fan, "Wideband cyclic MUSIC algorithms, *Signal Processing*, vol. 85, pp. 643-649, 2005.
[14] P. Charge, Y. Wang and J. Saillard, "An extended cyclic MUSIC algorithm, emphIEEE Trans. Signal Process. vol. 51, no. 7, pp. 1695-1701, 2003.