Regression

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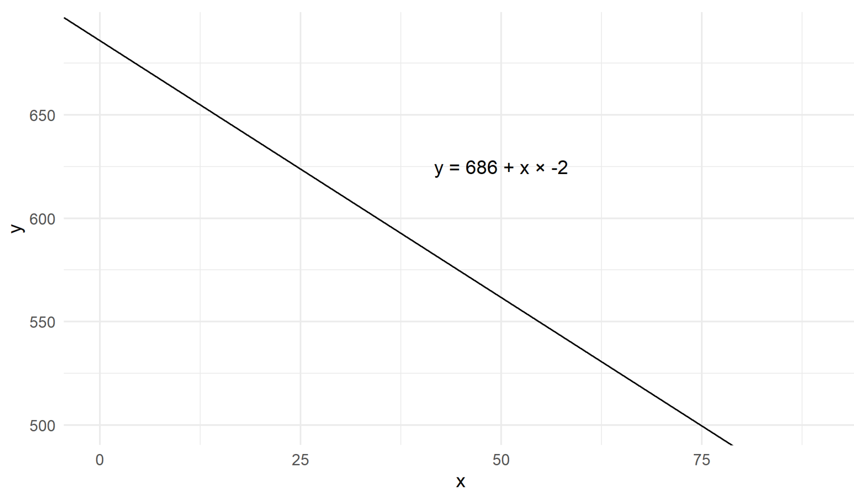
The regression model

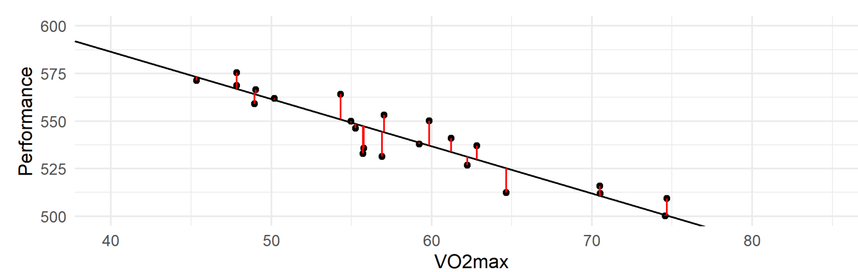


A univariate regression model can be expressed as $y = Intercept + x \times Slope$.

Building the model

A regression model built using observed data often contains some error:

$$y = Intercept + x \times Slope + Error$$



A more formal description of the model:

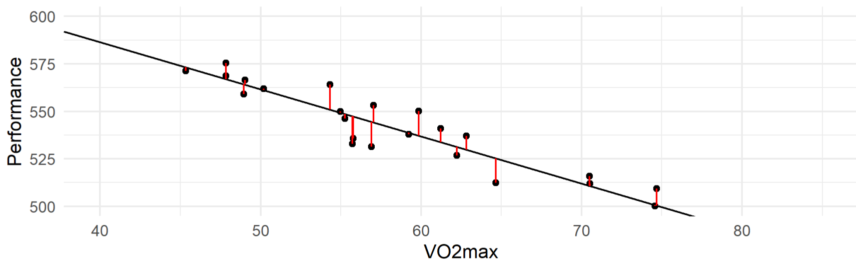
$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$

where y_i are the performance values for each participant $(i = 1, \ldots, n)$, β_0 is the intercept, β_1 is the slope and ϵ_i is the difference between each observation from its predicted values.

Fitting the model in R

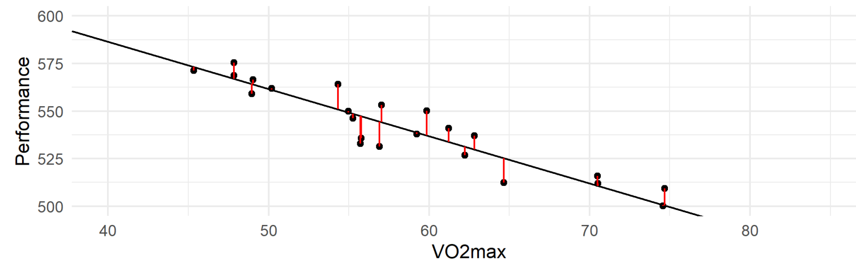
```
model <- lm(performance ~ vo2max, data = dat)
```

term	estimate	std.error	statistic	p.value
(Intercept)	685.80	11.60	59.10	< 0.001
vo2max	-2.48	0.19	-12.75	< 0.001

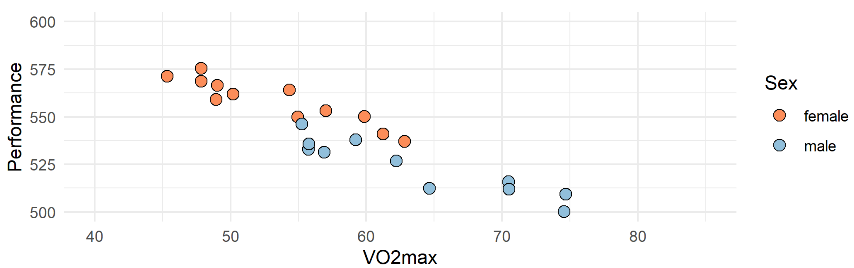


How is the regression model contructed?

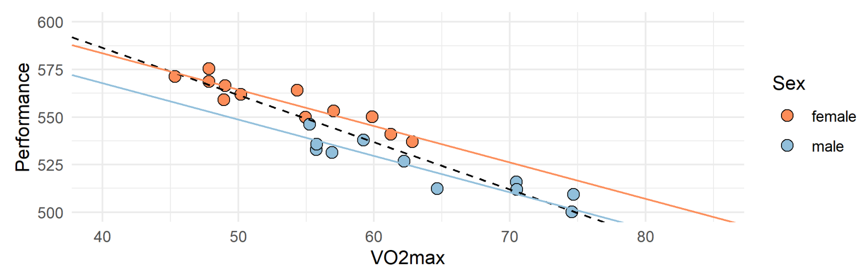
- We are trying to fit a line that most accurately predicts the observed points
- The "best fit line" minimizes distances from *predicted* values to *fitted values* (the best fit line).



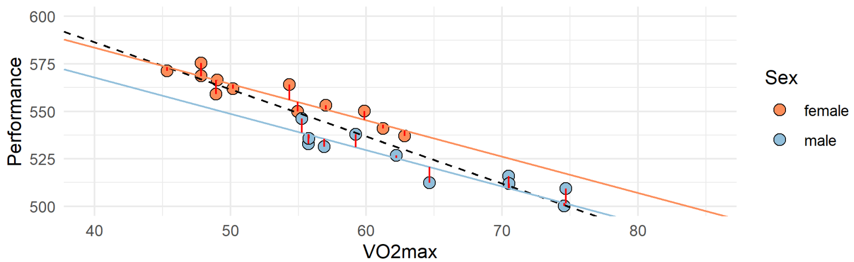
Additional information can improve the model



Additional information can improve the model

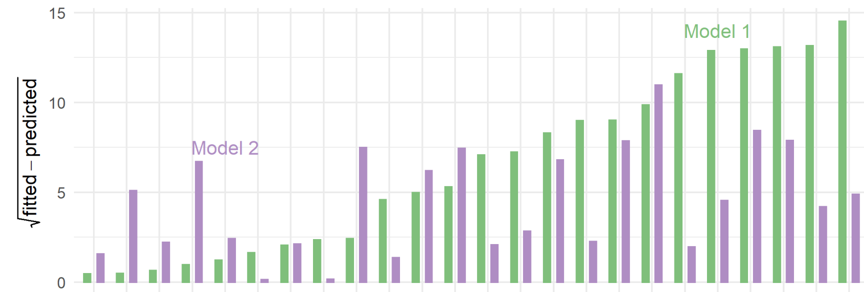


Additional information can improve the model



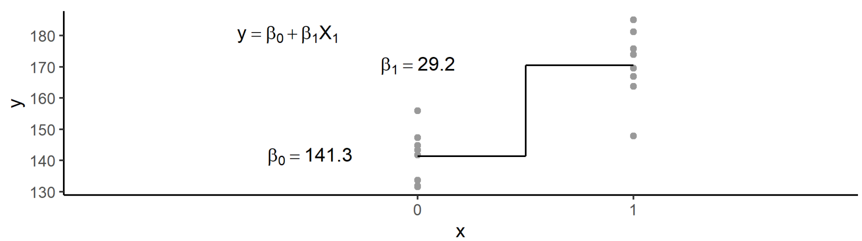
Minimizing the error of the model

Model	term	estimate	std.error	statistic	p.value
Model 1	(Intercept)	685.80	11.60	59.10	< 0.001
Model 1	vo2max	-2.48	0.19	-12.75	< 0.001
Model 2	(Intercept)	660.02	9.35	70.63	< 0.001
Model 2	vo2max	-1.91	0.17	-11.08	< 0.001
Model 2	sexmale	-15.64	3.03	-5.15	< 0.001



Dummy variables

- A dummy variable can be used in a regression model representing a qualitative variable (e.g. Male and Female) where the first **level** of the variable is **coded** 0 and the second level is **coded** 1
- In the regression model, the numerical coded variable is used, a simple uni-variate example:



Dummy variables

- In the case of Female and Male the dummy variable for sex is coded

if $sex = Female$ then $X = 0$

if $sex = Male$ then $X = 1$

Mean values for women:

$$y = \beta_0 + \beta_1 \times 0 = \beta_0$$

Mean values for men:

$$y = \beta_0 + \beta_1 \times 1 = \beta_0 + \beta_1$$

Dummy variables can used to code more levels than 2

- Using dummy variables, more **levels** can be coded into the model
- More parameters will have to be estimated, if three groups (A, B and C) are to be included in the model, 3-1 dummy variables are needed

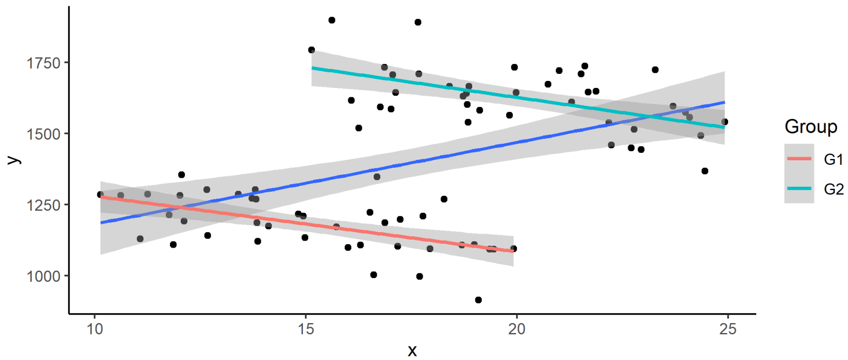
If $group = A$ then $X_1 = 0, X_2 = 0$

If $group = B$ then $X_1 = 1, X_2 = 0$

If $group = C$ then $X_1 = 0, X_2 = 1$

Dummy variables can be used to control for group effects

- Simpson's paradox is when **marginal** and **partial** relationships in the data set have different signs, i.e. a positive relationship in the whole data-set and negative relationships within subgroups



Dummy variables can be used to control for group effects

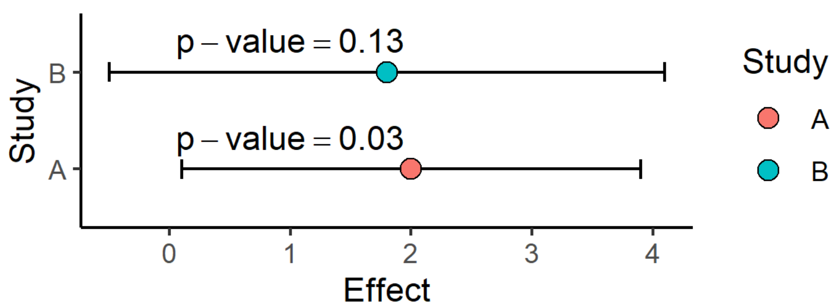
Simple model

	Estimate	Std. Error	t value
(Intercept)	895.83	121.07	7.40
x	28.67	6.71	4.27

Controlling for groups

	Estimate	Std. Error	t value
(Intercept)	1489.18	56.69	26.27
x	-20.45	3.62	-5.64
GroupG2	546.72	27.03	20.22

Estimation, an example



- What conclusions can be drawn from the two studies (using NHST vs. estimation)?

Example from: Cumming, G. (2012). **Understanding the new statistics : effect sizes, confidence intervals, and meta-analysis**. New York, Routledge.

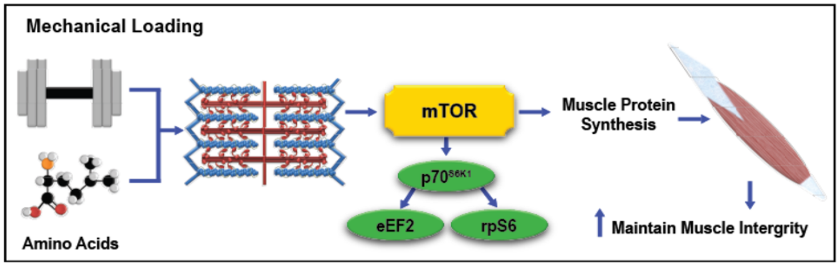
Estimation

- In addition to giving a interval representing the precision of the estimate, the confidence interval can be used to assess the clinical importance of a study.
- Are values inside the confidence interval large (or small) enough to care about in a clinical sense (e.g. weight gain study)

Issues in studies of association

- Influential data points
- Correlation does not imply causation
- Regression towards the mean

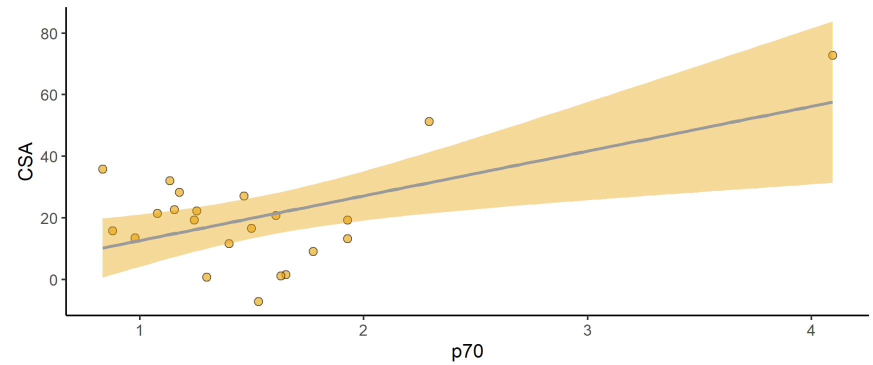
Influential data points -- mTOR signaling and exercise induced muscle hypertrophy



Mechanical loading and amino acids maximize mTORC1 signaling and muscle protein synthesis, thus contributing to the maintenance of skeletal muscle mass. Abbreviations: mTOR, mammalian target of rapamycin; p70[S6K1], 70-kDa ribosomal protein S6 kinase 1; eEF2, eukaryotic elongation factor 2; rpS6, ribosomal protein S6.

Pasiakos, S. M. (2012). "Exercise and Amino Acid Anabolic Cell Signaling and the Regulation of Skeletal Muscle Mass." Nutrients 4(7).

Exercise induced P70 S6-kinase phosphorylation predicts muscle hypertrophy (Mitchell et al. 2013)



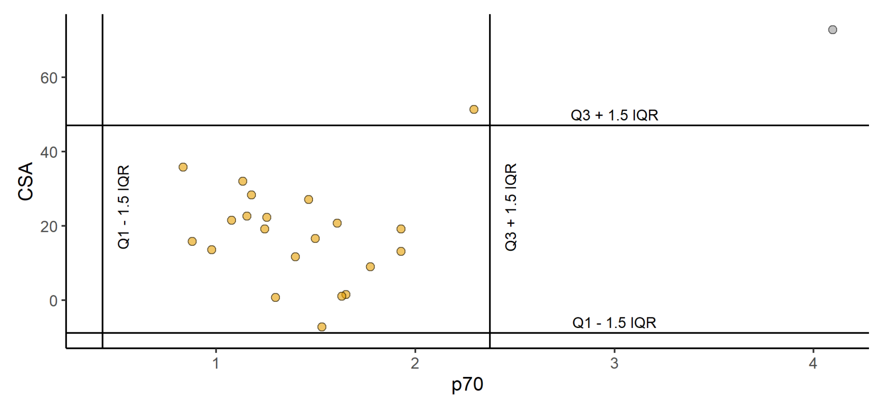
Mitchell, C. J., et al. (2013). "Muscular and Systemic Correlates of Resistance Training-Induced Muscle Hypertrophy." PLoS One 8(10): e78636.

Influential data points

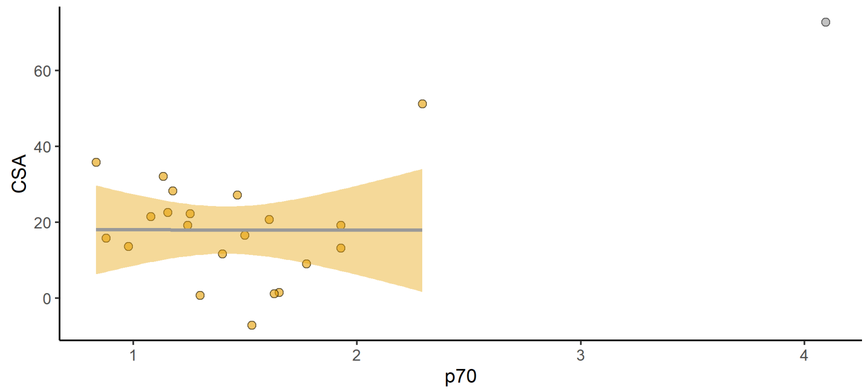
- Data points that substantially deviates from the rest of the data may affect the interpretation of regression models.
- "Leverage" is the effect each data point has on the model, unusual X-values produces larger leverage
- This can be assessed by looking at the graph, and numerically
- A tool in simple regression would be to assess outliers (in the X-axis) on model characteristics

Detect outliers

- An outlier is defined as $Q3/Q1 \pm 1.5 \times IQR$



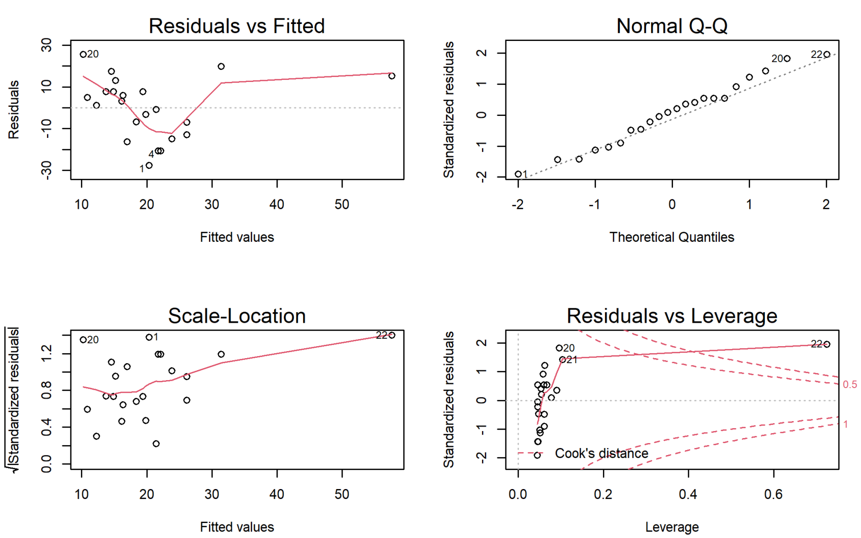
Re-do analysis without outlier



What can we conclude from the Mitchell data-set?

Graphical evaluation of regression models

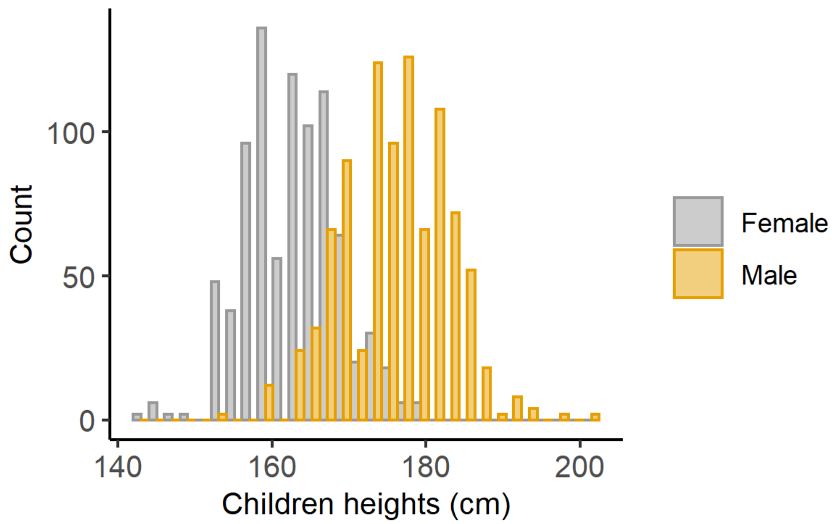
```
dat <- read_excel("./data/Mitchell2013.xlsx") # Import the data
m <- lm(CSA ~ p70, data = dat) # Fitting the model
plot(m) # Create diagnostic plots of the model
```

Regression towards the mean

- Francis Galton analyzed parents and children heights to study heritability (how much of a trait can be explained by genetics?)
- Does parents heights determine children heights?

Regression towards the mean



- Do tall parents have tall children?

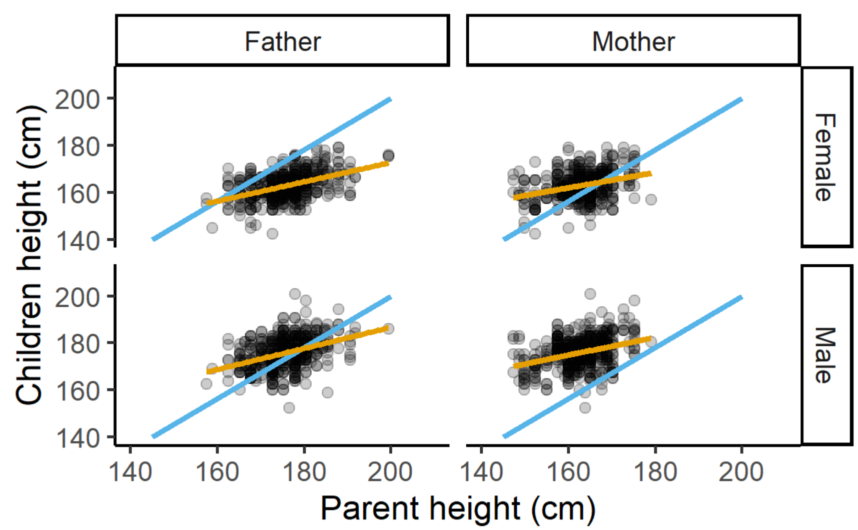
Regression towards the mean

- If parents heights would predict children heights, what would the regression line look like?

Regression towards the mean

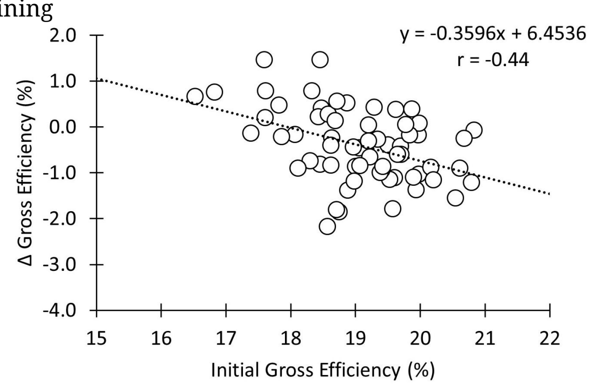
- Regression towards the mean predicts that upon repeated sampling from a normal distribution, extreme values will be less frequent than values close to the mean.
- An extreme value **within** a family will be "replaced" by a less extreme.
- How would the regression line look?

Regression towards the mean



Regression towards the mean

- This poses a problem when analyzing baseline characteristics and change due to training



When using a correction "*...to minimize the effect* [of regression to the mean], *the correlations in the present study were weakened.*"

Skovereng, K., et al. (2018). "Effects of Initial Performance, Gross Efficiency and O~2peak~ Characteristics on Subsequent Adaptations to Endurance Training in Competitive Cyclists." Front Physiol 9(713).