Statistical power, p-values and effect sizes

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Null hypothesis significance testing (NHST)

- NHST is the most common way of making *decisions* about **effects** within the sport sciences.
- NHST can be used to assess if e.g. groups are different or regression parameters are different than zero.
- NHST can be performed using the following steps:

1. Choose a *null*-hypothesis, e.g. there is no differences between groups $H_0 : \mu_1 = \mu_2$, and a alternative hypothesis e.g. $H_1 : \mu_1 - \mu_2 \neq 0$
2. Specify a **significance level**, usually 5% (or $\alpha = 0.05$).
3. Perform an appropriate test, in the case of differences between means, a t test and calculate the p-value
4. If the p-value is less than the stated α-level we declare the result as statistically significant and reject H_0.

NHST is a special flavour of hypothesis testing

- Two competing views on hypothesis testing were originally presented by Ronald A. Fisher on the one hand and Jerzy Neyman and Egon Pearson on the other hand.

Fisher	Neyman-Pearson
1. State H_0	1. State H_0 and H_1
2. Specify test statistic	2. Specify α (e.g. 5%)
3. Collect data, calculate test statistic and p-value	3. Specify test statistics and critical value
4. Reject H_0 if p is small	4. Collect data, calculate test statistic, determine p
	5. Reject H_0 if favor of H_1 if $p < \alpha$

Kline, R. B. (2013). *Beyond significance testing: Statistics reform in the behavioral sciences*, 2nd ed. Washington, DC, US, American Psychological Association

The p-value

- The p-value is the probability of obtaining a value of a **test statistic** (t) as extreme as the one obtained or more extreme under the condition that the null-hypothesis is true:

$$p(t|H_0)$$

- We assume that the **null is true** and we calculate how often a results such as the one obtained would occur as a result of chance. However, using $\alpha = 0.05$ we simply declare *significant* when $p < \alpha$ (and accept that we will be wrong in 5% of repeated studies), **this is the Neyman-Person approach**,

- The α-level is the Type 1 error rate, the probability of rejecting H_0 when it is actually true.

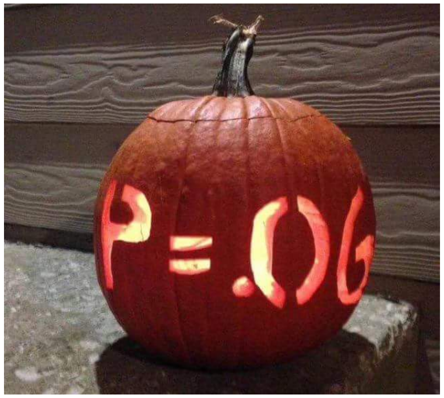
Interpreting p-values

- There are two distinct ways of looking at the p-value, one where the p-value is a pre-specified threshold for decision (Neyman-Pearson), and one where the p-value is thought of as a **meassure of strength of evidence** against the null-hypothesis (Fisher).

- It is common practice to combine the two approaches in analysis of scientific experiments. Examples:

  - "There was not a significant difference between groups but the p-values suggested a trend towards ..."
  - "The difference between group A and B was significant, but the difference between A and C was highly significant"
- According to the original frameworks, the mix (Fisher combined with Neyman-Pearson) leads to abuse of NHST

More on p-value interpretation



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Statistical power

- Neyman and Pearson extended Fishers hypothesis testing procedure with the concept of power.
- An alternative hypothesis can be stated for a specific value of e.g. a difference $H_1 : \mu_1 - \mu_2 = 5$
- Using this alternative hypothesis we can calculate the statistical power: The probability of **rejecting** H_0 if the alternative hypothesis is true.
- The probability of failing to reject H_0 if H_1 is true is the Type 2 error rate β.
- Statistical power is therefore: $1 - \beta$.

Errors in NHST

- There are two scenarios where we make mistakes, by rejecting H_0 when it is actually true and not rejecting H_0 when it is false.

	Accept H_0	**Reject H_0**
H_0 is true	Correct!	Type I error
H_0 is false	Type II error	Correct!

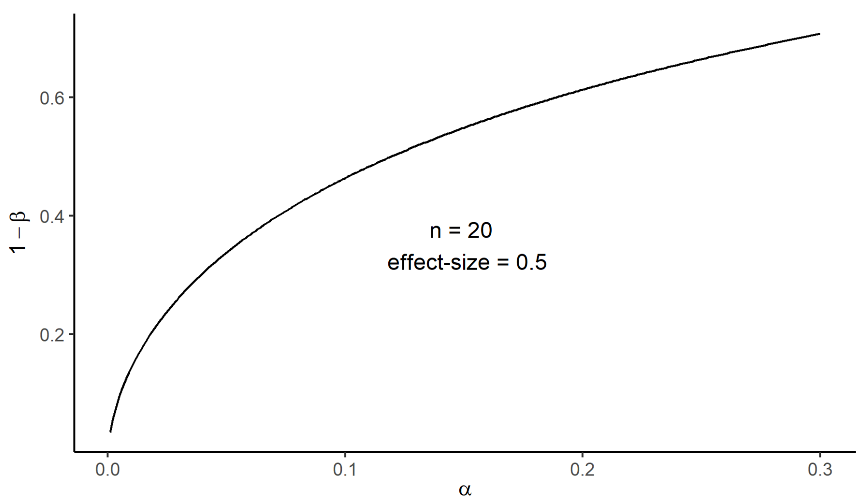
Error rates in NHST

- We usually specify the level of Type I errors to 5%
- Another convention is to specify the power to 80%, this means that the risk of **failing to reject** H_0 when H_0 is **false** is 20%.
- These levels are chosen by tradition(!), but a well designed study is planned using well thought through Type I and II error rates.
- In the case of $\alpha = 0.05$ and $\beta = 0.2$, Cohen (1988) pointed out that this can be thought of as Type I errors being a mistake four times more serious than Type II errors.

$$\frac{0.20}{0.05} = 4$$

- Rates could be adjusted to represent the relative seriousness of respective errors.

The α-error and statistical power are related



Error rates in NHST, an example

- If a study tries to determine if a novel treatment with no known side-effects should be implemented, **failure to detect a difference** compared to placebo when **there is a difference** (Type II error) would be more serious than to detect a difference that is not true (Type I error).

- In this case error rates could be adjusted to reflect this, decrease possibility Type II errors by increasing the possibility of Type I errors.

Power analysis in NHST

- When planning a study within human exercise physiology, we want to know *how many participants to recruit.*
- This is a question of **cost** as more participants means **more work**
- It is a question of **ethics** as more participants means that more people are subjected to risk/discomfort.
- We aim to recruit as many participants as is necessary to answer our question.
- We state our H_0 and H_1 (according to the Neyman-Pearson tradition).
- The H_1 has a special function, this can be seen as the smallest meaningful difference between conditions under study, the difference we want to be able to detect.
- When we have specified H_1 we can perform power analysis and **sample size estimation**.

Power analysis, an example

- We want to compare the muscle mass gains as a result of two resistance training protocols.
- A 1 kg difference in lean mass increases is considered a meaningful difference after 12 weeks of training.
- The standard deviation from previous studies is used to estimate the expected variation in responses to 12 weeks of resistance training.
- We plan to perform our experiment with equal sized groups and assume they will have the same variation ($\sigma = 2.5$).
- To calculate the required sample size we first must calculate a standardized effect size, also known as Cohen's d.
- We can standardize our "effect size" of 1 kg by dividing by the SD.

$$d = \frac{1}{2.5} = 0.4$$

Effect sizes

- The effect size is the primary aim of an experiment, we wish to know the difference, correlation, regression coefficient, percentage change...
- The effect size can be standardized (e.g. divided by the standard deviation or calculated as e.g. a correlation).

Power analysis, an example cont.

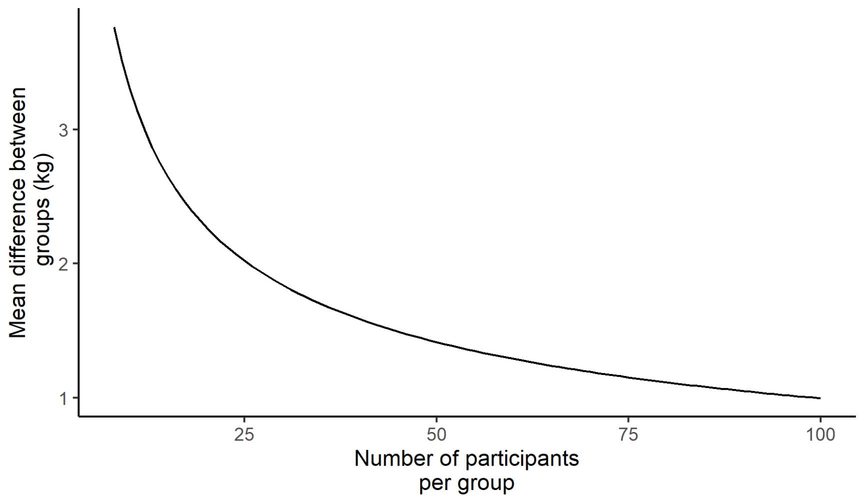
- We must specify α and $1-\beta$ to calculate the required sample size, let's say that the Type I error is four times more serious than the Type II error, and that we would accept to be wrong in rejecting H_0 at a rate of 5%.

$$\alpha = 0.05, \beta = 0.2, d = 0.4$$

```
power <- pwr.t.test(d = 0.4, power = 0.8,
                            sig.level = 0.05, type = "two.sample",
                            alternative = "two.sided")
```

- Given these specifics we would require 100 in each group to be able to show a meaningful difference with the power set to 80%.
- This is a **big study** what if we examine the smallest difference we can detect using a set sample size

Mean difference between groups vs. number of participants per group



Power analysis

Statistical power is influenced by:

- The α level
- The direction of the hypothesis (negative, positive or both ways different from H_0)
- Experimental design (within- or between-participants)
- The statistical test
- Reliability of test scores

Power analysis in R

- The `pwr` package can be used to calculate sample size, power, effect sizes or α-levels.
- The relationship between these values can be used to calculate one unknown.
- We simply must "guess" values for some of the values using data from previous studies

```
pwr.t.test(power = 0.8,
           d = 0.4,
           sig.level = 0.05,
           type = "two.sample",
           alternative = "two.sided")
```


Critique of NHST

- NHST with p-values tend to create an "either-or" situation, gives no answer about the size of an effect
- Test statistics are related to sample size, small effects can be detected using big sample sizes
- Built into the NHST framework is the acceptance of a proportion of tests being false positive, the likelihood of getting false positives increases with the number of tests.

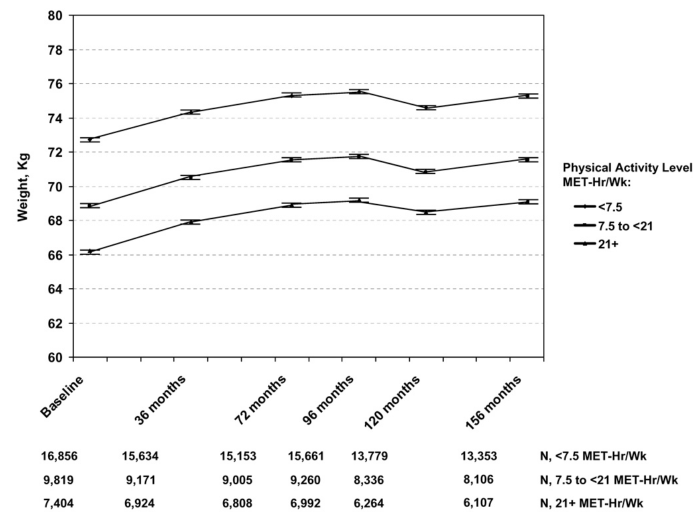
Statistical significance and clinical significance

Large sample sizes can make small effect sizes statistically significant. Example, Lee, I-Min et al (2010):

- Objective: To examine the association of different amounts of physical activity with long-term weight changes among women consuming a usual diet.
- Design: Prospective cohort study, following 34,079 healthy, US women (mean age, 54.2 years) from 1992–2007. At baseline, 36-, 72-, 96-, 120-, 144- and 156-months' follow-up, women reported their physical activity and body weight.
- Results: Women gained a mean of 2.6 kg throughout the study.

Comparison	Mean difference	*p*-value
7.5 - <21 MET vs. $\geq$ 21 MET	0.11 kg	0.003
< 7.5 MET vs. $\geq$ 21 MET	0.12 kg	0.002

Results



*Lee, I-Min et al. "Physical Activity and Weight Gain Prevention" JAMA: the journal of the American Medical Association 303.12 (2010): 1173–1179. PMC. Web. 24 Sept. 2018.

Making the wrong decision 5% of the time

- Given that NHST accepts mistakes at a rate of α, e.g. every $\frac{1}{\alpha_{0.05}} = 20^{th}$ test result will be false.
- The Neyman-Pearson approach is to only do NHST with an pre-specified α -level
- One must also avoid making up hypotheses after the test.
- If you do multiple tests, family-wise corrections can be made, e.g. the Bonferroni correction:

$$\alpha_{Bonferroni} = \frac{\alpha}{n\ tests}$$

- For statistical significance to be reached, the $\alpha_{Bonferroni}$ threshold must be reached.

Summary

- The p-value is the probability of the observed test result (or a more extreme) when the null hypothesis (H_0) is true
- p-values can be seen as a threshold for decision about H_0, or the degree of evidence against H_0.
- As p-values are related to sample size, null-hypothesis testing should be performed in studies where sample sizes are selected based on a power analysis.