Statistical power, p-values and effect sizes

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Null hypothesis significance testing (NHST)

- NHST is the most common way of making *decisions* about **effects** within the sport sciences.
- NHST can be used to assess if e.g. groups are different or regression parameters are different than zero.
- NHST can be performed using the following steps:

1. Choose a *null*-hypothesis, e.g. there is no differences between groups $H_0 : \mu_1 = \mu_2$, and a alternative hypothesis e.g. $H_1 : \mu_1 - \mu_2 \neq 0$
2. Specify a **significance level**, usually 5% (or $\alpha = 0.05$).
3. Perform an appropriate test, in the case of differences between means, a *t* test and calculate the *p*-value
4. If the *p*-value is less than the stated α-level we declare the result as statistically significant and reject H_0.

NHST is a special flavour of hypothesis testing

- Two competing views on hypothesis testing were originally presented by Ronald A. Fisher on the one hand and Jerzy Neyman and Egon Pearson on the other hand.

Fisher	Neyman-Pearson
1. State H_0	1. State H_0 and H_1
2. Specify test statistic	2. Specify α (e.g. 5%)
3. Collect data, calculate	
test statistic and p-value	3. Specify test statistics and critical value
4. Reject H_0 if p is small	4. Collect data, calculate test statistic, determine p
	5. Reject H_0 if favor of H_1 if $p < \alpha$

Kline, R. B. (2013). *Beyond significance testing: Statistics reform in the behavioral sciences*, 2nd ed. Washington, DC, US, American Psychological Association

The *p*-value

- The *p*-value is the probability of obtaining a value of a **test statistic** (t) as extreme as the one obtained or more extreme under the condition that the null-hypothesis is true:

$$p(t|H_0)$$

- We assume that the **null is true** and we calculate how often a results such as the one obtained would occur as a result of chance. However, using $\alpha = 0.05$ we simply declare *significant* when $p < \alpha$ (and accept that we will be wrong in 5% of repeated studies), **this is the Neyman-Person approach**,
- The α-level is the Type 1 error rate, the probability of rejecting H_0 when it is actually true.

Interpreting p-values

- There are two distinct ways of looking at the p-value, one where the p-value is a pre-specified threshold for decision (Neyman-Pearson), and one where the p-value is thought of as a **meassure of strength of evidence** against the null-hypothesis (Fisher).
- It is common practice to combine the two approaches in analysis of scientific experiments. Examples:
- "There was not a significant difference between groups but the p-values suggested a trend towards ..."
- "The difference between group A and B was significant, but the difference between A and C was higly significant"
- According to the original frameworks, the mix (Fisher combined with Neyman-Pearson) leads to abuse of NHST

Statistical power

- Neyman and Pearson extended Fishers hypothesis testing procedure with the concept of power.
- An alternative hypothesis can be stated for a specific value of e.g. a difference $H_1 : \mu_1 - \mu_2 = 5$
- Using this alternative hypothesis we can calculate the statistical power: The probability of **rejecting** H_0 if the alternative hypothesis is true.
- The probability of failing to reject H_0 if H_1 is true is the Type 2 error rate (β).
- Statistical power is therefore: $1 - \beta$.

Errors in NHST

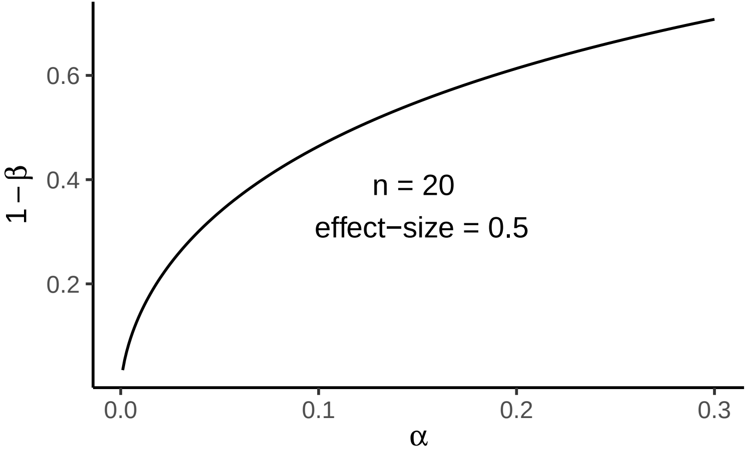
- There are two scenarios where we make mistakes, by rejecting H_0 when it is actually true and not rejecting H_0 when it is false.

	Accept H_0	Reject H_0
H_0 is true	Correct!	Type I error
H_0 is false	Type II error	Correct!

Error rates in NHST

- We usually specify the level of Type I errors to 5%
- Another convention is to specify the power to 80%, this means that the risk of **failing to reject** H_0 when H_0 is **false** is 20%.
- These levels are chosen by tradition(!), but a well designed study is planned using well thought through Type I and II error rates.
- In the case of $\alpha = 0.05$ and $\beta = 0.2$, Cohen (1988) pointed out that this can be thought of as Type I errors being a mistake four times more serious than Type II errors. ($\frac{0.20}{0.05} = 4$)
- Rates could be adjusted to represent the relative seriousness of respective errors.

The α-error and statistical power is related



Error rates in NHST, an example

- If a study tries to determine if a novel treatment with no known side-effects should be implemented, **failure to detect a difference** compared to placebo when **there is a difference** (Type II error) would be more serious than to detect a difference that is not true (Type I error).
- In this case error rates could be adjusted to reflect this, decrease possibility Type II errors by increasing the possibility of Type I errors.

Power analysis in NHST

- When planning a study within human exercise physiology, we want to know *how many participants to recruit.*
- This is a question of **cost** as more participants means **more work**
- It is a question of **ethics** as more participants means that more people are subjected to risk/discomfort.
- We aim to recruit as many participants as is necessary to answer our question.
- We state our H_0 and H_1 (according to the Neyman-Pearson tradition).
- The H_1 has a special function, this can be seen as the smallest meaningful difference between conditions under study, the difference we want to be able to detect.
- When we have specified H_1 we can perform power analysis and **sample size estimation**.

Power analysis, an example

- We want to compare the muscle mass gains as a result of two resistance training protocols.
- A 1 kg difference in lean mass increases is considered a meaningful difference after 12 weeks of training.
- The standard deviation from previous studies is used to estimate the expected variation in responses to 12 weeks of resistance training.
- We plan to perform our experiment with equal sized groups and assume they will have the same variation ($\sigma = 2.5$).
- To calculate the required sample size we first must calculate a standardized effect size, also known as Cohen's d.
- We can standardize our "effect size" of 1 kg by dividing by the SD.

$$d = \frac{1}{2.5} = 0.4$$

Effect sizes

- The effect size is the primary aim of an experiment, we wish to know the difference, correlation, regression coefficient, percentage change. . .
- The effect size can be standardized (e.g. divided by the standard deviation or calculated as e.g. a correlation).

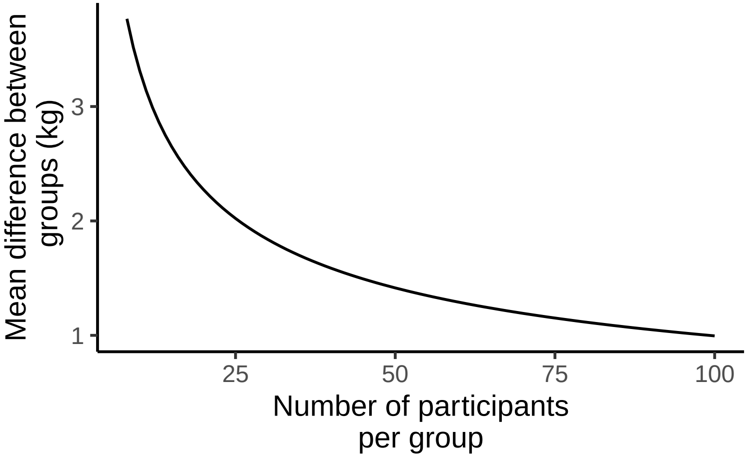
Power analysis, an example cont.

- We must specify α and $1-\beta$ to calculate the required sample size, let's say that the Type I error is four times more serious than the Type II error, and that we would accept to be wrong in rejecting H_0 at a rate of 5%.

$$\alpha = 0.05, \beta = 0.2, d = 0.4$$

- Given these specifics we would require 100 in each group to be able to show a meaningful difference with the power set to 80%.
- This is a **big study** what if we examine the smallest difference we can detect using a set sample size

Mean difference between groups vs. number of participants per group



Power analysis

Statistical power is influenced by:

- The α level
- The direction of the hypothesis (negative, positive or both ways different from H_0)
- Experimental design (within- or between-participants)
- The statistical test
- Reliability of test scores

Power analysis in R

- The `pwr` package can be used to calculate sample size, power, effect sizes or α-levels.
- The relationship between these values can be used to calculate one unknown.
- We simply must "guess" values for some of the values using data from previous studies

```
pwr.t.test(power = 0.8,
           d = 0.4,
           sig.level = 0.05,
           type = "two.sample",
           alternative = "two.sided")
```

Critique of NHST

- NHST with *p*-values tend to create an "either-or" situation, gives no answer about the size of an effect
- Test statistics are related to sample size, small effects can be detected using big sample sizes
- Built in to the NHST framework is the acceptance of a proportion of tests being false positive (α), the likelihood of getting false positives increases with the number of tests.

NHST and magnitude of an effect

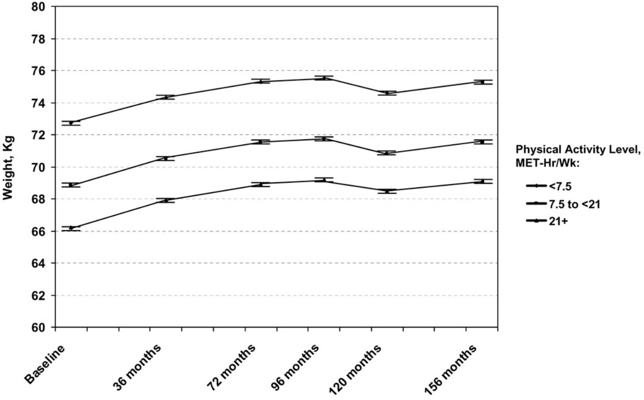
- “The training group gained 3 kg in muscle mass ($p<0.05$)”

Statistical significance and clinical significance

Large sample sizes can make small effect sizes statistically significant. Example, Lee, I-Min et al (2010):

- Objective: To examine the association of different amounts of physical activity with long-term weight changes among women consuming a usual diet.
- Design: Prospective cohort study, following 34,079 healthy, US women (mean age, 54.2 years) from 1992–2007. At baseline, 36-, 72-, 96-, 120-, 144- and 156-months' follow-up, women reported their physical activity and body weight.
- Results: Women gained a mean of 2.6 kg throughout the study. In multivariate analysis, compared with women expending ≥ 21 MET-hr/week, those expending 7.5-$<$21 and $<$7.5 MET-hr/week **gained 0.11 kg (SD=0.04; P=0.003) and 0.12 kg (SD=0.04; P=0.002)**, respectively, over a mean interval of 3 years.

Results



16,856	15,634	15,153	15,661	13,779	13,353	N, <7.5 MET-Hr/Wk
9,819	9,171	9,005	9,260	8,336	8,106	N, 7.5 to <21 MET-Hr/Wk
7,404	6,924	6,808	6,992	6,264	6,107	N, 21+ MET-Hr/Wk

*Lee, I-Min et al. "Physical Activity and Weight Gain Prevention" JAMA: the journal of the American Medical Association 303.12 (2010): 1173–1179. PMC. Web. 24 Sept. 2018.

Making the wrong decision 5% of the time

- Given that NHST accepts mistakes at a rate of α, every $\frac{1}{\alpha} = 20^{th}$ test result will be false.
- The Neyman-Pearson approach is to only do NHST with an pre-specified α-level
- One must also avoid making up hypotheses after the test.
- If you do multiple tests, family-wise corrections can be made, e.g. the Bonferroni correction:

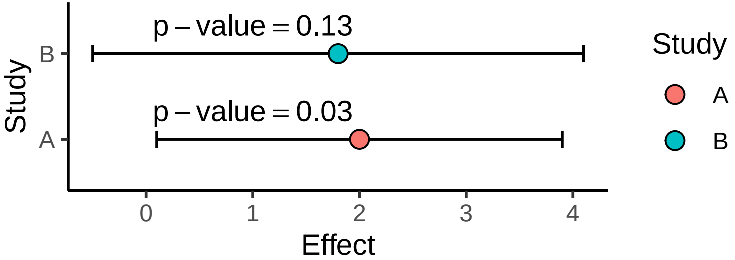
$$\alpha_{Bonferroni} = \frac{\alpha}{n\ tests}$$

- For statistical significance to be reached, the $\alpha_{Bonferroni}$ threshold must be reached.

A complement or alternative to NHST: Estimation

- Instead of testing against a null-hypothesis, estimation aims at finding a point estimate of the parameter of interest
- Secondly we want to find an interval estimate of the parameter
- This can be done using confidence intervals.
- Confidence intervals provides an point-estimate together with a range of plausible values of the population parameter.

Estimation, an example



- What conclusions can be drawn from the two studies (using NHST vs. estimation)?

Example from: Cumming, G. (2012). **Understanding the new statistics : effect sizes, confidence intervals, and meta-analysis**. New York, Routledge.

Estimation

- In addition to giving a interval representing the precision of the estimate, the confidence interval can be used to assess the clinical importance of a study.
- Are values inside the confidence interval large (or small) enough to care about in a clinical sense (e.g. weight gain study)