Substitution With Definite Integrals

SUGGESTED REFERENCE MATERIAL:

As you work through the problems listed below, you should reference Chapter 5.9 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Be able to evaluate definite integrals using a substitution of variables.

PRACTICE PROBLEMS:

For problems 1-3, use the given substitution to express the given integral (including the limits of integration) in terms of the variable u. Do not evaluate the integrals.

1. $\displaystyle\int_1^5 (3x-4)^{10}\,dx,\ u = 3x-4$

   $\boxed{\displaystyle\frac{1}{3}\int_{-1}^{11} u^{10}\,du}$

2. $\displaystyle\int_{\frac{1}{e}}^{e} \frac{(\ln x)^3}{x}\,dx,\ u = \ln x$

   $\displaystyle\int_{-1}^{1} u^3\,du$; Detailed Solution: Here

3. $\displaystyle\int_0^4 \frac{1}{2x+1}\,dx,\ u = 2x+1$

   $\boxed{\displaystyle\frac{1}{2}\int_1^9 \frac{1}{u}\,du}$

For problems 4-19, evaluate the following integrals.

4. $\displaystyle\int_0^{\frac{1}{2}} \left(\frac{x^3}{\sqrt{1-x^4}}\right) dx$

   $\dfrac{4-\sqrt{15}}{8}$; Detailed Solution: Here

5. $\displaystyle\int_0^{\frac{\pi}{4}} \sin^2(3x)\cos(3x)\,dx$

$\boxed{\dfrac{\sqrt{2}}{36}}$

6. $\displaystyle\int_1^{\ln 10} e^{4x}\,dx$

$\boxed{2500 - \dfrac{1}{4}e^4}$

7. $\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (\csc^2 x - \sin x\cos x)\,dx$

$\dfrac{1}{\sqrt{3}} - \dfrac{1}{8}$; Detailed Solution: Here

8. $\displaystyle\int_{-1}^{1} \frac{1}{1+3x^2}\,dx$

$\dfrac{2\pi\sqrt{3}}{9}$; Detailed Solution: Here

9. $\displaystyle\int_0^{\frac{\pi}{12}} \sec^2(4x)\,dx$

$\boxed{\dfrac{\sqrt{3}}{4}}$

10. $\displaystyle\int_{-1}^{10} \frac{1}{2+x}\,dx$

$\boxed{\ln 12}$

11. $\displaystyle\int_{-3}^{2} (2x+2)(x^2+2x-3)\,dx$

$\boxed{\dfrac{25}{2}}$

12. $\displaystyle\int_{0}^{\frac{\pi}{6}} \cos^4(3x)\sin(3x)\,dx$

$\boxed{\dfrac{1}{15}}$

13. $\displaystyle\int_{1}^{16} \frac{1}{\sqrt{x}}e^{\sqrt{x}}\,dx$

$-2e+2e^4$; Video Solution: http://www.youtube.com/watch?v=0plDXxeFNvQ

14. $\displaystyle\int_0^{\frac{1}{\sqrt{2}}} \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

$\boxed{\dfrac{\pi^2}{32}}$

15. $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \tan(x)\,dx$

$\dfrac{1}{2}(\ln 3-\ln 2)$; Detailed Solution: Here

16. $\displaystyle\int_{\sqrt[3]{5}}^{2} \frac{x^2}{(x^3-4)^3}\,dx$

$\boxed{\dfrac{5}{32}}$

17. $\displaystyle\int_{-1}^{1} \frac{1}{x^2-4x+4}\,dx$

$\boxed{\dfrac{2}{3}}$

18. $\displaystyle\int_4^5 x\sqrt{x-4}\,dx$

$\dfrac{46}{15}$; Video Solution: https://www.youtube.com/watch?v=LE52v9SV_po

19. $\displaystyle\int_{-1}^{6} \sqrt{3+|x|}\,dx$

$\boxed{\dfrac{70}{3} - 4\sqrt{3}}$

20. For each of the following, express the given definite integral (including the limits of integration) in terms of u. Then, evaluate the "new" integral by using an appropriate formula from geometry.

   (a) $\displaystyle\int_{0}^{\sqrt[4]{2}} x^3\sqrt{4-x^8}\,dx$ (HINT: Express x^8 as the square of some term).

   $\boxed{\dfrac{\pi}{4}}$

   (b) $\displaystyle\int_{1}^{e^4} \frac{\sqrt{16-(\ln x)^2}}{x}\,dx$

   $\boxed{4\pi}$

21. It can be shown that $\dfrac{8}{4x^2+4x-15} = \dfrac{1}{2x-3} - \dfrac{1}{2x+5}$.

   (a) Let t be a fixed constant such that $t > 2$. Use these facts to evaluate $\displaystyle\int_{2}^{t} \frac{8}{4x^2+4x-15}\,dx$.

   $\boxed{\dfrac{1}{2}\ln\left|\dfrac{2t-3}{2t+5}\right| + \ln 3}$

   (b) Evaluate $\displaystyle\lim_{t\to+\infty} \int_{2}^{t} \frac{8}{4x^2+4x-15}\,dx$

   $\boxed{\ln 3}$