Reg. No. : | | | | | | | | | | | | |

# Question Paper Code: 1143200

B.E. / B.Tech. DEGREE EXAMINATIONS, NOV/ DEC 2024

Third Semester

Mechanical Engineering

MA8353 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Regulation 2017)

Time: Three Hours Maximum: 100 Marks

Answer ALL questions

PART – A (10 x 2 = 20 Marks)

1. Form the PDE by eliminating the arbitrary constants a and b from the relation $z = \left(x^2 + a^2\right)\left(y^2 + b^2\right)$ .

2. Find the PI from the homogenous PDE $(D^2 - DD')z = \sin(x + y)$ .

3. State the Dirichlet's condition or sufficient conditions for the existence of Fourier series of f(x).

4. Find the Fourier constant $a_0$ for f(x)=k in $(0,2\pi)$.

5. Write the possible solutions of one-dimensional wave equation.

6. Classify the following partial differential equation:

$$3\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 6\frac{\partial^2 u}{\partial y^2} - 2\frac{\partial u}{\partial y} + u = 0$$

7. Define Fourier transform pair.

8. Find the Fourier sine transform of 1/x.

9. Find $Z[n]$.

10. State Initial and Final Value Theorem on Z-Transform.

PART – B (5 x 13 = 65 Marks)

11\. (a) (i) Form the PDE by eliminating arbitrary function from $z = y^2 + 2f(\frac{1}{x} + logy)$ (7)

(ii) Solve $z = px + qy + \sqrt{1 + p^2 + q^2}$. (6)

(OR)

(b) (i)Solve the PDE $x(y - z)p + y(z - x)q = z(x - y)$. (7)

(ii)Solve $(D^2 - 4DD' + 4D'^2)z = e^{2x+y} + 16$. (6)

12\. (a) Find the Fourier series of the function $f(x) = \begin{cases} x & \text{for } 0 < x < \pi \\ 2\pi - x & \text{for } \pi < x < 2\pi \end{cases}$. Also deduce the series $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$. (13)

(OR)

(b) (i) Find the half range sine series for $y = x$ in $(0, l)$ and deduce that

$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots = \frac{\pi^2}{6}$ (7)

(ii)Using harmonic analysis, find the Fourier series expansion upto two harmonics from the data. (6)

| x | 0 | $\pi/3$ | $2\pi/3$ | $\pi$ | $4\pi/3$ | $5\pi/3$ | $2\pi$ |
|---|---|---|---|---|---|---|---|
| f(x) | 10 | 12 | 15 | 20 | 17 | 11 | 10 |

13\. (a) A tightly stretched string with fixed end points x=0 and x=$\ell$ is initially at rest in its equilibrium position. If it is set to vibrate by giving each point a velocity $v_0 \sin^3\left(\frac{\pi x}{\ell}\right)$. Find the displacement of the string at any distance x from one end at any time t. (13)

(OR)

(b) A rectangular plate with insulated surface is 10 cm wide and so long when compared to its width that it may be considered infinite in length without introducing appreciable error. The temperature at the short edge y=0 is given by $u(x,0) = \begin{cases} 20\,x & \text{for } 0 \le x \le 5 \\ 20\,(10 - x) & \text{for } 5 \le x \le 10 \end{cases}$ and all the other three edges are kept at $0^\circ$C. Find the steady state temperature $u(x, y)$ at any point in the plate. (13)

14. (a) Using Fourier sine and cosine transform, evaluate $\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$ and $\int_0^\infty \frac{x^2\,dx}{(x^2+4^2)(x^2+5^2)}$. (13)

(OR)

(b) Find the Fourier Transform of given $f(x) = \begin{cases} a^2 - x^2 ; & |x| < a \\ 0 ; & |x| > a > 0 \end{cases}$ and hence deduce

(i) $\int_0^\infty \frac{\sin t - t\cos t}{t^3} dt = \frac{\pi}{4}$ , (ii) $\left(\frac{\sin x - x\cos x}{x^3}\right)^2 dt = \frac{\pi}{15}$. (13)

15. (a) Find the inverse Z-Transform of $\frac{8z}{(z-1)(z-2)^2}$ using Residue Method. (13)

(OR)

(b) Solve the difference equation $y_{n+2} + 6y_{n+1} + 9y_n = 3^n$ given $y_0 = 0, y_1 = 0$ using Z-Transform. (13)

## PART – C (1 x 15 = 15 Marks)

16. (a) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = 10\sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement $y(x,t)$. (15)

(OR)

(b) A long rectangular plate with insulated surface is $l$ cm wide. If the temperature along one short edge $(y = 0)$ is $u(x,0) = 60\ (lx - x^2)$ degrees, for $0 < x < l$, while the other two long edges $x = 0$ and $x = l$ as well as the other short edges are kept at $0°C$, find the steady state temperature function $u(x,y)$. (15)

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