Homework 1 : Computational Economics
Search Models

Dhananjay Ghei

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Search Models and Labor Wedge

1. First show that there exists an equilibrium in the following transformed variables: $\tilde{c}(s^t) = c(s^t)z(s^t)^{\frac{-1}{1-\alpha}}$, $\tilde{k}(s^t) = k(s^t)z(s^t)^{\frac{-1}{1-\alpha}}$ and $\tilde{w}(s^t) = w(s^t)z(s^t)^{\frac{-1}{1-\alpha}}$. Then show that there exists a Markovian solution with k, n, s as state variables. Boil down the equations below to two functional equations in two unknown functions: $\theta(k, n, s)$ and $\tilde{c}(k, n, s)$.

Appendix A gives the calculations for the equations derived below with the appropriate transformations.
The Euler equation is given by:

$$1 = \beta \sum_{s^{t+1}|s^t} \pi(s_{t+1}|s_t) \frac{\tilde{c}(s^t)}{\tilde{c}(s^{t+1})} e^{-\frac{s_{t+1}}{1-\alpha}} \left(\alpha \left(\frac{\tilde{k}(s^{t+1})}{n(s^{t+1}) - \theta(s^{t+1})(1-n(s^{t+1}))} \right)^{\alpha-1} + 1 - \delta \right)$$

The condition w.r.t employment is given by:

$$(1-\alpha)\left(\frac{\tilde{k}(s^t)}{n(s^t) - \theta(s^t)(1-n(s^t))} \right)^{\alpha} = \beta\mu(\theta(s^t)) \sum_{s^{t+1}|s^t} \pi(s_{t+1}|s_t)$$
$$\times \left((1-\alpha)\left(\frac{\tilde{k}(s^{t+1})}{n(s^{t+1}) - \theta(s^{t+1})(1-n(s^{t+1}))} \right)^{\alpha} \left(1 + \frac{1-x}{\mu(\theta(s^{t+1}))} \right) - \tilde{w}(s^{t+1}) \right)$$

The wage equation evaluated in history s^{t+1} reduces to:

$$(1-\tau)\tilde{w}(s^{t+1}) = (1-\phi)\gamma\tilde{c}(s^{t+1}) + \phi(1-\tau)(1-\alpha)\left(\frac{\tilde{k}(s^{t+1})}{n(s^{t+1}) - \theta(s^{t+1})(1-n(s^{t+1}))} \right)^{\alpha} (1 + \theta(s^{t+1}))$$

Eliminating wage $\tilde{w}(s^{t+1})$ between the last two equations gives:

$$(1-\alpha)\left(\frac{\tilde{k}(s^t)}{n(s^t) - \theta(s^t)(1-n(s^t))} \right)^{\alpha} = \beta\mu(\theta(s^t)) \sum_{s^{t+1}|s^t} \pi(s_{t+1}|s_t) \frac{\tilde{c}(s^t)}{\tilde{c}(s^{t+1})}$$
$$\left(-\frac{(1-\phi)\gamma\tilde{c}(s^{t+1})}{1-\tau} + (1-\alpha)\left(\frac{\tilde{k}(s^{t+1})}{n(s^{t+1}) - \theta(s^{t+1})(1-n(s^{t+1}))} \right)^{\alpha} \left(\frac{1-x}{\mu(\theta(s^{t+1}))} + 1 - \phi - \phi\theta(s^{t+1}) \right) \right)$$

The resource constraint is given by:

$$\tilde{k}(s^{t+1})e^{\frac{s_{t+1}}{1-\alpha}} = (\tilde{k}(s^t))^{\alpha}(n(s^t) - \theta(s^t)(1-n(s^t)))^{1-\alpha} + (1-\delta)\tilde{k}(s^t) - \tilde{c}(s^t)$$

The law of motion for employment is given by:

$$n(s^{t+1}) = (1-x)n(s^t) + f(\theta(s^t))(1-n(s^t))$$

In order to show that there exists a Markovian solution, we first conjecture that the solution is given as constants $\bar{c}, \bar{k}, \bar{w}, \bar{w}, \bar{\theta}$. Then, we plug in these values into the 5 equations above. If there is such an equilibrium, the equations are given by:

$$\begin{aligned}
1 &= \beta \sum \pi(s_{t+1}|s_t) e^{-\frac{s_{t+1}}{1-\alpha}} \left(\alpha \left(\frac{\bar{k}}{\bar{n} - \bar{\theta}(1-\bar{n})} \right)^{\alpha-1} + 1 - \delta \right) \\
(1-\alpha)\left(\frac{\bar{k}}{\bar{n} - \bar{\theta}(1-\bar{n})} \right)^{\alpha} &= \beta \mu(\bar{\theta}) \sum \pi(s_{t+1}|s_t) \times \left((1-\alpha)\left(\frac{\bar{k}}{\bar{n} - \bar{\theta}(1-\bar{n})} \right)^{\alpha} \left(1 + \frac{1-x}{\mu(\bar{\theta})} \right) - \bar{w} \right) \\
\bar{n} &= (1-x)\bar{n} + f(\bar{\theta})(1-\bar{n}) \\
\bar{k} e^{\frac{s_{t+1}}{1-\alpha}} &= \bar{k}^{\alpha}(\bar{n} - \bar{\theta}(1-\bar{n}))^{1-\alpha} + (1-\delta)\bar{k} - \bar{c} \\
(1-\tau)\bar{w} &= (1-\phi)\gamma \bar{c} + \phi(1-\alpha)(1-\tau)\left(\frac{\bar{k}}{\bar{n} - \bar{\theta}(1-\bar{n})} \right)^{\alpha} (1+\bar{\theta})
\end{aligned}$$

Substituting out $\bar{c}, \bar{n}, \bar{k}$, we get an equation in $\bar{\theta}$ and $\bar{w}$ as: There is an equilibrium of this form if and only if this equation can be satisfied in any history s^t. Thus, this requires that there exists a number $\bar{s}$ satisfying:

Finally, let the functions Θ and C define the recruiter-employment ratio and equilibrium consumption relative to trend as functions of the current state $(s, n, \tilde{k})$ so, $\theta(s^t) = \Theta(s_t, n(s^t), \tilde{k}(s^t))$ and $\tilde{c}(s^t) = C(s_t, n(s^t), \tilde{k}(s^t))$. Substituting in the expressions and eliminating $\tilde{k}(s^{t+1})$ and $n(s^{t+1})$, we get two nonlinear equations in Θ and C. Appendix B gives the calculations for this transformation. The functional equations are given as:

2. Compute the non stochastic steady state. Log linearise around the non stochastic steady state to get a system of linear expectational difference equations and solve for the policy rules. For example:

$$\log\theta_{t+1} = \log\bar{\theta} + \theta_s(s_{t+1} - \bar{s}) + \theta_n(\log n_t - \log\bar{n}) + \theta_k(\log k_t - \log\bar{k})$$

Appendix C uses the equations in part 1) to derive the equations for the non stochastic steady state. The steady state equations are given by:

We put these equations along with the parameter values to solve for the steady state using a non-linear solver. The steady state is given by

3. Using the calibration in Table 3.2 to compute the IRF and the ergodic moments of all the relevant variables. Please check your results against figure 3.2 in the book.

For most of the parameters, Shimer uses the same calibration as in Table 3.2. However, he changes some of the parameters. He sets $\bar{s} = 0.0012$, $\rho = 0.4$ and $\zeta = 0.00325$. Table 3 gives the values of the remaining parameters.

4. Derive an expression for labor wedge in this economy using the consumption output ratio and the hours which here is a fraction of HH that are employed. Obtain a log linear expansion of the wedge using the log linear policy rules.

Table 1 Calibration of parameters

These are the same parameters as in Shimer's book from Table 3.2

Parameter	Description	Values
β	Discounting	0.996
α	Share of capital	0.33
τ	Tax rate	0.4
x	Employment exit probability	0.034
γ	Worker's bargaining power	0.5
δ	Depreciation	0.0028

5. What is the correlation of the labor wedge with output and employment? You can sign it by staring at the policy rules. What is the ergodic std. of the labor wedge? How does the correlations and volatility compare to data.

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6. Now extract a TFP series such that the simulate path of $y(s^t)$ is exactly as in the data. Start with the steady state level of capital and employment. Using the policy rule for output reverse engineer a shock such that detrended output matches exactly to the value in the data. Now update the state variables using your policy rules and keep iterating this procedure. How does the TFP series extracted compare to a Solow residual in the data?

A Transforming the variables on a balanced growth path

B Reducing the set of equations to a pair of nonlinear equations

C Steady state calculations