Homework 1: Labor Economics

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Note: The code for this exercise is written in `R`*. The code is available at* `www.github.com/dhananjayghei/labor_economics`

1 Preliminary analysis

1. Visit the BHPS website and familiarise yourself with the basic structure and contents of the BHPS data. What features make it a suitable data set for the estimation of the BM model?

The BHPS data is suitable as it provides us with spells from initial duration for both employed and unemployed workers. This helps us in estimating the sample distribution of wage offers for both employed and unemployed workers which is used to estimate the parameters of interest (separation rate, job offer arrival rate for employed and unemployed workers).

2. Open the file and answer the following questions:
   (a) What is the sample size? What is the sex ratio in the sample?
   (b) What is the sample unemployment rate? What is the sample unemployment rate of men? Of women? Or workers in each education category?
   (c) What proportion of initial spells are right-censored? Answer the same question for each type of first spell (job or unemployment spell).

We have a sample of 2263 workers, 54.18% of which are male and 45.82% are female. The sample unemployment rate is 5.92%. Table 1 shows the unemployment rate on the basis of sex. The unemployment rate is higher in males compared to females. Table 2 shows the unemployment rate on the basis of education. The unemployment rate is the highest in people with less than A-level education and lowest in people with some higher education.

Table 1 Unemployment rate (by sex)

The table shows the unemployment rate in the BHPS sample on the basis of sex.

Sex	Unemployed	Total	Unemployment Rate
Female	39	1037	3.76
Male	95	1226	7.75

3. Construct the initial (spell-1) cross-sectional CDF (`G`) and density of log wages `logw1`. Produce the plots of these two objects.

Figure 2 shows the empirical CDF of G estimated using the data on log wages of workers who were employed in the initial spell. Figure 3 shows the kernel density estimates (g and f) using the same data as above. The kernel density estimate is a non-parametric estimator of the density function.

Table 2 Unemployment rate (by education)

The table shows the unemployment rate in the BHPS sample on the basis of education.

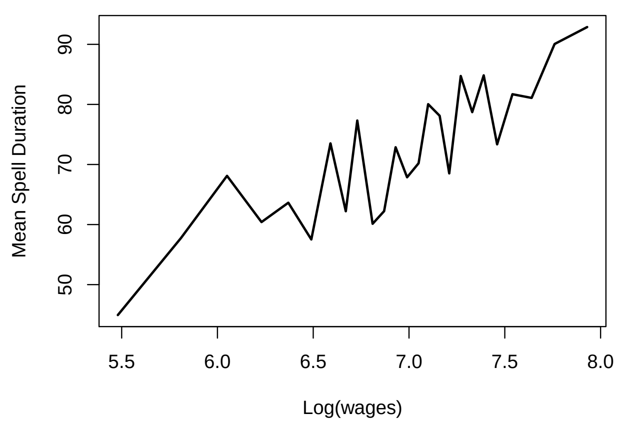
Education	Unemployed	Total	Unemployment Rate
less than A-levels	97	1453	4.29
A-levels	22	361	0.97
Some Higher Education	15	449	0.66

4. Create a variable categorising `logw1` into 25 bins (ie, percentiles 1-4, 5-8,9-12,. . . ,97-100) and a variable containing the mean spell-1 duration (`spelldur1`) within each of these 25 bins. Plot those mean durations against the wage percentiles. Is this consistent with the BM model?

Figure 1 shows the mean durations against the wage percentiles. It is evident from the figure that the duration has an increasing trend along the wage percentiles. Therefore, one would posit that as wages increase, the duration in the job also increases as it takes more time for firms to compensate the worker with the same wage offer as their current ones.

Figure 1 Variation in mean duration across wage percentiles

The figure shows the mean durations against the wage percentiles. The solid line shows the actual mean wage duration in the wage percentile.



5. Explain how one can obtain a non-parametric estimate of the wage sampling distribution F from the data. Construct this non-parametric estimate, and plot it on the same graph as `G`. Is this consistent with the theory? What else can you say about the estimate of F?

As mentioned in the slides, the cdf of wages accepted by workers who were just hired from unemployment provides a direct estimator $\hat{F}$ of the sampling distribution. I construct the empirical CDF of F using the wages of newly hired from unemployment. Figure 2 shows the empirical CDF of F and G estimated non-parametrically using the data on log wages. The solid gray line shows the empirical distribution function of $\hat{F}$ and the solid black line shows the empirical distribution function of $\hat{G}$. This is consistent with the theory that we learnt in class. Figure 2 shows that $\hat{G}$ first order stochastically dominates $\hat{F}$. This is expected given the selection of workers into better jobs. As explained in the lecture slides, the model is over-identified and hence, we can also estimate κ_1 from the estimates of F and G using the equation:

$$\kappa_1^{np} = \frac{\hat{F}(w) - \hat{G}(w)}{(1 - \hat{F}(w))\hat{G}(w)}$$

Figure 2 Non-parametric estimate of empirical CDF ($\hat{F}$ and $\hat{G}$)

The figure shows the empirical cdf of F and G estimated from the data on wages using BHPS survey data. The solid gray line shows the empirical distribution function of $\hat{F}$ and the solid black line shows the empirical distribution function of $\hat{G}$.

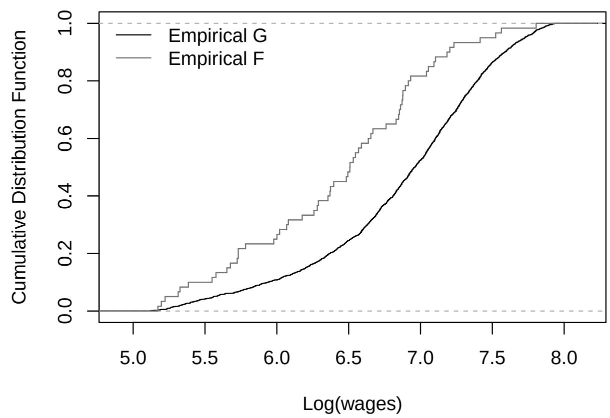
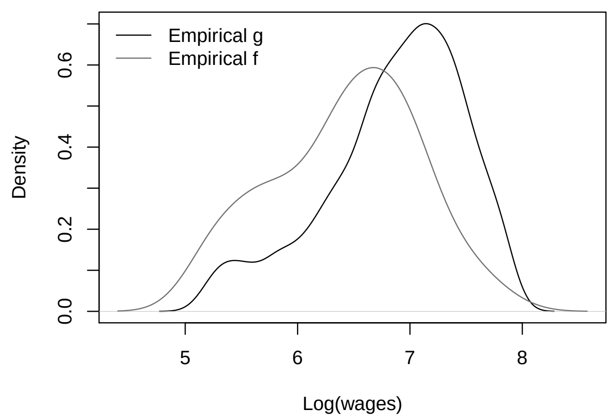


Figure 3 Kernel density estimate of PDF ($\hat{f}$ and $\hat{g}$)

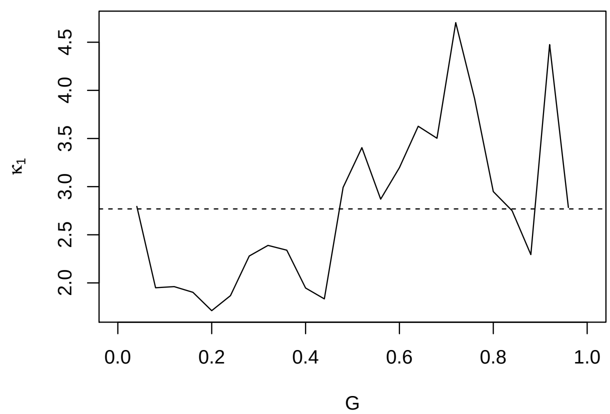
The figure shows the empirical pdf of f and g estimated from the data on wages using BHPS survey data. The solid gray line shows the kernel density estimate of $\hat{f}$ and the solid black line shows the kernel density estimate of $\hat{g}$.



I use this equation and estimate the average κ_1 across the quantiles of G. Figure 4 shows the estimated κ_1 across these quantiles. The mean of the estimate equals 2.76. The estimate should be roughly constant across the quantiles of $\hat{G}$. As we will see in the later part, the estimated κ_1 from MLE will be 1.35 (roughly half of this non-parametric estimate).

Figure 4 Non-parametric estimate of κ_1 using $\hat{F}$ and $\hat{G}$

The figure shows the non-parametric estimate of κ_1 across the quantiles of $\hat{G}$. The estimate is calculated using the empirical CDF of $\hat{F}$ and $\hat{G}$.



2 Estimation

1. Write code for the MLE estimation of the BM model following the two-step protocol of Bontemps, Robin and Van den Berg (1999).

I follow the two-step protocol as laid out in [Bontemps et al., 2000][1]. Given our estimates of $\hat{G}$ and $\hat{g}$, I write functions $\hat{F}(:,\kappa_1)$ and $\hat{f}(:,\kappa_1)$ as functions of $\kappa_1 = \frac{\lambda_1}{\delta}$. These expressions are given as:

$$\hat{F}(w) = \frac{(1+\kappa_1)\hat{G}(w)}{1+\kappa_1\hat{G}(w)} \qquad \text{and} \qquad \hat{f}(w) = \frac{(1+\kappa_1)\hat{g}(w)}{[1+\kappa_1\hat{G}(w)]^2}$$

Next, I write the likelihood function for the employed and unemployed using the expressions below:

$$L(x_i|e_1=0) = \lambda_0^{1-c_i} exp(-\lambda_0 d_i) f(y_{0i})^{1-c_i}$$

$$L(x_i|e_i=1) = g(y_{i1})(\delta+\lambda_1\bar{F}(y_{i1}))^{1-c_i} exp((-\delta+\lambda_1\bar{F}(y_{i1}))d_i)\left(\frac{\delta}{\delta+\lambda_1\bar{F}(y_{i1}}\right)^{\tau_{JUi}}\left(\frac{\lambda_1\bar{F}(y_{i1})}{\delta+\lambda_1\bar{F}(y_{i1})}\right)^{\tau_{JJi}}$$

The generic likelihood contribution of an observation x_i is therefore:

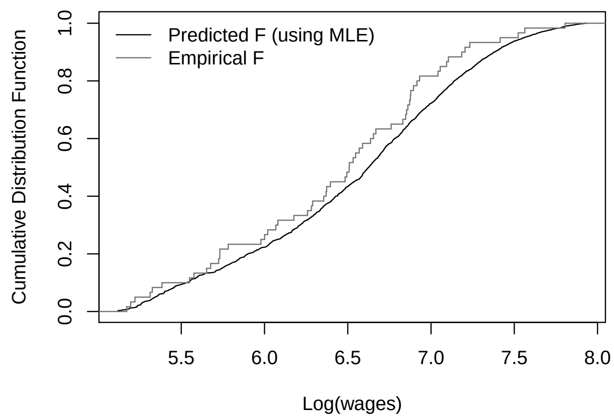
$$L(x_i) = \left(\frac{\lambda_0}{\delta+\lambda_0}L(x_i|e_i=1)\right)^{e_i}\left(\frac{\delta}{\delta+\lambda_0}L(x_i|e_i=0)\right)^{1-e_i}$$

which is a function of F, G and the parameters δ, λ_0, and λ_1 Finally, I take the natural logarithms of these, add them up and minimise the negative of the log likelihood using a routine optimisation in `R`. The solver converges and the values for these parameters are given by: $\lambda_0 = 0.1496$, $\lambda_1 = 0.01359$, and $\delta = 0.01$ which gives a value of $\kappa_1 = \frac{\lambda_1}{\delta} = 1.3591$.

[1] I think the paper you wanted to refer to was the 2000 one and not the 1999 one. The lecture slides say that it is the 2000 one.

Figure 5 Comparison of estimated $\hat{F}$ from direct estimation vs ML estimation

The figure shows a comparison of estimated $\hat{F}$ from direct estimation versus ML estimation. The solid black line shows the predicted $\hat{F}$ from the ML estimation and the solid gray line shows the estimated $\hat{F}$ from the sample of new hires who moved from unemployment to employment.



2. Write code for computing the standard errors of the estimates δ, λ_0 and λ_1, explaining the assumptions upon which those standard errors rely.

I compute the standard errors using two methods.
The first method is by inverting the Hessian matrix[2] and calculating the square root of the diagonal elements. This gives the asymptotic standard errors.

The second method is to compute it using bootstrap. I wrote the code for bootstrap but it is tremendously slow. It works for small samples although it does not give accurate results. I do 20 bootstraps on the data and estimate the standard errors from there. Table 3 shows the estimates of standard errors from the two methods. The asymptotic standard errors give a lower bound on the variance of the estimates and is lower than the bootstrap procedure.

Table 3 Estimates of standard errors

The table shows the estimate of standard errors calculated from the Hessian matrix and bootstrap. The number of bootstraps is 20.

	λ_0	λ_1	δ
Asymptotic	0.00918	0.00066	0.00041
Bootstrap	2.32346	0.99366	0.01298

3. The file `BM_data_simulated.csv` contains artificial data resulting from a simulation of 5,000 workers behaving according to the BM model with parameters $\delta = 0.01$, $\lambda_0 = 0.1$, $\lambda_1 = 0.05$ (monthly values). Run your ML estimation routine on the simulated data, and check that your estimates against the true parameter values.

Note that the simulated data does not have a column for wages of unemployed who were hired. But, from the lecture slides, one can see that if wage is unobserved, we can integrate out the wages from

[2]This is the Hessian matrix for the negative of log likelihood evaluated at the optimum.

the likelihood expression and re-write it as:

$$\int_{y}^{\bar{y}} L(x_i|e_i = 0)dy_{0i} = \lambda_0^{1-c_i} \exp(-\lambda_0 d_i)$$

I use this fact and then run the ML estimation routine as before on the simulated data set. My parameter estimates are $\lambda_0 = 0.09986$, $\lambda_1 = 0.05385$ and $\delta = 0.00999$. These are close to the true parameter values. Therefore, it gives us a sanity check as to if the ML estimation routine works well enough or not.

3 Playing around with the model

1. What is the predicted unemployment rate from the estimates obtained in Section II? Compare it with the sample unemployment rate, and discuss the possible reasons for any discrepancy.

Given our estimates of the parameters and the steady state assumption, the sample unemployment rate can be calculated as:

$$Pr\{e_i = 0\} = 1 - Pr\{e_i = 1\} = \text{Unemp. rate} = \frac{\delta}{\delta + \lambda_0}$$

For BHPS data, the sample unemployment rate is 5.92% whereas the predicted unemployment rate from the MLE estimation of parameters is 6.26%. For simulated data, the sample unemployment rate is 9.58% whereas the predicted unemployment rate from the MLE estimation is 9.09%.

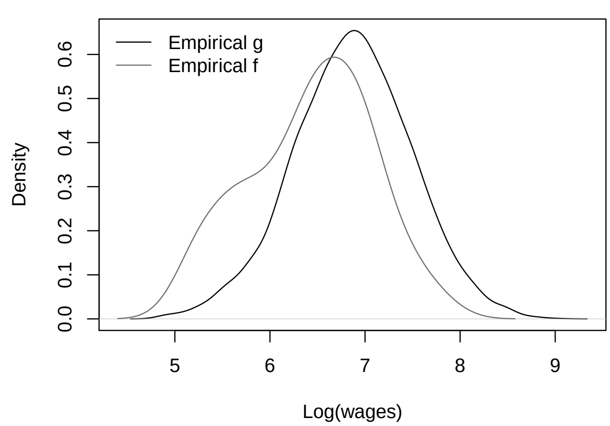
2. Construct kernel density estimates of the cross-section distribution of wages $g(w)$ and of the sampling distribution $f(w)$. Plot both densities on the same graph.

Figure 3 shows the kernel density estimates for the BHPS data. These kernel density estimates are from the sample itself and not from the predictions of our estimation routine.

Figure 6 shows the kernel density estimates of $\hat{g}$ and $\hat{f}$ from the sample of simulated data. Once again, these estimates are from the sample itself and not from the predictions of our estimation results.

Figure 6 Kernel density estimate of PDF ($\hat{f}$ and $\hat{g}$)

The figure shows the empirical pdf of f and g estimated from the simulated data. The solid gray line shows the kernel density estimate of $\hat{f}$ and the solid black line shows the kernel density estimate of $\hat{g}$.



3. Construct the distribution of firm productivity that rationalises the observed wage distribution within the BM model. Plot firm productivity against wages, and against the cross-section CDF of wages $G(w)$. Do you notice anything wrong?

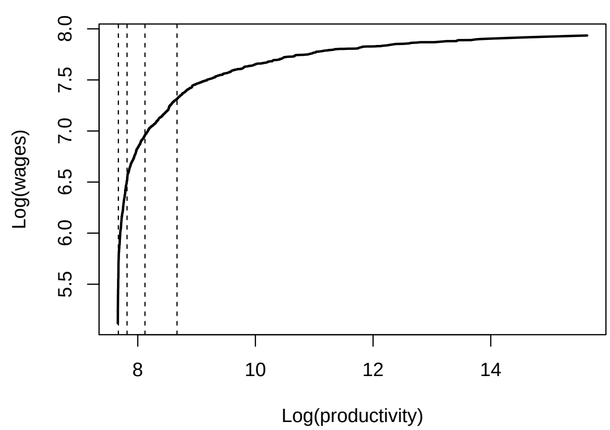
The firm productivity is given by:

$$p = w + \frac{1 + \kappa_1 \hat{G}(w)}{2\kappa_1 g(w)}$$

Using the above expression, I calculate the firm productivity and plot the (QQ - plots) log wages against log productivity. Figure 7 shows the QQ plot for log wage versus log productivity. For low productivity firms, the wage is increasing as a function of productivity, whereas the curve smooths out as we move towards the higher productivity firms. In particular, the vertical dotted lines show the 5th, 25th, 50th and 75th percentiles of log productivity. One can see that the wages are increasing over these percentiles but tend to smoothen out beyond the 75th percentile. Figure 8 shows the QQ-plot for empirical CDF (G) versus log productivity. The function is also concave and smooths out as we go beyond the 75th percentile.

Figure 7 QQ plot for log wage versus log productivity

The figure shows the quantile-quantile plot of log wage versus log productivity on the BHPS data set. The vertical lines represent the 5th, 25th, 50th and 75th percentile of log productivity.



4. Looking at the predicted profit rate of high-productivity firms, what else can you say about the BM model?

The profit rate is simply the surplus above the wage and is given by $\frac{(p-w)}{p}$. Figure 9 shows the QQ plot for profit rate versus log productivity. Since, wages are strictly increasing in the initial quantiles, the profits are roughly constant till the 25th percentile and take off beyond that. Higher productivity firms have steady wages and therefore, higher profit rates.

Figure 8 QQ plot for G versus log productivity

The figure shows the quantile-quantile plot of empirical CDF ($\hat{G}$) versus log productivity on the BHPS data set. The vertical lines represent the 5th, 25th, 50th and 75th percentile of log productivity.

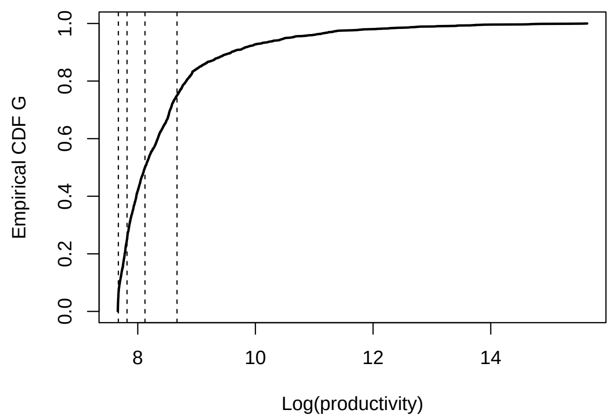
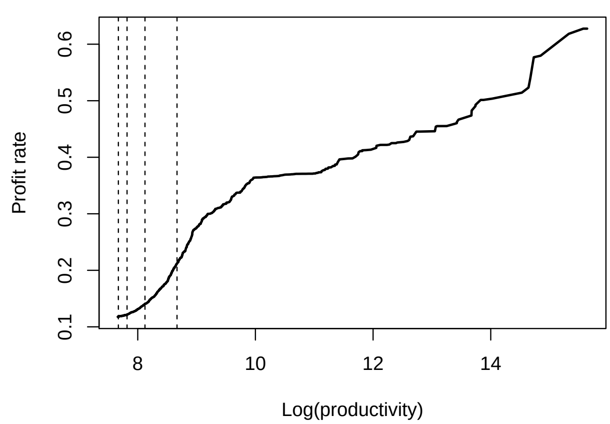


Figure 9 QQ plot for profit rate versus log productivity

The figure shows the quantile-quantile plot of profit rate versus log productivity on the BHPS data set. The vertical lines represent the 5th, 25th, 50th and 75th percentile of log productivity.



References

Christian Bontemps, Jean-Marc Robin, and Gerard J Van den Berg. Equilibrium search with continuous productivity dispersion: Theory and nonparametric estimation. *International Economic Review*, 41(2): 305–358, 2000.