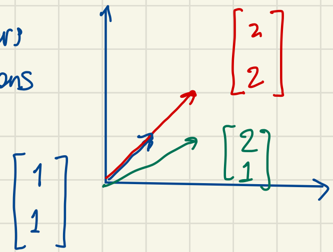
Matrix and vector

A vector is a list of numbers representing points and directions in the space



A matrix is a table used to represent multiple vectors

e.g. $\begin{bmatrix} 1 & 2 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

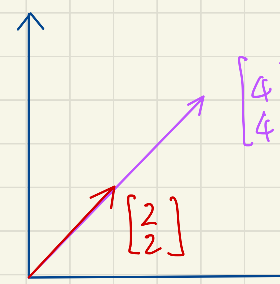
vector operations

1) vector addition / subtraction : each number is added / subtracted elementwise

$$\begin{bmatrix} 2 \\ 2 \end{bmatrix} - \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

2) vector scaling : each number is multiplied by a <u>scalar</u> (single number)

$$\begin{bmatrix} 2 \\ 2 \end{bmatrix} \cdot 2 = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$



Dot product

The sum of element-wise product of two vectors (of the same length/ number of dimensions)

$$a = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad b = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad a \cdot b = \begin{bmatrix} 1 \cdot 0 \\ 0 \cdot 1 \end{bmatrix} \Big\downarrow \text{sum}$$

$$= 0 + 0 \quad = 0$$

$$c = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \qquad c \cdot a = \begin{bmatrix} 2 \cdot 1 \\ 2 \cdot 0 \end{bmatrix} \Big\downarrow \text{sum}$$

$$= 2 + 0 \quad = 2$$

Consider three documents represented as feature vectors with bag-of-word features. Assume vocab is {good, bad}

d_1 = "good food" $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

d_2 = "good food good food" $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$

d_3 = "bad food" $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

- Find $d_1 \cdot d_2$
- Find $d_1 \cdot d_3$

Cosine similarity

cosine similarity measure how similar two vectors are.
based on the angle between them.
Dot product does not account for differences in length.

$$\text{Cosine}(a,b) = \frac{a \cdot b}{\|a\| \|b\|} \qquad \text{cosine}\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix} \right) = ?$$

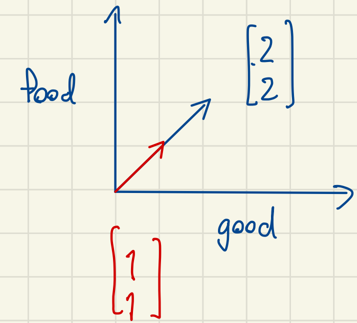
$$\|a\| = \sqrt{a_1^2 + a_2^2}$$

$$\|b\| = \sqrt{b_1^2 + b_2^2}$$

$$= \frac{2+2}{\sqrt{1^2+1^2} \cdot \sqrt{2^2+2^2}}$$

$$= \frac{4}{\sqrt{2} \cdot \sqrt{8}}$$

$$= \frac{\cancel{4}\,2}{\sqrt{2} \cdot \cancel{2}\sqrt{2}} = \frac{2}{2} = 1$$



What's the angle?
0 degree!

the value of cosine similarity is between -1 and 1

-1: dissimilar

1: identical

∴ These two documents are identical in this bag-of-word vector space.

Matrix multiplication

In NLP, we multiply a matrix with a vector to transform the vector into another form of vector (called linear transformation)

Note that

- The # of col of the matrix must match the dimension of the vector
- The dimension of the result vector is the number of rows.
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$$\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}\cdot\begin{bmatrix}1\\ 2\end{bmatrix}=\begin{bmatrix}2\cdot1+0\cdot2\\ 0\cdot1+2\cdot2\end{bmatrix}=\begin{bmatrix}2\\ 4\end{bmatrix}$$

This matrix stretchs out the vector

$$\begin{bmatrix}2 & 0\\ 0 & 1\\ 1 & 1\end{bmatrix}\cdot\begin{bmatrix}1\\ 2\end{bmatrix}=\begin{bmatrix}2\cdot1+0\cdot2\\ 0\cdot1+1\cdot2\\ 1\cdot1+1\cdot2\end{bmatrix}=\begin{bmatrix}2\\ 2\\ 3\end{bmatrix}$$

This matrix transforms the vector into a 3d vector.

$$\begin{bmatrix}2 & 0\\ 0 & 1\\ 1 & 1\end{bmatrix}\cdot\begin{bmatrix}1\\ 2\\ 1\end{bmatrix}$$

the dimensions don't match

Vector Addition Problems

1. Add the vectors $\mathbf{u} = [1, 2, 3]$ and $\mathbf{v} = [4, 0, -1]$.
2. Add the vectors $\mathbf{u} = [-2, 5, 1]$ and $\mathbf{v} = [3, -1, 0]$.
3. Add the vectors $\mathbf{u} = [0, 0, 5]$ and $\mathbf{v} = [3, 3, -2]$.

Dot Product Problems

1. Compute the dot product of $\mathbf{u} = [2, 3, 1]$ and $\mathbf{v} = [4, 0, -2]$.
2. Compute the dot product of $\mathbf{u} = [-1, 5, 0]$ and $\mathbf{v} = [2, -1, 3]$.
3. Compute the dot product of $\mathbf{u} = [0, 4, 3]$ and $\mathbf{v} = [1, 1, 1]$.

Matrix-Vector Multiplication Problems

1. Multiply:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & -1 \\ 2 & 1 & 1 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

2. Multiply:

$$A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 3 & 2 \\ 0 & 4 & -2 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

3. Multiply:

$$A = \begin{bmatrix} 0 & 2 & 1 \\ -1 & 1 & 0 \\ 3 & 0 & -2 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$