

$$C = -\frac{1}{m} \sum_i \left(y_i \log(y_p) + (1-y_i) \log(1-y_p) \right)$$

$$\frac{\partial C}{\partial \beta_i} = -\frac{1}{m} \left(\frac{y_i}{y_p} - \frac{(1-y_i)}{(1-y_p)} \right) \frac{\partial y_p}{\partial \beta}$$

$y_p = y_{\text{predicted}}$

$$y_p = \sigma(\beta \cdot x)$$

$$\frac{\partial y_p}{\partial \beta} = \frac{\partial \sigma}{\partial (\beta \cdot x)} \frac{\partial \beta \cdot x}{\partial \beta}$$

$\sigma = \text{sigmoid function}$

$$= \sigma(1-\sigma) \cdot x$$

$$= y_p(1-y_p) \cdot x$$

$$\frac{\partial C}{\partial \beta_i} = -\frac{1}{m} \left(\frac{y_i(1-y_p) - y_p(1-y_i)}{y_p(1-y_p)} \right) \cdot y_p(1-y_p) \cdot x$$

$$= \frac{1}{m} (y_p - y_i) x$$