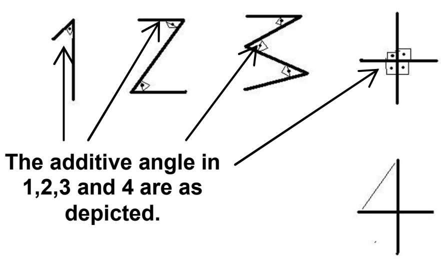
Chapter 1 EVOLUTION OF NUMBERS

Most of us may have wondered on the origin of the symbols for all numbers. How a particular notation came into existence, and why some particular notations found universal acceptance are exciting questions. All Arabic numbers we use today as international numerals, are ideograms created by Abu Ja'far Muhammad ibn Musa ***al-Khowarizmi*** (c.778 - c.850).

It is opined that he used the abacus notation for developing the manuscript for decimal system. Incidentally, for those who are unaware ABACUS is a calculating device, probably of Babylonian origin, that was long important in commerce. It is the ancestor of the modern calculating machine and computer. It is generally a board marked with lines and equipped with counters whose positions indicated numerical values—i.e., ones, tens, hundreds, and so on.

The numbers 1,2,3,4 were defined using additive angles.



Roots of the 1,2,3 and 4 digits

The number 1 has one angle.

The number 2 has two additives angles.

The number 3 has three additives angles.

The number 4 has four additive angles.

Probably due to cursive handwriting the number 4 gets closed.

Roots of the 5 to 10 numbers

The circle represented the hand which has five fingers. The number 5 was written under the line. Number 10 placed above the line, it meant the number on top acquired double value.

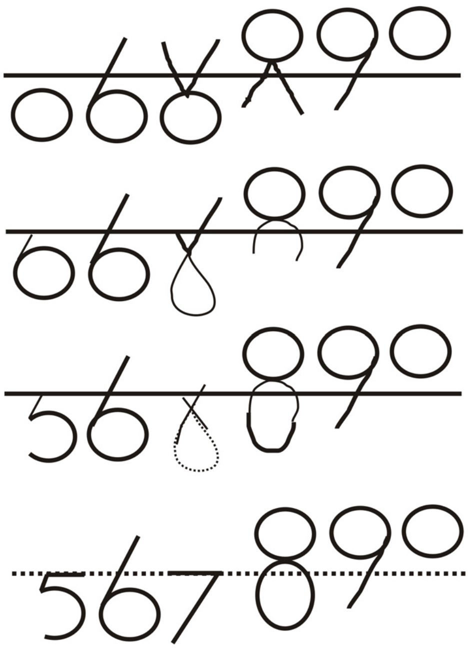
The circles, the up traces, the additive angles and the write line

To the circle five, one trace up was added, with one additive angle making the number six.
To the circle five were added two up traces, with two additive angles making the number seven.

The circles, the down traces, the diminutive angles and the write line

To the circle ten was added one down trace, with one diminutive angle making the number nine.

To the circle ten were added two down traces, with two diminutives angles making the number eight.



Circle below
the line represents
Hand with
5 closed fingers

so,
6 is 1 finger raised

7 is 2 fingers raised

Now circle above the line
acquires double value
10

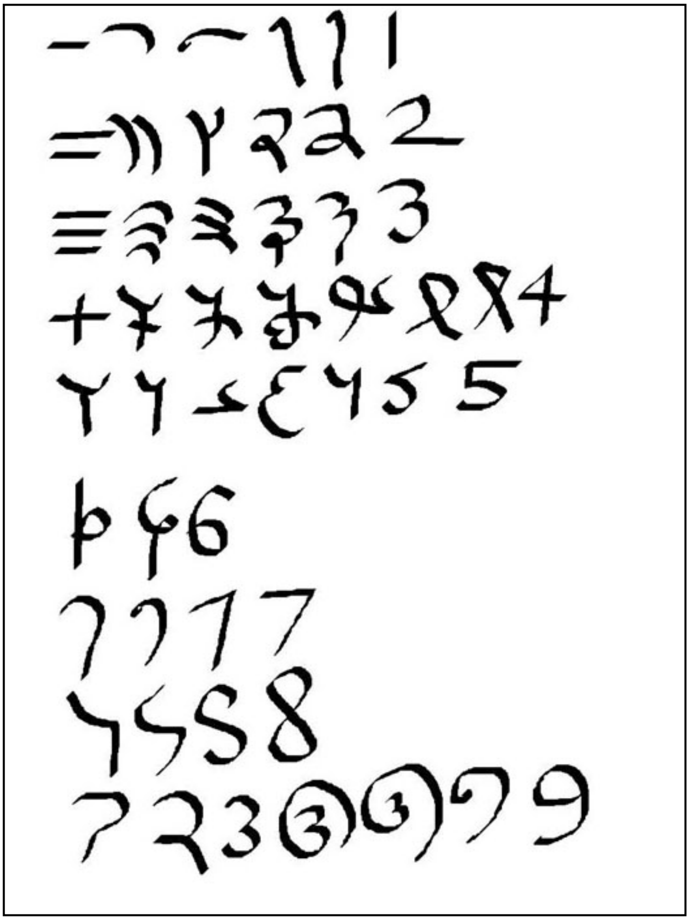
8 is 2 fingers
less than 10

9 is 1 finger
less than 10

Gradually cursive writing
led to present symbols

The cursive handwriting makes changes on the numbers format and aesthetic. The cursive numbers five and the number seven still uses the write line on its structures.

The number seven was placed totally under the write line, and was the most simplified during its cursive evolution. First the number seven was placed under the write line. The involution of the number seven was necessary due to the similarities that the cursive seven has with the numbers.



The above figure depicts a probable evolution of symbols for the numbers. It is interesting to find that across civilizations same types of symbols evolved, some were retained in their original form, but the Arabic system found universal acceptance and is now the International Numeral system.

Chapter 2 ORIGIN OF MATHEMATICAL TERMS AND SYMBOLS

School teachers break their heads in explaining the significance of $+ - \times \div =<>$ to the child. In the beginning it is all so confusing and gradually they are ingrained in the memory, and all the arithmetic is quietly done. The curious mind's journey does not end here, why = for equality and a lot many similar questions. The evolution of few most used symbols and terms were researched and some very interesting stories, ideas were established. The most important written source is the definitive *A History of Mathematical Notations by Florian Cajori.*

Symbol for equality

The equal symbol (=) was not really in print until early 17th century. It was previously abbreviated as *aeq*. It is contended by Cajori (CAJORI, FLORIAN "A History of Mathematics", The Macmillan Company 1926) that the symbol = was developed at Bologna. Robert Recorde first used the symbol in 1557 in *The Whetstone of Witte,* (1556), by Robert Recorde, a treatise on algebra. But why = in particular could not be established.

Symbols for plus and minus

The introduction of the + and - symbols seems to be due to the Germans. The arithmetic of John Widmann, brought out in 1489 in Leipzig, is the earliest printed book in which the + and - symbols have been found. The + sign is not restricted by him to ordinary addition; it has the more general meaning "et" or "and" as in the heading, "regula augmenti + decrementi." The - sign is used to indicate subtraction, but not regularly so. The word "plus" does not occur in Widmann's text; the word "minus" is used only two or three times. In 1521, the symbols + and – have been used for addition and subtraction by Heinrich Schreiber, a teacher at the University of Vienna, in the arithmetic of Grammateus,. Thus, by slow degrees, the adoption of the + and - symbols became universal. Several independent paleographic studies of Latin manuscripts of the fourteenth and fifteenth centuries make it almost certain that the + sign comes from the Latin *et*. As per Cajori, the origin of the sign - is still uncertain.

The first one to make use of these signs in writing an algebraic expression was the Dutch mathematician Vander Hoecke. These symbols seem to have been employed for the first time in arithmetic, to indicate operations, by Georg Walckl in 1536. Most of the English writers of this period reserved the + and - signs as symbols of operation for algebra. Robert Recorde used it in his 1557 book, *The Whetstone of Witte*.

There seems little doubt that the sign is merely a ligature for "et", much in the same way that we have the ligature "&" for the word "and". It may have emanated from the habit of early scribes of using it as a shorthand equivalent of "m."

Symbol for division

The Anglo-American symbol for division is of 17th century origin, and has long been used on the continent of Europe to indicate subtraction. Like most elementary combinations of lines and points, the symbol is old. It was used as early as the 10th century for the word ***est***. When written after the letter "i", it symbolized "id est." When written after the word "it", it symbolized "interest." It is possible that it denoted division when written after the word *"divisa"*, for *"divisa est"*. There is also some evidence that some Italian algebraists used it to indicate division. In a manuscript entitled *Arithmetica and Practtica* by *Giacomo Filippo Biodi dal Aucisco*, 1684, this symbol stands for division.

The symbol "÷" is called an *obelus*, and was first used for a division symbol around 1650.

Symbol for multiplication

William Oughtred (1574-1660) contributed vastly to the propagation of mathematical knowledge in English by his treatises, *The Clavis Mathematicae*, 1631, published in Latin (English edition 1647), *Circles of Proportion*, 1632, and *Trigonometrie*, 1657. Oughtred laid extraordinary emphasis upon the use of mathematical symbols, altogether he used over 150 of them. Three have survived to the modern times, namely the *cross* symbol for multiplication, **::** as that of proportion, and the symbol for "*difference between*".

Leibniz (1646-1715) had serious, logical doubts and reservations to the use of Oughtred's cross symbol because of possible confusion with the letter X. On 29 July 1698 he wrote in a letter to John Bernoulli : "*I do not like (the cross) as a symbol for multiplication, as it is easily confounded with x; .... often I simply relate two quantities by an* *interposed dot and indicate multiplication by ZC.LM.*"

Symbol for inequality

Thomas Harriot (1560-1621) was an English mathematician who lived the longer part of his life in the sixteenth century but whose outstanding publication appeared in the seventeenth century. His great work in this field, the *Artis Analyticae Praxis* was published in London posthumously in 1631, and deals largely with the theory of equations. In it he makes use of these symbols, ">" for "is greater than", and "<" for "is less than"

While Harriot was surveying North America, he saw a Native American with this symbol on his arm . it is likely he developed the two symbols from this symbol.

They were not immediately accepted, for many writers preferred the symbols (shown below), which another Englishman William Oughtred (1574-1660) had suggested in the same year in the popular Clavis Mathematicae, a work on arithmetic and algebra that did much toward spreading mathematical knowledge in that country.

Greater than

Less than

Symbol for percentage %

Percent has been used since the end of the fifteenth century in business problems such as computing interest, profit and loss, and taxes. However, the idea had its origin much earlier. When the Roman emperor Augustus levied a tax on all goods sold at auction, *centesima rerum venalium*, the rate was 1/100. Other Roman taxes were 1/20 on every freed slave and 1/25 on every slave sold. Without recognising percentages as such, they used fractions easily reduced to hundredths

$$\frac{o}{o}$$

The percent sign, %, has probably evolved from the symbol shown above introduced in an anonymous Italian manuscript of 1425. Instead of "per 100," "P cento," which were common at that time, this author used the symbol shown.

TERMS

ADDITION Fibonacci used the Latin additio, although he also used compositio and collectio for this operation. The arithmetic word *add* is from the Latin root *addere*, to give or to do. The *dere* part of the root is the same root that gives us Data/Datum and the name for dice. Donation and condone also share the same root. The first recorded use of the word in English is from "*The Crafte of Nombrynge*". The document was one of the first English language documents dealing with mathematics.

COMPUTE It is joining of the com (with) and the Latin root *putare*. This root is often cited as related to thinking or reckoning, but its meaning comes from an early word for cut or slice. The same root appears in amputate. This goes back to the earliest use of numbers in commerce and the idea of comparing values to a counting or tally stick. The sticks were notched to record values for future reference. Computing, then, was comparing the quantity to the marks on the tally stick

DIGIT The word digit refers both to the fingers (and toes) as well as the Arabic number symbols for 0 to 9. The root is the Indo-European word, *deik* and is related to many other words that reflect the use of the hands and fingers to "point" out objects. Index, indicate, dictate, indict, token, dice, judge and teach are all related to the same root.

CALCULATE / CALCULUS The origin of both these words is in the Greek word *kalyx* , for pebble or small stone. The manipulations of small stones on counting boards to do arithmetic operations led to the present mathematical meanings of calculate and calculus. The pebble root is still present in the medical use of the word calculus, a name for an accretion of mineral salts in the body into "stone" such as kidney stones.

DIFFERENCE It is the combination of two roots *dis* (away) and the second root, *ferre* is from the Latin for to carry. The difference between two numbers is the amount that one has been "carried away" from the other. The same root is present in fertile, but not in ferry.

DIVIDE shares its major root with the word **widow**. The root *vidua* refers to a separation. Widow is one who is separated from the spouse. The prefix, *di,* of divide is a contraction of *dis*, a two based word meaning apart or away, as in the process of division in which equal parts are separated from each other. The *vi* part of *vidua* is also derived from a two word, and is the same root as in vigesimal (two tens), for things related to twenty. An individual is one who cannot be divided.

ARITHMETIC was the Greek word for number, and is closely related to the root of **reckon**, which is an obsolete term for count. In the middle ages the best mathematicians of Germany were called *Reichenmeister* and their arithmetic texts were *Reichenbucher.* The beginning of the word is drawn from the Indo-European root *ar* that is related to "fitting together" and gives us words like army, and art. Order, adorn, and rate all come from variants of the same root.

AVERAGE The meaning of **average,** as it is used in math today, comes from a commercial practice of the shipping age. The root, *aver,* means to declare, and the shippers of goods would declare the value of their goods. When the goods were sold, a deduction was made from each persons share, based on their declared value, for a portion of the loss, their **AVERAG**E.

FRACTION comes from the Latin word *frangere,* to break. A fraction represented the broken portion of whole.

HUNDRED is from the German root *hundt.* The quantity that it represents has not been consistent over the years and has ranged from its present value, 100, to 112, 120, 124, and 132 at different times in different areas. The remnants of these old measures still persist in the *hundredweight* of some countries representing 112 or 120 pounds, depending on the country.

MULTIPLY comes from the combined roots of *multi,* many, and *pli,* for folds, as in a number folded on itself many times.

THOUSAND Our number for one thousand comes from an extension of hundred. The roots are from the Germanic roots *teue* and *hundt. Teue* refers to a thickening or swelling, and *hundt* is the root of our present day hundred. A thousand, then, literally means a swollen or large hundred. The root *teue* is the basis of such common words today as thigh, thumb, tumor, and tuber.

Chapter 3 ETYMOLOGY OF ALGEBRA

The word *algebra* is a Latin variant of the Arabic word *al-jabr*. This came from the title of a book, *Hidab al-jabr wal-muqubala*, written in Baghdad about 825 A.D. by the Arab mathematician *Mohammed ibn-Musa al-Khowarizmi*.

> The words *jabr* (JAH-ber) and *muqubalah* (moo-KAH-ba-lah) were used by al-Khowarizmi to designate two basic operations in solving equations. *Jabr* was to transpose subtracted terms to the other side of the equation. *Muqubalah* was to cancel like terms on opposite sides of the equation. In fact, the title has been translated to mean "science of restoration (or reunion) and opposition" or "science of transposition and cancellation" and "The Book of Completion and Cancellation" or "The Book of Restoration and Balancing."

Jabr is used in the step where $x - 4 = 16$ becomes $x = 20$. The left-side of the first equation, where x is lessened by 4, is "restored" or "completed" back to x in the second equation.

Muqabalah takes us from $x + y = y + 7$ to $x = 7$ by "cancelling" or "balancing" the two sides of the equation.

Eventually the *muqabalah* was left behind, and this type of math became known as algebra in many languages.

It is interesting to note that the word *al-jabr* used non-mathematically made its way into Europe through the

Moors of Spain. There an *algebrista* is a bonesetter, or "restorer" of bones. A barber of medieval times called himself an algebrista since barbers often did bone-setting and bloodletting on the side. Hence the red and white striped barber poles of today. The first use of the word "algebra" in English was by the Welsh mathematician and textbook writer, Robert Recorde in his Pathway of Knowledge written about 1550 .

Algebra is the heart of problem solving almost all problems except for the strictly geometric or logical, uses equations.

Chapter 4 WHY IS ELEVEN NOT ONETEEN AND TWELVE NOT TWOTEEN?

There would not be many mothers who did not have to face the music while making their wards memorize the count to twenty. The curious child often wondered on eleven and twelve. The most obvious thing was to call them *oneteen* and *twoteen.* Then why this strange notation.

Ten is the number of fingers on both hands so it was natural to develop it as the base for counting, since the most basic counting and calculation is done using the finger and thumb. But the decimal system has a small drawback that there are not many proper fractions of 10, just 2 and 5. It is felt that the duodecimal system of 12 had this advantage of having 2,3,4 and 6 as fractions.

Moreover the choice of the number twelve may have had the following reasons

- the approximate number of lunar months in an Earth year;
- the sum of ten fingers on human hands and two feet; or
- the number of phalanx bones in the four fingers of one hand, with the thumb used as an indicator.

The last reason seems to be the most authentic reason. Thus one after ten and two after ten developed as eleven and twelve. Eleven in Old English is *endleofan*, and related forms in the various Germanic languages point back to an original Germanic *ainlif*, “eleven.” *Ainlif* is composed of *ain–*, “one,” the same as our one, and the suffix *–lif* from the Germanic root *lib–, “remain left over.”* Thus, eleven is literally “one-left” (over, that is, past ten), and twelve is “two-left” (over past ten).

In many civilsations, 12 the duodecimal system was adopted probably because there are 12 signs of the zodiac. There are 12 hours in a day or night. All traditional Chinese calendars, clocks and compasses have their basis in 12.

Many European languages have special words for 11 and 12. Being a versatile denominator in fractions may explain why we have 12 inches in a foot, 12 ounces in a troy pound, 12 old British pence in a shilling, 12 items in a dozen.

That the number twelve was important in ancient times is given evidence by the fact that, in the Germanic languages at any rate, number elements are not repeated until after the number twelve. For, of course, the literal meanings of the words eleven and twelve are "one left" and "two left" from the Germanic compounds *"ain-lif"* and *"twa-lif."* So its just a tradition based on some requirement that 11 and 12 are what they are and not what we thought they should be.